DEEP INELASTIC SCATTERING AND QCD

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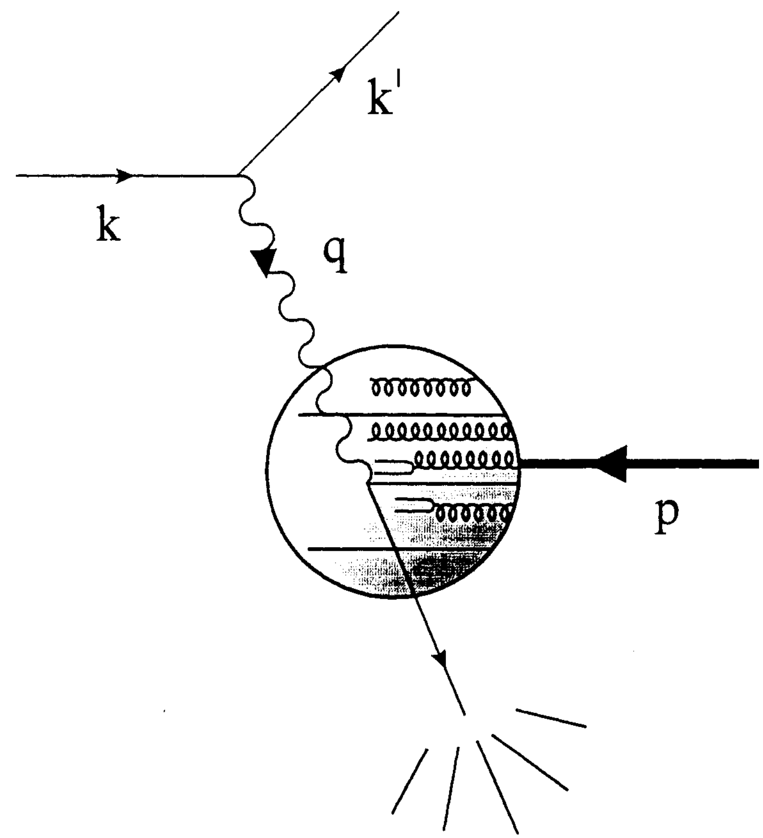
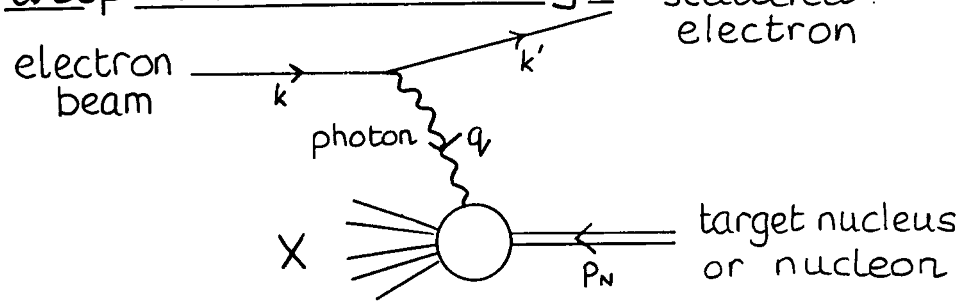
some introductory and background material for the lecture course

Deep Inelastic Scattering



$$Q^2 = -q^2, \quad x_{Bj} = \frac{Q^2}{2p \cdot q}$$

deep inelastic scattering



momentum transferred into target by photon : $Q^2 = -q^2 > 0$

$\therefore$ resolution : $\lambda = \frac{\hbar}{Q}$

$= \frac{2\times 10^{-16}\text{ m GeV}}{Q}$

inelasticity : $x = \frac{Q^2}{Q^2 + M_x^2 - M_N^2}$ $(0 < x \leq 1)$

hence $\underset{\substack{\uparrow \\ Q^2\to\infty}}{\text{deep}}$ $\underset{\substack{\uparrow \\ x<1}}{\text{inelastic}}$ scattering

experiments up to $\sqrt{s} = 300$ GeV (HERA)

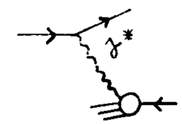
$\Rightarrow \quad Q^2 \lesssim 10^5\ \text{GeV}^2$

$\leftrightarrow \quad \lambda \gtrsim 10^{-18}\ \text{m} \simeq \frac{1}{1000} \times r_{proton}$

othe : $s = (p_1 + k)^2$. $u = \frac{Q^2}{\;}$ $(0 < u < 1)$

— in general, one can write

$$\frac{d^2\sigma}{dx\,dQ^2} = \frac{2\pi\alpha^2}{Q^4}\left\{ y^2 F_1 + 2(1-y) F_2 \right\}$$



structure functions $F_i(x, Q^2)$

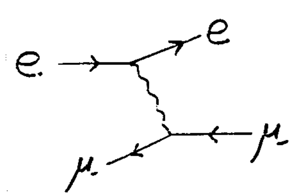
note : need two F_i because of γ_L^*, γ_T^*

convent

experimentally :

- $F_i(x, Q^2) \xrightarrow[Q^2 \gtrsim 1]{} F_i(x)$ 'scaling
- $F_1 \simeq F_2$

note



$\uparrow$

$\overline{\Sigma}|M|^2 = 2e^4 \frac{s^2+u^2}{t^2}$

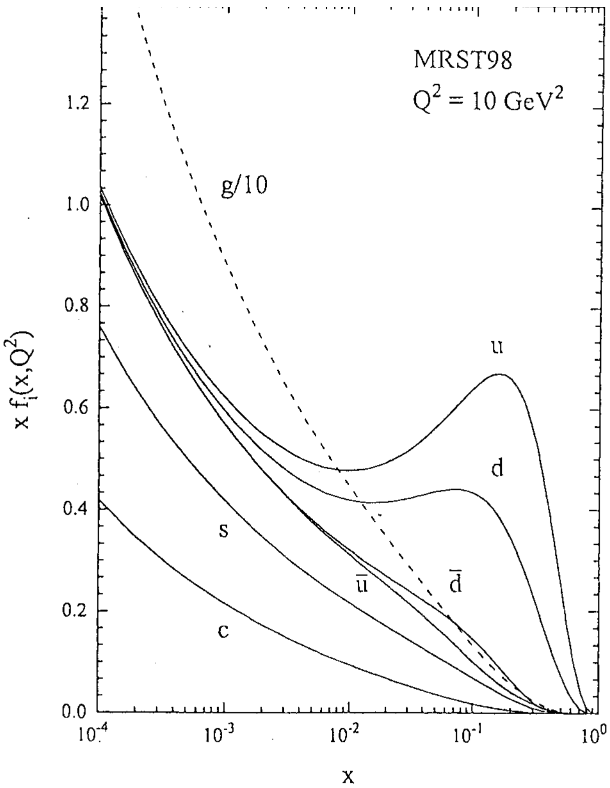
$$\frac{d\sigma^{e\mu\to e\mu}}{dQ^2} = \frac{2\pi\alpha^2}{Q^4}\left\{ y^2 + 2(1-\right.$$

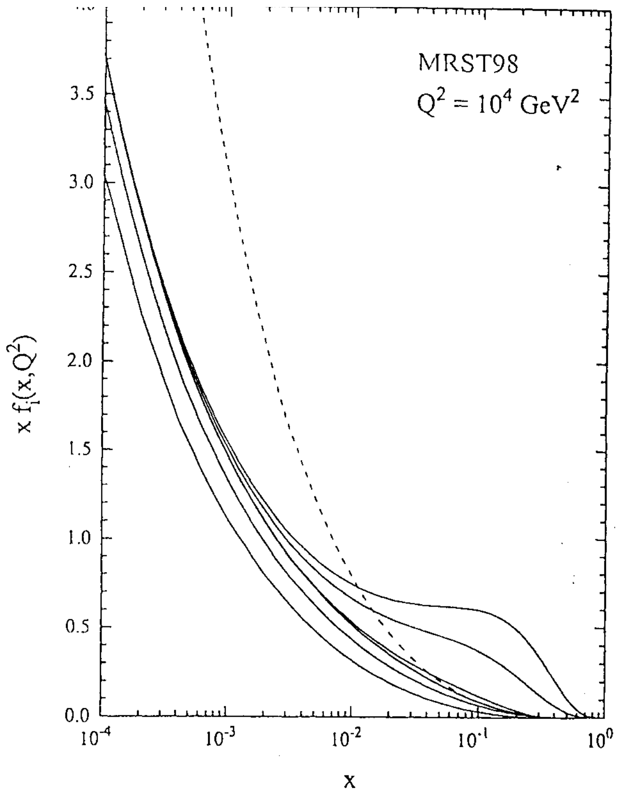
$\uparrow \sin^4\frac{\theta}{2}$

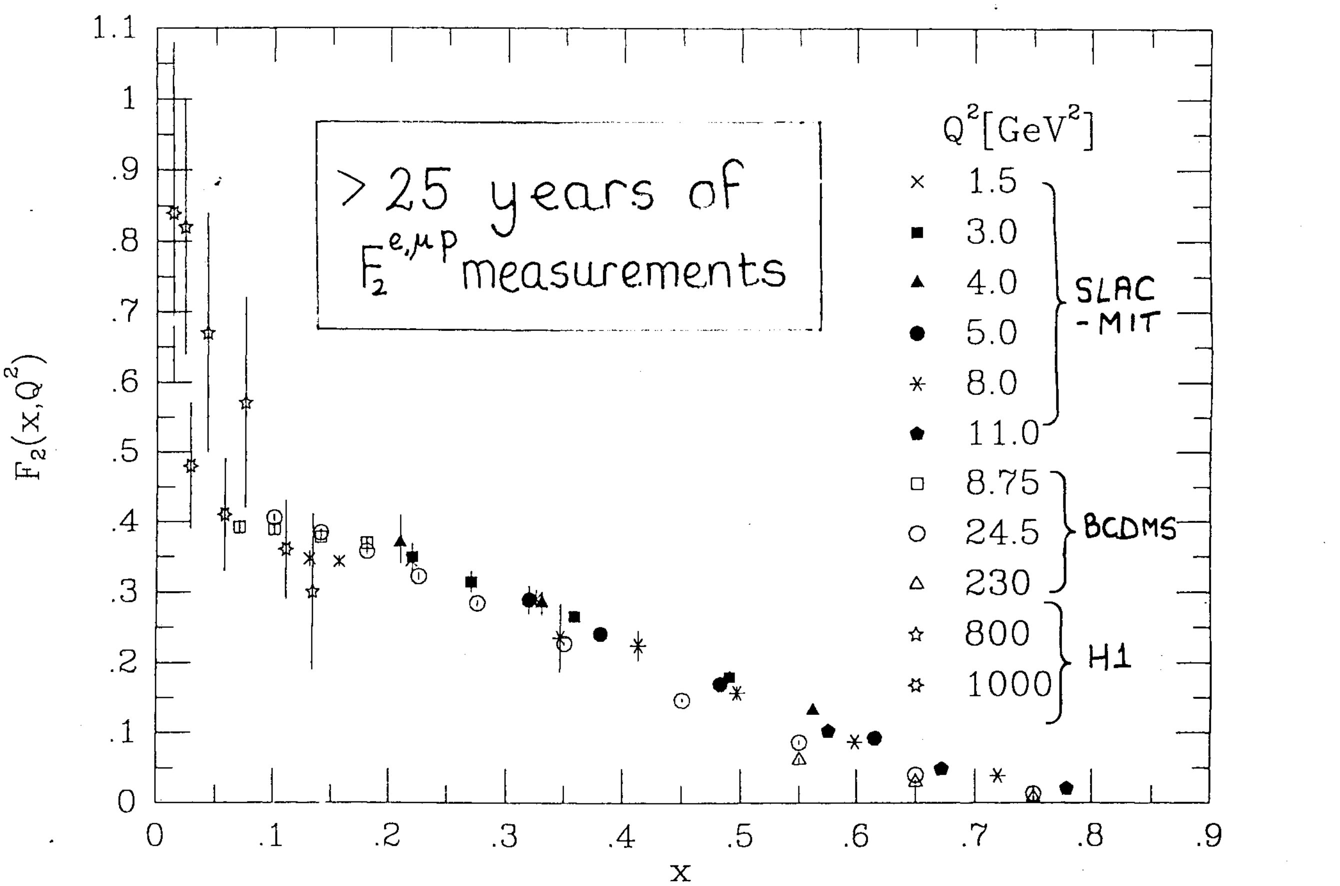
i.e. $F_1 = F_2 = \delta(1-x)$

$\uparrow$ because of spin = 1/2

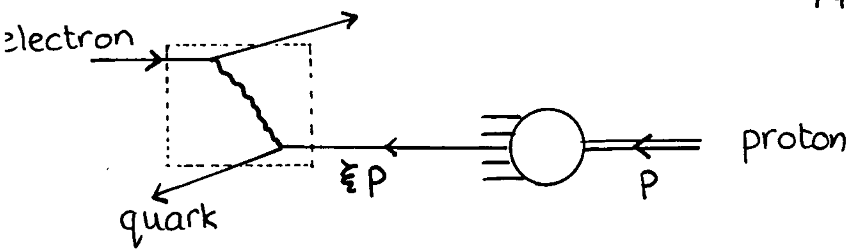
$\uparrow$ elas scattering


MRST98
$Q^2 = 10\ GeV^2$
g/10
u
d
s
$\bar{u}$
$\bar{d}$
c
$x\ f_i(x,Q^2)$
x


MRST98
$Q^2 = 10^4\ GeV^2$
$x\ f_i(x,Q^2)$
x


> 25 years of
$F_2^{e,\mu p}$ measurements
$Q^2[GeV^2]$
1.5
3.0
4.0
5.0
8.0
11.0
SLAC -MIT
8.75
24.5
230
BCDMS
800
1000
H1
$F_2(x,Q^2)$
x

<u>the parton model</u> (Feynman) 1969

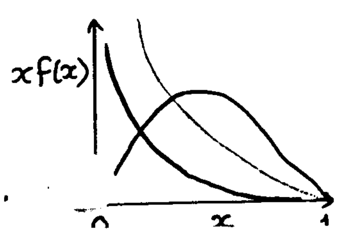


— photon scatters incoherently off pointlike, massless, spin $\frac{1}{2}$ QUARKS

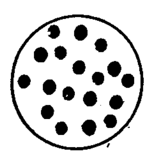
— probability that a quark carries fraction ξ of parent proton's momentum is $\leftarrow F_2^{eq}$

then
$$F_2(x) = \sum_q \int_0^1 d\xi\, q(\xi)\, e_q^2\, \delta(x-\xi)$$
$$= \sum_q e_q^2\, q(x)$$
$$= \tfrac{4}{9}u(x) + \tfrac{1}{9}d(x) + \dots$$

— the functions $u(x), d(x), s(x), \dots, g(x)$ are called parton distribution functions (pdf's)



$\Longleftrightarrow$



● = valence quarks (7\
● = sea quarks + antiquarks

— different beams, targets measur
different combinations of quark pc

$$\tfrac{1}{x}F_2^{ep} = \tfrac{4}{9}u + \tfrac{1}{9}d + \tfrac{1}{9}s + \dots$$
$$\tfrac{1}{x}F_2^{en} = \tfrac{1}{9}u + \tfrac{4}{9}d + \tfrac{1}{9}s + \dots$$
$$\tfrac{1}{x}F_2^{\nu N} = 2\,[d + s + \bar{u} + \bar{c}\,]$$
$$\tfrac{1}{x}F_2^{\bar{\nu} N} = 2\,[u + c + \bar{d} + \bar{s}\,]$$

N = he is ta

... thus the individual $q(x)$ car be extracted from measurements of a set of such F.

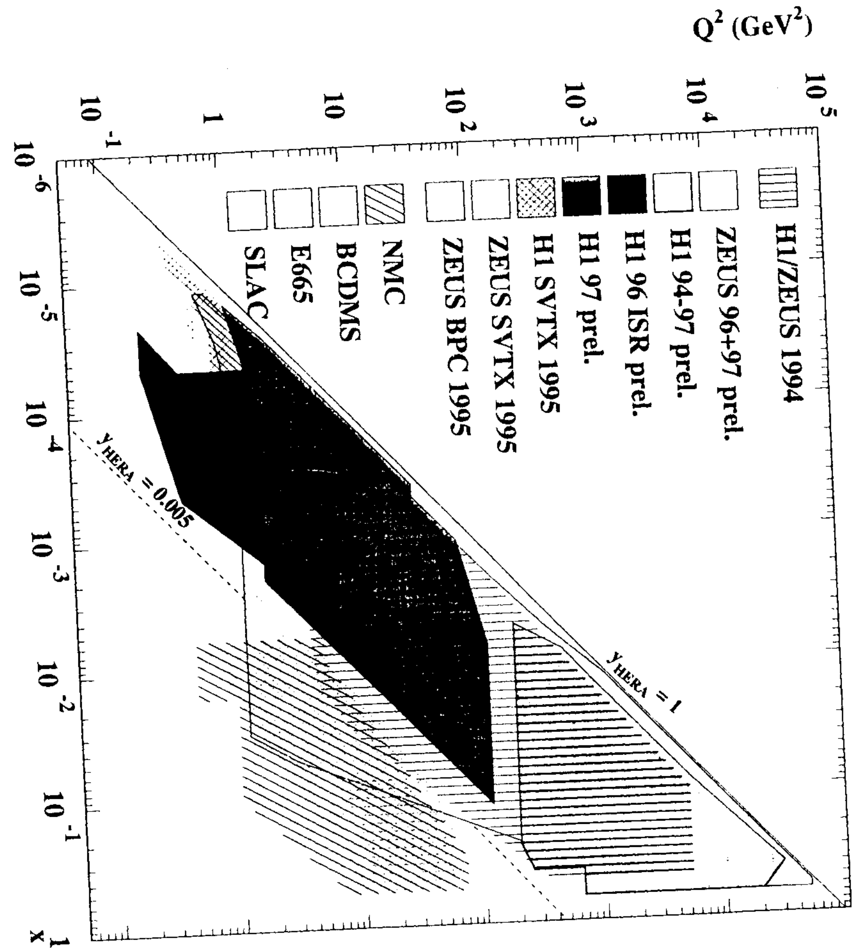
e.g. MRS CTE GR\ : serie

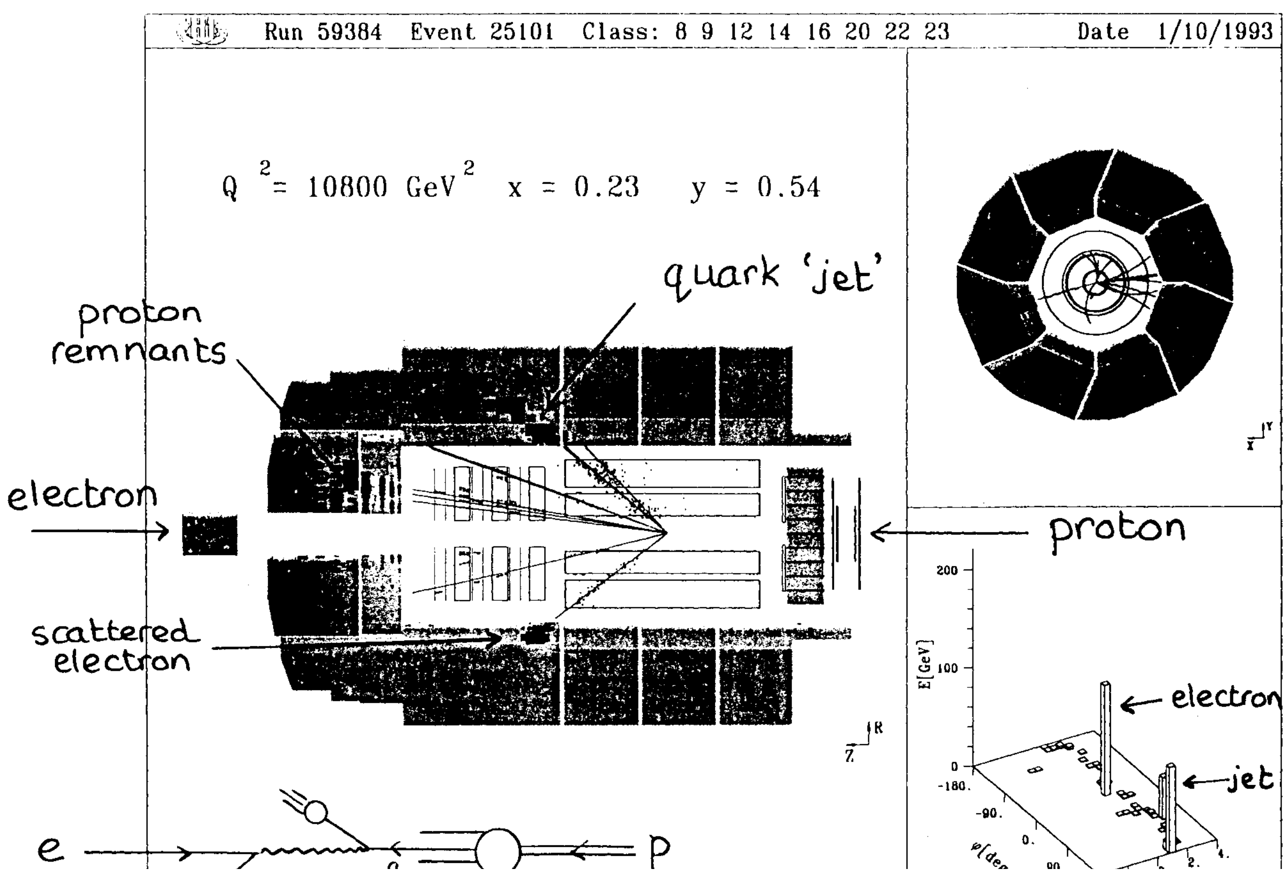
<u>note</u>

- the measured range is
$$10^{-5} \lesssim x \lesssim 1$$
→ fig
- don't measure $g(x)$ directly, bu

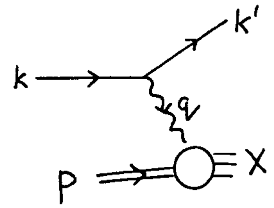
$$\sum_q \int_0^1 dx\, x\, q(x) = 0.55 \implies \int_0^1 dx\, x\, g(x) \simeq 0$$

↑ momentum conservation


Q^2 (GeV^2)
x
H1/ZEUS 1994
ZEUS 96+97 prel.
H1 94-97 prel.
H1 96 ISR prel.
H1 97 prel.
H1 SVTX 1995
ZEUS SVTX 1995
ZEUS BPC 1995
NMC
BCDMS
E665
SLAC
y_HERA = 1
y_HERA = 0.005


Run 59384 Event 25101 Class: 8 9 12 14 16 20 22 23 Date 1/10/1993
Q^2 = 10800 GeV^2 x = 0.23 y = 0.54
quark 'jet'
proton remnants
electron
proton
scattered electron
electron
jet
E[GeV]
e
p

† ## derivation of the structure function equation



$$\sigma = \underset{\text{flux}}{\frac{1}{4ME}} \sum_X' \int d\Phi \; \frac{1}{4} \sum_{\text{spins}} |M|^2_{ep\to eX}$$

phase space

amplitude squared

now $\int d\Phi = \frac{1}{(2\pi)^3} \int \frac{d^3k'}{2E'} \, d\Phi_X$

$$\frac{d^3k'}{(2\pi)^3 2E'} = \frac{1}{8\pi^2} ME \, y \, dy \, dx$$

and we can write

$$\frac{1}{4}\sum |M|^2 = \frac{e^4}{Q^4} \, L^{\mu\nu} \, h_{X\,\mu\nu}$$

photon propagator; leptonic part; hadronic part

where $L^{\mu\nu} = \frac{1}{4} Tr(\not{k}\gamma^\mu \not{k}'\gamma^\nu) = k^\mu k'^\nu + k'^\mu k^\nu - g^{\mu\nu} k\cdot k'$

so if we define

$$H_{\mu\nu} = \sum_X' \int d\Phi_X \, h_{X\,\mu\nu}$$

<u>nc.</u> a function of $q^2, p\cdot q$ ($\equiv x, Q^2$) only

then ...

$$\frac{d^2\sigma}{dx\,dy} = \frac{1}{4ME} \frac{MEy}{8\pi^2} \frac{(4\pi\alpha)^2}{Q^4} L^{\mu\nu} H_{\mu\nu}$$

$$= \frac{y\alpha^2}{2Q^4} L^{\mu\nu} H_{\mu\nu}$$

— all the information on the had structure "seen" by the virtual p is contained in the tensor $H_{\mu\nu}$ (:

— now in general we can write

$$H_{\mu\nu} = -g_{\mu\nu} H_1 + \frac{p^\mu p^\nu}{Q^2} H_2 + \text{terms } \ldots$$

terms w vanish contra with L

whence

$$L^{\mu\nu} H_{\mu\nu} = 2\, k\cdot k' \; H_1 + \frac{1}{Q^2}(2p\cdot k\, p\cdot k' - M^2 k\cdot k') \, H_2$$

$$= Q^2 H_1 + \frac{Q^2}{2}\left[\frac{1-y}{x^2y^2} - \frac{M^2}{Q^2}\right] H_2$$

$$\Rightarrow \frac{d^2\sigma}{dx\,dy} = \frac{y\alpha^2}{2Q^2}\left[H_1 + \frac{1}{2}\left\{\frac{1-y}{x^2y^2} - \frac{M^2}{Q^2}\right\}H_2\right]$$

$$\equiv \frac{4\pi\alpha^2}{Q^2xy}\left[xy^2\,F_1(x,Q^2) + \left\{1-y-\frac{M^2}{Q^2}x^2y^2\right\}F_2(x,Q^2)\right]$$

where

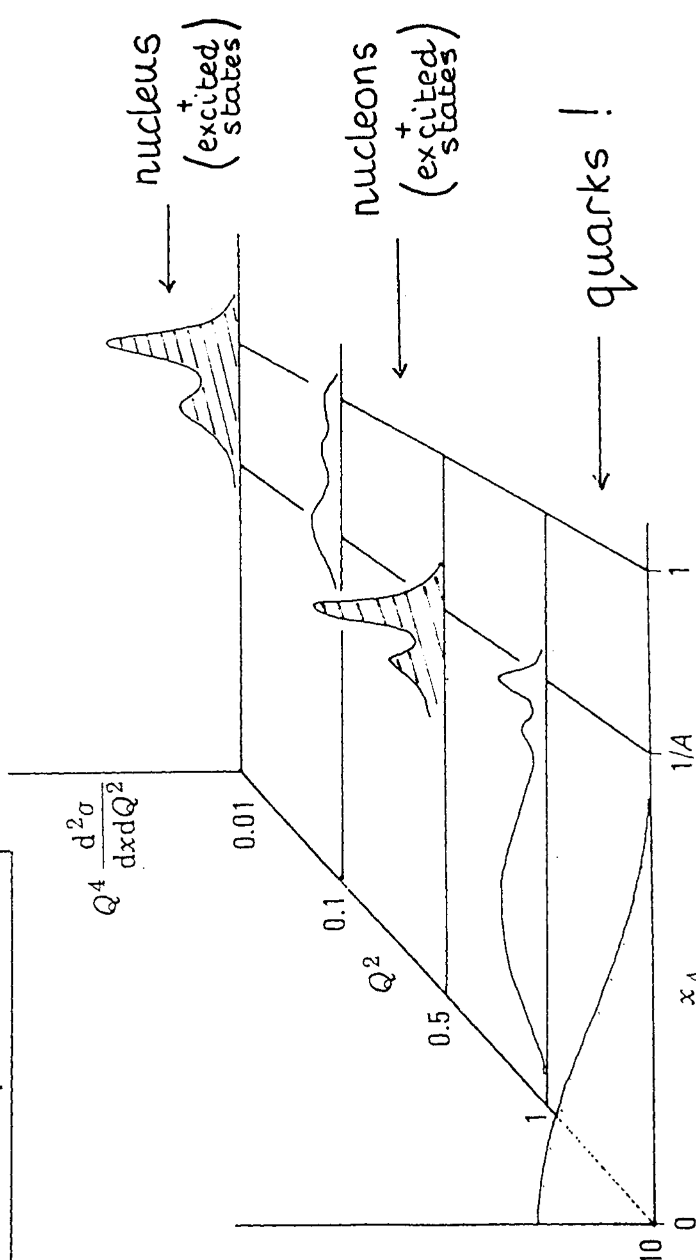
$$F_1 = \frac{1}{8\pi}H_1 \quad , \quad F_2 = \frac{1}{16\pi x}H_2$$

— this is the <u>standard form</u> for the two structure functions F_1 and F_2.

