DEEP-INELASTIC SCATTERING
I. Elements of Experiment and Theory

2000 CTEQ Summer School

on QCD Analysis and Phenomenology

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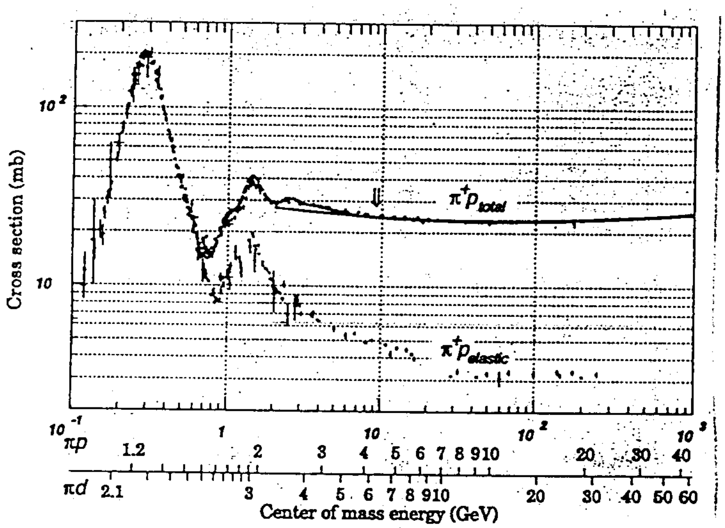
Electroweak Scattering of Nucleons

Why we say hadrons are composite

- Resonances in Hadronic Scattering [1]

$$\pi^+ + p \to \Delta^{++}(1232) \to \pi^+ + p \ldots$$



Baryon Summ

This short table gives the name, the quantum numbers (where known), and the status of baryons in the Review. Only the or 4-star status are included in the main Baryon Summary Table. Due to insufficient data or uncertain interpretation, the oth short table are not established as baryons. The names with masses are of baryons that decay strongly. See our 1986 edition (1708) for listings of evidence for Z baryons (KN resonances).

p	P_{11}	****	$\Delta(1232)$	P_{33}	****	Λ	P_{01}	****	Σ^+	P_{11}	****	Ξ^0
n	P_{11}	****	$\Delta(1600)$	P_{33}	***	$\Lambda(1405)$	S_{01}	****	Σ^0	P_{11}	****	Ξ^-
$N(1440)$	P_{11}	****	$\Delta(1620)$	S_{31}	****	$\Lambda(1520)$	D_{03}	****	Σ^-	P_{11}	****	$\Xi(1530)$
$N(1520)$	D_{13}	****	$\Delta(1700)$	D_{33}	****	$\Lambda(1600)$	P_{01}	***	$\Sigma(1385)$	P_{13}	****	$\Xi(1620)$
$N(1535)$	S_{11}	****	$\Delta(1750)$	P_{31}	*	$\Lambda(1670)$	S_{01}	****	$\Sigma(1480)$		*	$\Xi(1690)$
$N(1650)$	S_{11}	****	$\Delta(1900)$	S_{31}	**	$\Lambda(1690)$	D_{03}	****	$\Sigma(1560)$		**	$\Xi(1820)$
$N(1675)$	D_{15}	****	$\Delta(1905)$	F_{35}	****	$\Lambda(1800)$	S_{01}	***	$\Sigma(1580)$	D_{13}	**	$\Xi(1950)$
$N(1680)$	F_{15}	****	$\Delta(1910)$	P_{31}	****	$\Lambda(1810)$	P_{01}	***	$\Sigma(1620)$	S_{11}	**	$\Xi(2030)$
$N(1700)$	D_{13}	***	$\Delta(1920)$	P_{33}	***	$\Lambda(1820)$	F_{05}	****	$\Sigma(1660)$	P_{11}	***	$\Xi(2120)$
$N(1710)$	P_{11}	***	$\Delta(1930)$	D_{35}	***	$\Lambda(1830)$	D_{05}	****	$\Sigma(1670)$	D_{13}	****	$\Xi(2250)$
$N(1720)$	P_{13}	****	$\Delta(1940)$	D_{33}	*	$\Lambda(1890)$	P_{03}	****	$\Sigma(1690)$		**	$\Xi(2370)$
$N(1900)$	P_{13}	**	$\Delta(1950)$	F_{37}	****	$\Lambda(2000)$		*	$\Sigma(1750)$	S_{11}	***	$\Xi(2500)$
$N(1990)$	F_{17}	**	$\Delta(2000)$	F_{35}	**	$\Lambda(2020)$	F_{07}	*	$\Sigma(1770)$	P_{11}	*	
$N(2000)$	F_{15}	**	$\Delta(2150)$	S_{31}	*	$\Lambda(2100)$	G_{07}	****	$\Sigma(1775)$	D_{15}	****	Ω^-
$N(2080)$	D_{13}	**	$\Delta(2200)$	G_{37}	*	$\Lambda(2110)$	F_{05}	***	$\Sigma(1840)$	P_{13}	*	$\Omega(2250)^-$
$N(2090)$	S_{11}	*	$\Delta(2300)$	H_{39}	**	$\Lambda(2325)$	D_{03}	*	$\Sigma(1880)$	P_{11}	**	$\Omega(2380)^-$
$N(2100)$	P_{11}	*	$\Delta(2350)$	D_{35}	*	$\Lambda(2350)$	H_{09}	***	$\Sigma(1915)$	F_{15}	****	$\Omega(2470)^-$
$N(2190)$	G_{17}	****	$\Delta(2390)$	F_{37}	*	$\Lambda(2585)$		**	$\Sigma(1940)$	D_{13}	***	
$N(2200)$	D_{15}	**	$\Delta(2400)$	G_{39}	**				$\Sigma(2000)$	S_{11}	*	Λ_c^+
$N(2220)$	H_{19}	****	$\Delta(2420)$	$H_{3,11}$	****				$\Sigma(2030)$	F_{17}	****	$\Lambda_c(2593)^+$
$N(2250)$	G_{19}	****	$\Delta(2750)$	$I_{3,13}$	**				$\Sigma(2070)$	F_{15}	*	$\Lambda_c(2625)^+$
$N(2600)$	$I_{1,11}$	***	$\Delta(2950)$	$K_{3,15}$	**				$\Sigma(2080)$	P_{13}	**	$\Sigma_c(2455)$
$N(2700)$	$K_{1,13}$	**							$\Sigma(2100)$	G_{17}	*	$\Sigma_c(2520)$
									$\Sigma(2250)$		***	Ξ_c^+
									$\Sigma(2455)$		**	Ξ_c^0
									$\Sigma(2620)$		**	$\Xi_c(2645)$
									$\Sigma(3000)$		*	Ω_c^0
									$\Sigma(3170)$		*	
												Λ_b^0
												Ξ_b^0 Ξ_b^-

- Photon-electron Compton

$$\gamma + e \to e \to \gamma + e \ \ldots$$

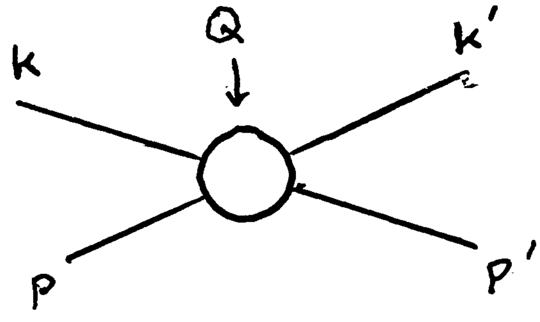
$$\text{Klein-Nishina: } \frac{d\sigma}{d\Omega_\gamma^{\rm lab}} = \frac{\alpha^2}{2m_e^2}\left(\frac{k'}{k}\right)^2\left[\frac{k'}{k} + \frac{k}{k'} - 4(\epsilon\cdot\epsilon')^2 - 2\right]$$

$$\begin{pmatrix}\nu_e \\ e\end{pmatrix}_L \quad \begin{pmatrix}\nu_\mu \\ \mu\end{pmatrix}_L \quad \begin{pmatrix}\nu_\tau \\ \tau\end{pmatrix}_L \qquad (e_R) \quad (\mu_R) \quad (\tau_R)$$

- Use the simple to study the complex ...

[1] Particle Data Group, 1998

Electron-Proton Elastic Scattering



- Rosenbluth formula:

$$\frac{d\sigma}{d\Omega_e^{\mathrm{lab}}} = \frac{4\pi\alpha^2}{Q^4}\left(\frac{E'}{E}\right)\left[\frac{|G_E(Q^2)|^2 + |G_M(Q^2)|^2}{1+\tau} + 2\tau|G_M(Q^2)|^2 \ \tan^2(\theta/2)\right.$$

  - Q^2 – momentum transfer
  - $\tau \equiv Q^2/4m_p^2$
  - G_E, G_M – Q^2-dependent form factors

- Dipole form factors (SLAC, 1960's)

$$G_{E,M} \sim \frac{1}{\left[1 + \frac{Q^2}{0.71\,\mathrm{GeV}^2}\right]^2}$$

- Diminishing returns? Hides too much? Elastic scattering & resonance production is back at JLAB

- BUT: suggestive of point substructure ! [2]

$$\sigma(Q,\theta) = \sigma(Q,\theta)_{\mathrm{point}} \times G(Q)_{\mathrm{wave\ function}}$$

- In late 60's attention turned to ...