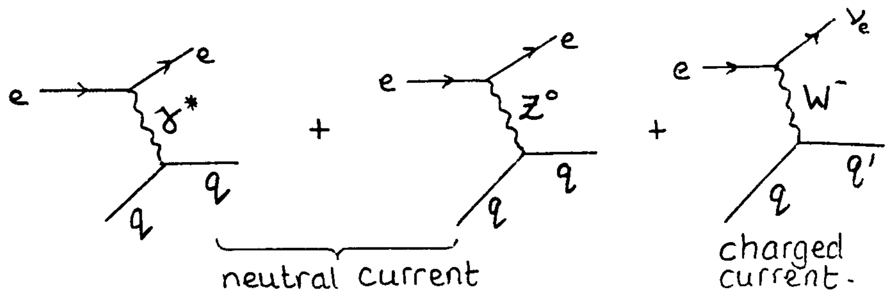
— note that the [] can be rewritten

$$\frac{d^2\sigma}{dx\,dy} = \frac{4\pi\alpha^2}{Q^2xy}\left[\frac{1+(1-y)^2}{2}\cdot 2xF_1 + (1-y)(F_2-2xF_1) - \frac{M^2}{Q^2}x^2y^2F_2\right]$$

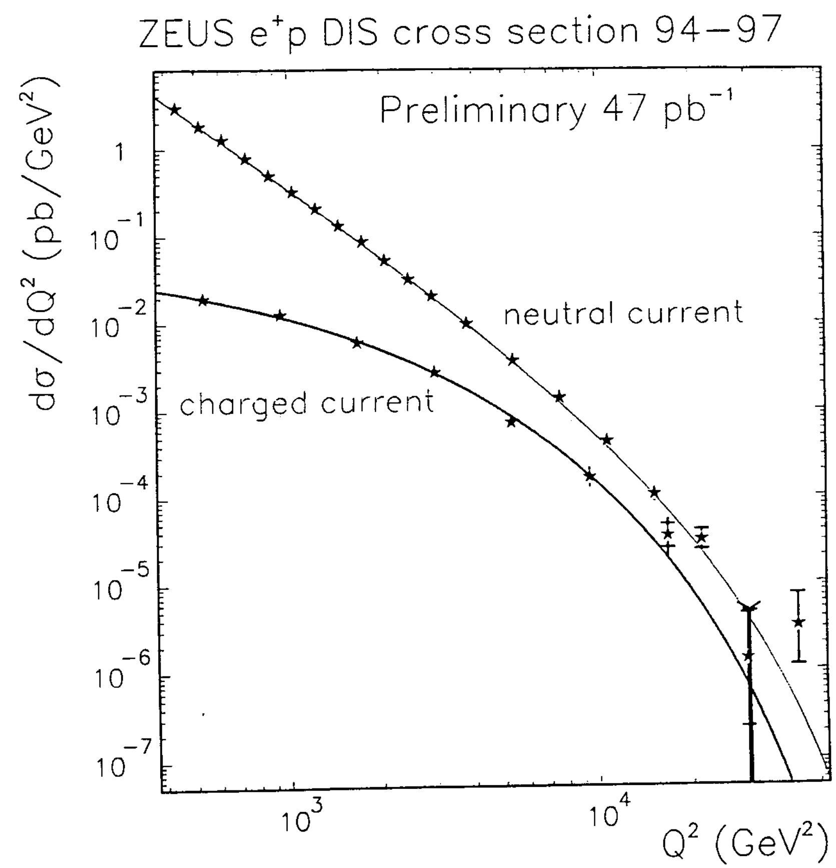
scattering cross section as a function of x and Q^2



— Note that if $Q^2 \gtrsim 10^4\ \mathrm{GeV}^2$ (e.g. at HERA) we must also include $W^\pm$ and Z^0 exchange* in DIS ep scattering :



... which generalizes the result for $\frac{d\sigma}{dx\,dQ^2} \sim [F_1, F_2]$ obtained with photon exchange only → fig.



$$* \quad \frac{1}{Q^4} \longrightarrow \frac{1}{(Q^2 + M_V^2)^2}$$

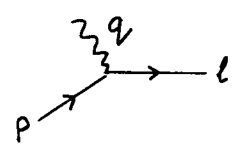
Scaling violations and QCD

— in fact, Bjorken scaling is only approximate ... there are systematic '<u>scaling violations</u>', which, as we shall see, are consistent with the predictions of perturbative QCD

→ fig.

— first, we compute $\gamma^* q \to X$ to $O(\alpha_s)$

lowest order



$$M_\alpha = -ie_q \bar{u}(\ell)\gamma^\alpha u(p)$$

$$d\Phi_1 = 2\pi\, \delta((p+q)^2)$$

to project out F_2, we introduce a light-like vector n^μ such that

$$p^2 = n^2 = 0, \quad n \cdot p = 1, \quad n \cdot q = 0$$

$\{$e.g. $p = (P,0,0,P)$, $n = (\frac{1}{2P}, 0, 0, \frac{-1}{2P})$ $\}$

whence F_2 is projected out by $n^\alpha n^\beta$

i.e.

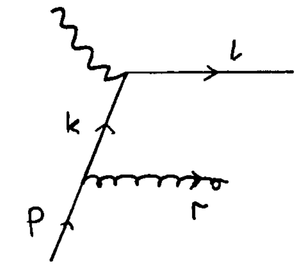
lenotes uark n.fn. →

$$\hat{F}_2(x) = \frac{1}{4\pi}\int d\Phi_1\; n^\alpha n^\beta\; \overline{\sum} |M|^2_{\alpha\beta}$$

↑ "flux"

$$= e_q^2\, \delta(1-x)$$

— now consider



phase space: $d\Phi_2 = \frac{1}{4\pi^2}\int d^4k\; \delta^+((p-k)^2)\; \delta^+((k+\ldots$

write $k^\mu = \xi p^\mu + \frac{k_T^2 - k^2}{2\xi} n^\mu + k_T^\mu$ $k^2 =$

$$\longrightarrow \quad d^4k = -\frac{d\xi}{2\xi}\, dk^2\, d^2k_T$$

then

$$d\Phi_2 = \frac{1}{16\nu\pi^2}\int d\xi\, dk^2\, dk_T^2\, d\theta$$

$$\times\; \delta\left(k_T^2 - (1-\xi)k^2\right)$$

$$\times\; \delta\left(\xi - x - \frac{k^2 + 2Qk_T\cos\theta}{2\nu}\right)$$

matrix element: $M^\alpha = -ig_s e_q\, \bar{u}(\ell)\gamma^\alpha \frac{\not{\epsilon}}{\not{k}}\, T^a u(p)$ ← col ma

and use

$$\sum_{\text{pol}} \epsilon_\mu(r)\epsilon_\nu^*(r) = -g_{\mu\nu} + \frac{n_\mu r_\nu + n_\nu r_\mu}{n \cdot r}$$

light-cone gauge

$$\longrightarrow \frac{1}{4\pi} n^\alpha n^\beta \overline{\sum} |M|^2_{\alpha\beta} = \frac{8 e_q^2 \alpha_s}{k^2} \xi \, P(\xi)$$

note: $\frac{k^2}{k^4}$

where $\quad P(\xi) = C_F \frac{1+\xi^2}{1-\xi}$

and so

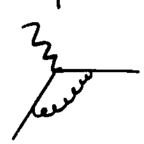
$$\hat{F}_2 = e_q^2 \frac{\alpha_s}{2\pi^2} \int_0^{2\nu \;\leftarrow\; \frac{Q^2}{x}} \frac{dk^2}{k^2} \int_{\xi_-}^{\xi_+} d\xi \frac{\xi \, P(\xi)}{[(\xi_+ - \xi)(\xi - \xi_-)]^{1/2}}$$

— introduce lower limit κ^2 to regularize divergence at $k^2 = 0$, then

$$\hat{F}_2 = e_q^2 \frac{\alpha_s}{2\pi} x \left[P(x) \ln \frac{Q^2}{\kappa^2} + C_1(x) \right]$$

— other real emission diagrams? no logarithms, only contribute $C_2(x)$, $C_3(x)$, ... in this gauge!

— loop diagrams?



$$+ \cdots \longrightarrow \hat{F}_2\Big|_{\text{virt.}} = \alpha_s K \, \delta(1-x)$$

and the logarithmic piece gives

$$P(x) \longrightarrow C_F \left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2} \delta(1-x) \right]$$

$$\int_0^1 dx \frac{f(x)}{(1-x)_+} = \int_0^1 dx \frac{f(x) - f(1)}{1 \cdot}$$

so finally ...

$$\boxed{\hat{F}_2(x, Q^2) = e_q^2 x \left[\delta(1-x) + \frac{\alpha_s}{2\pi} \left\{ P(x) \ln \frac{Q^2}{\kappa^2} + C(x \right. \right.}$$

— to obtain F_2, we fold in the 'bare' quark distribution $q_0(y)$

$$F_2(x, Q^2) = x \sum_q e_q^2 \left[q_0(x) + \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} q_0(y) \left\{ P\left(\frac{x}{y}\right) \ln \frac{Q^2}{\kappa^2} + C\left(\frac{x}{y}\right) \right\} \right.$$

— dependence on κ^2 ('collinear diver since it comes from) indicates sensitivity to long-ranc part of the strong interaction, ther incalculable in pert^n theory

... but can factorize (MASS FACTORIZATION) the divergence in a 'renormalized' quark distribut

$$q_0(x) \longrightarrow q(x, \mu^2)$$

faci ... ation scale

$$\frac{1}{x}F_2 = \sum_q e_q^2 \int_x^1 \frac{dy}{y}\, q(y,\mu^2)\Big\{\delta(1-\tfrac{x}{y}) + \underbrace{\tfrac{\alpha_s}{2\pi}\big(P(\tfrac{x}{y})\ln\tfrac{Q^2}{\mu^2} + C_q(\tfrac{x}{y})\big)}_{\text{finite by construction}}\Big\}$$

finite observable

where

$$q(x,\mu^2) = q_0(x) + \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y} q_0(y)\Big\{P(\tfrac{x}{y})\ln\frac{\mu^2}{\kappa^2} + \bar{C}(\tfrac{x}{y})\Big\}$$

$$\bar{C} = C - C_q$$

<u>note</u>: arbitrariness of $C_q \Leftrightarrow$ 'factorization scheme dependence'

— the μ^2 dependence of $q(x,\mu^2)$ <u>is</u> calculable in pert^n. theory:

$t = \mu^2$

$$\boxed{t\frac{\partial}{\partial t} q(x,t) = \frac{\alpha_s(t)}{2\pi}\int_x^1 \frac{dy}{y} q(y,t) P\left(\frac{x}{y}\right)}$$

— the Dokshitzer Gribov-Lipatov-Altarelli-Parisi equation*

... so pQCD predicts <u>scaling violations</u>, i.e. $\frac{\partial F_2}{\partial \log Q^2} \neq 0$

* 'DGLAP equation' 'Altarelli-Parisi equation'

<u>DGLAP equation - physical pictu</u>

— a fast-moving quark loses momen by emitting a gluon

$$d\mathcal{P} \simeq \frac{\alpha_s(k_T^2)}{2\pi}\frac{dk_T^2}{k_T^2} P(\xi)$$

'splitti functi

with phase space $k_T^2 \lesssim Q^2$, hence

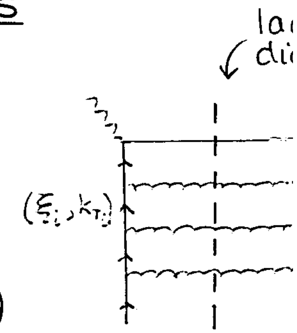
$$d\mathcal{P} \simeq \frac{\alpha_s}{2\pi}\ln Q^2\, P(\xi)\, d\xi$$

$\Rightarrow$ <u>scaling violations</u>

<u>multigluon emission</u>?

can calculate and resum leading logarithms to all orders ...

(<u>note</u> : axial gauge $\rightarrow$ ladder diagrams)



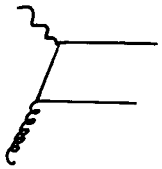
$$q(x,Q^2) = \delta(1-x) + \sum_{n=1}^{\infty}\int_{\kappa^2}^{k_{T_2}^2}\frac{dk_{T_1}^2}{k_{T_1}^2}\frac{\alpha_s(k_{T_1}^2)}{2\pi}\cdots\cdots\int_{k_{T_{n-1}}^2}^{Q^2}\frac{dk_{T_n}^2}{k_{T_n}^2}$$

strongly ordered transverse momenta

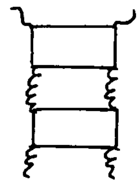
$$\times \int_x^1 \frac{d\xi_{n-1}}{\xi_{n-1}} P\left(\frac{x}{\xi_{n-1}}\right)\cdots\int_{\xi_2}^1 \frac{d\xi_1}{\xi_1} P\left(\frac{\xi_2}{\xi_1}\right) P($$

then $\frac{\partial}{\partial \ln Q^2}$ gives the DGLAP equatio

... other $O(\alpha_s)$ contributions :



... or more generally, ladder diagrams with quark rungs $\longrightarrow$



$\Longrightarrow$ additional contributions to the DGLAP eq^n.

$$\boxed{\begin{aligned} t\frac{\partial}{\partial t} q_i(x,t) &= \frac{\alpha_s(t)}{2\pi}\int_x^1 \frac{dy}{y}\left\{ P^{qq}\left(\tfrac{x}{y}\right) q_i(y,t) + P^{qg}\left(\tfrac{x}{y}\right) g(y,t)\right\} \\ t\frac{\partial}{\partial t} g(x,t) &= \frac{\alpha_s(t)}{2\pi}\int_x^1 \frac{dy}{y}\left\{ P^{gq}\left(\tfrac{x}{y}\right) \sum_j q_j(y,t) + P^{gg}\left(\tfrac{x}{y}\right) g(y,t)\right\} \end{aligned}}$$

i.e. $(2n_f+1)$ equations for $\{q_i, \bar{q}_i, g\}$

$\longrightarrow$ 3 generic types, for

$$q_{NS} = q_i - q_j \qquad (i \neq j)$$
$$q_S = \sum_i (q_i + \bar{q}_i)$$

then

convolution: $\int_x^1 \frac{dy}{y} f\left(\frac{x}{y}\right) g(y)$

$$t\frac{\partial}{\partial t}\begin{pmatrix} q_{NS} \\ q_S \\ g \end{pmatrix} = \frac{\alpha_s(t)}{2\pi}\begin{pmatrix} P^{qq} & 0 & 0 \\ 0 & P^{qq} & P^{qg} \\ 0 & P^{gq} & P^{gg} \end{pmatrix} \circledast \begin{pmatrix} q_{NS} \\ q_S \\ g \end{pmatrix}$$

Splitting functions (leading order)

$$P^{qq} = C_F\left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\delta(1-x)\right] \qquad P^{qg} = \frac{1}{2}\left[x^2 + (1-x)^2\right.$$

$$P^{gq} = C_F\,\frac{1+(1-x)^2}{x} \qquad P^{gg} = 2C_A\left[\frac{x}{(1-x)_+} + \frac{1-x}{x}\right.$$
$$+\delta(1-x)\left(\frac{11C_A - 2r}{6}\right.$$

with properties...

$$\int_0^1 dx\, P^{qq}(x) = 0 \qquad \text{conservation o[f] quark (baryon) nu[mber]}$$

$$\left.\begin{aligned} \int_0^1 dx\, x\left[P^{qq}(x) + P^{gq}(x)\right] &= 0 \\ \int_0^1 dx\, x\left[P^{qg}(x) + P^{gg}(x)\right] &= 0 \end{aligned}\right\} \text{momentum conservation}$$

Solution of the DGLAP equations

(A) numerical

boundary conditions : $u(x,Q_0^2), \bar{u}(x,Q_0^2), \ldots, g(x, Q$

'starting scale' : $\alpha_s(Q_0^2) \ll 1$

typically : $A x^{\alpha}(1-x)^{\beta}\left[1 + \epsilon\sqrt{x} + \gamma x\right]$

Ⓑ semi-analytic (moments)

define $f(n,Q^2) = \int_0^1 dx\, x^{n-1} f(x,Q^2)$, $f = q_i, g$

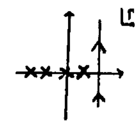
$\gamma_n^{ab} = \int_0^1 dx\, x^{n-1}\, P^{ab}(x)$

then

$$t\frac{\partial}{\partial t} q_{NS}(n,t) = \frac{\alpha_s(t)}{2\pi}\, \gamma_n^{qq}\, q_{NS}(n,t)$$

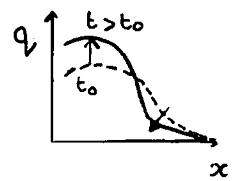
$$\Longrightarrow \quad \underline{q_{NS}(n,t) = q_{NS}(n,t_0)\left[\frac{\alpha_s(t_0)}{\alpha_s(t)}\right]^{d_n^{qq}}} \quad \xleftarrow{} \frac{\gamma_n^{qq}}{2\pi\beta_0}$$

then inverse transform : $q_{NS}(x,t) = \frac{1}{2\pi i}\oint dn\, x^{-n}\, q_{NS}(n,t)$



note

(i) $\left.\begin{array}{l} d_1^{qq} = 0 \\ d_{n\geq 2}^{qq} < 0 \end{array}\right\}$



(ii) $$t\frac{\partial}{\partial t}\begin{pmatrix} q_s(2) \\ g(2)\end{pmatrix} = \frac{\alpha_s(t)}{2\pi}\begin{pmatrix} -\frac{4}{3}C_F & \frac{1}{3}n_f \\ \frac{4}{3}C_F & -\frac{1}{3}n_f\end{pmatrix}\begin{pmatrix} q_s(2) \\ g(2)\end{pmatrix}$$

$$t\frac{\partial}{\partial t}\begin{pmatrix} q_s(2)+g(2) \\ q_s(2) - \frac{n_f}{4C_F}g(2)\end{pmatrix} = \frac{\alpha_s(t)}{2\pi}\begin{pmatrix} 0 & 0 \\ 0 & -\left(\frac{4}{3}C_F + \frac{n_f}{3}\right)\end{pmatrix}\begin{pmatrix} q_s(2)+g(2) \\ q_s(2) - \frac{n_f}{4C_F}g(2)\end{pmatrix}$$

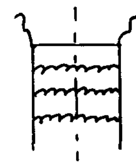
↙ momentum conserved $\forall t$

and asymptotically $(t\to\infty)$ . . .

$$\frac{q_s(2)}{g(2)} = \frac{n_f}{4C_F} = \frac{3}{16}n_f \quad \left\{ \begin{array}{l}\text{momentum shared} \\ \text{between quarks} \\ \text{and gluons}\end{array}\right.$$

Beyond leading order . . .

(i) leading order (leading log approx^{n})



$$\to \sum_n \alpha_s^n \log^n Q^2 f_n(x) \to \left\{ \begin{array}{l} F(x,Q^2) = \sum_i c_i \\ Q^2\frac{\partial}{\partial Q^2} q = \frac{\alpha_s}{2\pi} P\otimes \end{array}\right.$$

(ii) next-to-leading order

↙ lose one log from non-e

$$\to \sum_n \alpha_s^n\{\log^n Q^2 f_n(x) + \log^{n-1}Q^2 \bar{f}_n(x) + \ldots\}$$

now

$$\frac{1}{x}F_2(x,Q^2) = \sum_q e_q^2 \int_x^1 \frac{dy}{y}\left[q\left(\frac{x}{y},Q^2\right)\left\{\delta(1-y) + \frac{\alpha_s(}{2\pi}\right.\right.$$

$$+ \frac{\alpha_s(Q^2)}{2\pi}\, g\left(\frac{x}{y},Q^2\right) C_g (\,$$

↑ coeffi func

and

$$P^{ab}(x) \longrightarrow P^{(0)ab}(x) + \frac{\alpha_s}{2\pi} P^{(1)ab}(x)$$

. . . where now the coefficient functi and NLO splitting functions carry factorization scheme and renormalisatic scheme labels

e.g. DIS , $\overline{MS}$, . . .

↑ $C_q = C_g \equiv 0$ ↑ most widely used in pr

note

— in the limit $x \to 0$, $Q^2 \to \infty$ the behaviour of F_2 can be calculated analytically:

$$Q^2 \frac{\partial}{\partial Q^2} \begin{pmatrix} q \\ g \end{pmatrix} \simeq \frac{\alpha_s}{\pi} \begin{pmatrix} 0 & 0 \\ \frac{4/3}{x} & \frac{3}{x} \end{pmatrix} \otimes \begin{pmatrix} q \\ g \end{pmatrix}$$

$$\Rightarrow \quad F_2 \approx x \sum_i q \sim \exp\left[2 \sqrt{\frac{3\alpha_s}{\pi} \ln\frac{Q^2}{Q_0^2} \ln\frac{1}{x}} \right] \quad \alpha_s \text{ fixed}$$

double leading log approximation (DLLA) — De Rujula et al. (1974)

... which is a <u>reasonable</u> approximation to the HERA measurements

{ developed as 'double asymptotic scaling' + corrections by Ball and Forte → fig. }

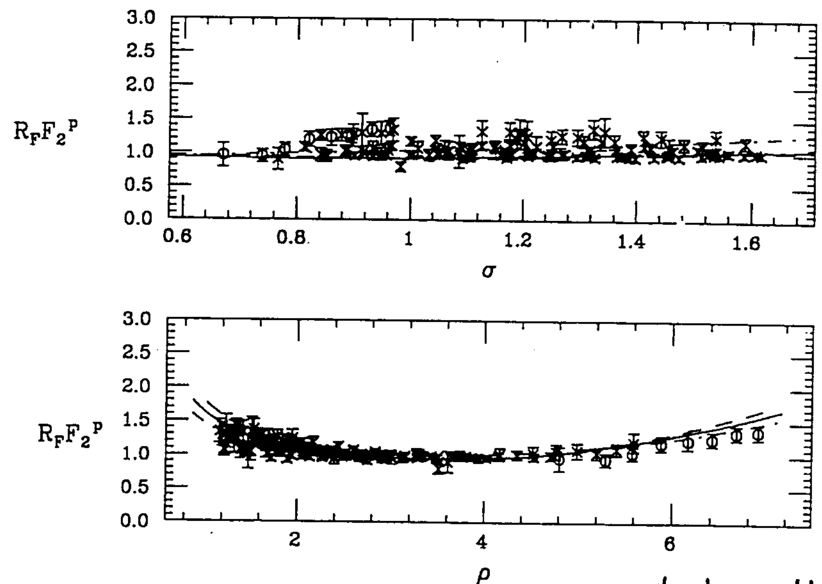
double asymptotic scaling

$$\sigma = \left[\ln\frac{t}{t_0} \ln\frac{x_0}{x} \right]^{\frac{1}{2}}$$

$$\rho = \left[\ln\frac{x_0}{x} \Big/ \ln\frac{t}{t_0} \right]^{\frac{1}{2}}$$

$(t = \ln Q^2/\Lambda^2)$

Ball, Forte



data: H1

Parton Distribution Functions (pdf's)

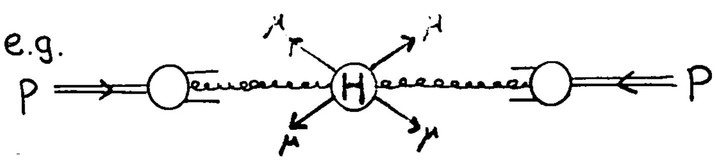
— the bulk of the information on pdf's comes from fitting DIS structure functions ....

... although other processes can also be useful e.g. the Drell-Yan process for constraining the sea (anti)quarks

— the pdf's are useful in two ways:

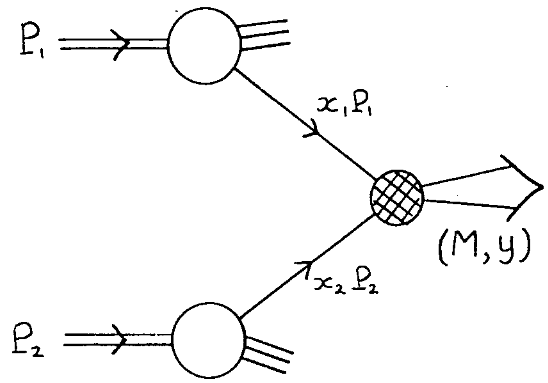
- they are important for predicting hadron-collider cross sections †
- they give us detailed information on the flavour content of the nucleon

† e.g.



$$\sigma_{Higgs} \propto [g(x)]^2 \quad , \quad x \simeq \frac{M_H}{\sqrt{s}}$$

hadron - hadron collisions



X = { Jets, W, Z, WW, ZZ, $t\bar{t}$, $b\bar{b}$, Higgs, sparti..., Z', ⋮

$$d\sigma = \sum_{i,j} \int dx_1 dx_2 \, f_i(x_1,\mu^2) \, f_j(x_2,\mu^2)$$

$$\times \, d\hat{\sigma}_{ij}\left(p_1, p_2, \alpha_S(\mu^2), \frac{M^2}{\mu^2}\right)$$

"Factori... the...

where

$$x_{1,2} \simeq \frac{M}{\sqrt{s}} e^{\pm y}$$

$$\mu \simeq M$$

note: $\mu_F = \mu_R \equiv \mu$ as...

parton structure : f_i

tests of pQCD : $d\hat{\sigma}_{ij}$, f_i

- instead of having to laboriously integrate the 'Altarelli - Parisi' equations each time a distribution (e.g. $u(x, Q^2 = M_W^2)$) is needed, analytic and numerical approximations are provided in the literature, e.g.
  - Duke and Owens (DO), Gluck et al. (GHR), Eichten et al. (EHLQ), Tung et al. (CTEQ), Martin et al. (MRS) ⋮ — 1984 → 1999
- thus, for example,

```
SUBROUTINE MRST(X,Q,UV,DV,USEA,DSEA,
                STR,CHM,BOT,GLU)
```

(input: X, Q)

- the MRS series of fits (1987 ⟶)
  - at $Q_0 = 1$ GeV parameterise

$$\boxed{f_i(x,Q_0^2) = A_i\, x^{\lambda_i - 1}\left[1 + \epsilon_i \sqrt{x} + \gamma_i x\right](1-x}$$

with $i = u, d, \ldots\ldots, \bar{c}, \bar{b}, g$, then evolve using NLO-DGLAP and fit for $\{A_i, \ldots \eta_i, \alpha_s\}$

  - as the data improve, the fits are fine-tuned ....