[2] Perl, *High energy hadron physics* (Wiley, New York, 1974), p. 448

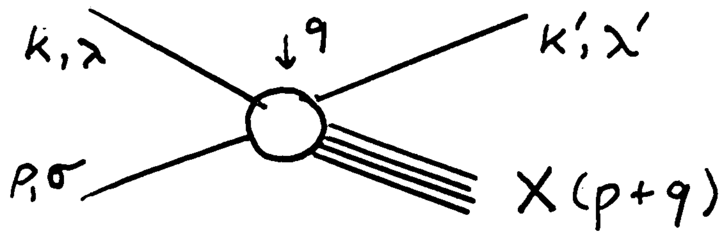
Inclusive eN Scattering in QED

- Assume QED Lagrangian :

$$\mathcal{L}_{\mathrm{QED}} = \bar{e}\,(i\partial\!\!\!/ - m_e)\; e - \bar{e}\,(\mathrm{e}\gamma_\mu)\, eA^\mu + A_\mu J^\mu_{\mathrm{hadron}} + \mathcal{L}_{\mathrm{photon}}(A)$$

- J_{hadron} arbitrary but *local*

- Cross Section $e + p \to e + X$ $(M_X \geq m_p)$:



$$A_X(p,k,q;\sigma,\lambda,\lambda') \;=\; \bar{u}_{\lambda'}(k')\,(-i\mathrm{e}\gamma_\mu)\,u_\lambda(k) \times \frac{-i}{Q^2} \times \langle X|\mathrm{e}J_\mu(0)|p,\sigma\rangle$$

- Unpolarized inclusive cross section $(S \gg m_e^2)$

$$2E_{k'}\frac{d\sigma}{d^3k'} \;=\; \frac{1}{2S}\frac{1}{2^3}\sum_{\lambda',\lambda,\sigma}\,\sum_X \int_{\mathrm{PS}} |A_X|^2 \;=\; \frac{1}{SQ^4}\, L^{\mu\nu}\, W_{\mu\nu}$$

- Leptonic and hadronic tensors (unpolarized)

$$L^{\mu\nu}(k',k) \;=\; \frac{\mathrm{e}^2}{8\pi^2}\sum_{\lambda\lambda'}\,[\bar{u}_{\lambda'}(k')\gamma^\mu u_\lambda(k)]^*\;\bar{u}_{\lambda'}(k')\gamma^\nu u_\lambda(k)$$

$$W_{\mu\nu}(p,q) \;=\; \frac{1}{8\pi}\sum_\sigma\sum_X [\,\langle X|\mathrm{e}J^\mu(0)|p,\sigma\rangle\,]^*\,\langle X|\mathrm{e}J^\nu(0)|p,\sigma\rangle$$

Structure Functions[3]

- Dirac equation $[\not{k} - m_e]u_\lambda(k) = [\not{k}' - m_e]u_{\lambda'}(k') = 0 \Rightarrow$

  $$q^\nu\, L_{\mu\nu}(k',k) \sim \left(\bar{u}_{\lambda'}(k')\,\gamma_\mu\, u_\lambda(k)\right)^*\bar{u}_{\lambda'}(k')\,(\not{k} - \not{k}')\,u_\lambda(k) = 0$$

  – current conservation: $\partial \cdot J_{\text{electron}} = 0$

$$L_{\mu\nu}(k',k) = \frac{\mathbf{e}^2}{2\pi^2}\left(k'^\mu k^\nu + k^\mu k'^\nu - g^{\mu\nu}k'\cdot k\right)$$

- Assume conserved hadronic current: $\partial \cdot J_{\text{hadronic}} = 0$
  $\Rightarrow q^\mu W_{\mu\nu}(p,q) = 0$

- Assume parity conservation ($\bar{k} \equiv (k^0, -\vec{k})$)
  $\Rightarrow W^{\mu\nu}(\bar{p},\bar{q}) = W_{\mu\nu}(p,q)$
  $\Rightarrow$

$$\begin{aligned} W_{\mu\nu}(p,q) \;&=\; -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) W_1(x,Q^2) \\ &\quad + \left(p_\mu - q_\mu\frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu\frac{p\cdot q}{q^2}\right)\frac{1}{m_p^2}\,W_2(x,Q^2) \end{aligned}$$

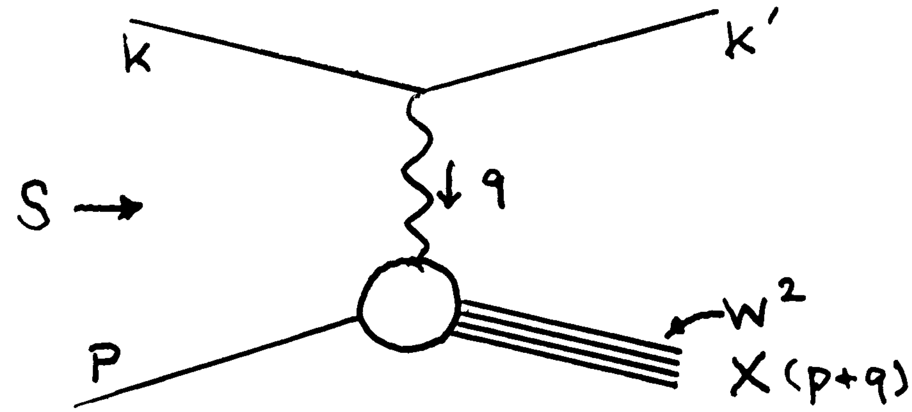
- Bjorken scaling variable: $x \equiv 2p\cdot q/Q^2$

$$\begin{array}{ccc} 0 \le & (p+q)^2 & \le S \\ & \Downarrow & \\ 0 \le & x & \le 1 \end{array}$$

[3] I try to be consistent with the conventions of *Handbook of Perturbative QCD*, *Rev. Mod. Phys.*, 57(1995) 157.

Kinematics of DIS

$$e(k) + N(p) \to e(k') + X(p+q)$$



$$Q^2 = -q^2 = -(k' - k)^2 \qquad \text{momentum transfer}$$

$$x = \frac{Q^2}{2p \cdot q} \qquad \text{scaling variable}$$

$$y = \frac{p \cdot q}{p \cdot k} \qquad \text{fractional energy transfer}$$

$$W^2 = (p+q)^2 \qquad \text{mass squared of X}$$

- Relations (neglect m_N)

$$\begin{aligned} xy &= \frac{Q^2}{S} \\ W^2 &= \frac{Q^2}{x}(1-x) \end{aligned}$$

Electron-Proton DIS Cross Section (QED)

- Alternate dimensionless structure functions

$$\begin{aligned} F_1(x,Q^2) &= W_1(x,Q^2) \\ F_2(x,Q^2) &= \frac{p\cdot q}{m_p^2}\,W_2(x,Q^2) \end{aligned}$$

- Relation to γ^* polarizations

$$W^{\mu\nu}(p,q) = \sum_{\lambda=\ell,r,L} \epsilon^{\mu}_{\lambda}(q)^* \epsilon^{\nu}_{\lambda}(q)\, F_\lambda(x,Q^2)$$

$$F_{\ell,r} = F_1 \pm F_2$$
$$2xF_L = F_2 - 2xF_1$$

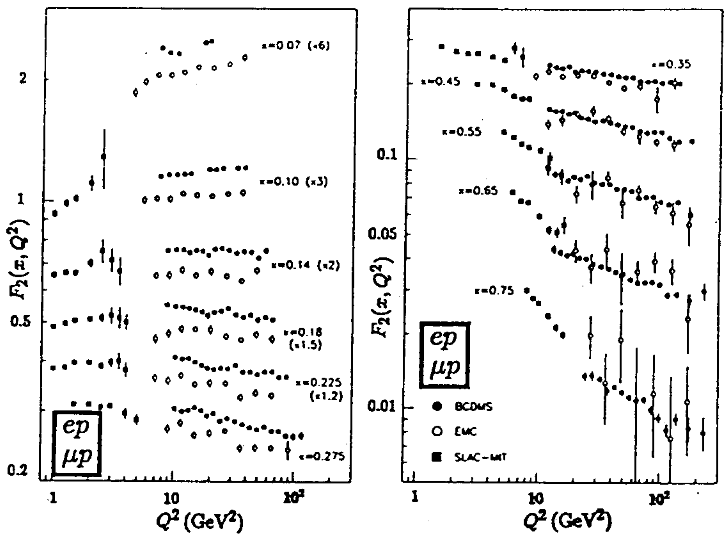
- Compute $L^{\mu\nu}W_{\mu\nu} \rightarrow$

$$\frac{d\sigma^{\gamma p}}{dx dQ^2} = \frac{2\pi\alpha^2}{xQ^4}\left[Y_+ F_2(x,Q^2) - 2xy^2 F_L(x,Q^2)\right]$$

$$Y_\pm \equiv 1 \pm (1-y)^2$$

- When $xy^2 = Q^4/xS^2 \ll 1 \rightarrow$ "read off" F_2 from data in x, Q

Pre-HERA ep and μp Data [4]



The proton structure function F_2^p measured in electromagnetic scattering of electrons (SLAC-MIT) and muons (BCDMS, EMC) on hydrogen targets, versus Q^2, for fixed bins of x. The data have been multiplied by the factors shown on the left-hand figure for convenience in plotting. Only statistical errors are shown. $R = \sigma_L/\sigma_T = 0.21$ is assumed in the SLAC-MIT data, $R = 0$ in the EMC data, and a QCD prediction for R in the BCDMS data. Where necessary, the SLAC-MIT and EMC data were interpolated to the x bins of the BCDMS data. Note that there are no SLAC-MIT data in the lowest x bin. References: SLAC-MIT—A. Bodek *et al.*, Phys. Rev. D20, 1471 (1979); EMC—J.J. Aubert *et al.*, Nucl. Phys. B259, 189 (1985); BCDMS—A.C. Benvenuti *et al.*, Phys. Lett. B223, 485 (1989).

- Slow Q^2 dependence $F_2(x, Q^2) \sim F_2(x)$ for much of data
- Suggests (compare elastic)

$$\sigma_{\rm el}(Q,\theta) \sim \sigma_{\rm el}(Q,\theta)_{\rm point} \times G(Q)_{\rm wave\ function}$$
$$\Downarrow$$
$$\sigma_{\rm incl}(Q,\theta) \sim \sigma_{\rm incl}(Q,\theta)_{\rm point}$$

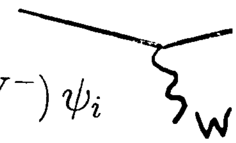
- cf. Parton Model $\rightarrow F_2(x)$ "scaling"
- cf. QCD evolution $\rightarrow F_2(x, Q^2)$ "scale breaking"
- First, generalize: $\mathcal{L}_{\rm QED} \rightarrow \mathcal{L}_{\rm EW}$

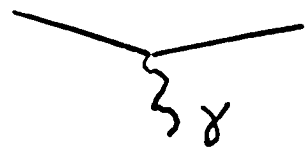
[4] PDG, *Phys. Lett.* B219 (1990) 1.

DIS in the Standard Model

- Interactions from

$$
\begin{aligned}
\mathcal{L}_{\text{EW}}^{\text{lep}} &= \sum_i \bar{\psi}_i \,(i\partial\!\!\!/ - m_i)\, \psi_i \\
&\qquad - \frac{g}{s\sqrt{2}} \sum_i \bar{\psi}_i \,\gamma_\mu (1-\gamma_5)\, (T^+ W^+ + T^- W^-)\, \psi_i \\
&\qquad - \mathrm{e}\; Q_i \sum_i \bar{\psi}_i \gamma_\mu \psi_i A^\mu \\
&\qquad - \frac{g}{2\cos\theta_W} \sum_i \bar{\psi}_i \gamma_\mu (g_V^i - g_A^i \gamma_5) \psi_i A^\mu \\
&\qquad + \mathcal{L}_{\text{boson}}(W, Z, \gamma, H)
\end{aligned}
$$





$$
\begin{aligned}
g_V^i &= t_{3L}^i - 2q_i \sin^2\theta_W \qquad & T^+ &= \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \\
g_A^i &= t_{3L}^i & T^- &= \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}
\end{aligned}
$$

- ψ_i summed over fields (L & R separately):

$$
\begin{array}{l} t_{3L} = 1/2 \\ t_{3L} = -1/2 \end{array} \qquad \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \; \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \; \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L \qquad (e_R) \quad (\mu_R) \quad (\tau_R)
$$

- Parity violation $\rightarrow$ additional structure function

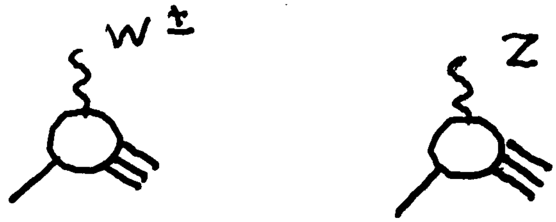
Charged Currents and Neutral Currents

- NC: exchange of γ, Z CC: exchange of $W^\pm$
- Include ψ_i = quark fields in $\mathcal{L}_{\rm EW}$

$$\begin{array}{l} t_{3L}=1/2 \\ t_{3L}=-1/2 \end{array} \quad \begin{pmatrix} u \\ d' \end{pmatrix}_L \quad \begin{pmatrix} c \\ s' \end{pmatrix}_L \quad \begin{pmatrix} t \\ b' \end{pmatrix}_L \qquad (q_R)$$

- General form in $\mathcal{L}_{\rm EW}$:

$$W^+_\mu J^{+}_{\rm hadron}{}^{\mu} + W^-_\mu J^{-}_{\rm hadron}{}^{\mu} + Z_\mu J^{Z}_{\rm hadron}{}^{\mu}$$



- Pre-standard model: analysis based on four-fermion interactions
- Examples[5]
  - $e+p\to e+X$: SLAC, H1, ZEUS
  - $\mu+p\to\mu+X$: BCDMS, EMC, E665
  - $e+p\to\nu_e+X$: H1, ZEUS
  - $\nu+A\to\mu+X$: CHARM, CDHS, CCFR, NuTeV
  - $\bar\nu+A\to\bar\mu+X$: CHARM, CDHS, CCFR, NuTeV
  - $\mu+A\to\mu+X$: BCDMS, EMC, NMC, E665
  - $e+A\to e+X$: SLAC, JLAB, HERMES

[5]Unsystematic list, from plots of PDG, 1990, 1998.

Cross Sections

- Cross Sections ($Y_\pm \equiv 1 \pm (1-y)^2$)

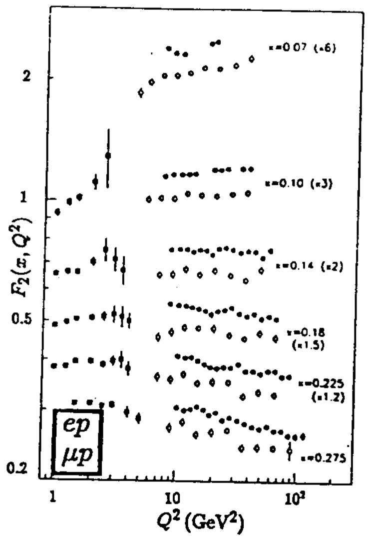
$$\frac{d\sigma^{(NC)}}{dxdQ^2} = \frac{2\pi\alpha^2}{xQ^4}$$
$$\times \Big[Y_+ F_2^{(NC)}(x,Q^2) - Y_- xF_3^{(NC)}(x,Q^2) - 2xy^2 F_L^{(NC)}(x,Q^2$$

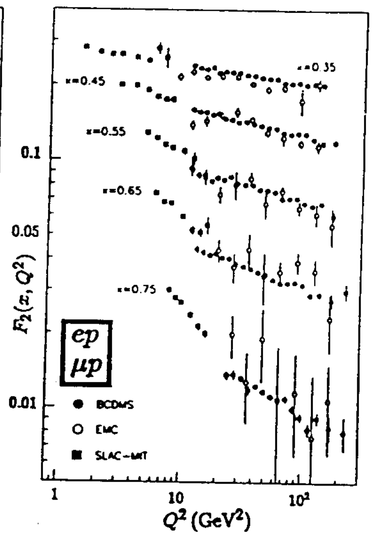
$$\frac{d\sigma^{(CC)}}{dxdQ^2} = \frac{G_F^2}{4\pi x}\left(\frac{M_W^2}{M_W^2+Q^2}\right)^2$$
$$\times \Big[Y_+ F_2^{(CC)}(x,Q^2) - Y_- xF_3^{(CC)}(x,Q^2) - 2xy^2 F_L^{(CC)}(x,Q^2)$$

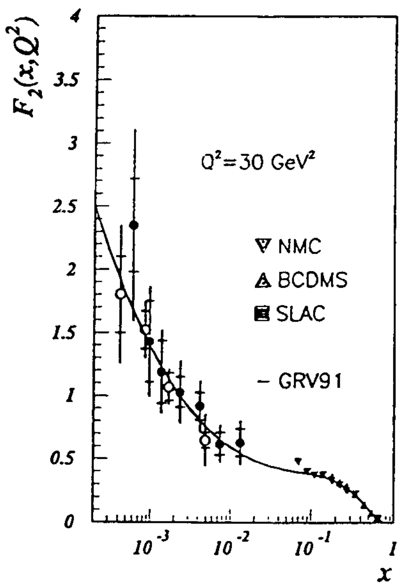
- EW couplings $g_A,\ g_V$ absorbed into structure functions

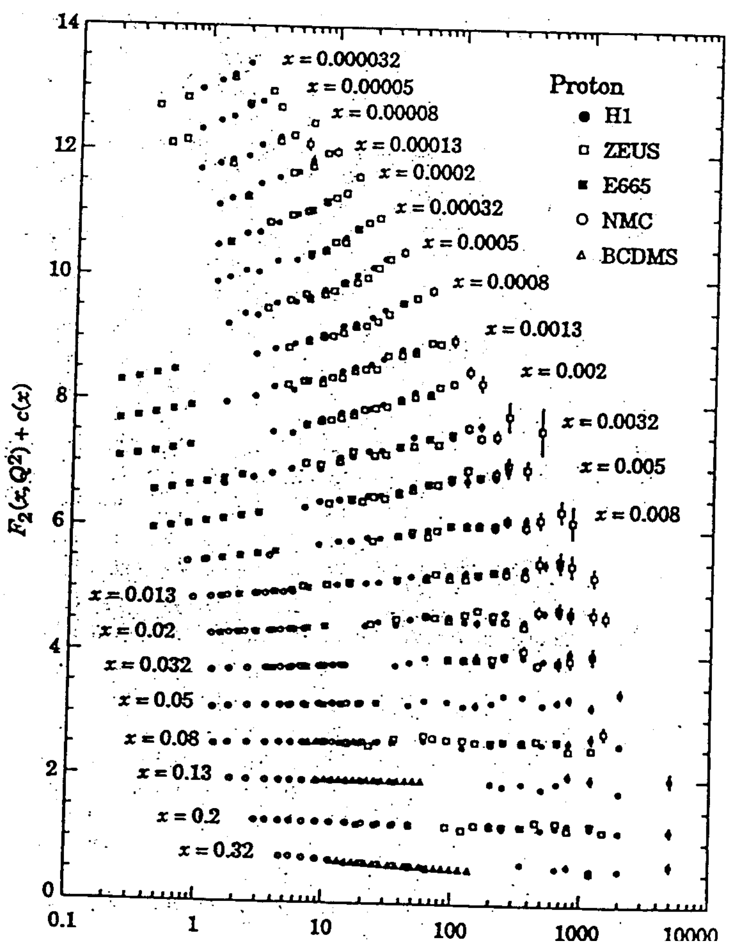
The HERA Transformation: PDG F_2^{ep}, 1990 and 1998

- $e(27\ \mathrm{GeV}) + p(820 \to 920\ \mathrm{GeV})$, $S \sim 50,000\ \mathrm{GeV}^2$