- most recent MRS analysis

A. D. Martin
R.G. Roberts
R.S. Thorne
W. J. Stirling

$\Longrightarrow$ MRST (1998) sets*

[hep-ph/9803445]

[code from: http://durpdg.dur.ac.uk/HEPDATA]

- <u>features</u>

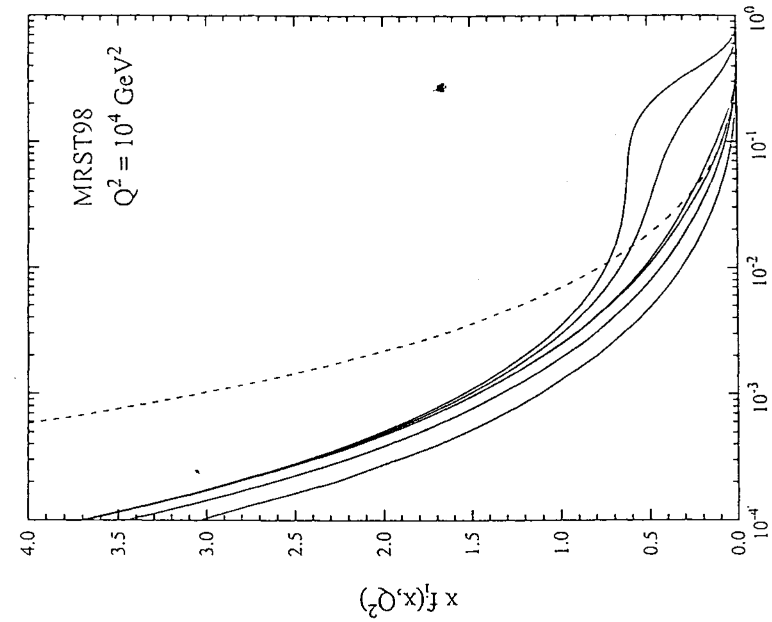
  — new and updated data sets

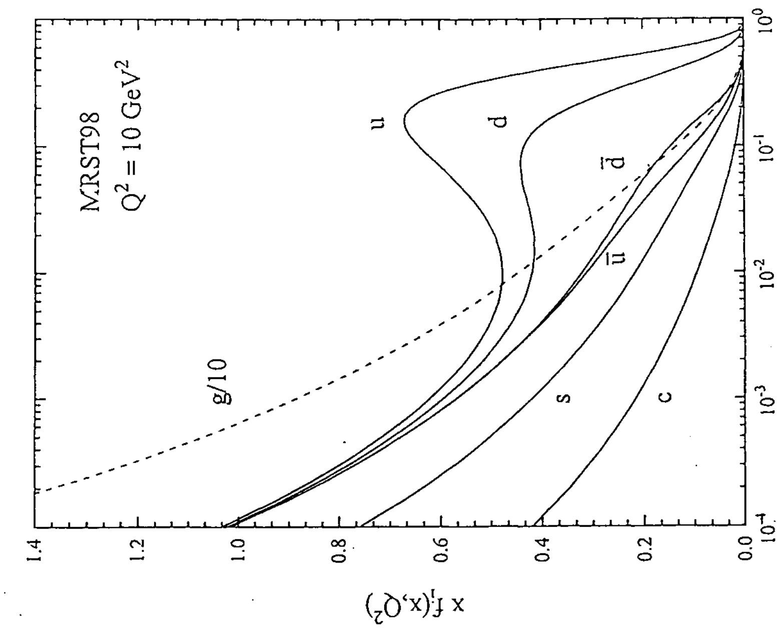
  — improved treatment of heavy flavour and prompt photon production

  — default + 4 sets:
    - variation in α_s ↑↓
    - variation in gluon ↑↓

* small bug found and corrected: ⟶ MRST99
see: hep-ph/9907231

Martin Roberts, Stirling Thorne





experimental data and errors

DIS str. fns + hadron-hadron

\+

theoretical framework

e.g. NLO-DGLAP $\overline{MS}$ scheme

\+

theoretical assumptions and prejudices

e.g. Q^2, W^2 cuts, omit datasets, npQCD effects, ⋮

$$= \boxed{f_i(x,Q^2) \quad Q^2 > Q_0^2 \quad i = u,d,\ldots,g} + \alpha_S$$

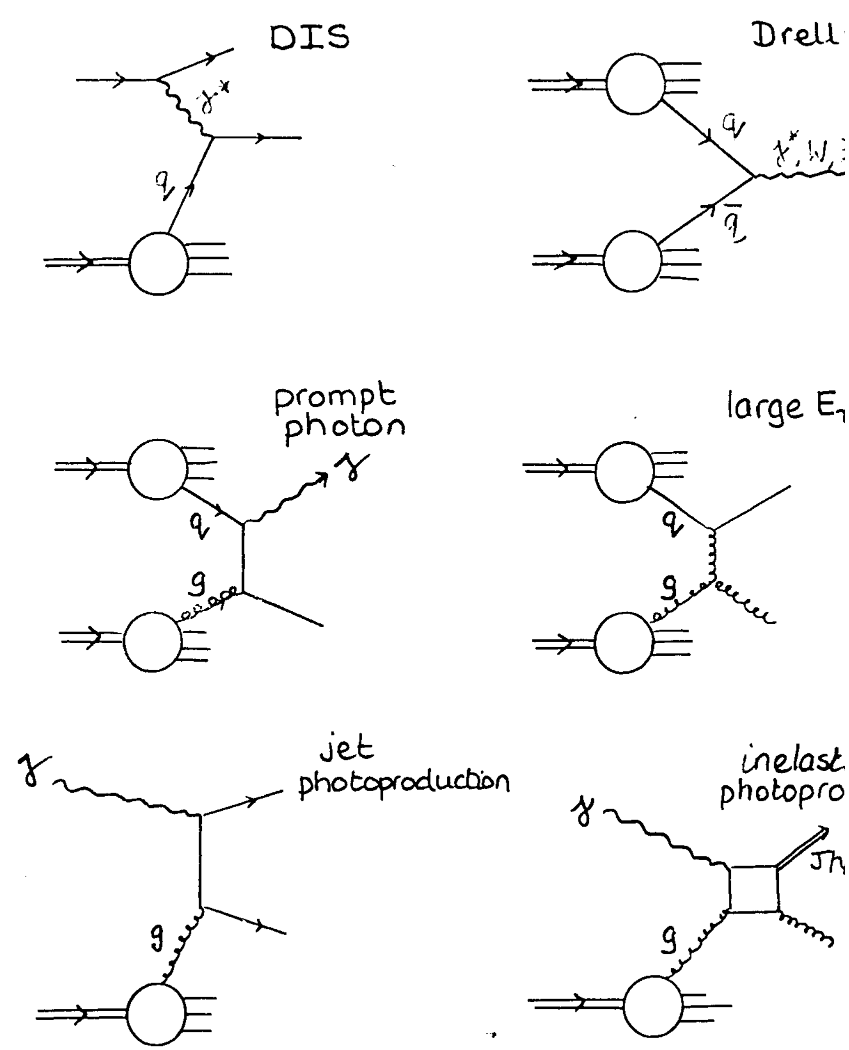
→ fig.

<u>summary</u>

$F_i \longrightarrow$ quarks (all x)
DY $\longrightarrow$ sea quarks (high x)
$\frac{\partial F_i}{\partial \ln Q^2} \longrightarrow$ gluon (small x)
hh → jets, $\gamma \longrightarrow$ gluon (high x)

→ fig.

<u>note</u> { MRST (1998), CTEQ5 (1999) } ... broadly similar



Process/ Experiment	Leading order subprocess	Parton determination
DIS ($\mu N \to \mu X$) $F_2^{\mu p}, F_2^{\mu d}. F_2^{\mu n}/F_2^{\mu p}$ (SLAC, BCDMS, NMC, E665)	$\gamma^* q \to q$	Four structure functions $\to$ $u + \bar{u}$, $d + \bar{d}$, $\bar{u} + \bar{d}$, s (assumed $= \bar{s}$), but only $\int xg(x, Q_0^2)dx \simeq 0.35$ and $\int(\bar{d} - \bar{u})dx \simeq 0.1$
DIS ($\nu N \to \mu X$) $F_2^{\nu N}, xF_3^{\nu N}$ (CCFR)	$W^* q \to q'$	
DIS (HERA) F_2^{ep} (H1, ZEUS)	$\gamma^*(Z^*)q \to q$	λ ($x\bar{q} \sim x^{-\lambda_s}$. $xg \sim x^{-\lambda_g}$)
$\ell N \to c\bar{c}X$ F_2^c (EMC, H1, ZEUS)	$\gamma^* c \to c$	c ($x \gtrsim 0.01$, $x \lesssim 0.01$)
$\nu N \to \mu^+\mu^- X$ (CCFR)	$W^* s \to c$ $\hookrightarrow \mu^+$	$s \approx \frac{1}{2}(\bar{u} + \bar{d})$
$pN \to \gamma X$ (WA70, UA6, E706, ...)	$qg \to \gamma q$	g at $x \simeq 2p_T^\gamma/\sqrt{s} \to$ $x \approx 0.2 - 0.6$
$pN \to \mu^+\mu^- X$ (E605, E772)	$q\bar{q} \to \gamma^*$	$\bar{q} = ...(1-x)^{\eta_S}$
$pp, pn \to \mu^+\mu^- X$ (E866, NA51)	$u\bar{u}, d\bar{d} \to \gamma^*$ $u\bar{d}, d\bar{u} \to \gamma^*$	$\bar{d}/\bar{u}$ at $x \approx 0.04 - 0.3$
$p\bar{p} \to WX(ZX)$ (UA1, UA2, CDF, D0) $\to \ell^\pm$ asym (CDF)	$ud \to W$	u, d at $x \simeq M_W/\sqrt{s} \to$ $x \approx 0.13, 0.05$ slope of u/d at $x \approx 0.05 - 0.1$
$p\bar{p} \to$ jet $+ X$ (CDF, D0)	$gg, qg, qq \to 2j$	q, g at $x \simeq 2E_T/\sqrt{s} \to$ $x \approx 0.05 - 0.5$

DIS ←

adron–
adron ←

MRST
1998

"Science", 22 January 1999

Each time physicists probe the teeming interior of the proton, aswarm with short-lived particles, they seem to turn up more surprises

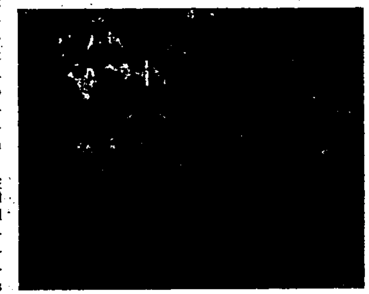
Exploring the Proton Sea

In the Dr. Seuss story "Horton Hears a Who!," Horton the elephant insists to his fellow animals, all deeply skeptical, that a speck of dust is teeming with life. With his sensitive ears, Horton can hear the chatter and buzz of its microscopic inhabitants—whole cities of them. Physicists studying the humble proton will understand his fascination. To most researchers, the proton is a workaday particle: the stuff that gives every atomic nucleus its positive charge, and the heart of the ubiquitous hydrogen atom. But recent studies probing deep into the proton are revealing a society as complex as the one on Horton's dust mote: a churning and bubbling sea of "virtual" particles that pop into existence for an instant, then disappear again, bathing more enduring components of the proton in a quantum flux.

The ephemeral nature of the sea's inhabitants, mass- and charge-carrying particles called quarks and force-carrying particles called gluons, belies their importance. "This virtual sea is responsible for many of the proton's properties, such as its mass, its structure, and its interaction with other particles and fields," says Michael Leitch of the Los Alamos National Laboratory in New Mexico. Charting the sea is also important for future experiments: The world's most powerful particle accelerator, the Large Hadron Collider now being built at the CERN particle physics lab near Geneva, will slam protons together at enormous energies. One aim is to create the Higgs boson, the particle thought to endow all others with mass, which has been on physicists' "most wanted" list for 3 decades. Knowing what is in the proton is essential for calculating what will come out of those collisions. "If new physics is to be discovered, we need to understand the predictions from the old physics with some precision," says Arie Bodek of the University of Rochester in New York.

Yet the normal theoretical apparatus used to describe the subatomic landscape can make few predictions at the energies found in the proton's interior. As a result, physicists found themselves in uncharted waters as they began exploring the interior of the proton by probing it with beams of other particles. Lately, a series of experiments at accelerators in Europe and the United States to measure the different types of quarks in the proton sea, compare the proportion of quarks to gluons, and identify differences in the quark sea of the proton and that of the proton's sister particle, the neutron, have delivered a string of surprises. As Anthony Thomas of the University of Adelaide in Australia puts it, "Every time we have tested a prejudice about the sea ... it has proven to be wrong."

When the proton was discovered by



Flavor enhancers. Fermilab's NuTeV team found a short-age of strange quarks in the proton.

Ernest Rutherford in 1919, it was thought to be an indivisible basic building block of matter. But that fundamental status did not last long. Early proton-proton collision experiments in the 1930s revealed that the proton was more than an infinitesimally small "point-charge": It had a finite size and presumably some kind of structure. Further experiments revealed a bewildering array of particles related to the proton, whose properties fell into patterns that cried out for an explanation in terms of more fundamental building blocks. A breakthrough came 1960s, when theoreticians Murray Mann of the California Institute of Tec gy in Pasadena and his ex-student C Zweig at CERN proposed that fundar particles called quarks make up proton trons, and the short-lived particles mesons. Protons and neutrons contain quarks each, and mesons a quark and a quark. In 1969, electron-proton collisi the Stanford Linear Accelerator Cente firmed the existence of pointlike nugg side the proton, which had to be quarks

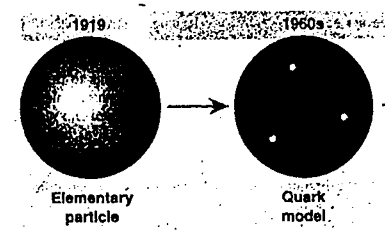
The new picture painted by Gell- and Zweig was simple: The proton is m two so-called "up" quarks and a "down" quark—its "valence" quarks quark has a fractional electric charge, a combination in the proton adds up to p its single positive charge. In the neutr numbers of ups and downs are reverse ing a one-up, two-down combinatio makes the neutron electrically neutr theory later found to govern these qua their interactions was dubbed quantum modynamics (QCD), now part of the dard Model by which physicists und the subnuclear world. QCD predic quarks carry a "color charge," mimick familiar electrical charge, which is the of the force binding them together and ried by gluons, force particles analog the photons of electromagnetism.

But even this tidy model of three quarks and a buzz of gluons holding together proved to be far from the story. Experiments at CERN in the 1970s probing protons with ghostly p called neutrinos revealed the presence tiquarks along with the three valence and soon researchers' image of the began to change. A proton "is not thing with three balls in it all hooked

The Evolving Proton

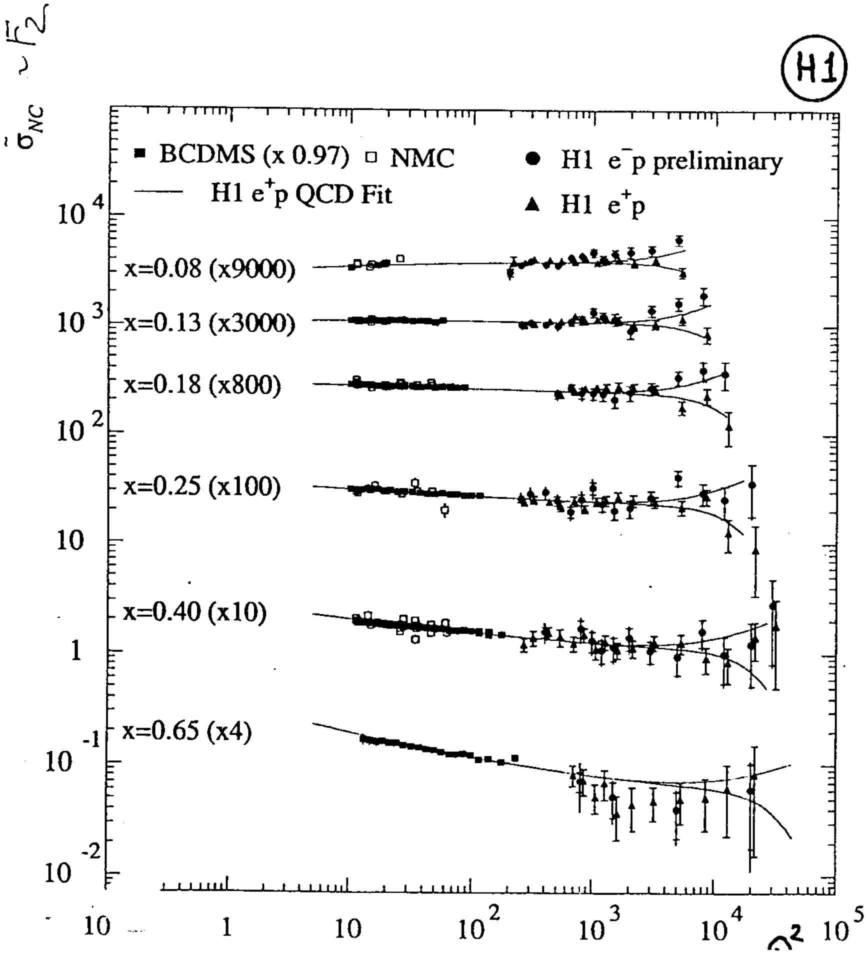
Probing ever deeper. Physicists' increasingly complex view of the proton.

- Valence quark
- Valence antiquark
- Sea quark
- Sea antiquark
- Gluons



— nothing unusual at high Q^2...

- quarks still pointlike
- DGLAP works
- electroweak effects visible. $(Q^2 \gtrsim M_Z^2)$



Physics World April 1997

DESY double offers high hopes for new physics

Particle physicists are eagerly awaiting new data from the HERA collider in Germany to confirm if they have detected a new particle known as a leptoquark, evidence for substructure in quarks, both leptoquarks and quark substructure or, possibly, none of these

From James Stirling in the Departments of Mathematical Sciences and Physics, University of Durham, UK

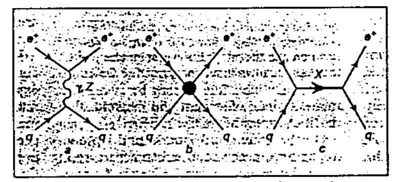
ASK any particle physicist to bet on which new particle will be discovered next and the most popular answer, at least until recently, would have been the Higgs boson. A less well known particle – the leptoquark – would have been well down most people's list. However, dramatic new results from HERA, the high-energy collider at the DESY Laboratory in Hamburg, could be the first evidence for these unusual new particles. Alternatively, the data might indicate that quarks, previously thought to be elementary particles, have substructure.

If the results are confirmed by further data from HERA, the cherished Standard Model of elementary particles and their interactions will have been stood on its head. The holy grail of late 20th century particle physics – experimental evidence for new physics beyond the Standard Model – will finally have been found.

The excitement at HERA centres on a handful of unusual events seen by both the H1 and ZEUS experiments. The two teams involved submitted their results to *Zeitschrift für Physik* in late February and their papers, plus a wealth of information on the HERA collider and the two experiments, are available on the DESY World Wide Web site at http://info.desy.de/. Both experiments are run by large international collaborations: H1 by 400 physicists from 12 countries; and ZEUS by 430 physicists from 12 countries.

The two experiments were studying what happens when a 27.5 GeV positron beam collides with a 820 GeV proton beam. (HERA can also collide electron beams with proton beams.) Such high-energy collisions have traditionally provided an enormous amount of information on fundamental particles and their interactions. In particular, when a positron scatters off a proton at a large angle, the energy transferred to the proton causes it to break up, revealing details of its innermost structure.

This type of "deep inelastic scattering" experiment was first performed, using electron beams at much lower collision energies, at the Stanford Linear Accelerator Center (SLAC) more than 25 years ago. The SLAC experiment showed for the first time that the proton was made up of quark constituents. Quarks are believed to be the fundamental building blocks of strongly interacting or "hadronic" matter, particles such as protons, neutrons and pions. In the 1970s theorists developed a quantum field theory for the strong interactions of quarks called quantum chromodynamics (QCD), which nowadays is part of the Standard Model.



1 Feynman diagrams illustrate how particles interact. In most collisions (a) a positron scatters off a quark inside the proton by exchanging a "virtual" photon or a neutral Z boson, as predicted by the Standard Model. In (b), a new type of interaction between leptons and quarks, perhaps indicating further substructure, gives an additional contribution to the scattering. In the leptoquark scenario (c), the positron and quark annihilate to make a new heavy particle, X, which subsequently decays back to a positron–quark pair.

According to the Standard Model, a high-energy positron scatters off a proton by exchanging a virtual (i.e. short-lived) photon. This photon has a wavelength $\hbar/Q$, where $\hbar$ is Planck's constant divided by 2π and Q is the momentum transferred from the positron, and the proton's substructure can be resolved on this scale. In the HERA experiments this wavelength can be several orders of magnitude smaller than the overall size of the proton (which is about 10^{-15} m). The virtual photon therefore travels deep inside the proton and scatters off a quark (figure 1*a*). The quark is knocked out of the proton, emerging not as a free particle (the strong force always confines quarks in hadrons) but rather as a "jet" of pions, kaons and other hadrons.

The number of deep inelastic scattering events seen in the detectors varies with the energy loss of the scattered positron anc the angle through which it is scattered These two measured quantities determin two variables that are useful for makin the connection with theory – the square o the momentum transfer, Q^2, and the frac tion of the proton's momentum, x, that i carried by the quark struck by the virtua photon. The dependence of the event rat on x provides important information o how the quarks share the momentum o the proton.

On the other hand, the dependence o Q^2 is precisely predicted by the theory under two important assumptions: tha the interactions of the quarks an positrons are exactly as specified i the Standard Model; and that th quarks are "point-like" – that is the have no discernible size on the scal of the resolution $\hbar/Q$. If both thes assumptions are satisfied, the even rate is predicted to fall like $1/Q^4$ (fo fixed x), subject to small calculabl corrections. One such correctio comes from the possibility of th positron and the quark exchanging heavy neutral Z boson rather tha photon (figure 1*a*).

Over the last two years the H and ZEUS experiments have col lected a large number of dee inelastic scattering events over wide range in Q^2 and x. The numbe of small and medium Q^2 event agrees beautifully with the Standar Model predictions. However, at high Q there is an apparent excess of events instead of falling off rapidly as predicte by theory, the event rate seems to level of The H1 experiment observes 12 events fo $Q^2 > 15000$ GeV^2 where 4.71 ± 0.76 ar expected in the Standard Model, while th ZEUS experiment observes 2 events fo $Q^2 > 35000$ GeV^2 where 0.145 ± 0.01 events are expected.

The high Q^2 events have a very distinc tive signature, each containing a ver energetic positron and jet of hadrons (se figure 2). Because the number of exces events is not particularly large, the effec could be a statistical fluctuation. Howeve the experimenters estimate the probabil ities of such a fluctuation to be very sma – 1% for H1 and 6% for ZEUS. Take together, this is about as likely as gettin

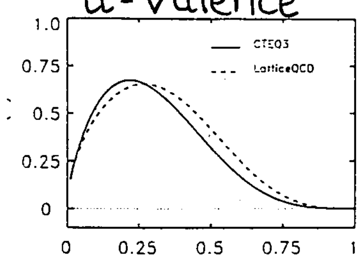
— pdfs from npQCD (Lattice)

... Lattice calculations give first few moments of q and Δq for $p, \pi, \rho, \ldots$

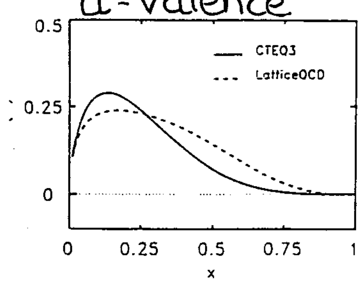
↑ polarised pdfs

non-singlet, leading-twist + QUENCHED approx.

u-valence



d-valence



↑ Mankiewicz [illegible]

proton moments

Moment	Lattice (quenched, $\mu^2 \approx 5\,GeV^2$)	Experiment ($\mu^2 = 4\,GeV^2$)
$\langle x\rangle^{(u)}$	0.410(34)	0.284
$\langle x\rangle^{(d)}$	0.180(16)	0.102
$\langle x\rangle^{(u)}-\langle x\rangle^{(d)}$	0.230(38)	0.182
$\langle x^2\rangle^{(u)}$	0.108(16)	0.083
$\langle x^2\rangle^{(d)}$	0.036(8)	0.025
$\langle x^3\rangle^{(u)}$	0.020(10)	0.032
$\langle x^3\rangle^{(d)}$	0.000(6)	0.008
$\langle x\rangle^{(g)}$	0.53(23)	0.441

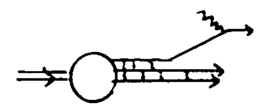
Schierholz et al. (1997)

+ $x \to 0, 1$ assumptions

other non-perturbative models

dimensional counting $(x \to 1)$

- $f_i(x) \sim (1-x)^{2n_s - 1}$



e.g. $f_v \sim (1-x)^3$

- Regge $(x \to 0)$

$$f(x) \sim x^{-\alpha_{\mathbb{R}}(0)} \sim \begin{cases} x^{-1/2} & \text{valence} \\ x^{-1} & \text{sea, glue} \end{cases}$$

- 2DQCD

valence quark structure of Baryon soliton

Krishnas[illegible] Rajeev ⋮

- dynamical partons

evolve from pure 3-quark valence state at very small $Q^2 = \mu_0^2$ $(= 0.3\,GeV^2\,!)$

Glü[illegible] Rey[illegible] Vog[illegible] ⋮

— **problem**: at what Q^2 do these npQCD models apply?

James Stirling

Lecture II

CTEQ SUMMER SCHOOL 2000

scaling violations — summary

$$\frac{\partial F_2}{\partial \log Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 dy\, P_{qq}(y) F_2\left(\frac{x}{y}, Q^2\right) + \sum_q e_q^2 P_{qg}(y) \frac{x}{y} g\left(\frac{x}{y}\right.$$

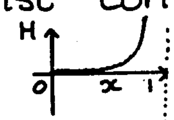
→ large x : $F_2 \gg xg$ ∴ 1^{st} term dominat

⇒ precision α_s measurement

(…, BCDMS, CCFR, …)

{ careful! "higher twist" contributio

$F_2 \to F_2\left(1 + \frac{H(x)}{Q^2}\right)$



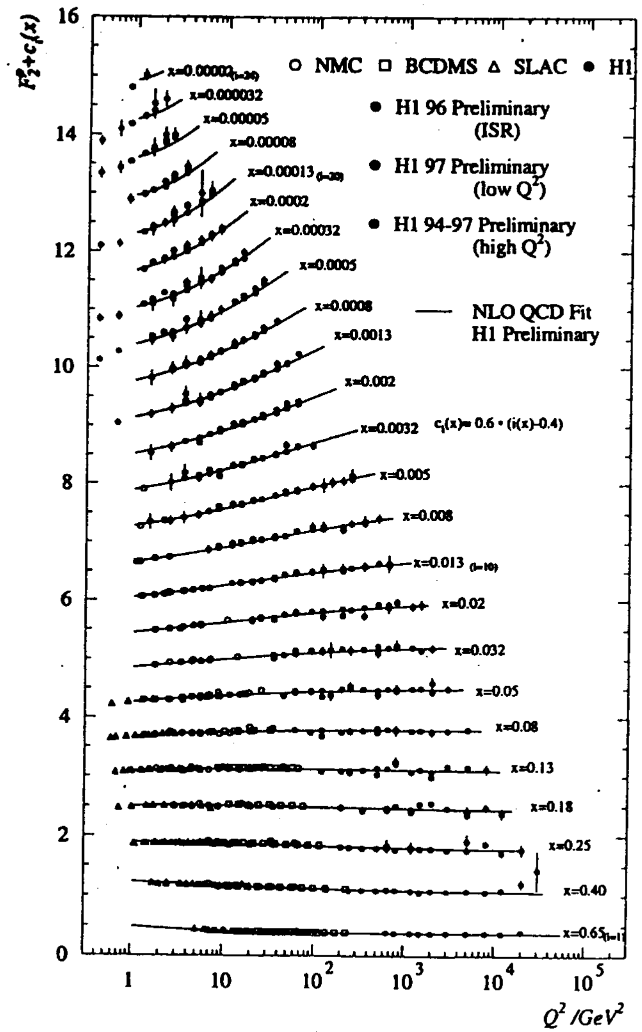
see e.g. Alekhin, Kataev (1999) }

→ small x : $F_2 \ll xg$ ∴ 2^{nd} term domina

⇒ precision gluon measurement

(H1, ZEUS, NMC)

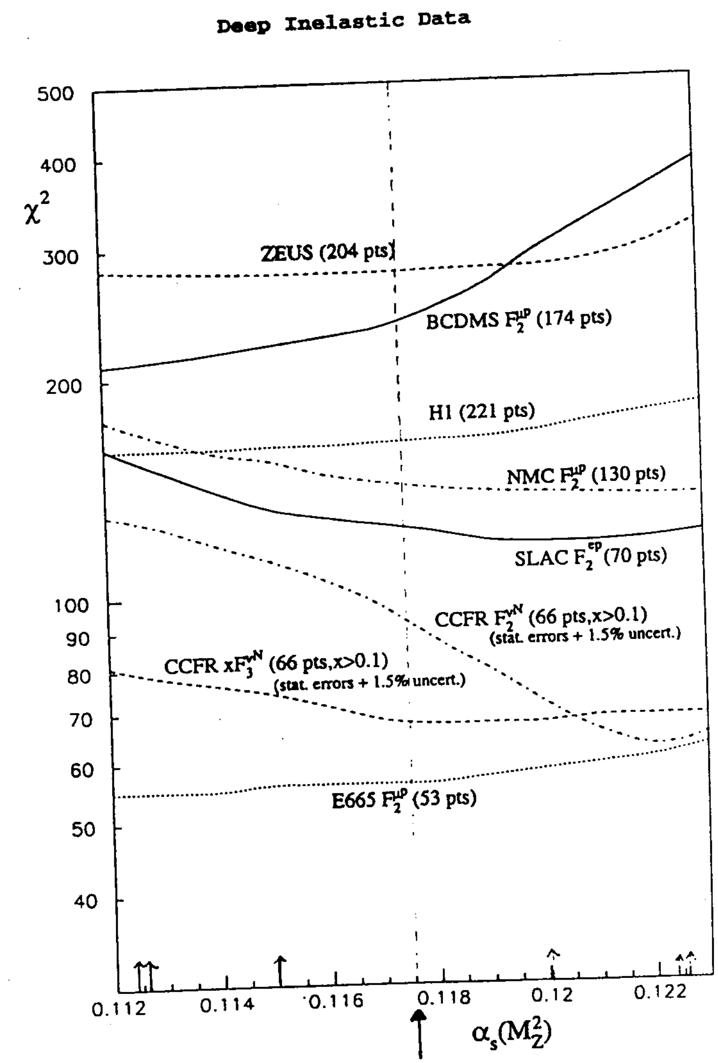
2



3

α_s from global fit

MRST



global fit: $\alpha_s = 0.1175$

cf. world average: $\alpha_s = 0.118 \pm 0.004$

CCFR 1997

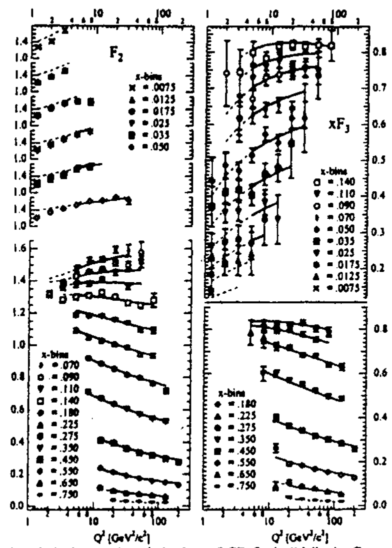


FIG. 1. The F_2 and xF_3 data (statistical errors) and the best QCD fit (solid line). Cuts of $Q^2 > 5\ GeV^2$, $W^2 > 10\ GeV^2$, and $x < 0.7$ were applied for the NLO-QCD fit which include target mass corrections. The dashed line extrapolates the QCD fit into the data regions excluded by the cuts. Deviations of the data from the extrapolated fit are partly due to non-perturbative effects.

1997 CCFR analysis

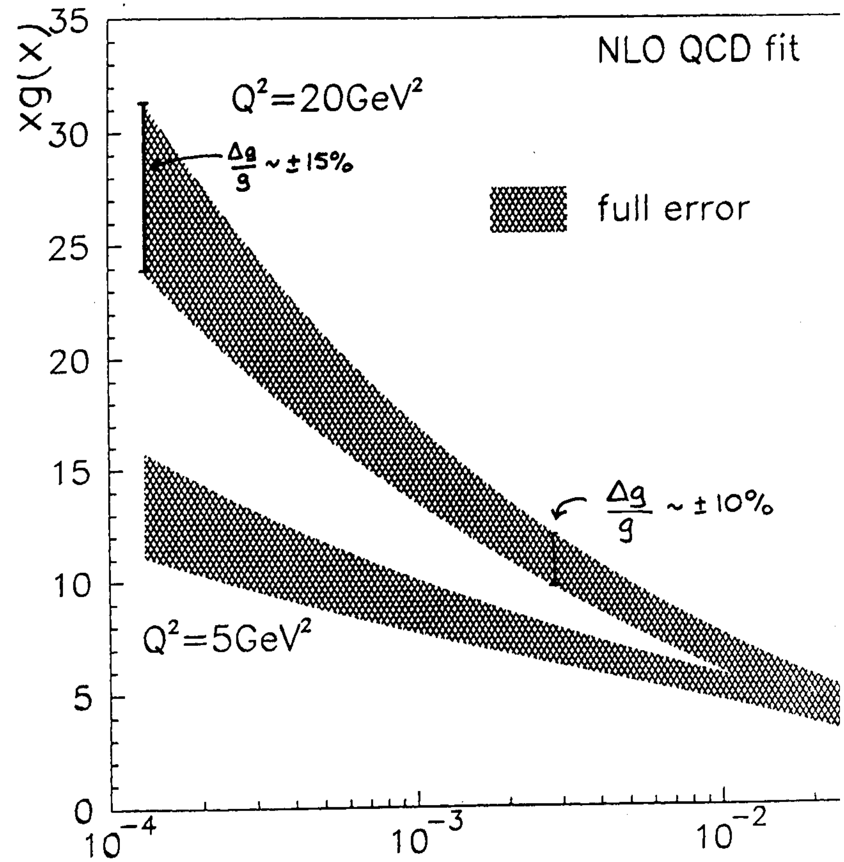
$$\alpha_s(M_Z^2) = 0.119 \pm \underset{\substack{\text{exp.} \\ F_2,\, xF_3}}{0.002} \pm \underset{\substack{\text{HT} \\ (\sim \frac{1}{Q^2})}}{0.001} \pm \underset{\substack{\text{thy. scale} \\ (\mu^2/Q^2 = \frac{1}{3} \to 3)}}{0.004}$$

cf. 1993 analysis:

$\alpha_s = 0.111 \pm 0.002\,(\text{stat.}) \pm 0.003\,(\text{sys.})$

"new energy calibrations"

H1, 1996



te

in the limit $x \to 0$, $Q^2 \to \infty$ the behaviour of F_2 can be calculated analytically:

$$\frac{\partial}{\partial Q^2}\begin{pmatrix} q \\ g \end{pmatrix} \simeq \frac{\alpha_s}{\pi}\begin{pmatrix} 0 & 0 \\ \frac{4/3}{x} & \frac{3}{x} \end{pmatrix} \otimes \begin{pmatrix} q \\ g \end{pmatrix}$$

$$F_2 \sim x\sum q \sim \exp\left[2\sqrt{\frac{3\alpha_s}{\pi}\ln\frac{Q^2}{Q_0^2}\ln\frac{1}{x}}\right] \quad \alpha_s \text{ fixed}$$

double leading log approximation (DLLA)

De Rujula et al (1974)

.. which is a reasonable approximation to the HERA measurements

[developed as 'double asymptotic scaling' + corrections by Ball and Forte → fig.]

Method 1 — moments

- note $P^{gg} = \frac{6}{x} \Rightarrow \langle P^{gg}\rangle_n = \frac{6}{n-1}$

$$t\frac{\partial}{\partial t} g(n,t) = \frac{\alpha_s}{2\pi}\frac{6}{n-1}\, g(n,t)$$

$$\Rightarrow g(n,t) = g(n,t_0)\left[\frac{t}{t_0}\right]^{\frac{3\alpha_s}{\pi(n-1)}} \qquad t_0 = Q_0^2$$

$$\Rightarrow xg(x,t) = \frac{1}{2\pi i}\oint dn\, g(n,t_0)\, x^{-(n-1)}\left[\frac{t}{t_0}\right]^{\frac{3\alpha_s}{\pi(n-1)}}$$

$$\equiv \frac{1}{2\pi i}\oint dn\, g(n,t_0)\, e^{J(n)}$$

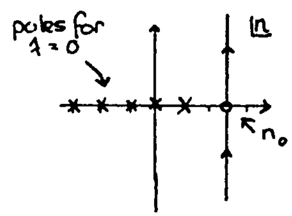
$$J(n) = (n-1)\ln\frac{1}{x} + \frac{1}{n-1}\frac{3\alpha_s}{\pi}\ln\frac{t}{t_0}$$

- when $\ln\frac{1}{x}, \ln\frac{t}{t_0} \gg 1$, can estimate the integral using a saddle-point approximation

$$J'(n) = \ln\frac{1}{x} - \frac{1}{(n-1)^2}\frac{3\alpha_s}{\pi}\ln\frac{t}{t_0}$$

$$= 0 \quad \text{for} \quad n = 1 + \sqrt{\frac{3\alpha_s}{\pi}\frac{\ln(t/t_0)}{\ln(1/x)}} \equiv n_0$$

What about $g(n,t_0)$? If $g(x,t_0) \sim x^{-1-\lambda}(1-x)^m$ then $g(n,t_0)$ has poles at $n = 1+\lambda, \lambda, -1+\lambda, \ldots$



...so if $\lambda \leq 0$, behaviour is dominated by the saddle point

$$xg(x,t) \sim e^{J(n_0)}$$

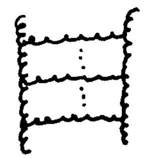
$$\sim \exp\left\{2\left[\frac{3\alpha_s}{\pi}\ln\frac{1}{x}\ln\frac{t}{t_0}\right]^{\frac{1}{2}}\right\}$$

i.e. as $x \to 0$, xg rises faster than any power of $\ln\frac{1}{x}$ but slower than any power of x

(De Rujula et al., 1974)

Method 2 — diagrams

— recall ladder diagrams for leading $\log Q^2$ contributions to structure function.



now
$$g(x,t) = \delta(1-x) + \sum_{n=1}^{\infty} \int_{K^2}^{K_{T_2}^2} \frac{dk_{T_1}^2}{k_{T_1}^2} \frac{\alpha_s(k_{T_1}^2)}{2\pi} \cdots \int_{K_{T_{n-1}}^2}^{t} \frac{dk_{T_n}^2}{k_{T_n}^2} \frac{\alpha_s(k_{T_n}^2)}{2\pi}$$
$$\times \int_x^1 \frac{d\varepsilon_{n-1}}{\varepsilon_{n-1}} P\left(\frac{x}{\varepsilon_{n-1}}\right) \cdots \int_{\varepsilon_2}^1 \frac{d\varepsilon_1}{\varepsilon_1} P\left(\frac{\varepsilon_2}{\varepsilon_1}\right) \underset{\uparrow\, P^{gg}}{P(\varepsilon_1)}$$

set $P^{gg}(x) = \frac{6}{x}$ (small x approx^m)
and take α_s fixed ...

$$g(x,t) \simeq \delta(1-x) + \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{2\pi}\right)^n \frac{\log^n \frac{t}{K^2}}{n!} \frac{6^n}{x} \frac{\log^{n-1}\frac{1}{x}}{(n-1)!}$$

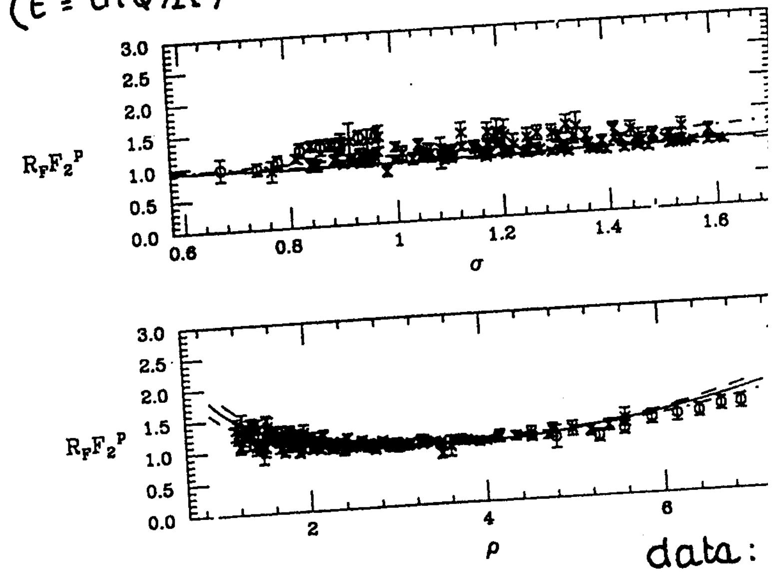
... set $K^2 = Q_0^2$ and fold with $g(y, Q_0^2)$

$$xg(x,t) \simeq \underset{\uparrow\, xg(x,t_0)|_{x=0}}{G_0} \sum_{n=0}^{\infty} \left(\frac{3\alpha_s}{\pi}\right)^n \frac{1}{n!^2} \log^n \frac{t}{t_0} \log^n \frac{1}{x}$$
$$= G_0 I_0(z) \leftarrow \text{modified Bessel function with } z^2/4 = \frac{3\alpha_s}{\pi} \log\frac{t}{t_0} \log\frac{1}{x}$$
$$\sim G_0 \frac{e^z}{\sqrt{2\pi z}} \quad \text{as } z \to \infty, \quad \text{as before!}$$

double asymptotic scaling

$$\sigma = \left[\ln\frac{t}{t_0} \ln\frac{x_0}{x}\right]^{\frac{1}{2}}$$
$$\rho = \left[\ln\frac{x_0}{x} \Big/ \ln\frac{t}{t_0}\right]^{\frac{1}{2}}$$
$(t = \ln Q^2/\Lambda^2)$

Ball, Forte



data:

each gluon emission in the ladder generates two large logarithms, hence " double leading logarithm approximation " (DLLA)

refinements

unning α_s

$$\frac{\alpha_s}{2\pi}\log\frac{Q^2}{Q_0^2} \longrightarrow \int_{Q_0^2}^{Q^2}\frac{dq^2}{q^2}\frac{\alpha_s(q^2)}{2\pi}$$
$$= \frac{1}{2\pi\beta_0}\log\frac{\log Q^2/\Lambda^2}{\log Q_0^2/\Lambda^2}$$

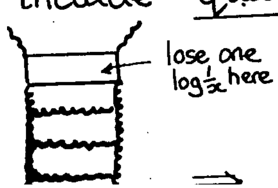
subleading terms from saddlepoint approx

replace $\log\frac{1}{x}$ by $\log\frac{x_0}{x}$ and restrict to $x < x_0 << 1$

subleading term in splitting function

$$\langle P^{gg}\rangle_n \underset{(n\simeq 1)}{\simeq} \frac{a}{n-1} + b$$

include quarks



note : only P^{gg} and P^{gq} have $\frac{1}{x}$ singularity

$$\Longrightarrow xq_0 \sim xq \sim e^{2\sqrt{\ }}$$

High–Q^2 Structure Functions for HERA

- neutral current

$$\frac{d^2\sigma_{NC}(e^\pm p)}{dxdQ^2} = \frac{2\pi\alpha^2}{xQ^4}\Big[[1+(1-y)^2]F_2(x,Q^2) - y^2F_L(x,Q^2) \mp 2y(1-y)xF_3(x,Q^2)\Big]$$

$$F_2(x,Q^2) = \sum_q [xq(x,Q^2) + x\bar{q}(x,Q^2)]\ A_q(Q^2)$$
$$xF_3(x,Q^2) = \sum_q [xq(x,Q^2) - x\bar{q}(x,Q^2)]\ B_q(Q^2)$$

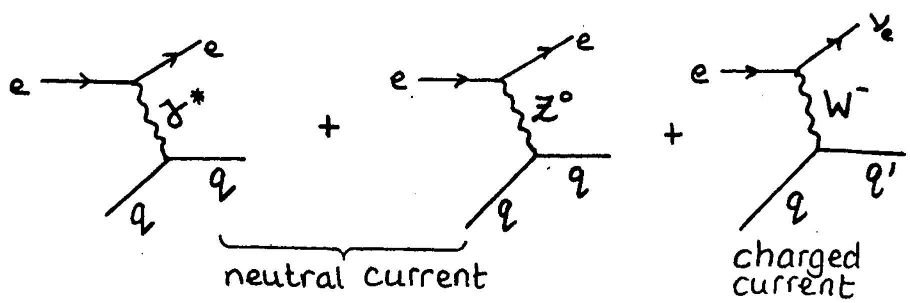
$$A_q(Q^2) = e_q^2 - 2e_q v_e v_q P_Z + (v_e^2 + a_e^2)(v_q^2 + a_q^2)P_Z^2$$
$$B_q(Q^2) = \ - 2e_q a_e a_q P_Z + 4v_e a_e v_q a_q P_Z^2$$
$$P_Z = \frac{Q^2}{Q^2 + M_Z^2}\ \frac{\sqrt{2}G_\mu M_Z^2}{4\pi\alpha}$$

- charged current

$$\frac{d^2\sigma_{CC}(e^-p)}{dxdQ^2} = [1-\mathcal{P}_e]\frac{G_\mu^2}{2\pi}\left(\frac{M_W^2}{Q^2+M_W^2}\right)^2 \times \sum_{i,j}\Big[|V_{u_id_j}|^2 u_i(x,Q^2) + (1-y)^2|V_{u_jd_i}|^2\bar{d}_i(x,Q^2)\Big]$$

$$\frac{d^2\sigma_{CC}(e^+p)}{dxdQ^2} = [1+\mathcal{P}_e]\frac{G_\mu^2}{2\pi}\left(\frac{M_W^2}{Q^2+M_W^2}\right)^2 \times \sum_{i,j}\Big[|V_{u_id_j}|^2 \bar{u}_i(x,Q^2) + (1-y)^2|V_{u_jd_i}|^2 d_i(x,Q^2)\Big]$$

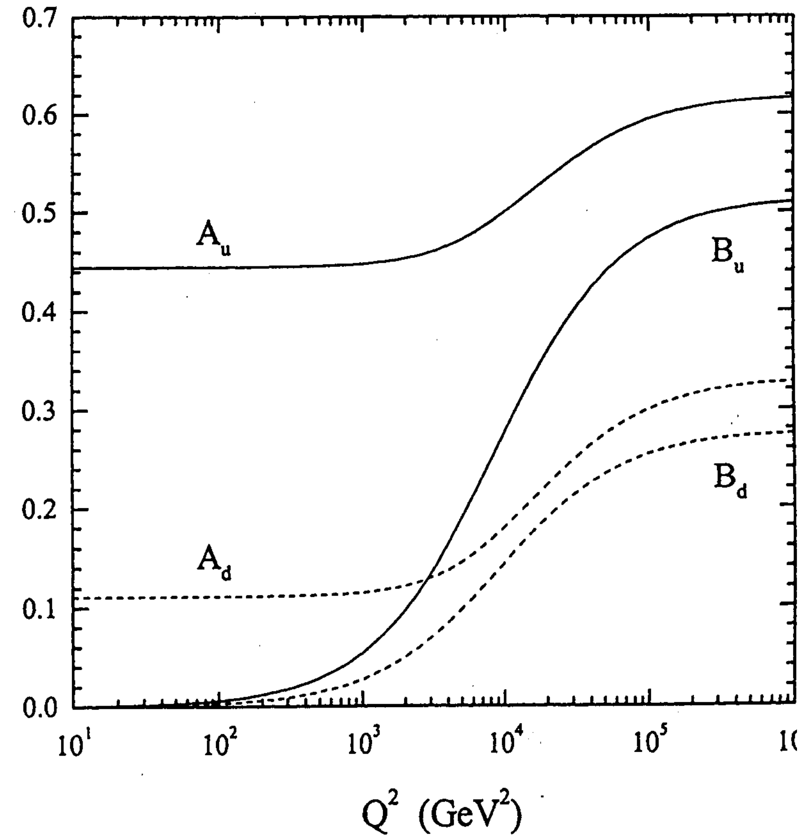
— Note that if $Q^2 \gtrsim 10^4\ GeV^2$ (e.g. at HERA) we must also include $W^{\pm}$ and Z^0 exchange* in DIS ep scattering :



... which generalizes the result for $\frac{d\sigma}{dx\,dQ^2} \sim [F_1, F_2]$ obtained with photon exchange only → fig.

$$* \quad \frac{1}{Q^4} \longrightarrow \frac{1}{(Q^2 + M_V^2)^2}$$

NC pdf coefficients



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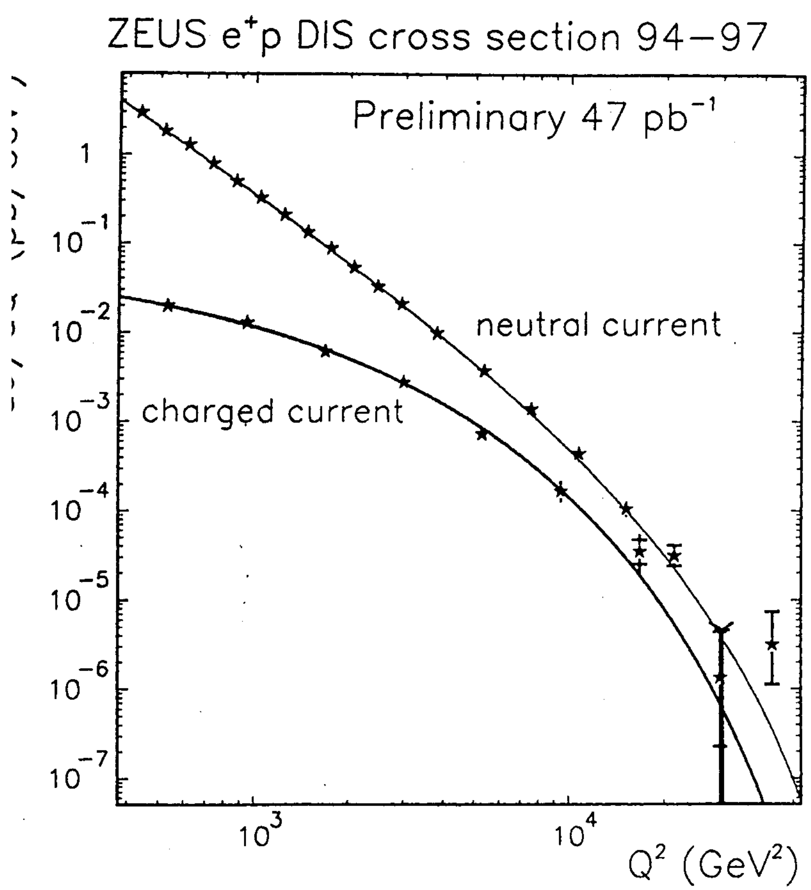

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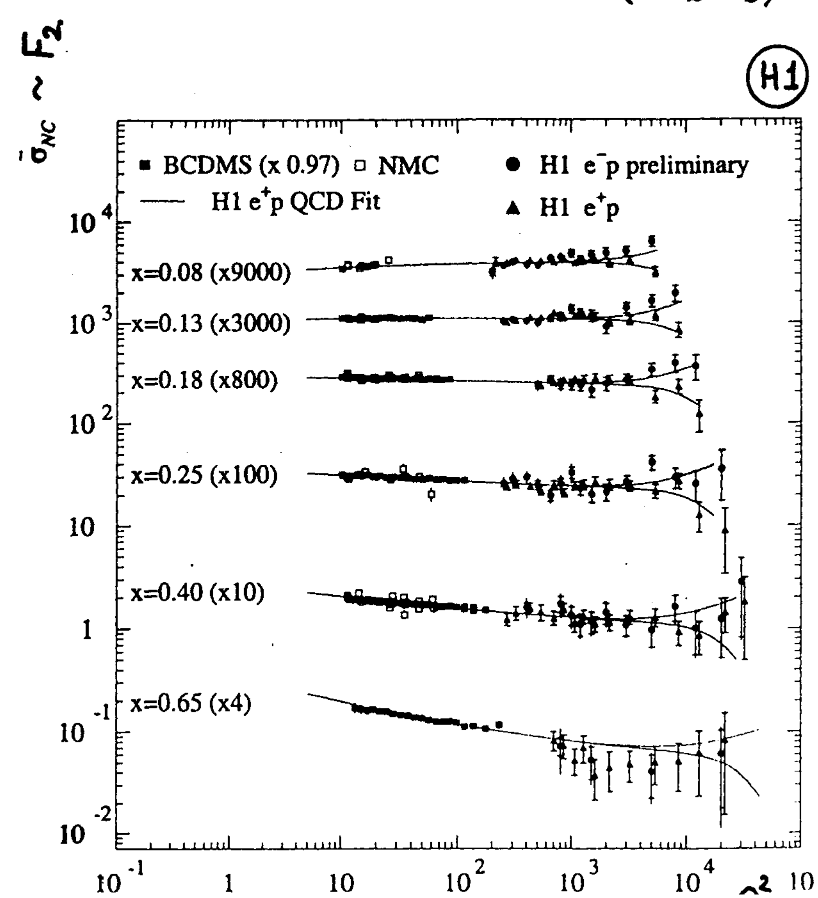
— nothing unusual at high Q^2...

- quarks still pointlike
- DGLAP works
- electroweak effects visible ($Q^2 \gtrsim M_Z^2$)



→ influence of F_L

→ d,s contributions

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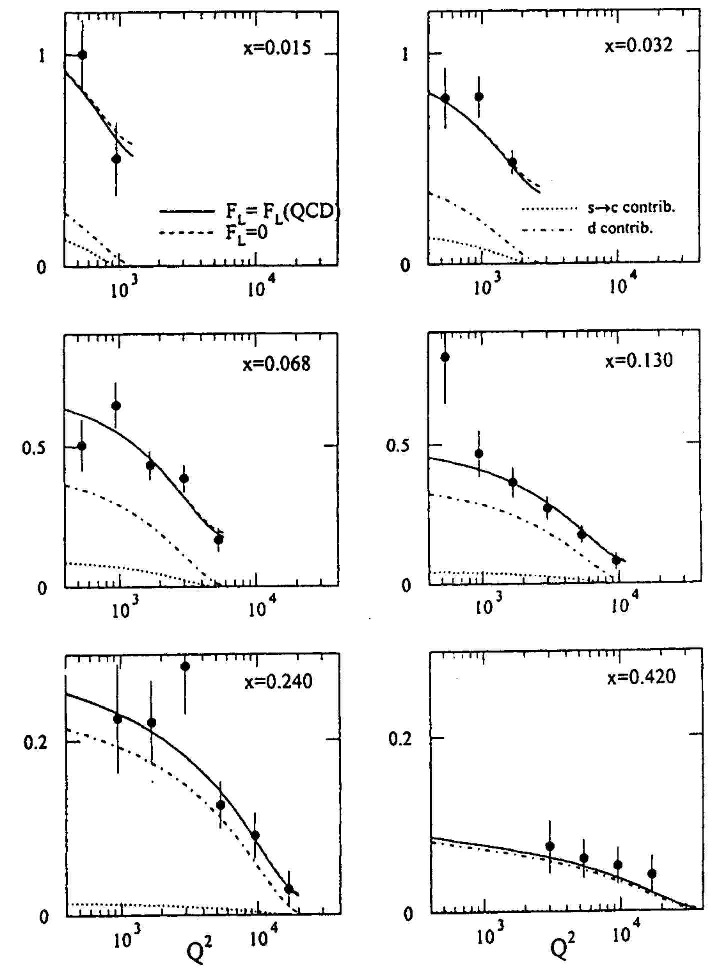

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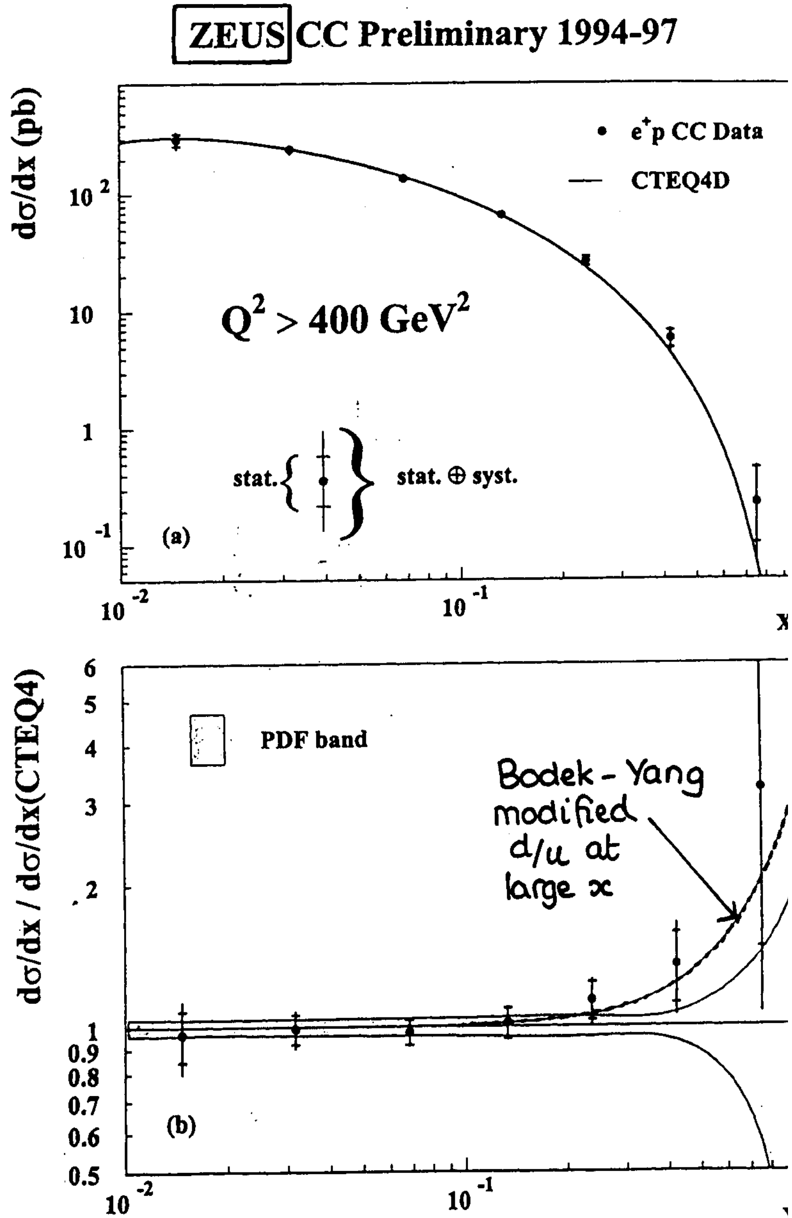

Bodek - Yang modified d/u at large x

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Physics World April 1997

›ESY double offers high ›opes for new physics

›ticle physicists are eagerly awaiting new data from the HERA collider in Germany ›confirm if they have detected a new particle known as a leptoquark, evidence for ›structure in quarks, both leptoquarks and quark substructure or, possibly, none of these

James Stirling in the Departments of ›matical Sciences and Physics, University of ›m, UK

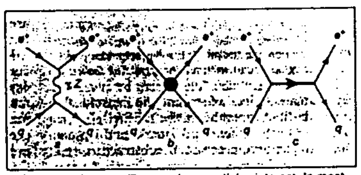
›any particle physicist to bet on which particle will be discovered next and most popular answer, at least until ›ntly, would have been the Higgs ›n. A less well known particle – the ›quark – would have been well down ›t people's list. However, dramatic results from HERA, the high-energy ›der at the DESY Laboratory in ›burg, could be the first evidence for ›e unusual new particles. Alternatively, data might indicate that quarks, ›iously thought to be elementary ›icles, have substructure.