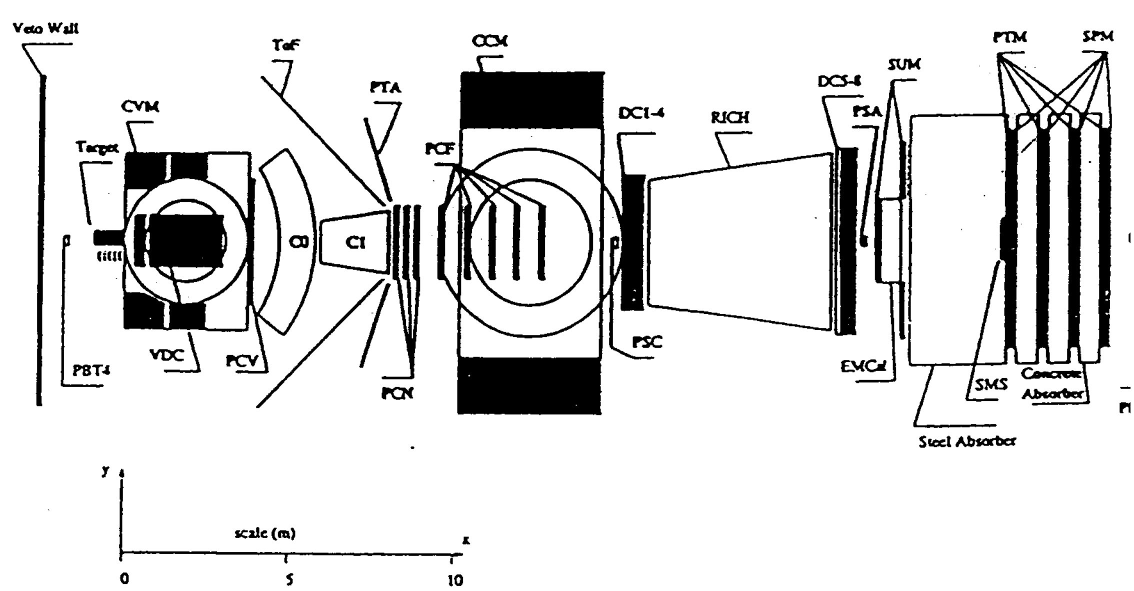






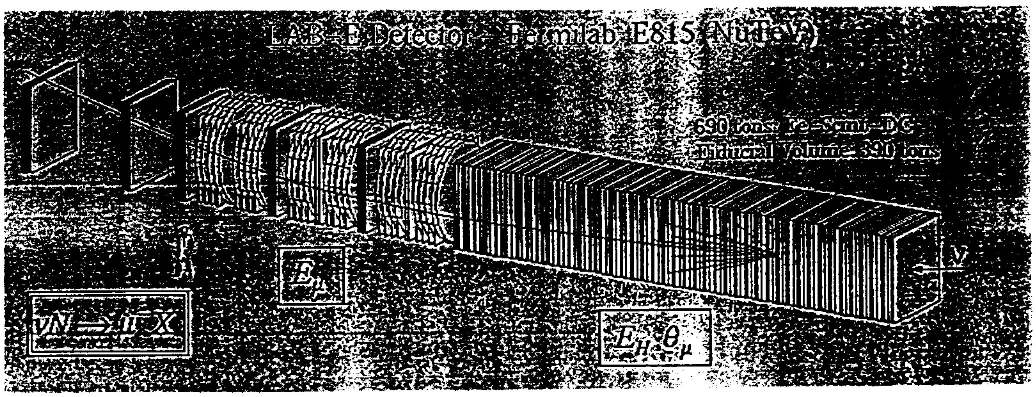
The E665 Detector

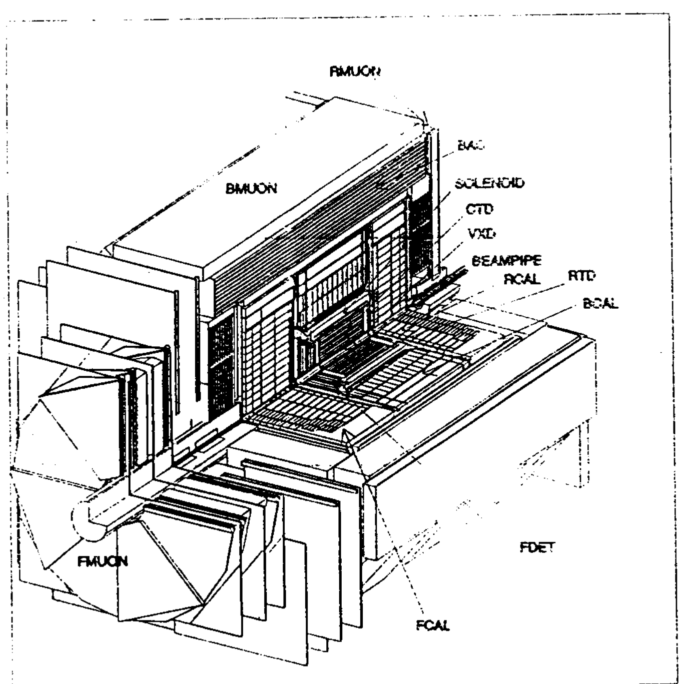
- Beam Energy: ~ 500 GeV
  Beam Spectrometer: 4 PWC stations, dipole magnet
  $\Delta p/p = 0.005$ at 500 GeV
- Hydrogen and Deuterium Targets (plus heavy targets)
- Charged Track Reconstruction:
  Vertex Drift Chambers & 56 PWC and DC planes,
  Two Dipole Magnets,
  $\Delta p/p = 0.00002p \to 1\%$ at 500 GeV.
- Electromagnetic Energy Measurement:
  20 planes separated by 5mm $(1\lambda_0)$ lead sheets,
  resolution: $38\%/\sqrt{E}$.
- Physics Triggers:
  Small angle muon scatters ($\theta > 1$ mrad),
  Large angle muon scatters ($\theta > 3$ mrad),
  Calorimeter Trigger ($\sim$5 GeV energy in multiple quadrants)



Collecting DIS

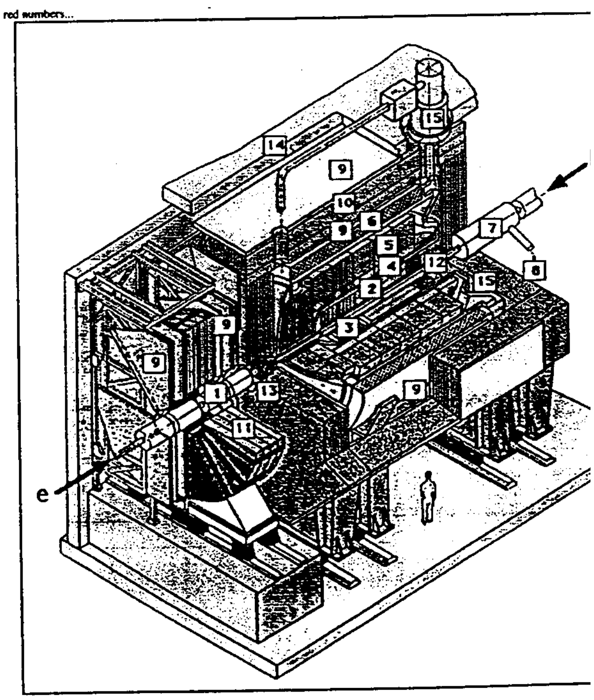
Detectors: Fixed target and collider



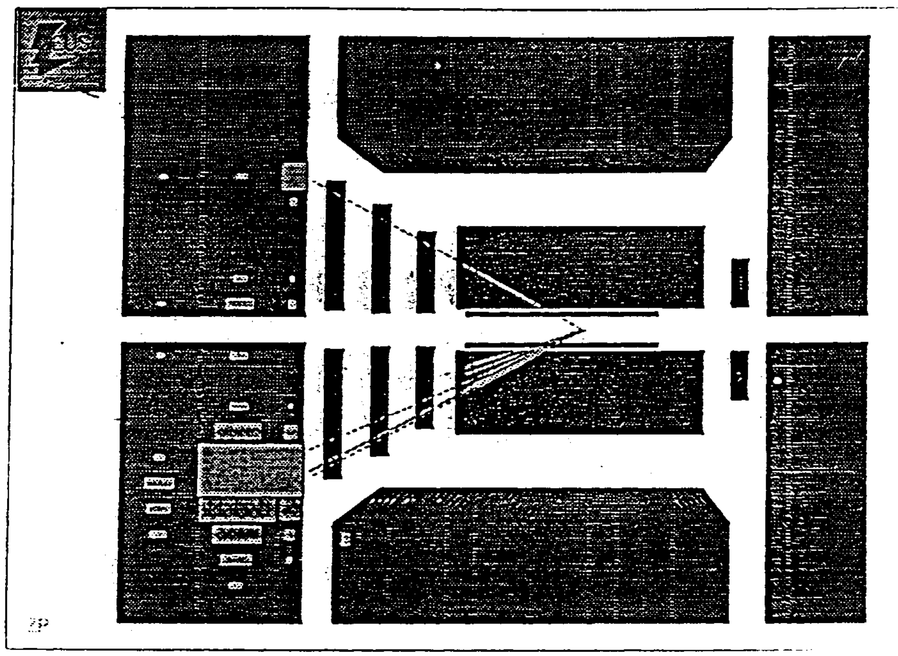


The H1 Detector

click to the red numbers...