›the results are confirmed by ›her data from HERA, the cher›d Standard Model of element›' particles and their interactions have been stood on its head. ›holy grail of late 20th century ›icle physics – experimental evid›e for new physics beyond the ›dard Model – will finally have ›n found.

›he excitement at HERA centres ›a handful of unusual events seen ›both the H1 and ZEUS experi›nts. The two teams involved sub›ted their results to *Zeitschrift für* ›*sik* in late February and their ›ers, plus a wealth of information ›the HERA collider and the two experi›nts, are available on the DESY World ›le Web site at http://info.desy.de/. Both ›eriments are run by large international ›aborations: H1 by 400 physicists from ›countries; and ZEUS by 430 physicists ›n 12 countries.

›he two experiments were studying ›it happens when a 27.5 GeV positron ›m collides with a 820 GeV proton ›m. (HERA can also collide electron ›ms with proton beams.) Such high›rgy collisions have traditionally pro›ed an enormous amount of information ›fundamental particles and their inter›ions. In particular, when a positron ›tters off a proton at a large angle, the ›rgy transferred to the proton causes it ›break up, revealing details of its inner›st structure.

›This type of "deep inelastic scattering" experiment was first performed, using electron beams at much lower collision energies, at the Stanford Linear Accelerator Center (SLAC) more than 25 years ago. The SLAC experiment showed for the first time that the proton was made up of quark constituents. Quarks are believed to be the fundamental building blocks of strongly interacting or "hadronic" matter, particles such as protons, neutrons and pions. In the 1970s theorists developed a quantum field theory for the strong interactions of quarks called quantum chromodynamics (QCD), which nowadays is part of the Standard Model.



1 Feynman diagrams illustrate how particles interact. In most collisions (a) a positron scatters off a quark inside the proton by exchanging a "virtual" photon or a neutral Z boson, as predicted by the Standard Model. In (b), a new type of interaction between leptons and quarks, perhaps indicating further substructure, gives an additional contribution to the scattering. In the leptoquark scenario (c), the positron and quark annihilate to make a new heavy particle, X, which subsequently decays back to a positron–quark pair.

According to the Standard Model, a high-energy positron scatters off a proton by exchanging a virtual (i.e. short-lived) photon. This photon has a wavelength $\hbar/Q$, where $\hbar$ is Planck's constant divided by 2π and Q is the momentum transferred from the positron, and the proton's substructure can be resolved on this scale. In the HERA experiments this wavelength can be several orders of magnitude smaller than the overall size of the proton (which is about 10^{-15} m). The virtual photon therefore travels deep inside the proton and scatters off a quark (figure 1*a*). The quark is knocked out of the proton, emerging not as a free particle (the strong force always confines quarks in hadrons) but rather as a "jet" of pions, kaons and other hadrons.

The number of deep inelastic scattering events seen in the detectors varies with the energy loss of the scattered positron and the angle through which it is scattered. These two measured quantities determine two variables that are useful for making the connection with theory – the square of the momentum transfer, Q^2, and the fraction of the proton's momentum, x, that is carried by the quark struck by the virtual photon. The dependence of the event rate on x provides important information on how the quarks share the momentum of the proton.

On the other hand, the dependence on Q^2 is precisely predicted by the theory, under two important assumptions: that the interactions of the quarks and positrons are exactly as specified in the Standard Model; and that the quarks are "point-like" – that is they have no discernible size on the scale of the resolution $\hbar/Q$. If both these assumptions are satisfied, the event rate is predicted to fall like $1/Q^4$ (for fixed x), subject to small calculable corrections. One such correction comes from the possibility of the positron and the quark exchanging a heavy neutral Z boson rather than a photon (figure 1*a*).

Over the last two years the H1 and ZEUS experiments have collected a large number of deep inelastic scattering events over a wide range in Q^2 and x. The number of small and medium Q^2 events agrees beautifully with the Standard Model predictions. However, at high Q^2 there is an apparent excess of events – instead of falling off rapidly as predicted by theory, the event rate seems to level off. The H1 experiment observes 12 events for $Q^2 > 15000\ \mathrm{GeV}^2$ where 4.71 ± 0.76 are expected in the Standard Model, while the ZEUS experiment observes 2 events for $Q^2 > 35000\ \mathrm{GeV}^2$ where 0.145 ± 0.013 events are expected.

The high Q^2 events have a very distinctive signature, each containing a very energetic positron and jet of hadrons (see figure 2). Because the number of excess events is not particularly large, the effect could be a statistical fluctuation. However, the experimenters estimate the probabilities of such a fluctuation to be very small – 1% for H1 and 6% for ZEUS. Taken together, this is about as likely as getting

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quality of NLO DGLAP fit

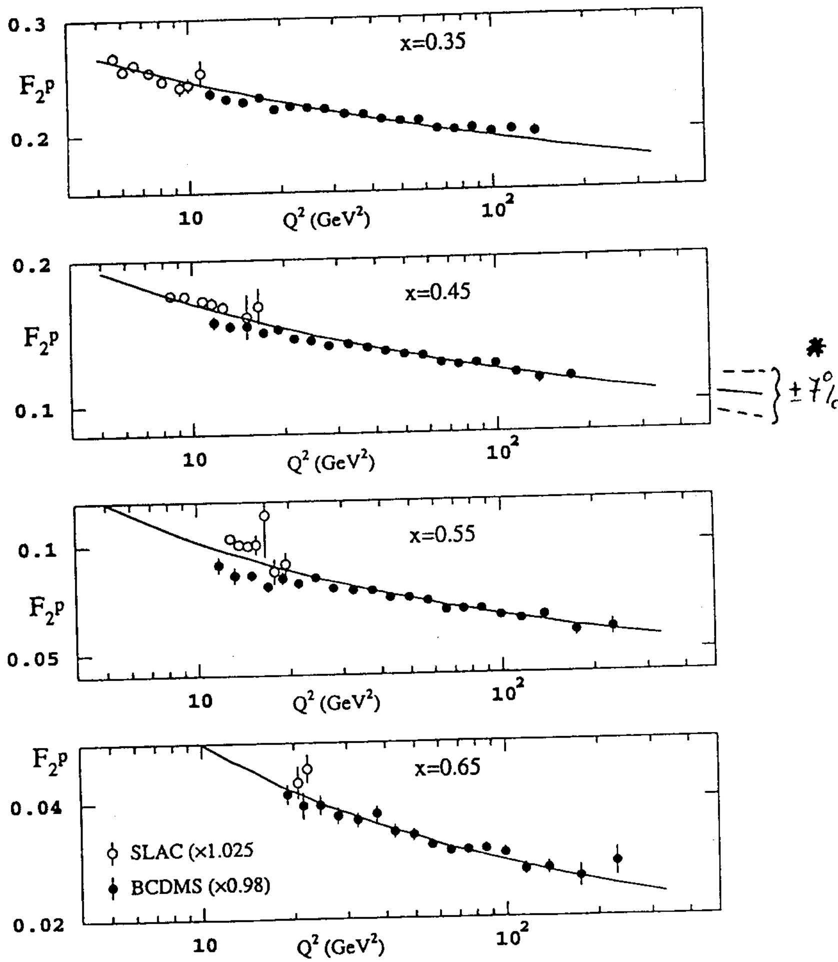
— overall, excellent agreement with DIS data : $Q^2 \gtrsim 2\ \mathrm{GeV}^2$, $x \gtrsim 10^{-5}$, ↝ fig.
however $\cancel{\chi^2}$

- BCDMS $\alpha_s = 0.113 \pm 0.005$
  cf. world average (inc. CCFR) $\alpha_s = 0.118$
- very small x HERA data
  $\Rightarrow$ "valence" gluon at Q_0^2 ↝ figs.
  note : not "cured" by HT contributions
- small x, high Q^2 (new) HERA data
  $\Rightarrow$ DGLAP undershoots high Q^2 points ↝
- $\left.\dfrac{\partial F_2}{\partial \log Q^2}\right|_{NMC}$ too large at $x \sim 0.05$ ↝
  note : cannot add much more glue here because of $\int_0^1 x g\, dx$ constraint

? $\Longrightarrow$ evidence for ...

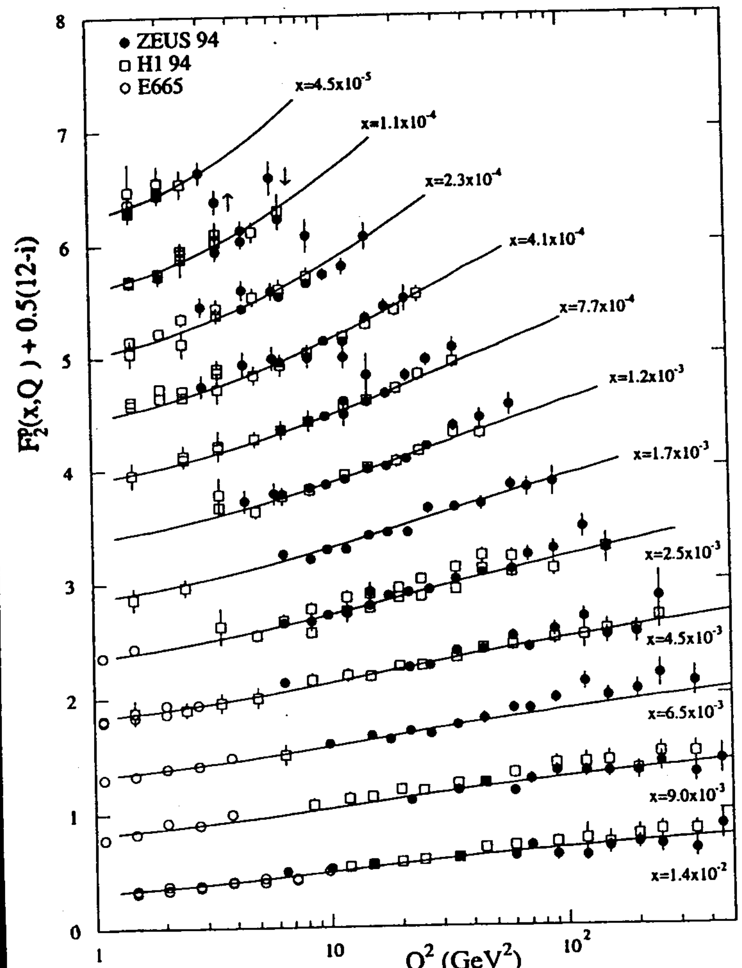
- higher twist contributions ? ↝
- log x resummation ?
  ↳ later

. . .



* extrapolation to $Q^2 \sim 10^5\,\text{GeV}^2$ (HERA)
error due to : $\delta\alpha_s = \pm 0.005$
combine with $\delta F_2|_{exp} \lesssim \pm 3\%$

— NLO - DGLAP "works" down to $x \gtrsim 10^{-5}$, $Q^2 \gtrsim 2\,\text{GeV}^2$
$\Rightarrow$ constraints on – BFKL physi
– higher twist



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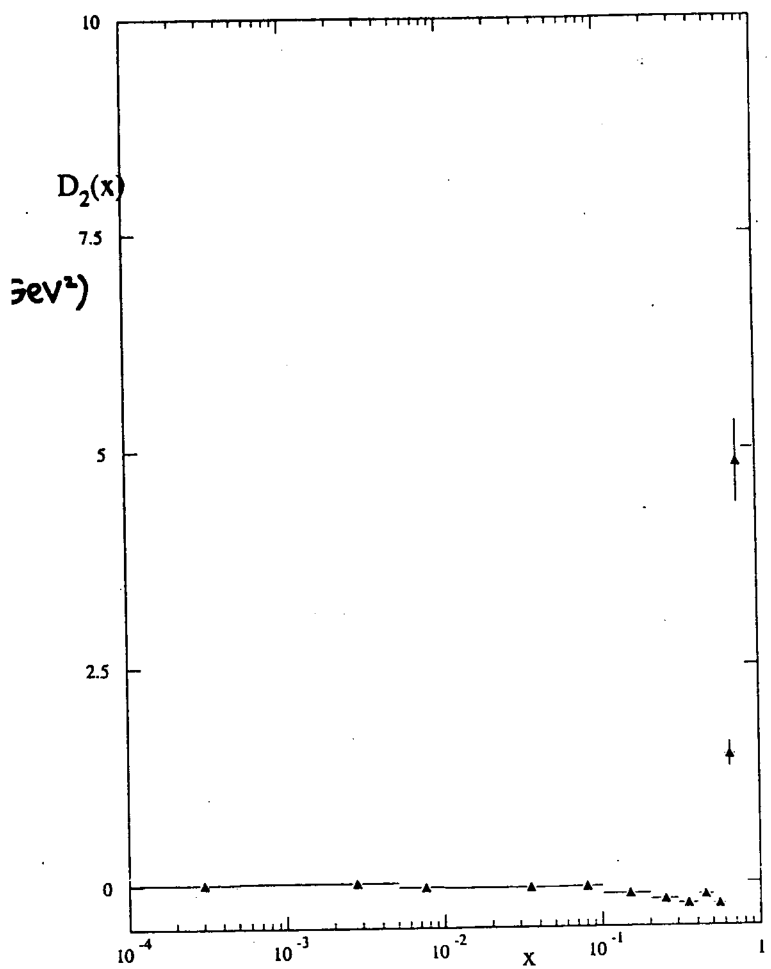
$$F_2 = F_2^{LT}\left[1 + \frac{D_2(x)}{Q^2}\right]$$



MRST

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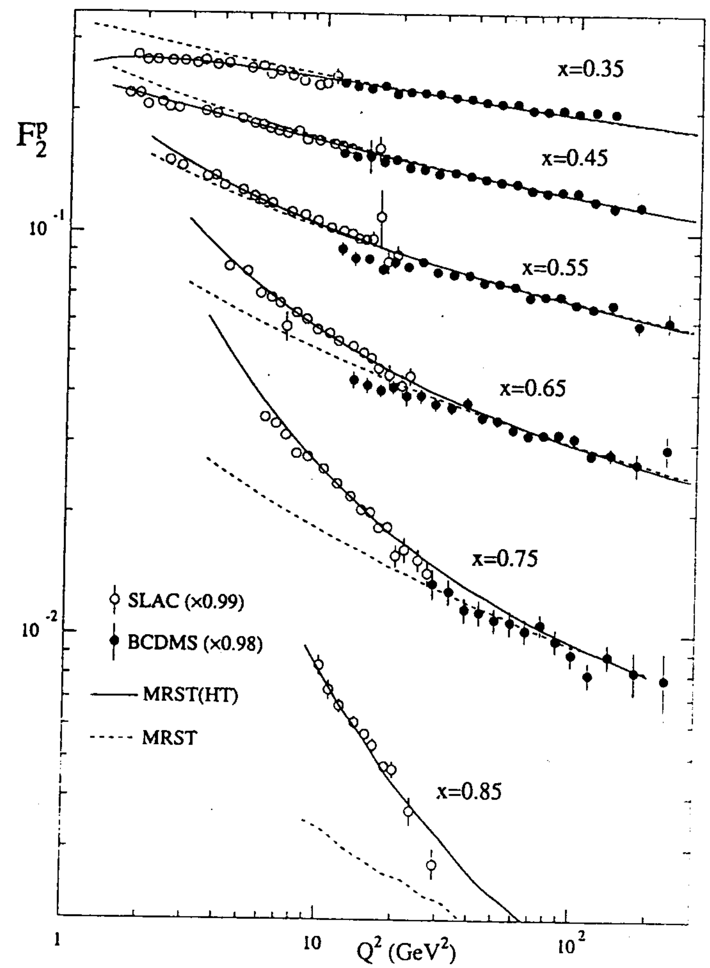
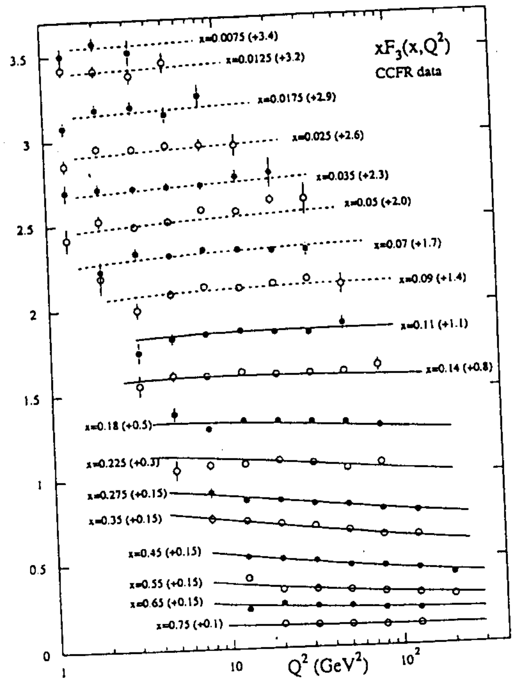
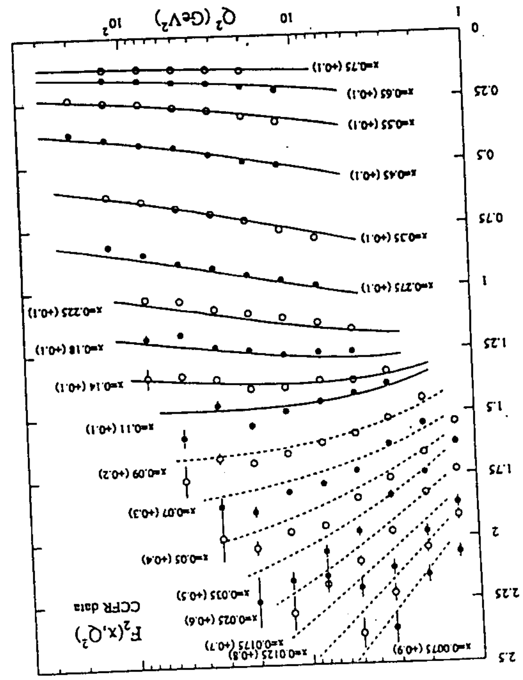
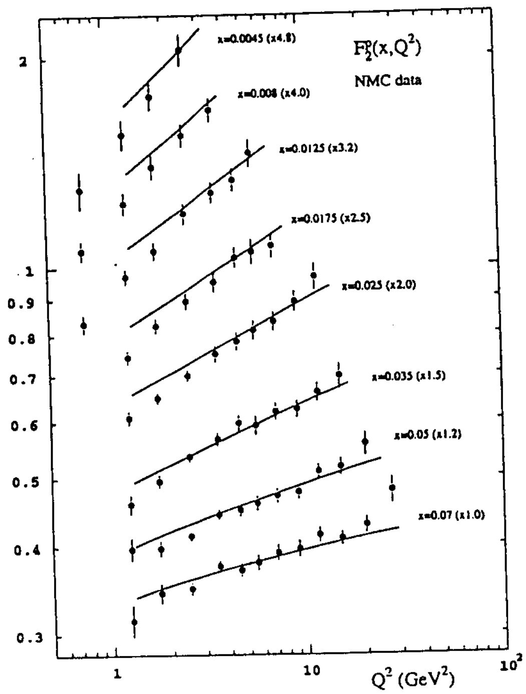


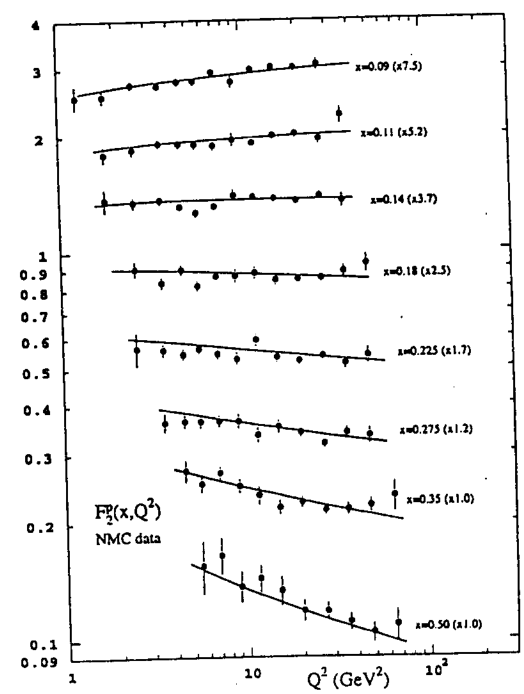




NMC proton data and MRST



NMC proton data and MRST



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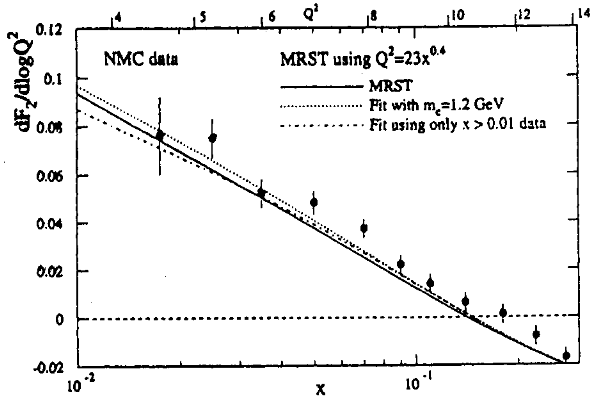

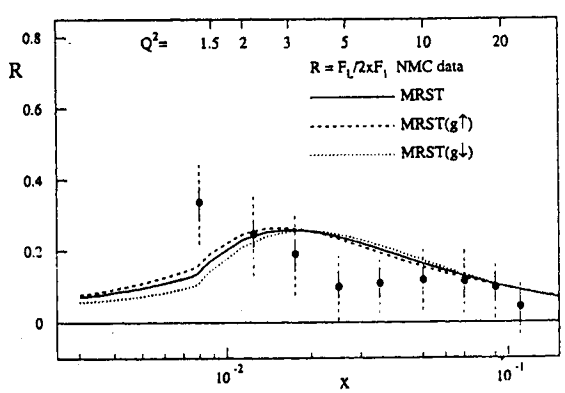

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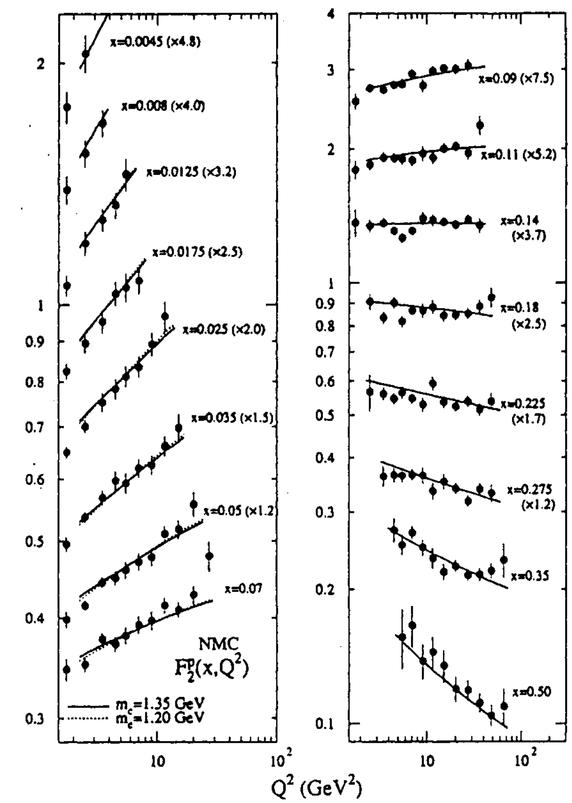
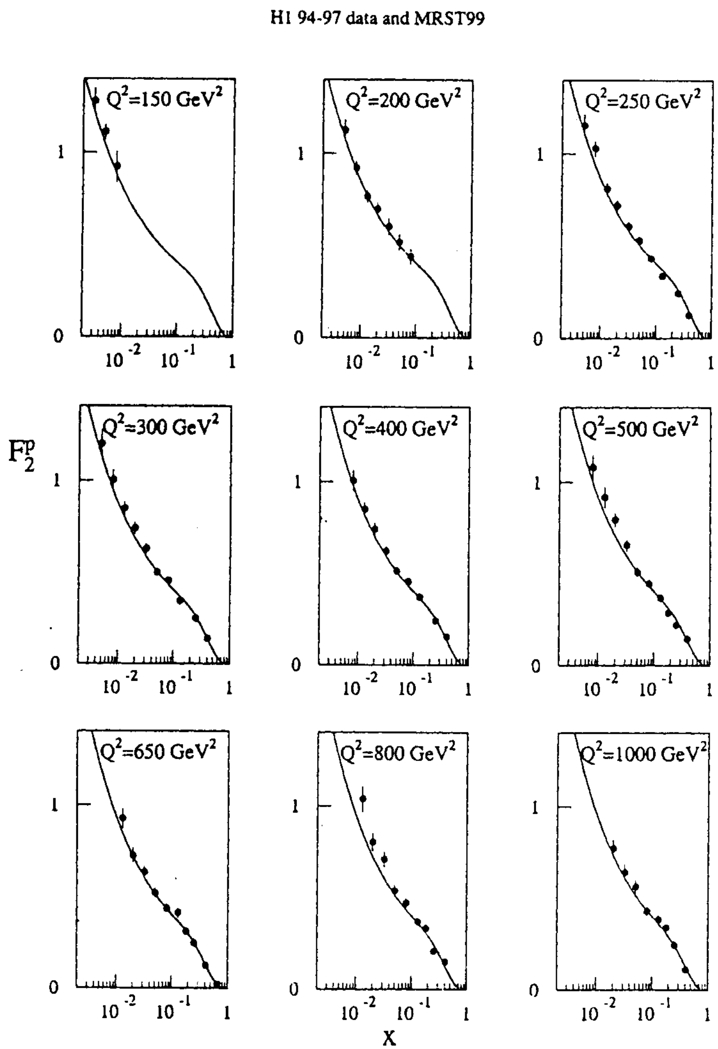
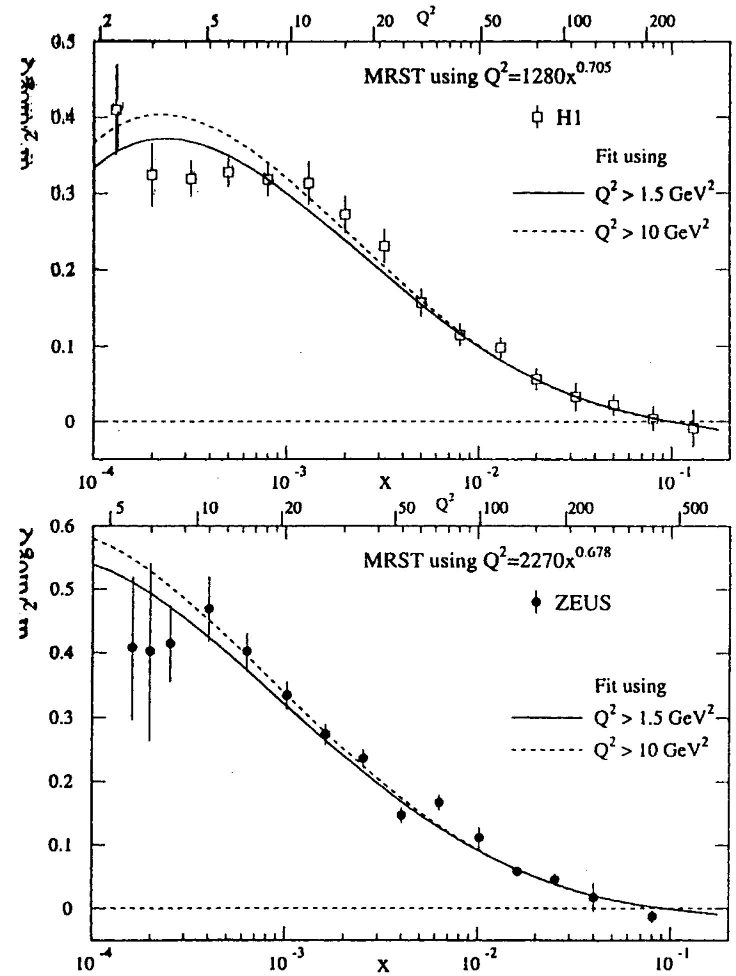


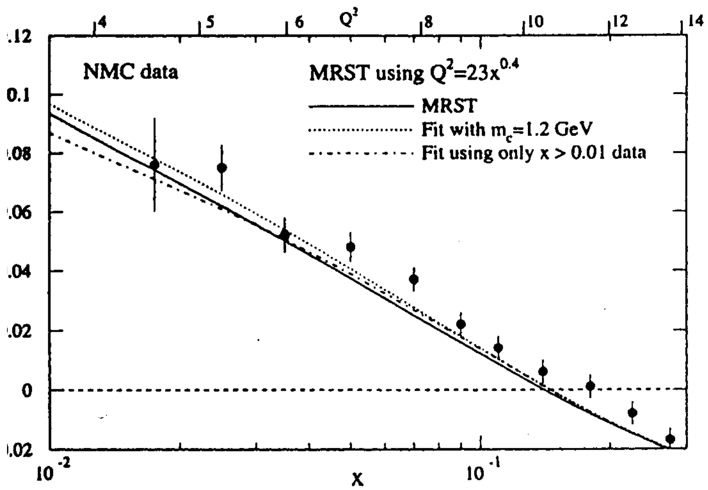
Description of the NMC F_2^p data [13] by the MRST partons. The effect of lowering the charm quark mass from 1.35 to 1.20 GeV is shown by the dotted curve. For display purposes we have multiplied F_2^p by the numbers shown in brackets.