A NC Event in the ZEUS Detector



Uranium-Scintillator Calorimeter

6000 Cells, each read out by 2 PMTs

$\sigma_{\theta_e} = 5$ mrad

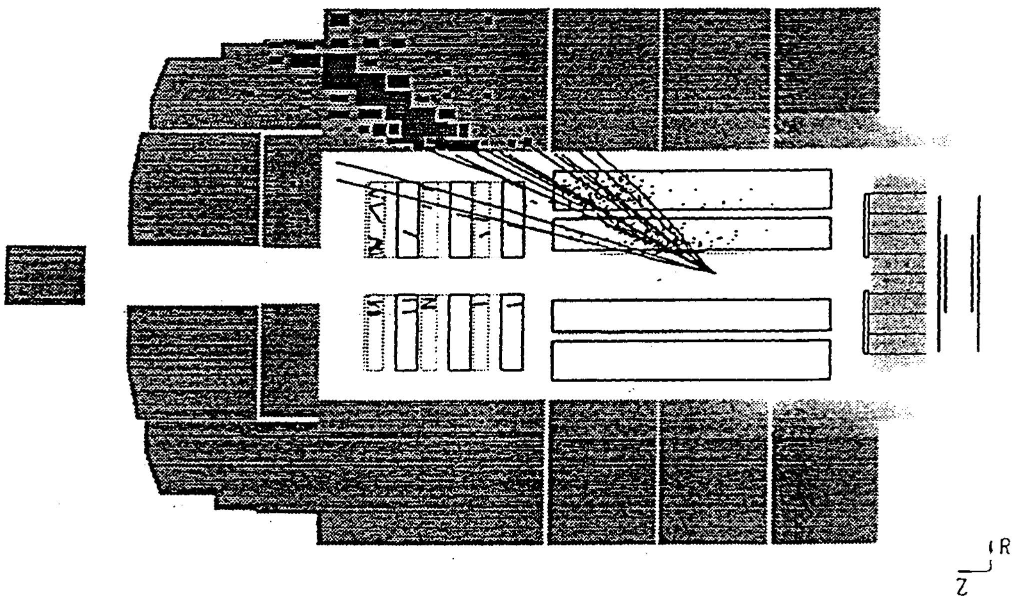
$\sigma/\sqrt{E}$ (e) = 18 %

$\sigma/\sqrt{E}$ (had) = 35 %

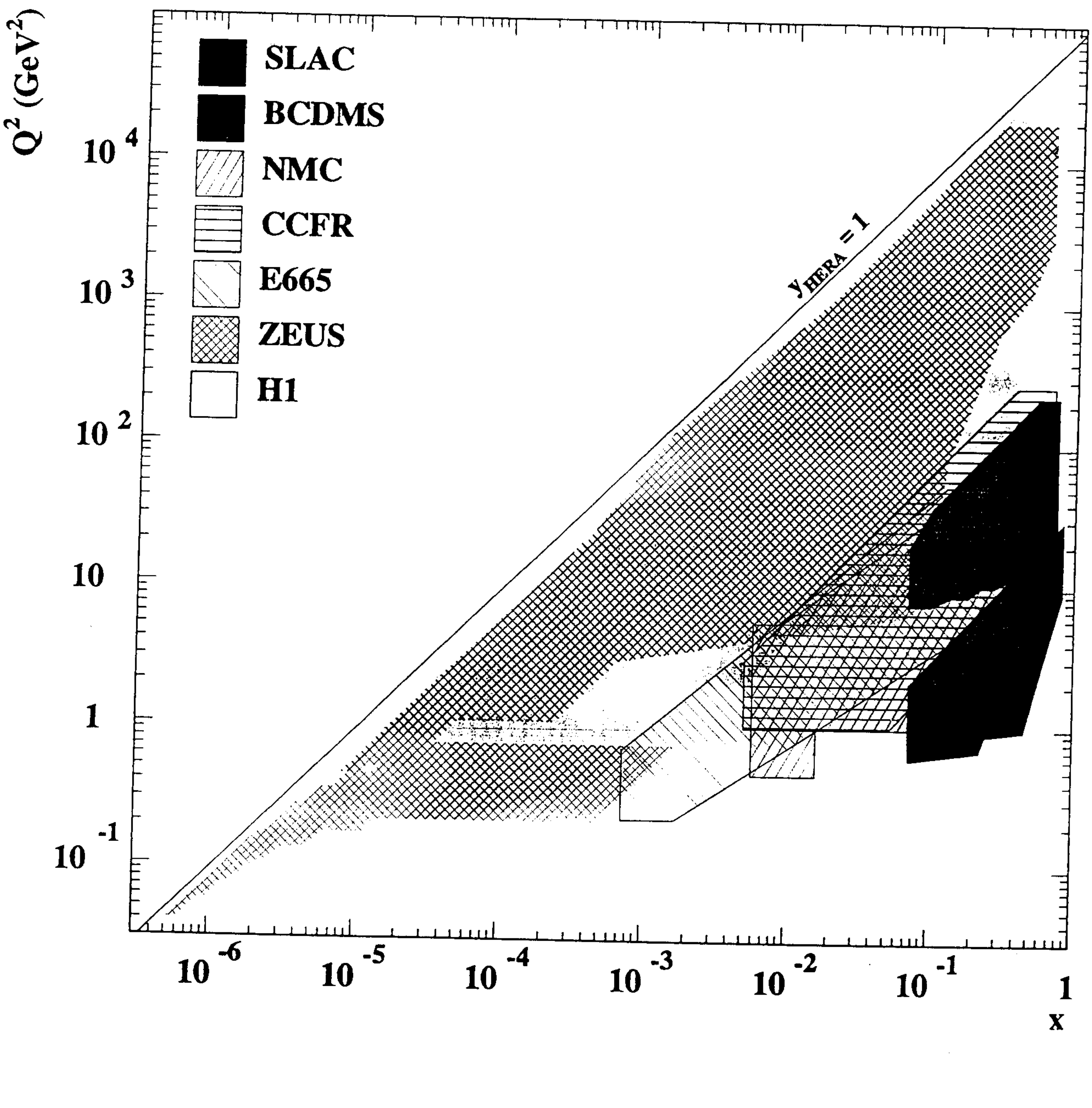
$\Delta E/E$ (syst) = 3 %

in situ calibration good to 1–2% (e)

A CC Event in the H1 Detector



Liquid Argon Calorimeter
44000 Cells
$\sigma_{\theta_e} = $ 2-5 mrad
$\sigma/\sqrt{E}$ (e) = 12 %
$\sigma/\sqrt{E}$ (had) = 50 %
$\Delta E/E$ (syst) = 3 - 4 %
in situ calibration good to 1 – 3% (e)



SLAC
BCDMS
NMC
CCFR
E665
ZEUS
H1
$y_{HERA} = 1$
Q^2 (GeV^2)
10^4
10^3
10^2
10
1
10^{-1}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
x

Kinematic Reconstruction [6]

- Light-cone variables

$$
\begin{aligned}
v^\mu &= (v^+, v^-, \mathbf{v}_T) \\
v^\pm &= \frac{1}{\sqrt{2}}\left(v^0 - v^3\right) \\
v^2 &= 2v^+v^- - v_T^2 \\
v \cdot u &= v^+u^- + v^-u^+ - \mathbf{v}_T \cdot \mathbf{u}_T
\end{aligned}
$$

- HERA: $p \sim (p^+, 0^-, \mathbf{0})$ $\qquad k \sim (0^+, k^-, \mathbf{0})$

- Rapidity

$$
\begin{aligned}
\eta(k) &= \frac{1}{2}\ln\left(\frac{k^+}{k^-}\right) \\
&= \frac{1}{2}\ln\left(\frac{2k^{+2}}{k^2 + k_T^2}\right) \\
&\to \frac{1}{2}\ln\left(\cot\frac{\theta_k}{2}\right)
\end{aligned}
$$

- Variables: S, Q^2, x, y, related by $xy = Q^2/S$
  - CC: must rely on hadronic measurements
    - ⋆ $p_{T\text{had}}^2 \equiv \not{p}_T^2 = Q_T^2$, $\quad p_{\text{had}}^- = -q^- \equiv \delta/\sqrt{2}$
    - ⋆ $\cos\gamma_h \equiv \frac{\not{p}_T^2 - \delta^2}{\not{p}_T^2 + \delta^2}$
  - NC: measure $E_{k'}, \theta_{k'}$ and/or hadronic $\to Q^2$, y

[6] ZEUS, Eur.Phys.J. C11 (1999) 427; *ibid.* C12 (2000) 411; H1 *ibid.* C13 (2000) 609

- CC Reconstruction
  - $\delta^-_{\text{had}} = \sqrt{2} q^- =$
  - $k^+ = \mathbf{k}_T = 0 \to k_0 = 2^{-1/2} k^-$
  - y from:

$$\frac{\delta}{2k_0} = \frac{q^-}{k^-} = \frac{q^- p^+}{k^- p^+} = \frac{p \cdot q}{p \cdot k} = y$$

  - Q^2 from:

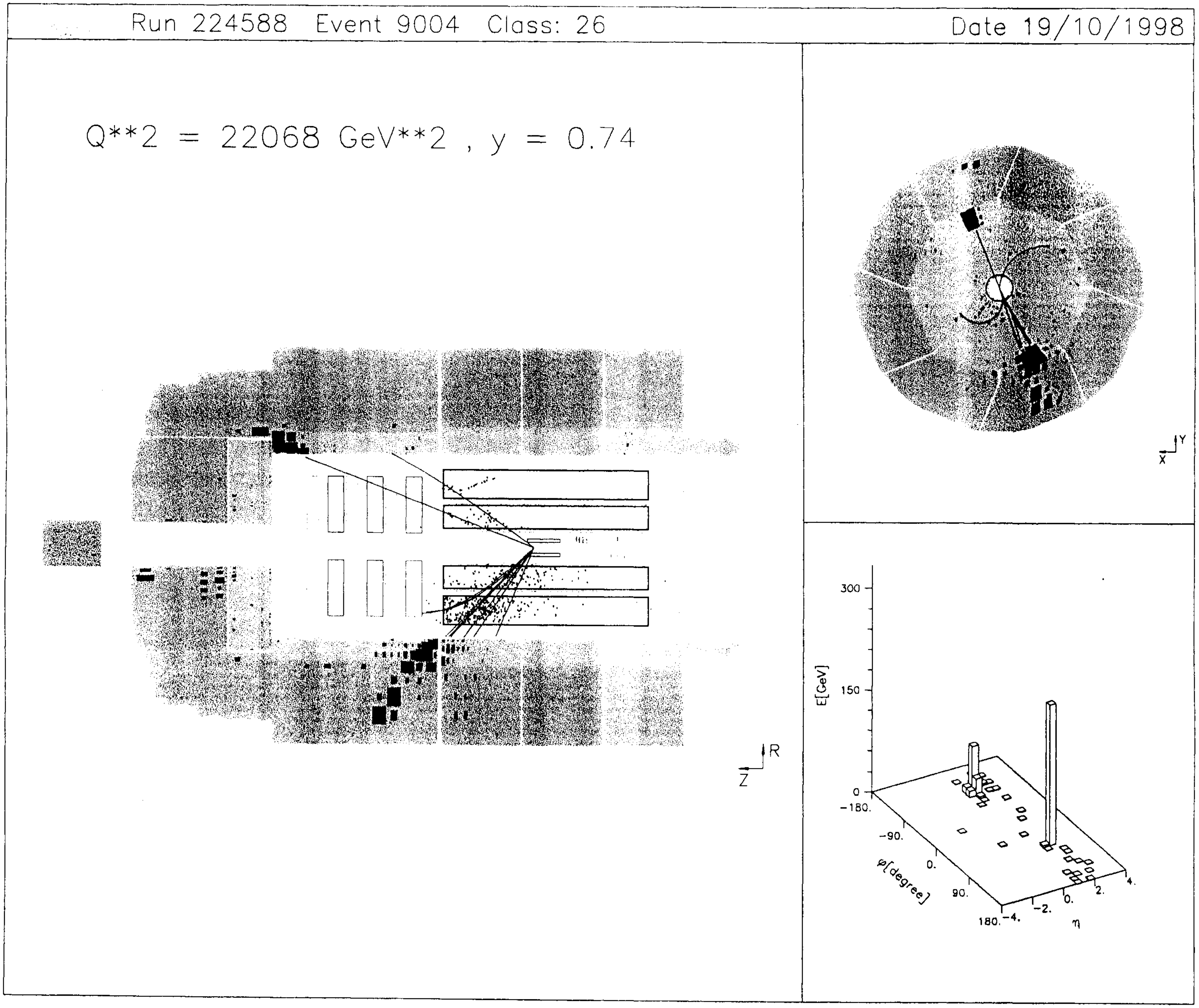
$$\frac{\not{p}_T^2}{1-y} = \frac{2k'^+ k'^-}{1-y} = 2k' \cdot k \; \frac{2k' \cdot p}{S(1-y)} = Q^2$$

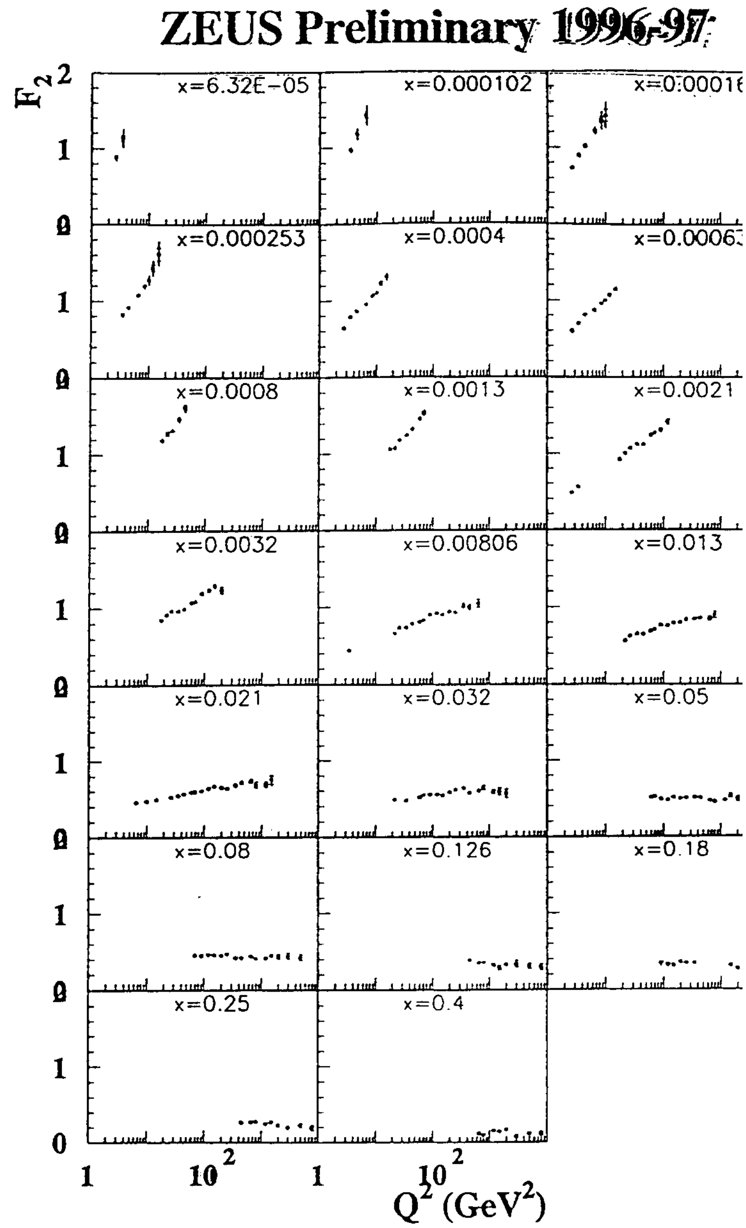
- NC Reconstruction
  - H1: Σ (=δ) method (reconstruct k^μ after IS radiation)

$$y = \frac{\Sigma}{\Sigma + E'_e(1 - \cos\theta_{e'})} = \frac{q^-}{q^- + k'^-} = \frac{p \cdot q}{p \cdot (q + k')}$$