$\frac{\partial F_2}{\partial \log Q^2}$ vs. x at HERA



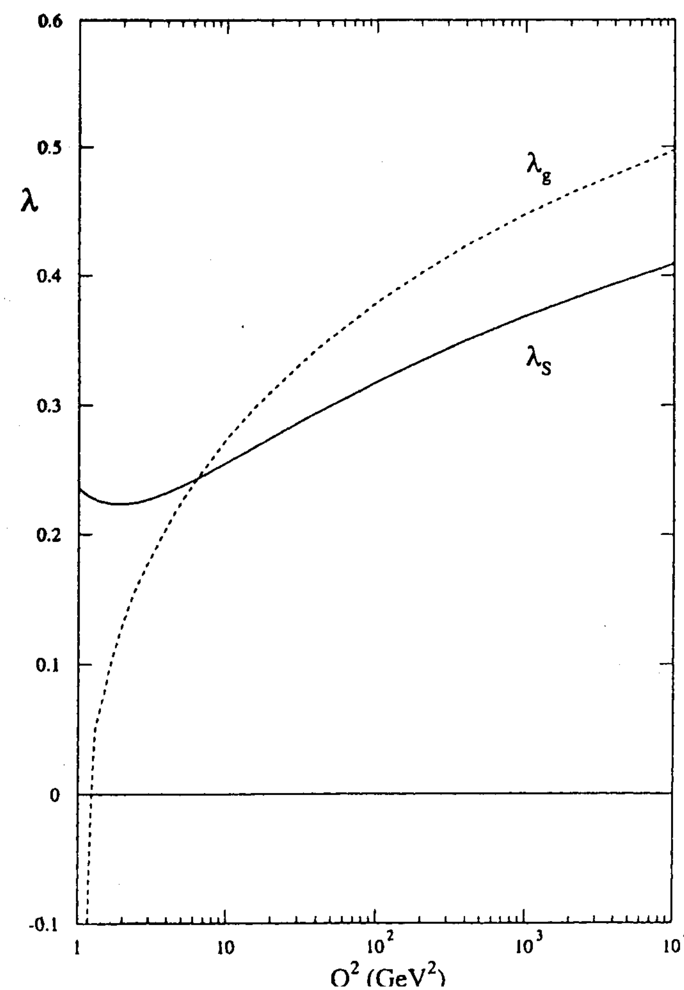
$$\frac{\partial F_2}{\partial \log Q^2} \quad vs. \quad x \quad (NMC)$$



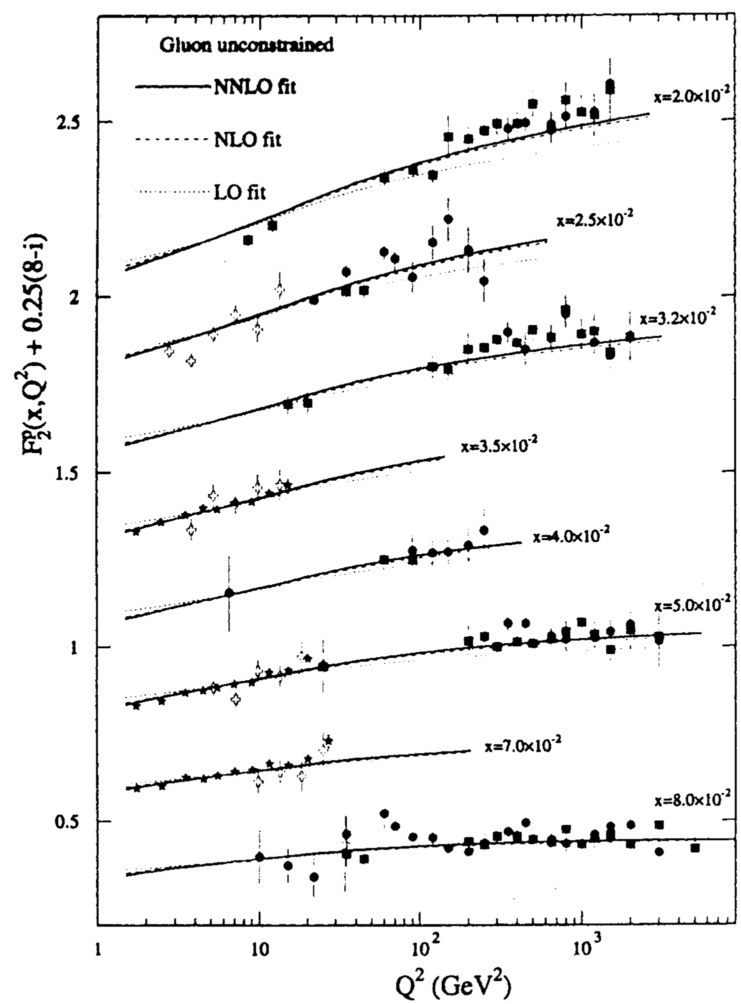
31

$$x g \sim x^{-\lambda_g}$$

$$x q_s \sim x^{-\lambda_s}$$



MRST NNLO and NLO fits , x=0.02 - 0.08



the DGLAP perturbation serie[s]

recall

$$P_{ab}(x,\alpha_s) = P_{ab}^{(0)}(x) + \alpha_s P_{ab}^{(1)}(x) + \alpha_s^2 P_{ab}^{(2)}(\ldots$$

$$C_q(x,\alpha_s) = e_q^2\,\delta(1-x) + \alpha_s C_q^{(1)}(x) + \alpha_s^2 C_q^{(2)}(x)$$

we can now begin to see how th[e] series converges and how the qu[ality] of the fit changes :

$$\left[\ \chi^2_{LO} > \chi^2_{NLO} > \chi^2_{NNLO} \ \cdots\ \right]$$

- LO vs. NLO

main difference is at small x

$$\frac{\partial F_2}{\partial \log Q^2} \simeq \sum_q e_q^2 \frac{\alpha_s}{2\pi}\, x\int_x^1 \frac{dy}{y}\, P_{qg}(y)\, g\!\left(\frac{x}{y},Q^2\right)$$

$$\underset{y\simeq 0}{\longrightarrow}\ \frac{1}{2} + \frac{\alpha_s}{2\pi}\cdot\frac{20}{3}\cdot\frac{1}{y}$$

n

$$\therefore\ \frac{1}{F_2}\frac{\partial F_2}{\partial \log Q^2}\Big|_{LO} < \frac{1}{F_2}\frac{\partial F_2}{\partial \log Q^2}\Big|_{NLO}$$

NLO vs NNLO

- NNLO splitting functions not yet fully calculated, however...

(2) $\left\{ \begin{array}{l} N = 2, 4, 6, 8, 10 \text{ moments} \quad \text{Larin et al.} \\ x \to 0, 1 \text{ leading logarithms} \\ \int P \text{ sum rules, symmetry relations} \end{array} \right.$

- Vogt and van Neerven have derived approximate $P^{(2)}(x)$ ↘

- improved $\alpha_s|_{DIS}$ determination

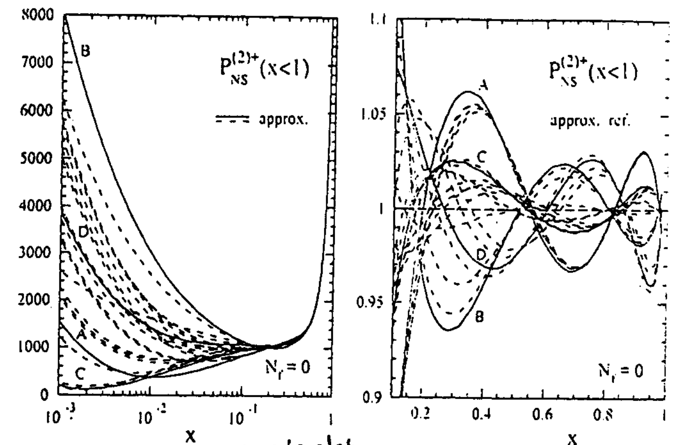
$$\Delta\alpha_s(M_Z)\Big|_{NLO \to NNLO} \simeq -0.002$$

and $\delta\alpha_s|_{scale} \downarrow$

- small-x fit improves

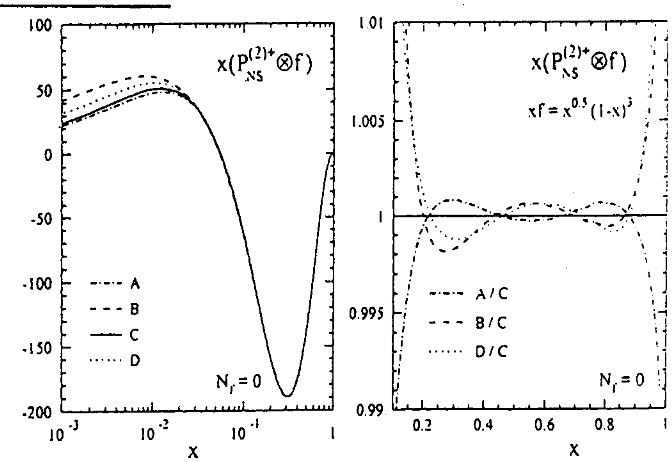
- can now do $\sigma_{p\bar{p} \to W+\cdots}$ consistently at NNLO ↘

Vogt
Van Neerven '99



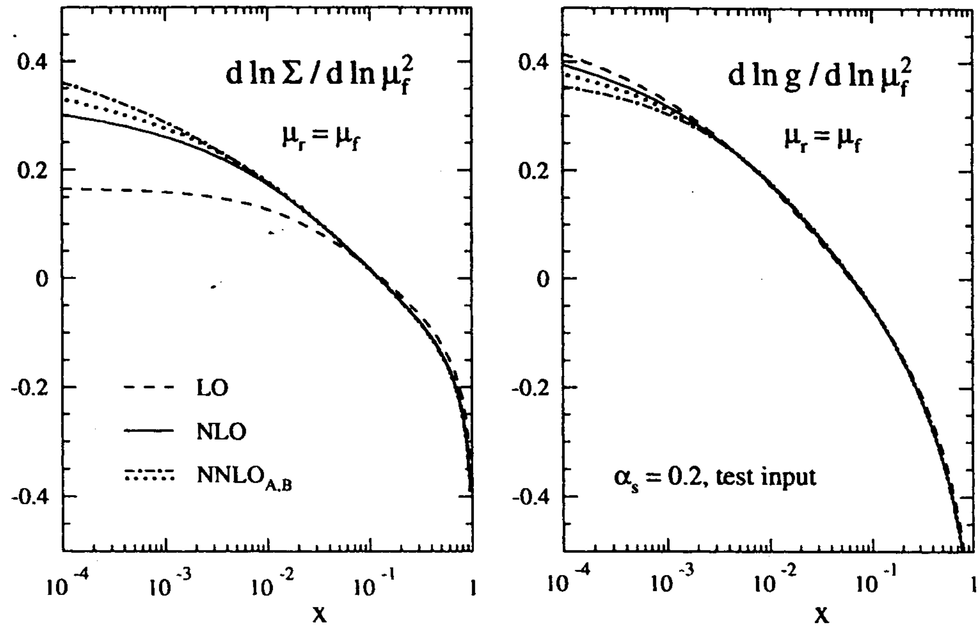
non-singlet

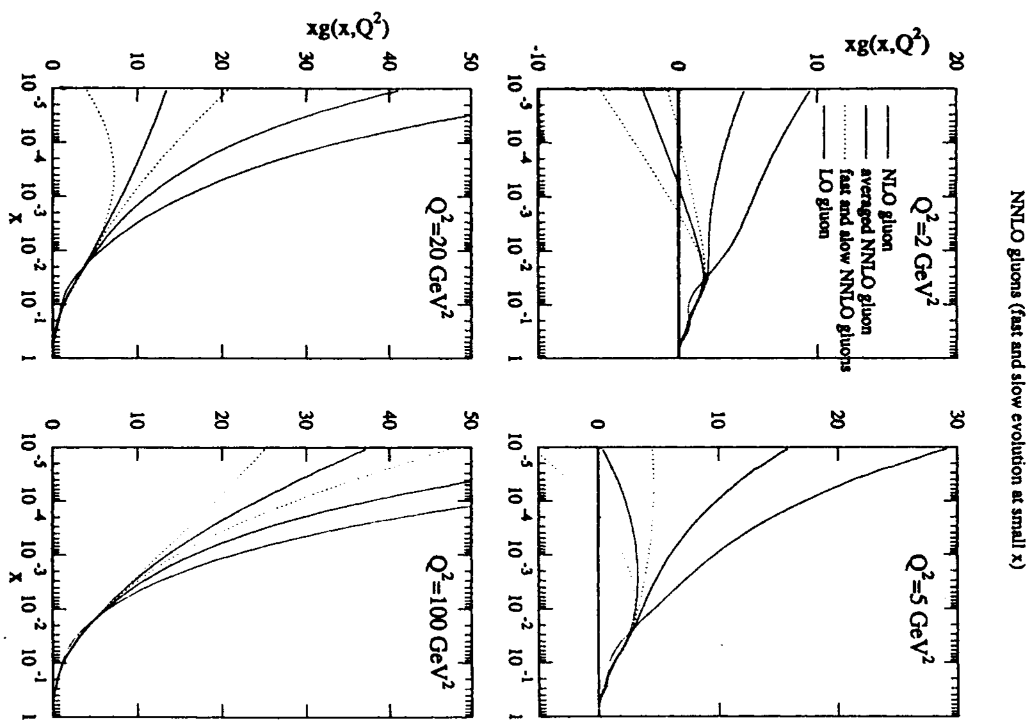
approximating NNLO splitting functions using known (moments, logarithmic) behaviour



singlet evolution
at "NNLO"

Vogt, van Neerven
(presentation at
LHC Workshop, Oct. '99





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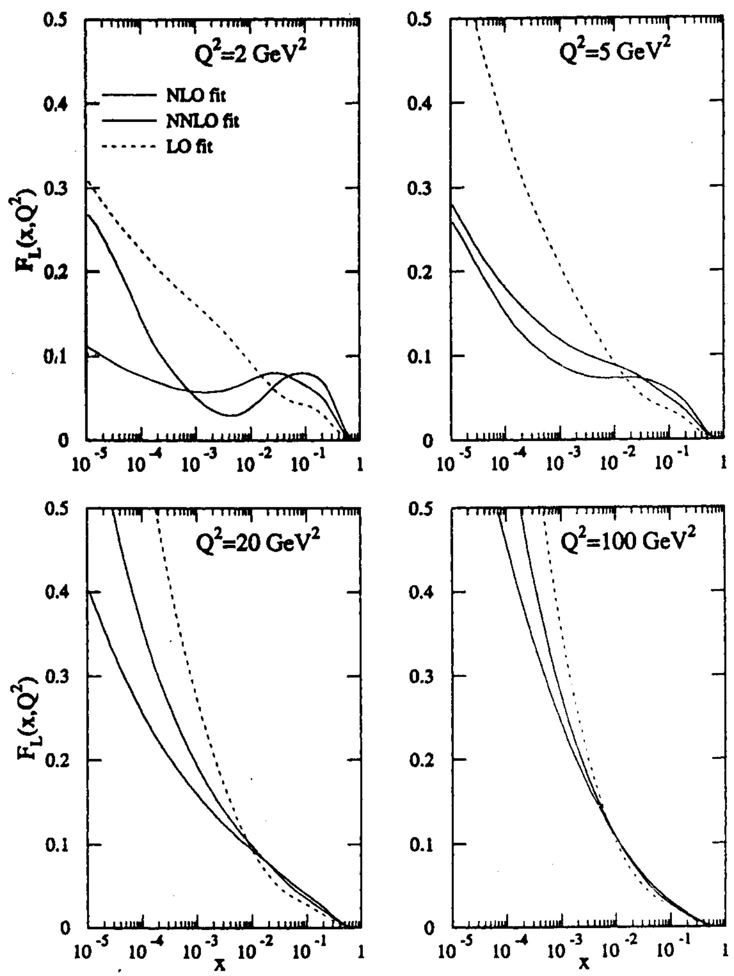
F_L LO , NLO and NNLO



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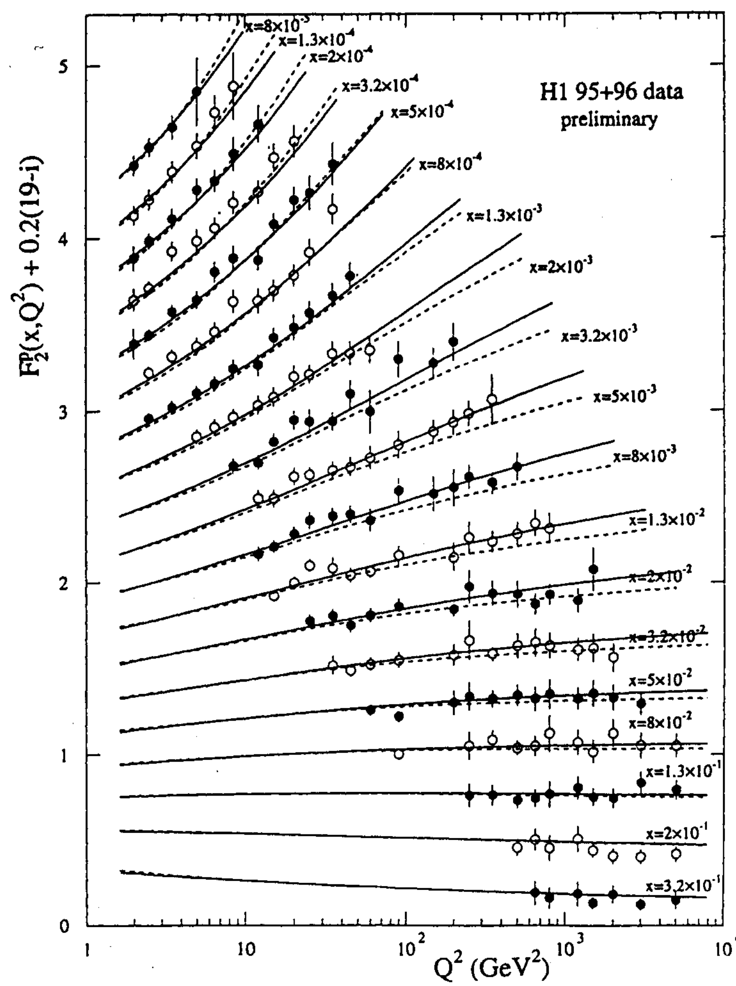
- - - - - standard MRST NLO-DGLAP fit

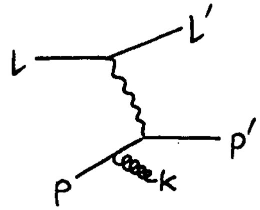
——— "L_x improved" DGLAP fit (Thorne)

$\chi^2_{\_\_\_} < \chi^2_{----}$



QED-corrected DGLAP

- $eq \to eqg$

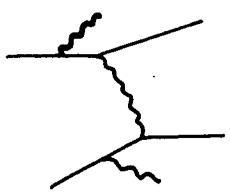


$$|M|^2 = 8e^4 g_s^2 e_q^2 \, \frac{s^2+s'^2+u^2+u'^2}{tt'} \times \left[\frac{p\cdot p'}{p\cdot k \; p'\cdot k}\right]$$

$\downarrow$ collinear singularity

$\hookrightarrow$ DGLAP: $P_{qq} \otimes q$

- $eq \to eq\gamma$



$$|M|^2 = 2e^6 e_q^2 \, \frac{s^2+s'^2+u^2+u'^2}{tt'} \times \left[e_q^2 \frac{p\cdot p'}{p\cdot k \; p'\cdot k} + \frac{l\cdot l'}{l\cdot k \; l'\cdot k}\right]$$

— lepton corrections, large near edge of phase space $(y \to 0, 1)$ $\sim \alpha \log^2 \frac{Q^2}{m_e^2}$

— quark corrections: additional QED contribution to DGLAP evolution

$$\sim e_q^2 \left(\frac{1+x^2}{1-x}\right)_+ \otimes q$$

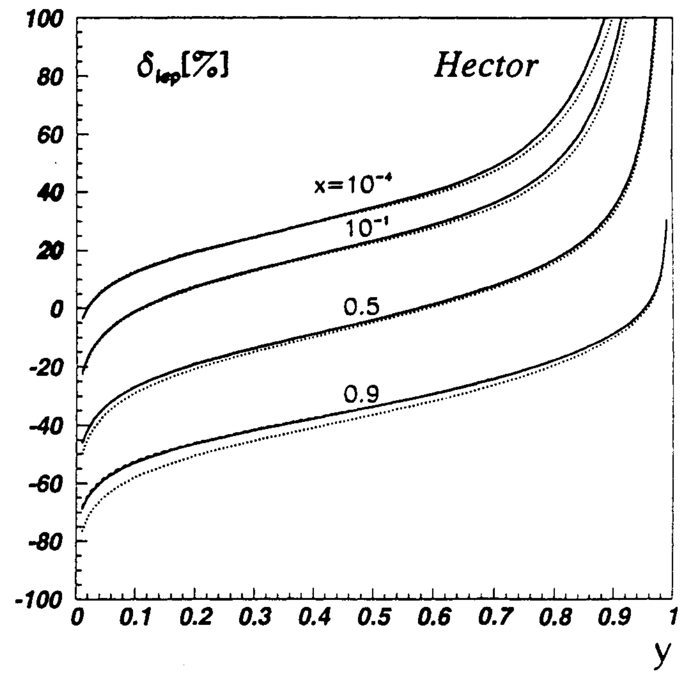


Figure 12: *Radiative corrections in leptonic variables for the default settings in percent. Dotted lines: $\mathcal{O}(\alpha)$, dashed lines: $\mathcal{O}(\alpha^2)$, solid lines: in addition soft photon exponentiation.*

H. Spiesberger, 1994

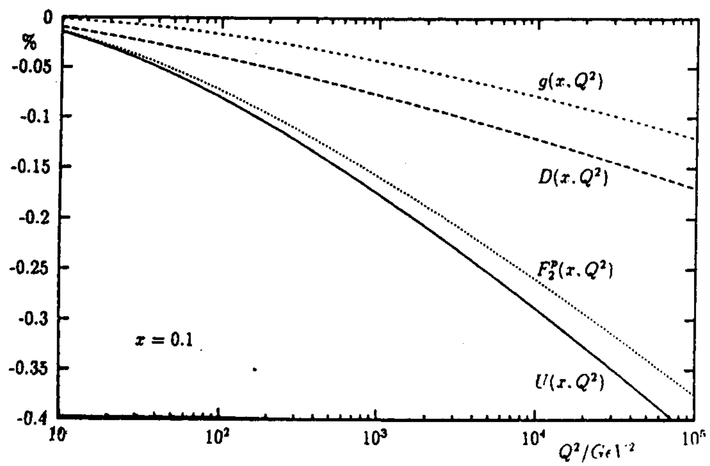


Figure 1: Q^2 dependence of the QED corrections (in per cent, see text) to parton distributions and the structure function F_2^p for deep inelastic lepton-proton scattering at $x = 0.1$. Input parton distributions were taken from [12].