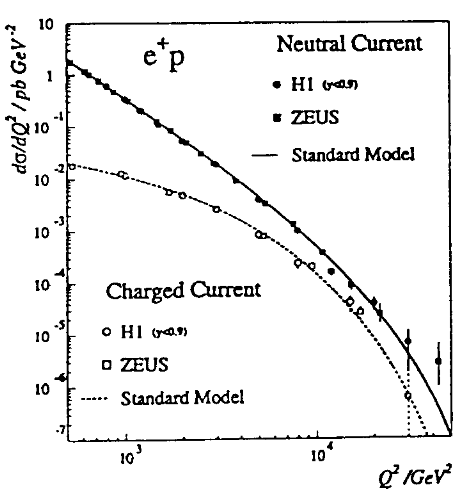
  - ZEUS: "double angle", using γ_h and $\theta_{e'}$

- Systematic Error: hadronic calorimetry most important

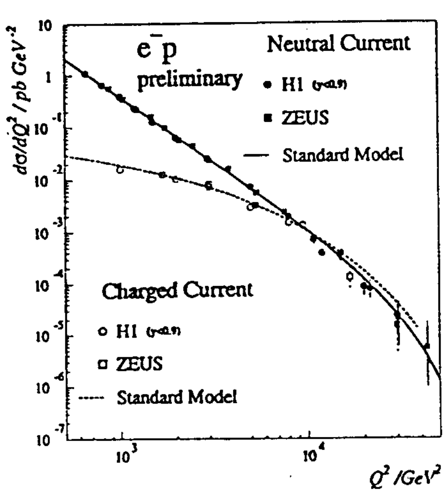

Run 224588 Event 9004 Class: 26
Date 19/10/1998
Q**2 = 22068 GeV**2 , y = 0.74
Z
R
X
Y
E[GeV]
300
150
0
-180.
-90.
0.
90.
180.
φ[degree]
-4.
-2.
0.
2.
4.
η



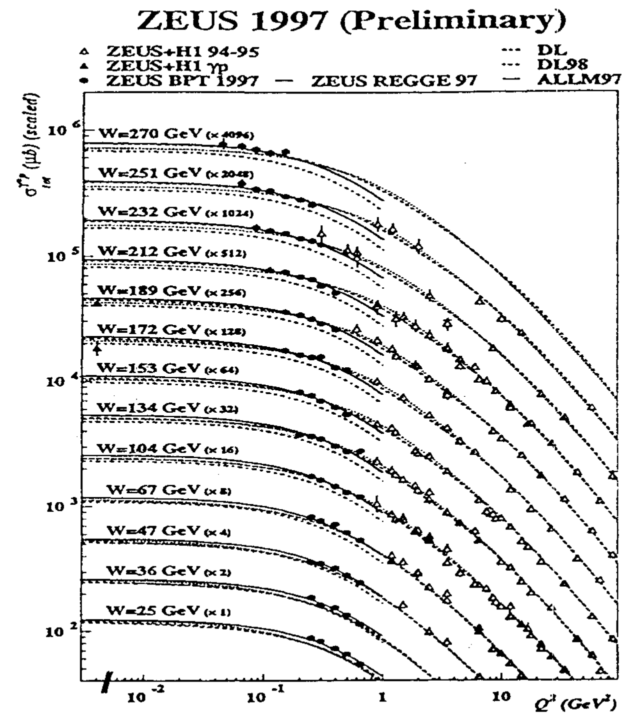
ZEUS Preliminary 1996-97
F_2
x=6.32E-05
x=0.000102
x=0.00016
x=0.000253
x=0.0004
x=0.00063
x=0.0008
x=0.0013
x=0.0021
x=0.0032
x=0.00806
x=0.013
x=0.021
x=0.032
x=0.05
x=0.08
x=0.126
x=0.18
x=0.25
x=0.4
Q^2 (GeV^2)



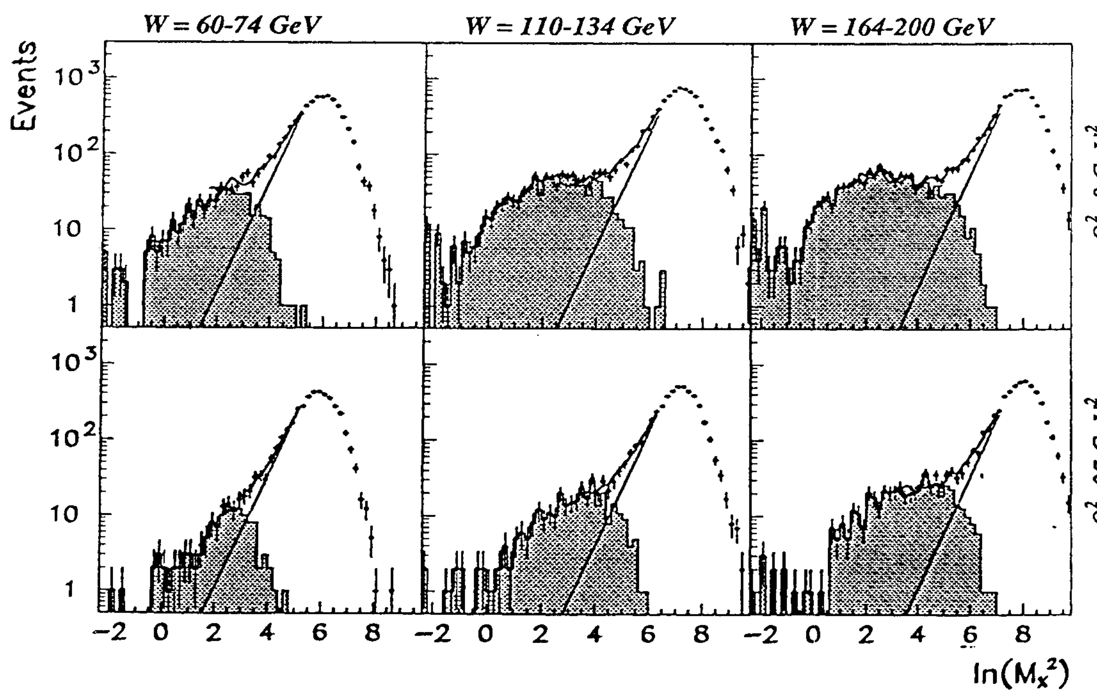
$d\sigma/dQ^2$ / pb GeV^{-2}
e^+p
Neutral Current
H1
ZEUS
Standard Model
Charged Current
H1
ZEUS
Standard Model
Q^2 /GeV^2



$d\sigma/dQ^2$ / pb GeV^{-2}
e^-p
preliminary
Neutral Current
H1
ZEUS
Standard Model
Charged Current
H1
ZEUS
Standard Model
Q^2 /GeV^2



ZEUS 1997 (Preliminary)
ZEUS+H1 94-95
ZEUS+H1 γp
ZEUS BPT 1997
ZEUS REGGE 97
DL
DL98
ALLM97
W=270 GeV (x 4096)
W=251 GeV (x 2048)
W=232 GeV (x 1024)
W=212 GeV (x 512)
W=189 GeV (x 256)
W=172 GeV (x 128)
W=153 GeV (x 64)
W=134 GeV (x 32)
W=104 GeV (x 16)
W=67 GeV (x 8)
W=47 GeV (x 4)
W=36 GeV (x 2)
W=25 GeV (x 1)
Q² (GeV²)



W = 60-74 GeV
W = 110-134 GeV
W = 164-200 GeV
Events
Q²=8 GeV²
Q²=27 GeV²
ln(Mx²)

The Parton Model for DIS

Fundamental Relation

$$\frac{d\sigma_{\ell h}(p,q)}{dxdQ^2} = \sum_{partons\ a} \int_0^1 d\xi\, \frac{d\hat{\sigma}^{\rm elastic}_{ea}(\xi p, q)}{dxdQ^2}\, \phi_{a/h}(\xi)$$

- $dxdQ^2$ same on both sides
- $\sigma_{ea}^{elastic}$ elastic

$$e(k) + a(\xi p) \to e(k') + a(p+q)$$

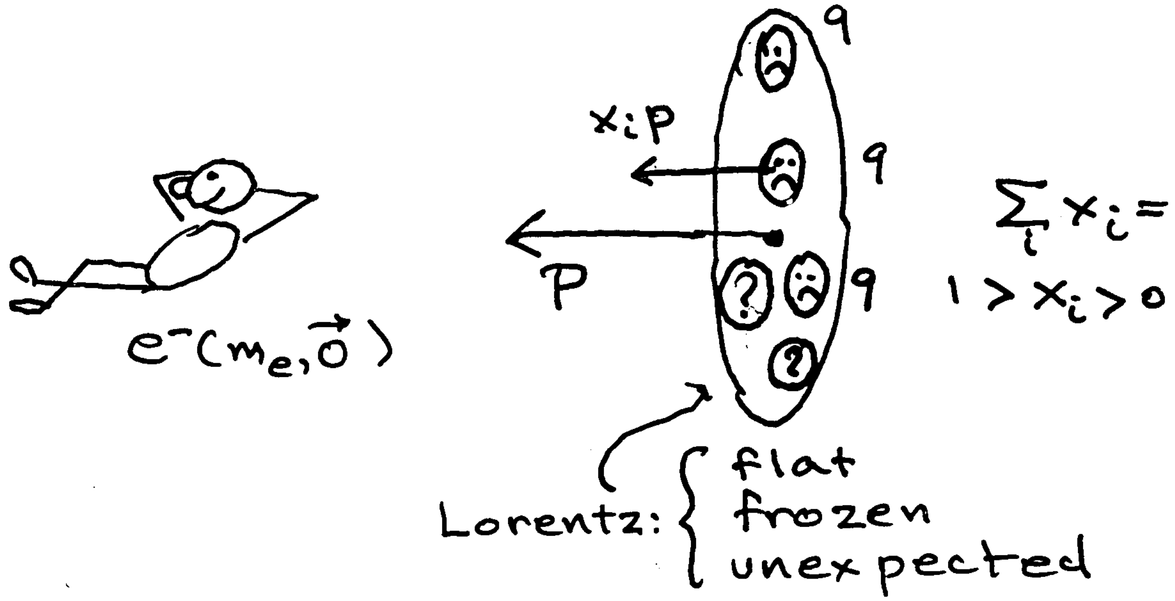
- $(p+q)^2 = 0 \Rightarrow d\sigma^{\rm el}/dxdQ^2 \sim \delta(\xi - x)$
- Fundamental assumption: quantum mechanical incoherence of large-Q scattering and partonic distribution
- In PM, cross sections, structure functions proportional to parton distributions → pdf's from data
- Realizes virtual states of proton by scattering one constituent