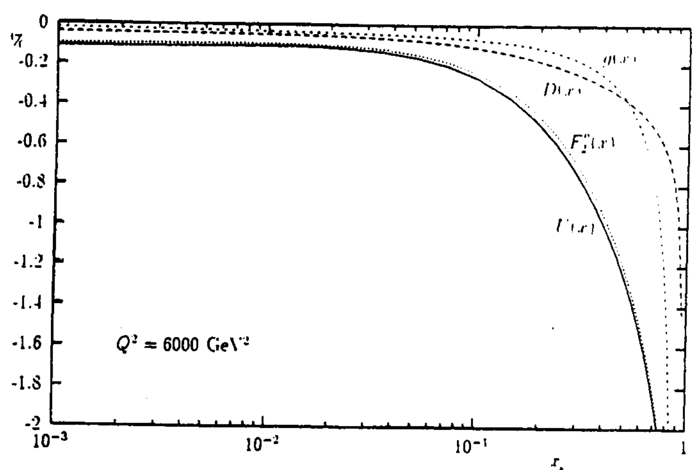
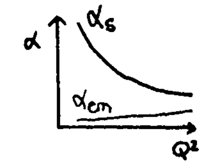


Figure 2: x dependence of the QED corrections (in per cent, see text) to parton distributions and the structure function F_2^p for deep inelastic lepton-proton scattering at $Q^2 = 6 \times 10^3$ GeV^2. Input parton distributions were taken from [12].

$$\frac{\partial q_i}{\partial \ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\left[P_{qq} \otimes q_i + P_{qg} \otimes g\right] + \frac{\alpha_{em}(Q^2)}{2\pi} P^{\gamma}_{qq} \otimes q_i$$



$$\frac{\partial g}{\partial \ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\left[P_{gq} \otimes q + P_{gg} \otimes g\right]$$

where $P^{\gamma}_{qq} = \frac{e_q^2}{C_F} P_{qq}$

— in principle $O(\alpha_{em}) \sim O(\alpha_s^2)$

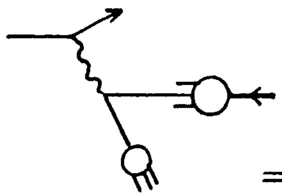
— in practice, negligible except at very large x

beyond LLA

- non-factorizable $O(\alpha)$ corrections
- factorization scheme dependence
- photon + jet events

the DIS final state

- properties of the current jet

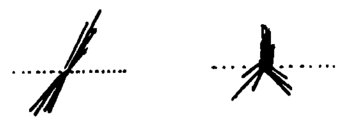


  - hadron multiplicity ⇝
  - jet width/profile ⇝

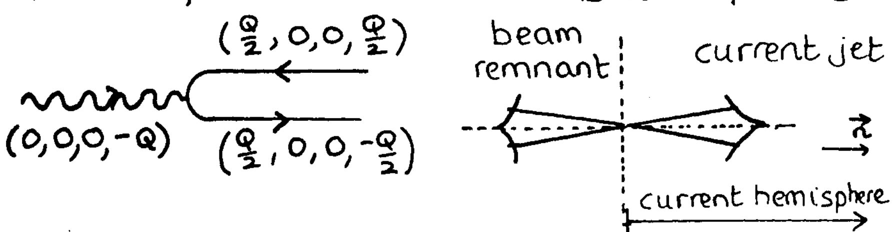
$\Rightarrow$ current jet is a quark jet!

- event shapes

cf thrust in e^+e^-

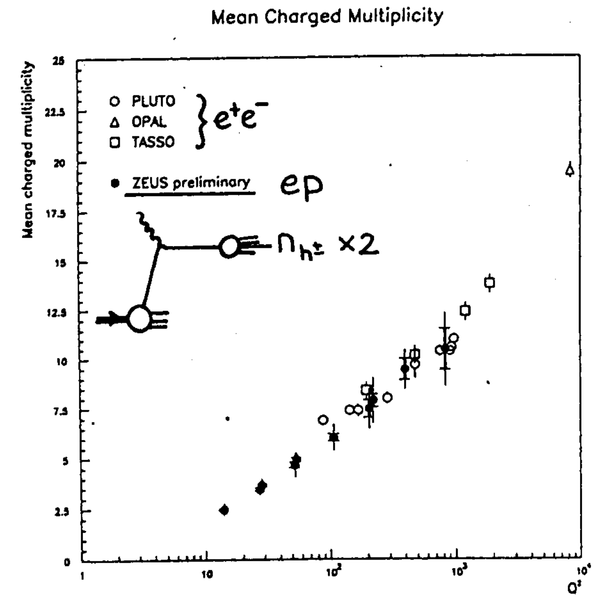


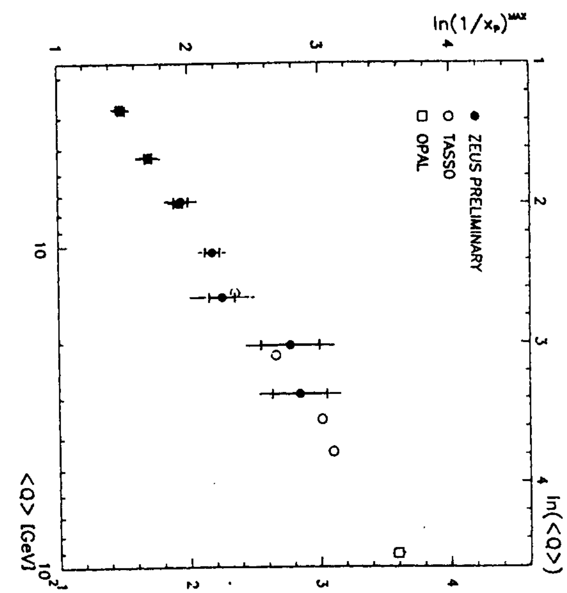
in DIS, introduce the BREIT frame



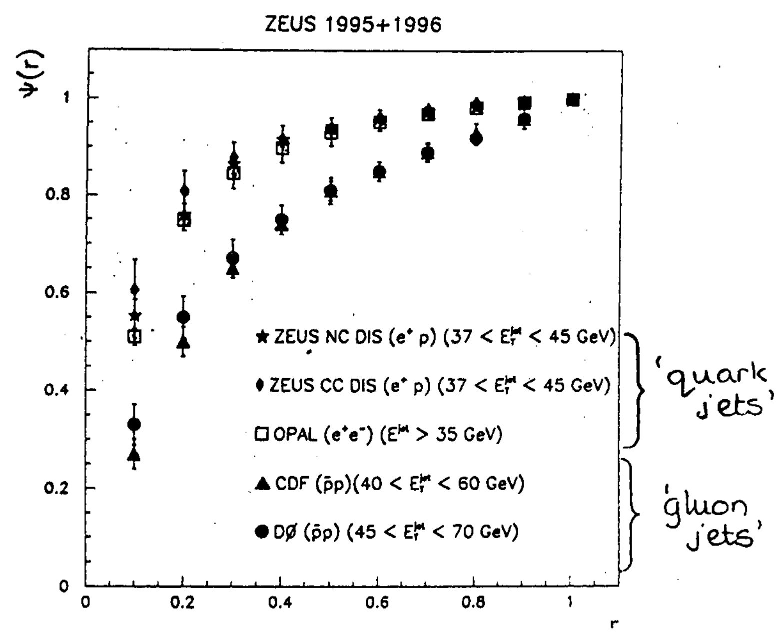
then

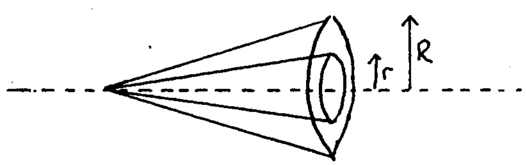
$$T = \frac{\sum_{h \in CH} |\vec{p}_h \cdot \vec{n}|}{\sum_{h \in CH} |\vec{p}_h|} \qquad \text{etc.}$$





comparison of jet profiles in e^+e^-, ep, $p\bar{p}$





$\psi(r) =$ (average) fraction of jet energy inside an 'inner cone' of radius $r < R$

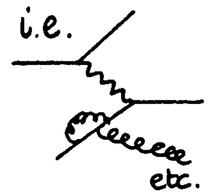
— pQCD NLO calculations:

MEPJET (Mirkes, Zeppenfeld)

DISENT (Catani, Seymour)

DISASTER (Graudenz)

i.e.



$$\frac{1}{\sigma}\frac{d\sigma}{dT} = \frac{\alpha_s}{2\pi} A_1(T) + \left(\frac{\alpha_s}{2\pi}\right)^2 A_2(T)$$

→ good agreement with data at high Q^2 ...

... at lower Q^2, evidence for (approximately) universal power corrections $\sim \frac{1}{Q^p}$, $p = 1, 2$

→

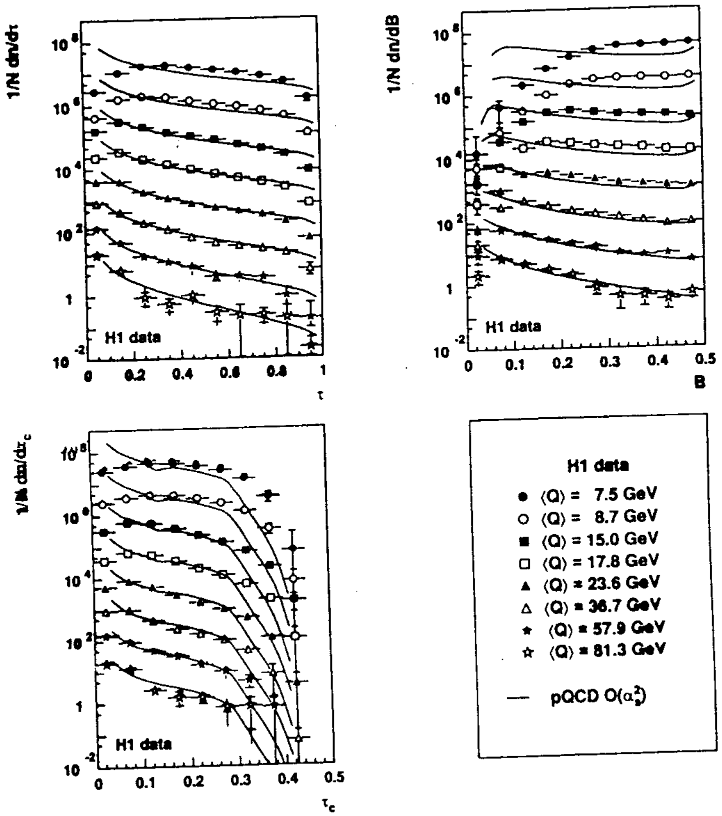
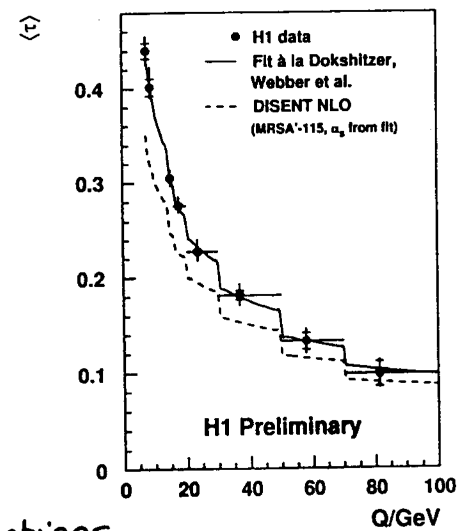
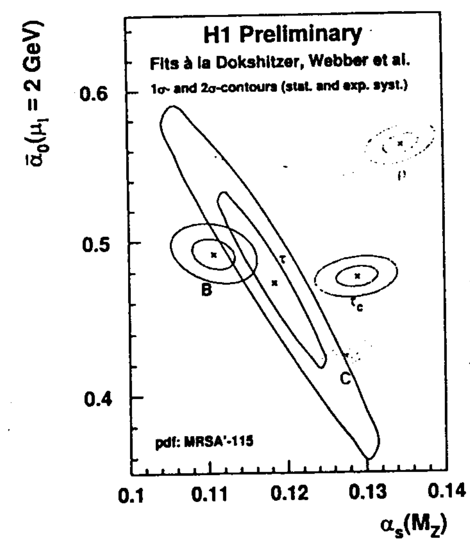


Figure 2: Normalized differential distributions of the event shapes τ, B and τ_C. H1 data (symbols) are compared with DISENT NLO calculations (curves) using the MRSA' parton density functions with $\alpha_s(M_Z) = 0.115$. The error bars represent statistical and systematic uncertainties. The spectra given at $\langle Q\rangle$ = 7.5 GeV, 8.7 GeV, 15.0 GeV, 17.8 GeV, 23.6 GeV, 36.7 GeV, 57.7 GeV and 81.3 GeV (from top to bottom) are multiplied by factors of 10^n ($n = 7, \ldots, 0$).



power corrections
in DIS event shapes (H1)



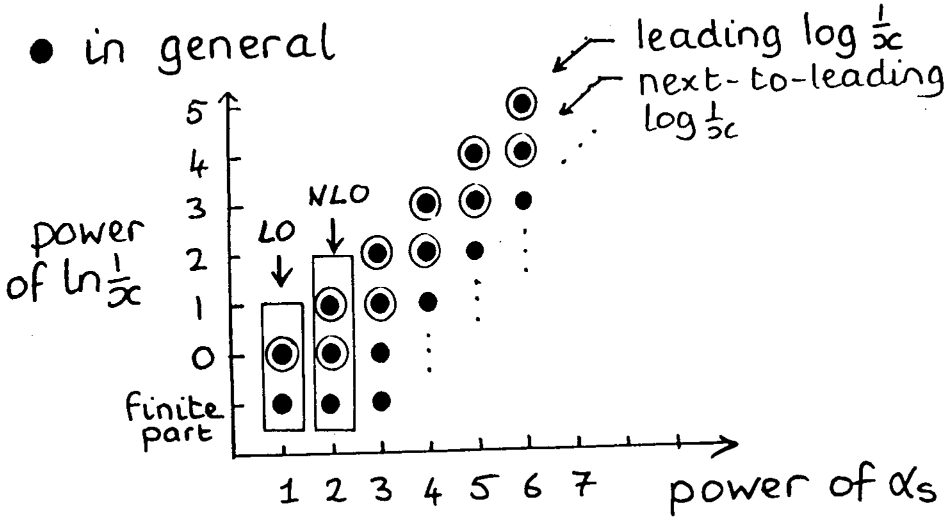
$\log\frac{1}{x}$ resummation as $x \to 0$

- for $x \ll 1$, large logarithms $\ln\frac{1}{x}$ appear in the splitting function and coefficient function perturbation series, generically

$$xP(x,\alpha_s) \sim \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{2\pi}\right)^{n+1} A_n \left(\ln\frac{1}{x}\right)^n + \dots$$

e.g. for P_{gg}: $A_0 = 6$, $A_1 = A_2 = 0$, $A_3 = 518.4, \dots$

- in general



- DGLAP equation

$$\frac{\partial}{\partial t}\begin{pmatrix}\Sigma \\ g\end{pmatrix} = \begin{pmatrix} P_{qq} & P_{qg} \\ P_{gq} & P_{gg}\end{pmatrix} \otimes \begin{pmatrix}\Sigma \\ g\end{pmatrix}$$

$$\downarrow$$

$$\begin{pmatrix} NL_x + \dots & NL_x + \dots \\ L_x + NL_x + \dots & L_x + NL_x + \dots \end{pmatrix}$$

— the L_x, NL_x, ... contributions can be resummed using th BFKL equation

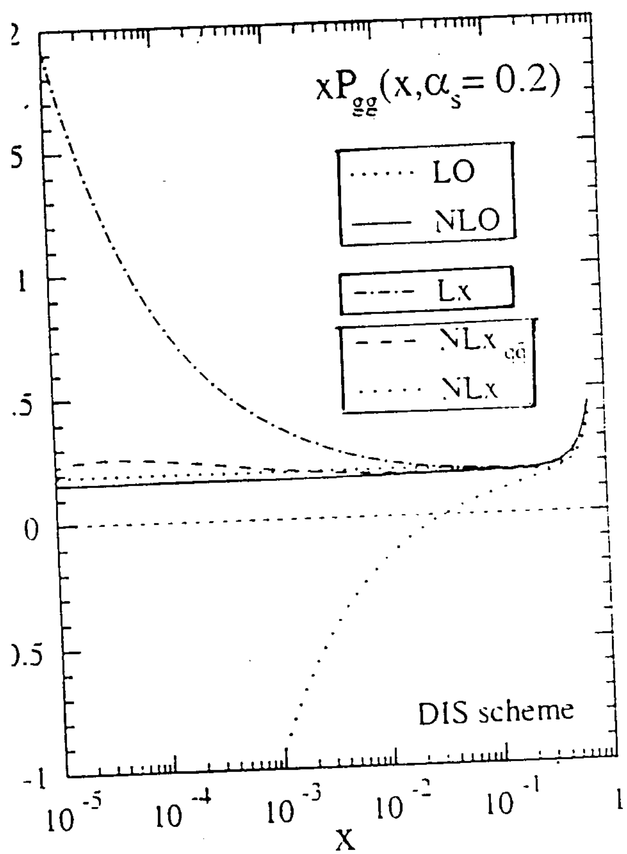
Bal
Fa
Ku
Li

... but the resummation leads to highly unstable resu

— this is a very hot theoretica topic !

/

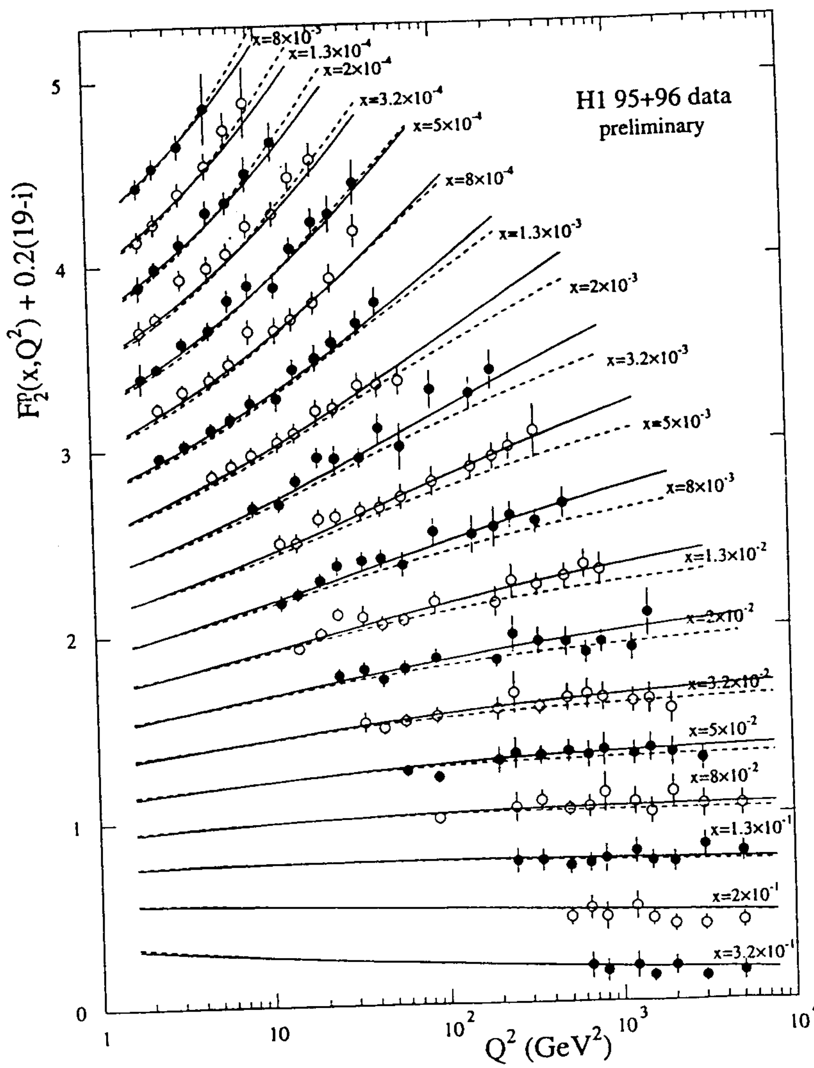
Blümlein et al.
hep-ph/9806368



$- - - -$ standard MRST NLO-DGLAP fit

—— "L_x improved" DGLAP fit (Thorne)

$\chi^2_{—} < \chi^2_{---}$



3

4

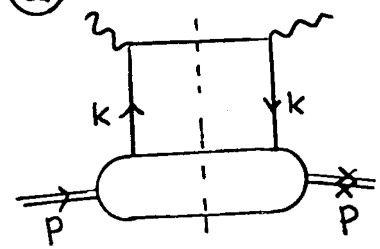
parton distribution functions

- first-principles calculations
- theoretical models
- extracted directly from data

theoretical expectations for the p

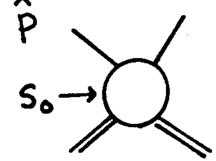
① 'first principles' calculation
e.g. from Lattice QCD —

② $x \to 0$: Regge theory —



$$s_0 = (p-k)^2 = \frac{-k^2(1-x) - k}{x}$$

$\to \infty$ as $x \to 0$ –



$$\sim \beta_{P}\, s_0^{\overbrace{\alpha_P - 1}^{0.08}} + \beta_{R}\, s$$

$$\Rightarrow \quad x q_s \, (xg) \underset{x \to 0}{\sim} x^{-0.08}$$

$$x q_v \underset{x \to 0}{\sim} x^{+0.45}$$

OK, but : — at which Q^2? (~1G
— in which scheme?
(LO, NLO, DIS,

note

- F_2 data ($Q^2 > 1\,\mathrm{GeV}^2$) – for qua

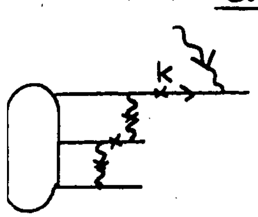
F_2 data ($Q^2 \lesssim 1$ GeV^2)

$\longleftrightarrow$ REGGE + 'higher twist'

e.g. Donnachie - Landshoff fit

$$F_2^{DL} \sim A\, x^{-0.08} \frac{Q^2}{Q^2+a^2} + B\, x^{0.45} \frac{Q^2}{Q^2+b^2} \quad \rightarrow \text{fig.}$$

$x \to 1$: <u>dimensional counting</u>



$$-k^2 = \frac{x s_0 + k_T^2}{1-x} \underset{x \to 1}{\sim} \frac{s_0}{1-x} \to \infty$$

Blankenbecler
Brodsky
Gunion
Farrar
Jackson
...

lines x off-shell by $O\left(\frac{1}{1-x}\right)$

$f(x) \sim (1-x)^{2n_s - 1}$ ← number of spectator partons

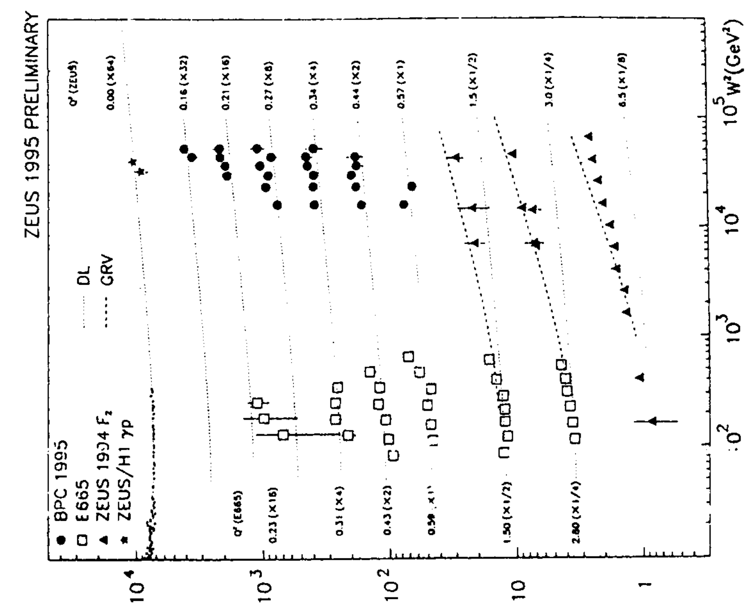
$u_v, d_v \sim (1-x)^3$ → also $\frac{d}{u} \to \frac{1}{5}$ Farrar Jackson

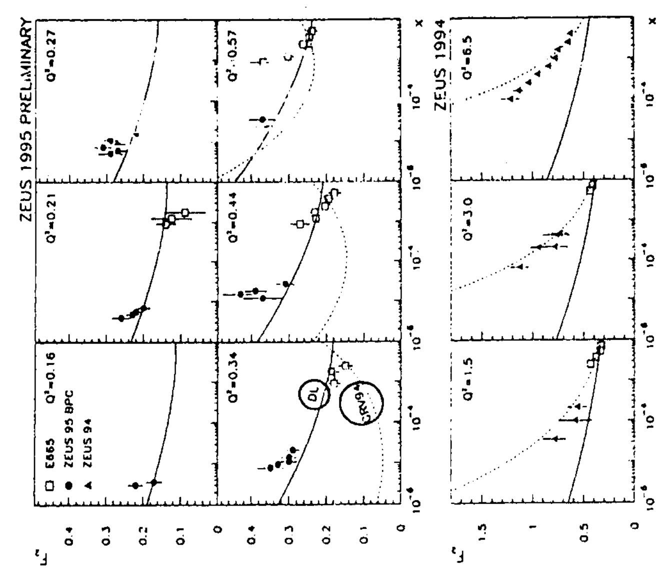
$q_s \sim (1-x)^7$

$g \sim (1-x)^5$

– qualitative agreement with pdf's fitted to data
(Q^2? scheme? ...)

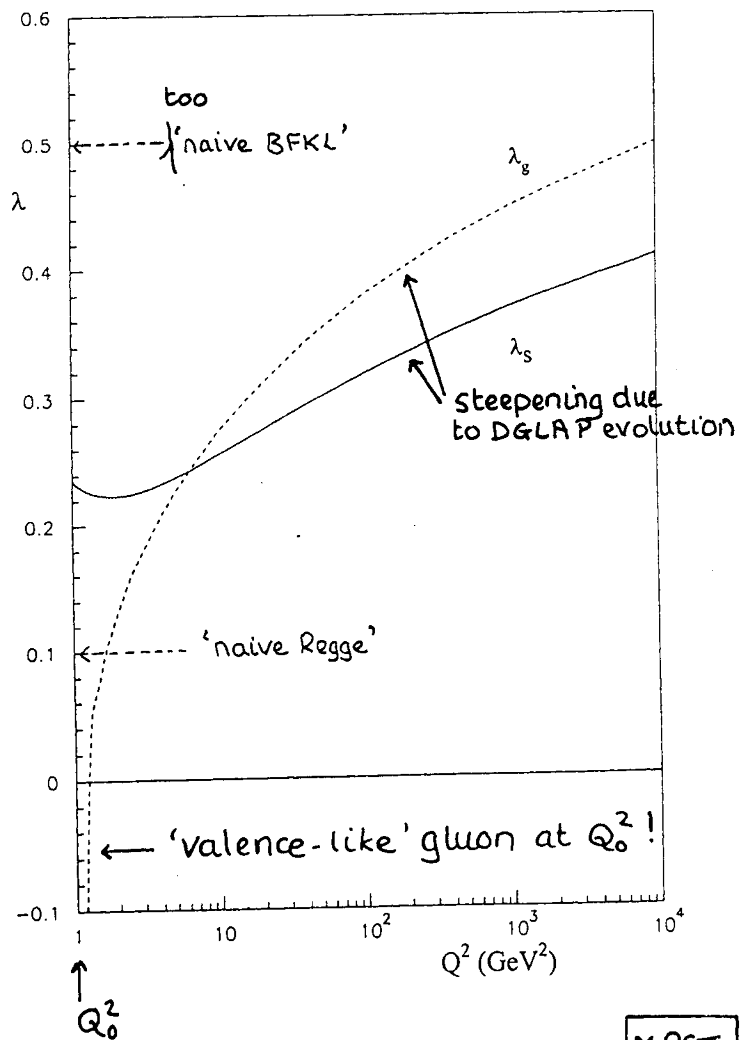
structure functions at low Q^2





small x behaviour of quarks, gluons

$$xq_S \simeq A_S\, x^{-\lambda_S} \quad ; \quad xg \simeq A_g\, x^{-\lambda_g}$$

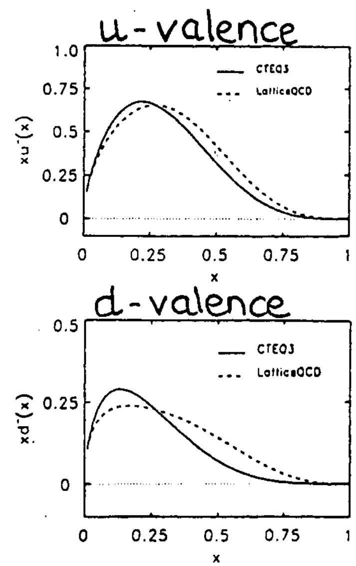


— pdfs from npQCD (Lattice)

... Lattice calculations give first few moments of q and Δq for $p, \pi, \rho, \ldots$

polar pdf

non-singlet, leading -t...
+
QUENCHED approx.