Parton interpretation

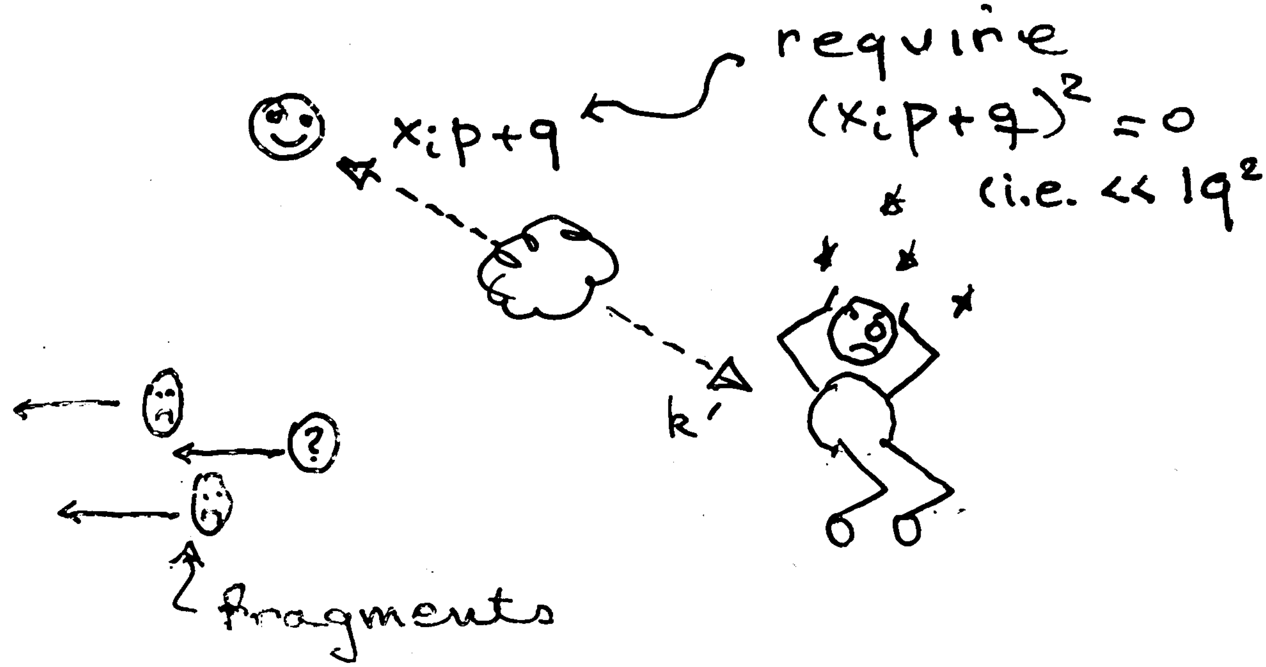
(Feynman 1969, 1972)

Look in e^- rest frame:

i) before

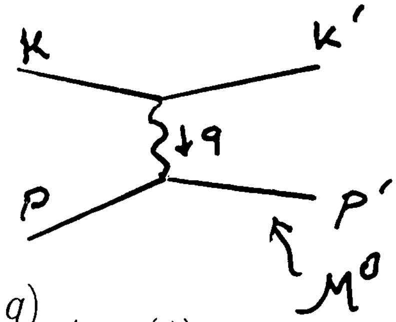


ii) after



'Deeply Inelastic Scattering'

PM for F_2 in QED



- $a = f$; quark flavor f; PM relation:

$$\frac{d\sigma_{\ell N}(p,q)}{dx dQ^2} = \sum_f \int_0^1 d\xi \, \frac{d\hat{\sigma}^{\text{elastic}}_{ef}(\xi p, q)}{dx dQ^2} \, \phi_{f/N}(\xi)$$

- Elastic cross section

$$2E_{k'} \frac{d\hat{\sigma}^{\text{elastic}}_{ef}(\xi p, q)}{d^3k'} = \frac{1}{2(\xi S)Q^4} \, L^{\mu\nu}(k',k) \, W^{\gamma f}_{\mu\nu}(\xi p, q)$$

- Quark elastic

$$\begin{aligned} W^{\gamma f}_{\mu\nu}(\xi p, q) &= \frac{1}{8\pi} \sum_{spins} \int \frac{d^3p'}{(2\pi)^3 2\omega_{p'}} \, |\mathcal{M}^0(\xi p + q \to p')|^2 \\ &\qquad \times (2\pi)^4 \delta^4(p' - q - \xi p) \\ &= \frac{1}{4\pi} \frac{q_f^2}{2\xi p \cdot q} \delta(1 - x/\xi) \, \cdot \, 4\left[(\xi p + q)_\mu \xi p_\nu + \xi p_\mu (\xi p + q)_\nu - \xi p \cdot q g_{\mu\nu} \right. \\ &= -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) \left[\frac{q_f^2}{2} \, \delta(1 - x/\xi)\right] \\ &\qquad + \left(p_\mu - q_\mu \frac{p \cdot q}{q^2}\right) \left(p_\nu - q_\nu \frac{p \cdot q}{q^2}\right) \frac{1}{m_p^2} \left[\frac{q_f^2 m_p^2}{\xi p \cdot q} \, \delta(1 - x/\xi)\right] \end{aligned}$$

- (1) Insert in PM relation, (2) Use delta function, (3) Compare to σ_N in terms of $W^{\gamma N}$ $\Rightarrow$ tyical PM result

$$F_2^{\gamma N}(x) = \sum_f q_f^2 \, x \phi_{f/N}(x) = 2x F_1^{\gamma N}(x) \quad (C - G \text{ Reln.})$$

- PM relations for CC by exchanged VB (simple!)

$$F_2^{(W^+N)}(x) = 2x\left(\sum_{D=d,s}\phi_{D/N}(x) + \sum_{U=u,c}\phi_{\bar{U}/N}(x)\right)$$

$$F_2^{(W^-N)}(x) = 2x\left(\sum_{D=d,s}\phi_{\bar{D}/N}(x) + \sum_{U=u,c}\phi_{U/N}(x)\right)$$

$$F_3^{(W^+N)}(x) = 2\left(\sum_{D=d,s}\phi_{D/N}(x) - \sum_{U=u,c}\phi_{\bar{U}/N}(x)\right)$$

$$F_3^{(W^-N)}(x) = 2\left(-\sum_{D=d,s}\phi_{\bar{D}/N}(x) + \sum_{U=u,c}\phi_{U/N}(x)\right)$$

  - W^+N: νN, e^+N, etc.
  - extra (1/2) for e due to e_R

- Sample simplifying assumptions
  - Isospin symmetry: $\phi_{u/p} = \phi_{d/n}$, $\phi_{u/n} = \phi_{d/p}$
  - Drop heavy quarks: $\phi_{c,b,t/N} = 0$
  - SU(3) Symmetric "sea" $\phi_{\bar{u}/N} = \phi_{\bar{d}/N} = \phi_{\bar{s}/N}$ (Fails badly)

- With these, DIS data highly overdetermined → consistency checks

- But $a = f$ certainly not complete (from very early data):

$$\sum_f \int_0^1 dx\; x\phi_{f/N}(x) = \text{total momentum fraction} \sim 0.5$$

- Important role for the gluon ...

Further consistency checks: sum rules

- Valence content

$$N_{u/p} = \int_0^1 dx\, \left[\phi_{u/p}(x) - \phi_{\bar{u}/p}(x)\right] = 2$$

- Adler sum rule (CC)

$$\begin{aligned} 1 = N_{u/p} - N_{d/p} &= \int_0^1 dx\, \left[\phi_{d/n}(x) - \phi_{d/p}(x)\right] \\ &= \int_0^1 dx\, \left[\sum_D \phi_{D/n}(x) - \sum_U \phi_{\bar{u}/n}(x) - \sum_D \phi_{D/p}(x) + \sum_U \phi_{\bar{u}/p}(x)\right] \\ &= \int_0^1 \frac{dx}{2x} \left[F_2^{(\nu n)}(x) - F_2^{(\nu p)}(x)\right] \end{aligned}$$

- Gross-Llewellyn Smith SR (CC)

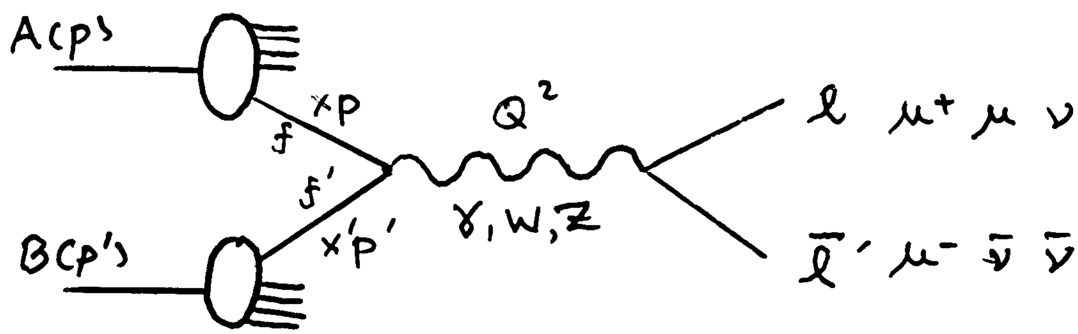
$$3 = N_u + N_d = \int_0^1 \frac{dx}{2x} \left[xF_3^{(\nu n)}(x) + xF_3^{(\nu p)}(x)\right]$$

- Gottfried SR (violation $\to$ asymmetric sea) (NC)

$$\frac{1}{3} = \int_0^1 \frac{dx}{x} \left[F_2^{(\mu p)}(x) - F_2^{(\mu n)}(x)\right]$$

Extending the PM

- Drell-Yan processes ($A + B \to \ell \bar{\ell}'$) in PM



$$\frac{d\sigma(p,p',Q)}{dQ^2} = \sum_{f,f'=q,\bar{q}} \int_0^1 dx\, dx'\ \phi_{f/A}(x)\ \frac{d\sigma^{\text{elastic}}_{ff'\to\ell\bar{\ell}'}(xp,x'p',Q)}{dQ^2}\ \phi_{f'/B}($$

Example: QED elastic ($f' = \bar{f}$ only)

$$\frac{d\sigma^{\text{elastic}}_{f\bar{f}\to\ell\bar{\ell}'}(xp,x'p',Q)}{dQ^2} = q_f^2\ \frac{4\pi\alpha^2}{9Q^2(xx'S)}\ \delta\left(1 - \frac{Q^2}{xx'S}\right)$$

- Other hard processes: Direct photon, jet ... global analysis; require

- Influence of gluons ...

QCD for DIS

From the Parton Model to Factorized DIS

- Accept QCD dynamics as source of composite hadrons
- Basic relation from parton model (arbitrary differential cross section)

$$d\sigma_{\ell h}(p,q) = \sum_{partons\ a} \int_0^1 d\xi \; d\hat{\sigma}_{ea}^{\rm elastic}(\xi p, q)\; \phi_{a/h}(\xi)$$

- Reinterpreted Assumption: incoherence of hard scattering from QCD interactions in nucleon
- QM: gluons of wavelength $< 1/Q$ "fit" in hard scattering
- But Q is variable $\Rightarrow$ $\phi_{a/N}(\xi)$ changes with Q
- Let $\phi_{a/N}(\xi,\mu)$ be pdf that "contains gluons with wavelength $\geq \mu$"
- Basic relation becomes

$$d\sigma_{\ell h}(p,q) = \sum_{partons\ a} \int_x^1 d\xi \; d\hat{\sigma}_{\ell a}\left(\frac{x}{\xi}, \frac{Q^2}{\mu^2}\right) \phi_{a/h}(\xi,\mu)$$

- $\hat{\sigma}$ gives physical final state $\rightarrow \xi > x$
- Now parton a can be a gluon
- Can compute $\hat{\sigma}$ in PT

Evolution

- μ arbitrary in

$$d\sigma_{\ell h}(p,q) = \sum_{partons\ a} \int_x^1 d\xi\ d\hat{\sigma}_{ea}\left(\frac{x}{\xi}, \frac{Q^2}{\mu^2}, \alpha_s\right) \phi_{a/h}(\xi,\mu)$$
$$\Downarrow$$
$$\mu\frac{d}{d\mu}\hat{\sigma} = 0$$

- Only variables $\hat{\sigma}$ and $\phi_{a/N}$ have in common are ξ, α_s

$$\Downarrow$$

- DGLAP evolution equation

$$\mu\frac{d}{d\mu}\phi_{a/N}(\xi,\mu) = \int_\xi^1 dz P_{ab}\left(\frac{\xi}{z}, \alpha_s\right) \phi_{b/N}(z,\mu) \qquad (A)$$

$$\mu\frac{d}{d\mu}\hat{\sigma}_{\ell a}\left(\frac{x}{z}, \frac{Q^2}{\mu^2}, \alpha_s\right) = -\int_x^z dz' \hat{\sigma}_{\ell c}\left(\frac{x}{z'}, \frac{Q^2}{\mu^2}, \alpha_s\right) P_{ca}\left(\frac{z'}{z}, \alpha_s\right) \qquad (B$$

- From (A) : extrapolate cross sections; determine pdf's from differe experiments

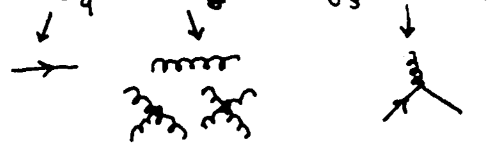
- From (B) : compute matrix of "splitting functions" P_{ab}

- Quantitative theory of Q^2-dependence of structure functions

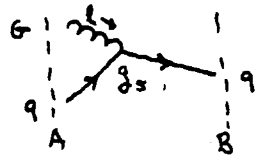
- cf. following lectures of Steve Ellis & James Stirling

SCHEMES & SCALES
(A MICROINTRODUCTION)

- $\mathcal{L}_{QCD} = \mathcal{L}_q + \mathcal{L}_G - g_s \bar{q}\, \gamma \cdot A\, q$

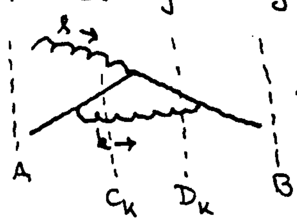


- **Every** Amplitude Tells a Story
  "state-to-state"



$M^{(0)}_{A\to B}(\ell) \sim g_s \ell$ (i.e. $|\vec{\ell}|$)

QM: "every story you can think of"



$\sum_k M^{(1)}_{A\to C_k \to D_k - B}(\ell, k) \equiv M^{(1)}_{A\to B}(\ell)$

$M^{(1)}_{k \gg \ell} \sim \int_\ell^\infty dk\, k^2 \cdot \frac{1}{k^3} \cdot (g_s k)^2 \cdot \frac{1}{k} \cdot \frac{1}{k} \cdot (g_s \ell)$

($dk\,k^2$: # paths; $\frac{1}{k^3}$: relative norm; $(g_s k)^2$: $M^{(0)}_{A\to C_k}$, $M^{(0)}_{D_k\to B}$; $\frac{1}{k}\cdot\frac{1}{k}$: lifetimes C_k, D_k; $(g_s\ell)$: L.O.)

$\sim (g_s \ell)\, g_s^2 \int_\ell^\infty \frac{dk}{k}$

UV Divergence

- Regularization
  Change the theory to cut down on high-energy paths!

Dimensional regularization:
change the counting of states!

$$\int_\ell^\infty \frac{dk}{k} \xrightarrow{DR} \int_\ell^\infty \frac{dk}{k}\left(\frac{k^2}{\mu^2}\right)^{-\epsilon} = \left(\frac{\ell^2}{\mu^2}\right)^{-\epsilon} \frac{1}{2\epsilon}$$

(μ: new scale)

$$= \frac{1}{2\epsilon} - \ln\frac{\ell}{\mu} + \mathcal{O}(\epsilon)$$

Or - just give $k > \Lambda$ weight zero!

$$\int_\ell^\infty \frac{dk}{k} \xrightarrow{\text{cutoff}} \int_\ell^\Lambda \frac{dk}{k} = \ln\frac{\Lambda}{\ell}$$

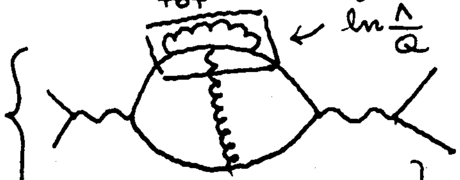
$\downarrow$ $\Lambda = e^{\frac{1}{2\epsilon}} \mu$
DR form

Relation:
$\left(\frac{k^2}{\mu^2}\right)^{-\epsilon} \approx 1$ for $k < \mu e^{\frac{1}{2\epsilon}} \sim \Lambda$

Coupling of regularized theory:

$g_s^0 \to \alpha_s^0 \equiv \frac{g_s^{0\,2}}{4\pi}$ ("bare")

- UV Divergence in σ_{tot}

$$\sigma_{tot} \sim \mathrm{Im}\Big\{$$  + friends $\Big\}$

In regularized theory:

$$\sigma_{tot} = \sigma_B \left(1 + \frac{\alpha_s^0}{\pi} + \left(\frac{\alpha_s^0}{\pi}\right)^2 \left[b_0 \ln\frac{\Lambda}{Q} + s_0\right] + \cdots \right)$$

- Enter the Renormalized Coupling

$$\alpha_s^0 \equiv \alpha_s^R \left(1 - b_0 \frac{\alpha_s^0}{\pi}\left[\ln\frac{\Lambda}{\mu} + c_0\right] + \cdots \right)$$

MUST have a new scale (μ); THIS is the scheme (c_0)

change $\mu \leftrightarrow$ change c_0

In renormalized theory:

$$\sigma_{tot} = \sigma_B \left(1 + \frac{\alpha_s^R}{\pi} + \left(\frac{\alpha_s^R}{\pi}\right)^2 \left[b_0 \ln\frac{\mu}{Q} - c_0 + s_0\right] + \cdots \right)$$

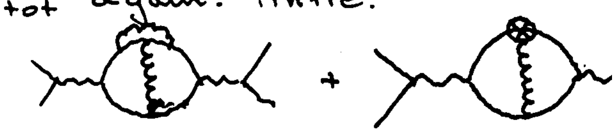
* Independent of Λ!
* $\alpha_s^R = \alpha_s^R(\mu_0, c_0) \equiv \alpha_s^R(\mu)$
* Generalizes to all orders

- Systematize: Renormalized ...

$$\mathcal{L}_R(\alpha_s^R) = \mathcal{L}(\alpha_s^0)$$
$$= \mathcal{L}_q + \mathcal{L}_G - g_s^R \bar{q}\gamma\cdot A q$$
$$- (g_s^0 - g_s^R)\bar{q}\gamma\cdot A q \leftarrow \text{"cou...}$$

$$g_s^0 - g_s^R = \sqrt{4\pi\alpha_s^0} - \sqrt{4\pi\alpha_s^R}$$
$$= \sqrt{4\pi\alpha_s^R\left(1 - \alpha_s^R\left[b_0 \ln\frac{\Lambda}{\mu} + c_0\right]\right)} - \sqrt{4 \ldots}$$
$$\cong -\frac{1}{2} g_s^R \alpha_s^R \left[b_0 \ln\frac{\Lambda}{\mu} + c_0\right] + \cdots$$

σ_{tot} again: finite!



- The β-function:

$$\beta(\alpha_s) \equiv \mu \frac{\partial}{\partial\mu} \alpha_s^R(\mu)$$
$$= \mu \frac{\partial}{\partial\mu} \frac{\alpha_s^0}{1 - b_0 \frac{\alpha_s^0}{\pi}\left(\ln\frac{\Lambda}{\mu} + c_0\right)}$$
$$= -\alpha_s^0 \frac{+b_0(\alpha_s^0/\pi)}{(\alpha_s^0/\alpha_s^R)^2} + \cdots$$
$$= -\frac{b_0}{\pi}\alpha_s^{R\,2} + \cdots$$

indep. of c_0 eve... to α_s^{R3} (if no μ i...