Mankiewicz Weigl

+ $x \to 0, 1$ assumptions

proton moments

Moment	Lattice (quenched, $\mu^2 \approx 5\,GeV^2$)	Experim... ($\mu^2 = 4\,G$...
$\langle x\rangle^{(u)}$	0.410(34)	0.28
$\langle x\rangle^{(d)}$	0.180(16)	0.10
$\langle x\rangle^{(u)}-\langle x\rangle^{(d)}$	0.230(38)	0.18
$\langle x^2\rangle^{(u)}$	0.108(16)	0.08
$\langle x^2\rangle^{(d)}$	0.036(8)	0.02
$\langle x^3\rangle^{(u)}$	0.020(10)	0.03
$\langle x^3\rangle^{(d)}$	0.000(6)	0.00
$\langle x\rangle^{(g)}$	0.53(23)	0.44

Schierholz et (1997)

'dynamical partons'

Altarelli
Cabibbo
Maiani
Petronzio

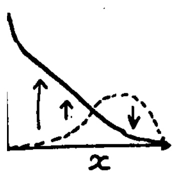
Gluck, Reya

⋮

NLO – DGLAP evolution from $\left\{\begin{array}{l}\text{Gluck}\\ \text{Reya}\\ \text{Vogt}\end{array}\right.$

$$\mu_0^2 = 0.34\ \text{GeV}^2$$

... where q_s, g are valence-like,
$xq_s, xg \to 0$ as $x \to 0$



– problem

size of q_s, g predicted at small x:

$$F_2 \longleftarrow q_s \Leftrightarrow g \longrightarrow \frac{\partial F_2}{\partial \ln Q^2}$$

i.e. large $F_2 \Leftrightarrow$ large $\frac{\partial F_2}{\partial \ln Q^2}$

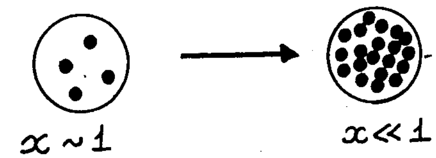
... recent HERA F_2 data indicate

$\frac{F_2}{\frac{\partial F_2}{\partial \ln Q^2}}$ overestimated by GRV partons

– ? higher twists?

parton saturation?

— if # of gluons (equiv. $g(x)$) becomes too large, they can saturate the proton and interact ⇒ breakdown of parton model

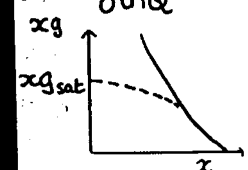


rough estimate: $\underset{\substack{\uparrow\\ \text{\#gluons/}\\ \text{unit rapidity}}}{N_g} \cdot \underset{\substack{\uparrow\\ gg\text{ cross}\\ \text{section}}}{\hat{\sigma}_{gg}} \simeq xg \cdot \frac{\alpha_s(Q^2)}{Q^2} = \pi \underset{\substack{\uparrow\\ \text{proton}\\ \text{radius}\\ R \simeq 5\,\text{GeV}^{-1}}}{R^2}$

$$\Longrightarrow \quad xg^{crit} \sim \frac{\pi R^2 Q^2}{\alpha_s(Q^2)}$$

— more rigorous implementation:
Gribov - Levin - Ryskin (GLR) equation

$$\frac{\partial g}{\partial \ln Q^2} = P^{gg} \otimes g - \frac{81\alpha_s^2}{16R^2Q^2}\int_x^1 dx' \, [g(x')]^2$$

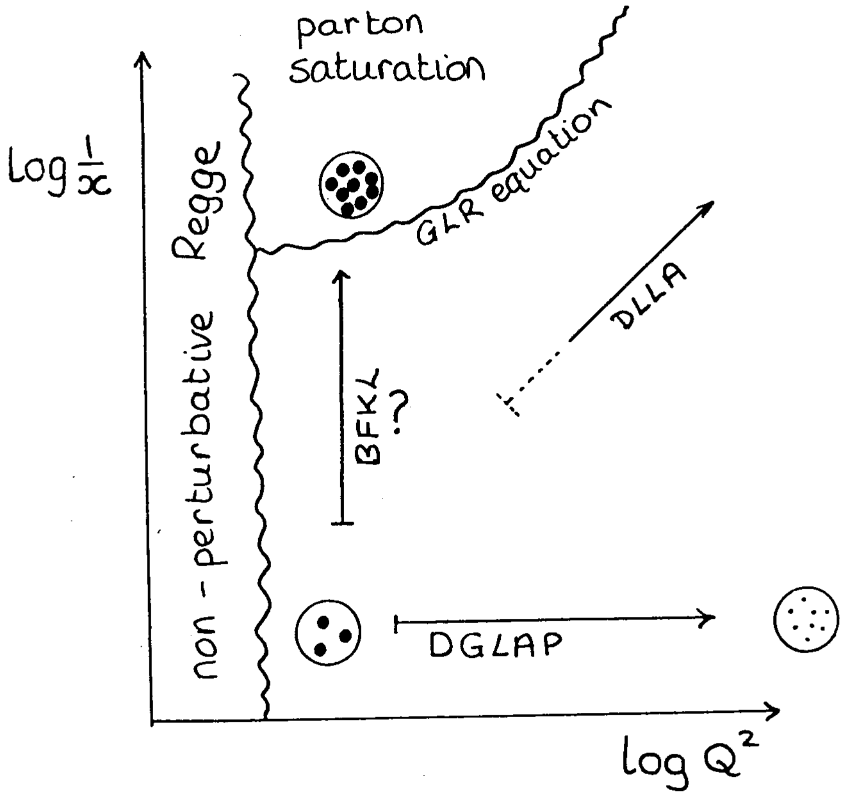


$$\Longrightarrow \quad xg^{sat} \simeq \frac{16}{27\pi} \frac{R^2Q^2}{\alpha_s(Q^2)}$$

$\simeq 500$ for $Q^2 \simeq 20\,\text{GeV}^2$ at HERA

BUT $xg^{measured} \lesssim 30$

∴ saturation probably irrelevant at HERA
(unless $R^{eff} \ll R \Rightarrow$ 'hot spots')



$$\boxed{\text{experimental data and errors}}$$ DIS str. fr
\+ hadron-h

\+ $\boxed{\text{theoretical framework}}$ e.g. NLO-DG $\overline{MS}$ sch

\+ $\boxed{\text{theoretical assumptions and prejudices}}$ e.g. Q^2, W^2 cu
omit data:
npQCD eff
⋮

$= \boxed{f_i(x,Q^2) \quad Q^2 > Q_0^2 \quad i = u,d,\ldots,g} + \alpha_s$

<u>summary</u>

F_i	$\longrightarrow$ quarks (all x)
DY	$\longrightarrow$ sea quarks (high x)
$\frac{\partial F_i}{\partial \ln Q^2}$	$\longrightarrow$ gluon (small x)
hh $\rightarrow$ jets, γ	$\longrightarrow$ gluon (high x)

<u>note</u> MRST (1998)
CTEQ5 (1999)

... broadly similar

ote

— instead of having to laboriously integrate the 'Altarelli – Parisi' equations each time a distribution (e.g. $u(x, Q^2 = M_W^2)$) is needed, analytic and numerical approximations are provided in the literature, e.g.

— Duke and Owens (DO), 1984
Gluck et al. (GHR),
Eichten et al. (EHLQ)
Tung et al. (CTEQ)
Martin et al. (MRS)
⋮ 1999

— thus, for example,

SUBROUTINE MRST($\overbrace{X, Q}^{\text{input}}$, UV, DV, USEA, DSEA, STR, CHM, BOT, GLU)

- the MRS series of fits (1987 ⟶) — Martin Roberts WJS + ...

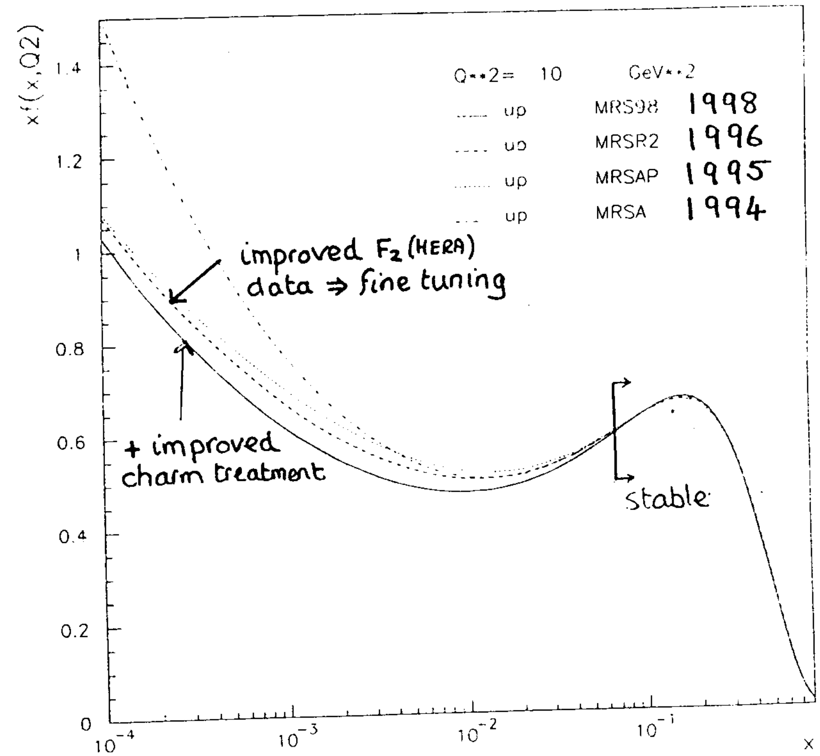
— at $Q_0 = 1$ GeV parameterise as

$$\boxed{f_i(x, Q_0^2) = A_i\, x^{\lambda_i - 1}\left[1 + \epsilon_i \sqrt{x} + \gamma_i x\right](1-x)^{\eta_i}}$$

with $i = u, d, \ldots, \bar{c}, \bar{b}, g$, then evolve using NLO-DGLAP and fit for $\{A_i, \ldots \eta_i, \alpha_s\}$

— as the data improve, the fits are fine-tuned ....

evolution of up quark distribution with time



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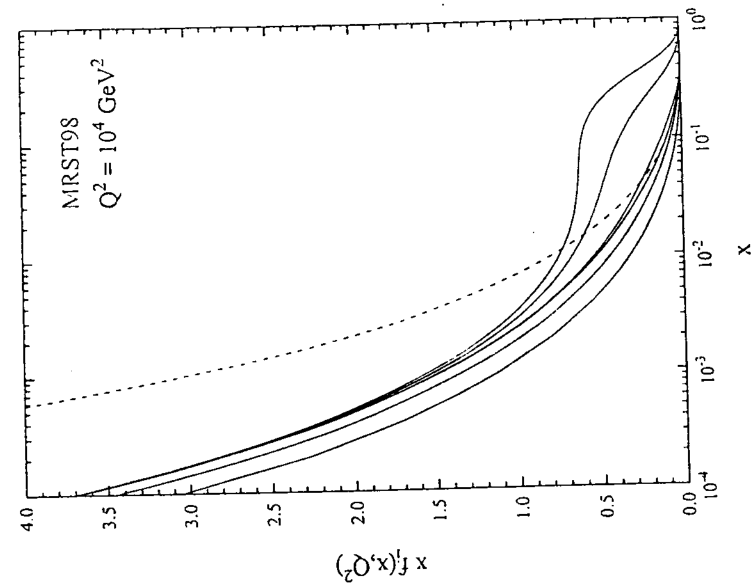
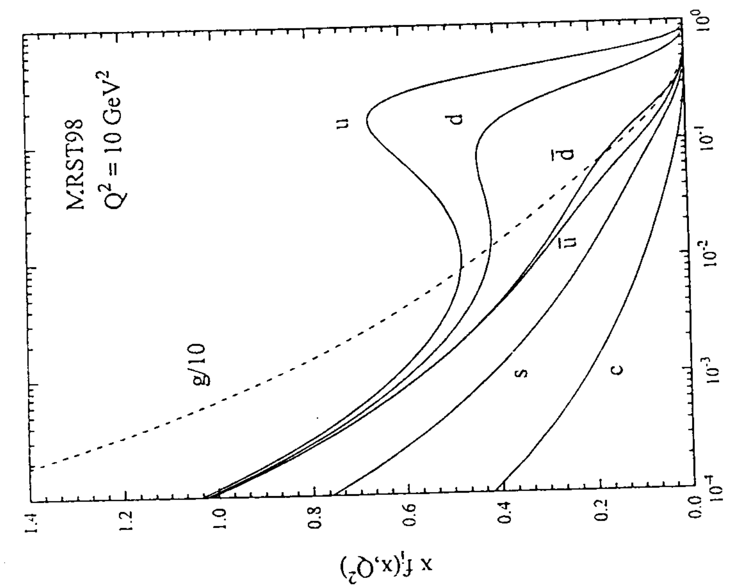
Martin Roberts '9?
Stirling Thorne





most recent MRS analysis

A.D. Martin
R.G. Roberts
R.S. Thorne
W.J. Stirling

$\Rightarrow$ MRST (1998) sets*

[hep-ph/9803445]

[code from: http://durpdg.dur.ac.uk/HEPDATA]

features

- new and updated data sets
- improved treatment of heavy flavour and prompt photon production
- default + 4 sets:
  - variation in α_s ↑↓
  - variation in gluon ↑↓

* small bug found and corrected: $\longrightarrow$ MRST99
see: hep-ph/9907231

- the CTEQ series of fits (1993 $\longrightarrow$)

Morfin
Tung
Lai
Huston
Kuhlmann
Olness
Owens
Pumplin

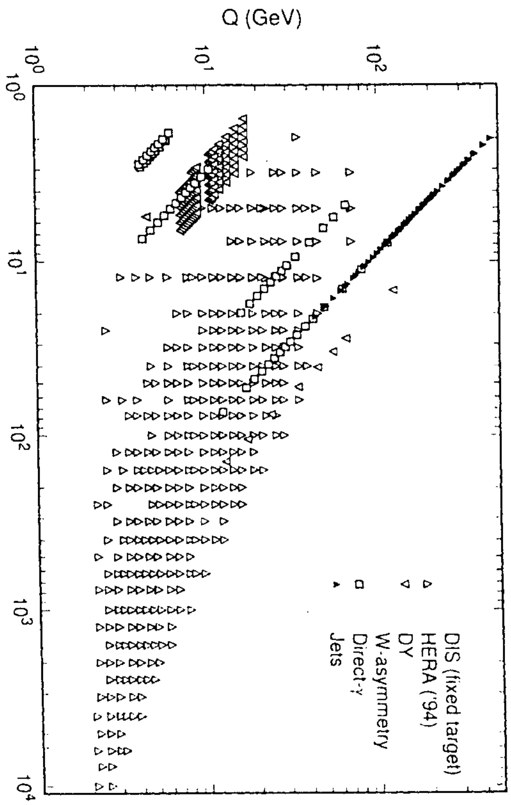
↑
CTEQ5

- parametrised at $Q_0 = 1$ GeV, uses NLO - DGLAP
- latest version:

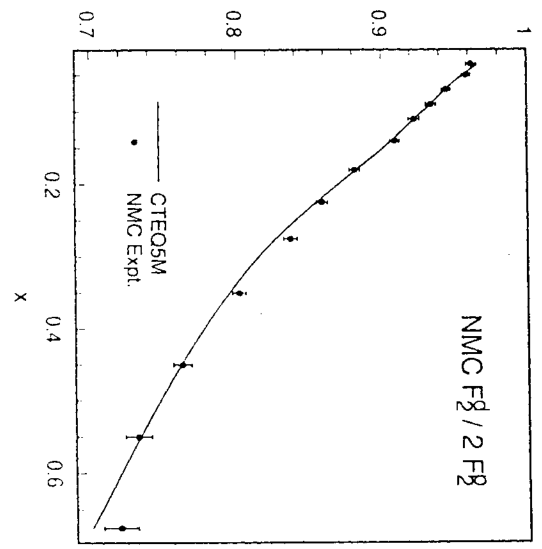
$\boxed{\text{CTEQ5 : hep-ph/9903282}}$

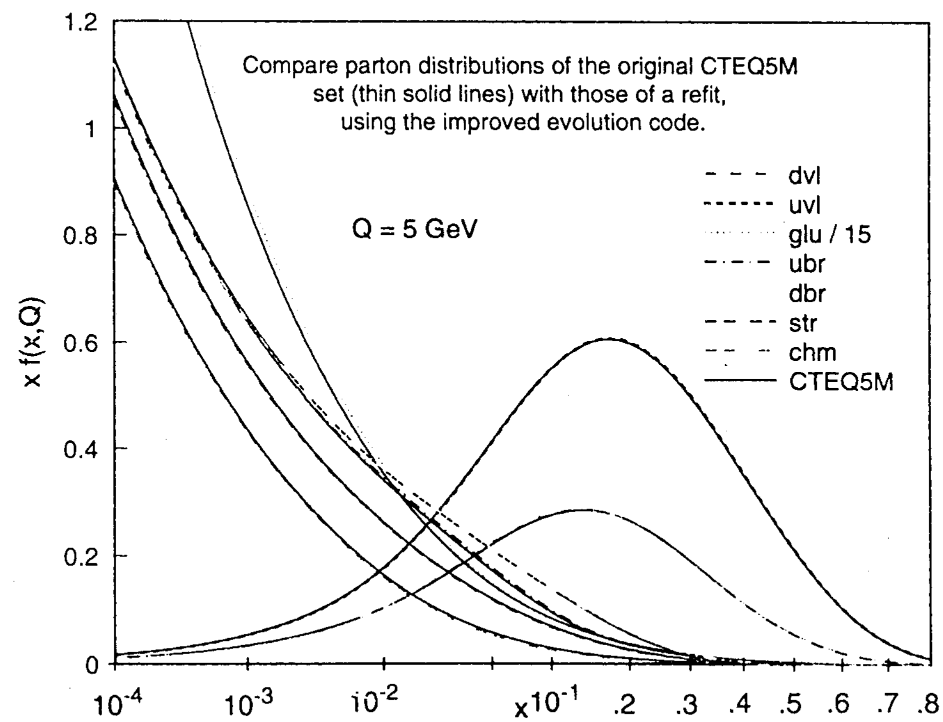
- various sets:

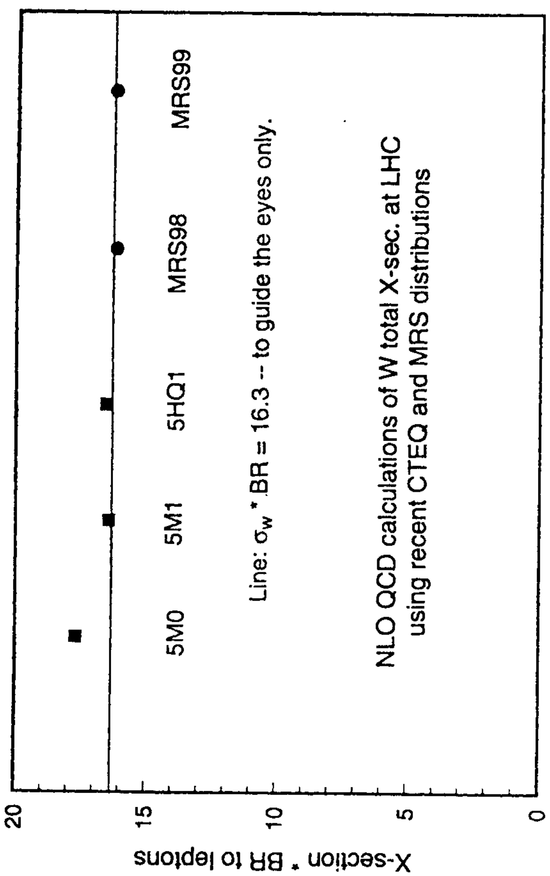
5M	$\overline{MS}$ scheme	zero mass partons
5D	DIS scheme	
5L	lowest-order	
5HJ	large-x gluon enhanced	
5HQ	$\overline{MS}$ ACOT scheme	on-shell heavy quarks
5F3	fixed 3 flavors	
5F4	fixed 4 flavors	


Q (GeV)
1/X
DIS (fixed target)
HERA ('94)
DY
W-asymmetry
Direct-γ
Jets

CTEQ


NMC F_2^d / 2 F_2^p
CTEQ5M
NMC Expt.
x


Compare parton distributions of the original CTEQ5M
set (thin solid lines) with those of a refit,
using the improved evolution code.
Q = 5 GeV
dvl
uvl
glu / 15
ubr
dbr
str
chm
CTEQ5M
x f(x,Q)



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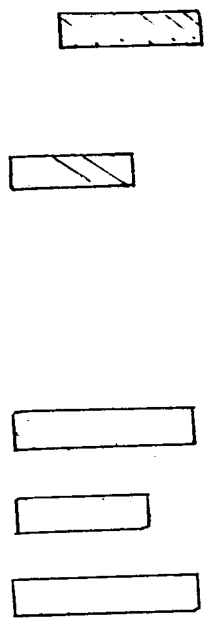
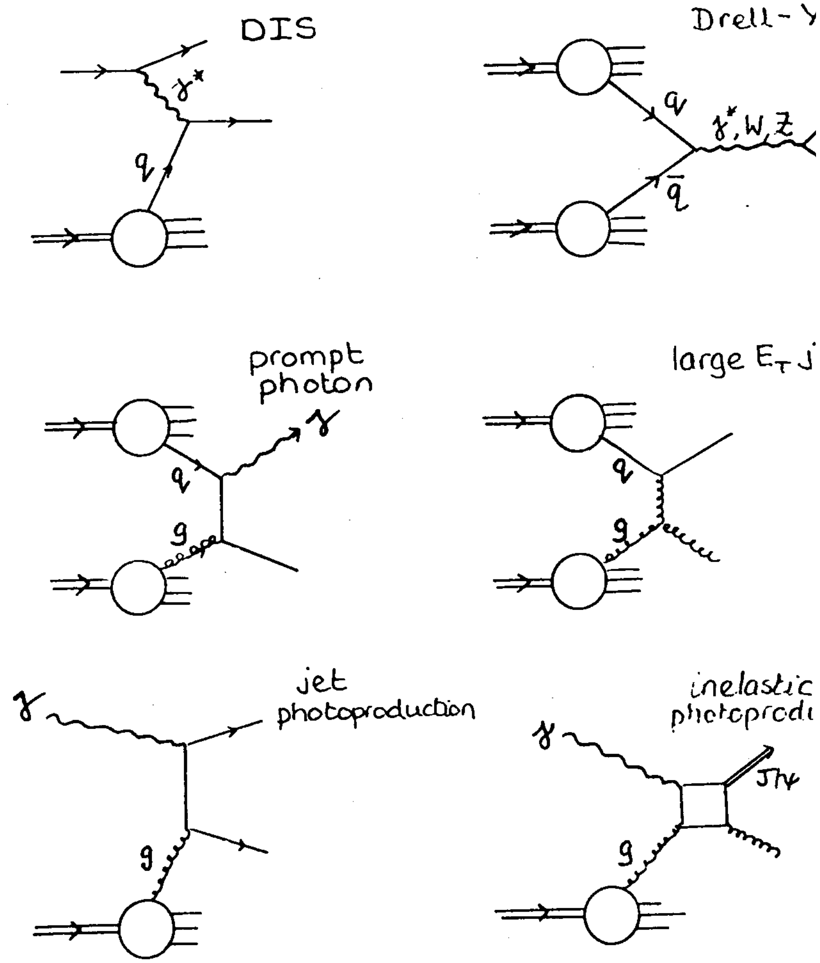
	Process/ Experiment	Leading order subprocess	Parton determination
DIS	**DIS ($\mu N \to \mu X$)** $F_2^{\mu p}, F_2^{\mu d}, F_2^{\mu n}/F_2^{\mu p}$ (SLAC, BCDMS, NMC, E665)	$\gamma^* q \to q$	Four structure functions $\to$ $u+\bar{u}$, $d+\bar{d}$, $\bar{u}+\bar{d}$, s (assumed $= \bar{s}$), but only $\int xg(x, Q_0^2)dx \simeq 0.35$ and $\int(\bar{d}-\bar{u})dx \simeq 0.1$
DIS	**DIS ($\nu N \to \mu X$)** $F_2^{\nu N}, xF_3^{\nu N}$ (CCFR)	$W^* q \to q'$	
DIS	**DIS (HERA)** F_2^{ep} (H1, ZEUS)	$\gamma^*(Z^*)q \to q$	λ ($x\bar{q} \sim x^{-\lambda_s}, xg \sim x^{-\lambda_g}$)
DIS	$\boldsymbol{\ell N \to c\bar{c}X}$ F_2^c (EMC, H1, ZEUS)	$\gamma^* c \to c$	c ($x \gtrsim 0.01, x \lesssim 0.01$)
DIS	$\boldsymbol{\nu N \to \mu^+\mu^- X}$ (CCFR)	$W^* s \to c \hookrightarrow \mu^+$	$s \approx \frac{1}{2}(\bar{u}+\bar{d})$
hadron-hadron	$\boldsymbol{pN \to \gamma X}$ (WA70, UA6, E706, ...)	$qg \to \gamma q$	g at $x \simeq 2p_T^\gamma/\sqrt{s} \to$ $x \approx 0.2 - 0.6$
hadron-hadron	$\boldsymbol{pN \to \mu^+\mu^- X}$ (E605, E772)	$q\bar{q} \to \gamma^*$	$\bar{q} = ...(1-x)^{\eta_S}$
hadron-hadron	$\boldsymbol{pp, pn \to \mu^+\mu^- X}$ (E866, NA51)	$u\bar{u}, d\bar{d} \to \gamma^*$ $u\bar{d}, d\bar{u} \to \gamma^*$	$\bar{d}/\bar{u}$ at $x \approx 0.04 - 0.3$
hadron-hadron	$\boldsymbol{p\bar{p} \to WX(ZX)}$ (UA1, UA2, CDF, D0) $\to \boldsymbol{\ell^\pm}$ asym (CDF)	$ud \to W$	u, d at $x \simeq M_W/\sqrt{s} \to$ $x \approx 0.13, 0.05$; slope of u/d at $x \approx 0.05 - 0.1$
hadron-hadron	$\boldsymbol{p\bar{p} \to}$ jet $+ \boldsymbol{X}$ (CDF, D0)	$gg, qg, qq \to 2j$	q, g at $x \simeq 2E_T/\sqrt{s} \to$ $x \approx 0.05 - 0.5$

MRST 1998



processes for which
'theoretical' uncertainties
are not precisely known

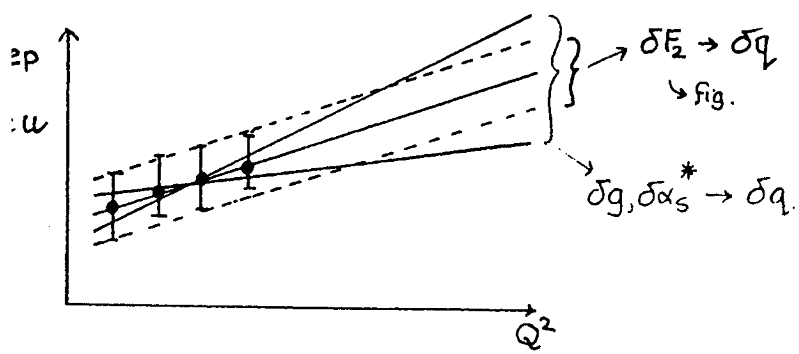


uarks

[u,d] measured directly by DIS structure functions at 'low' Q^2, then evolved to high Q^2 by DGLAP

$$\frac{\partial}{\partial \ln Q^2}\binom{q}{g} = \frac{\alpha_S(Q^2)}{2\pi}\int_x^1 \frac{dy}{y}\,[\bar{P}] * \binom{q}{g}$$

te — no significant evidence for breakdown of NLO DGLAP for $Q^2 \gtrsim 1-2\ \mathrm{GeV}^2$



te — global parton analyses entirely consistent with 'world average' $\alpha_S(M_Z) = 0.118 \pm 0.004$ values

arning! $\delta q \neq | q_{\dots} - q_{\dots} |$

flavour asymmetry of quark sea

- $\boxed{c, b, \dots}$ calculated perturbatively

e.g., scheme $\left\{\begin{array}{l} \mathrm{RT} \longrightarrow \mathrm{MRST} \\ \mathrm{ACOT} \longrightarrow \mathrm{CTEQ5} \\ \mathrm{FFN} \longrightarrow \mathrm{GRV} \end{array}\right.$ $\Big\}$ BGF ⊗ MPE

... and dependent on $m_{c,b,\dots}$

- $\boxed{s}$ from $\nu N \to \mu\mu X$ (CCFR, ...) $\simeq \frac{1}{2}\langle \bar{u} + \bar{d} \rangle$ at $Q_0^2 \sim 1\ \mathrm{GeV}^2$

note : $s(x) = \bar{s}(x)$?

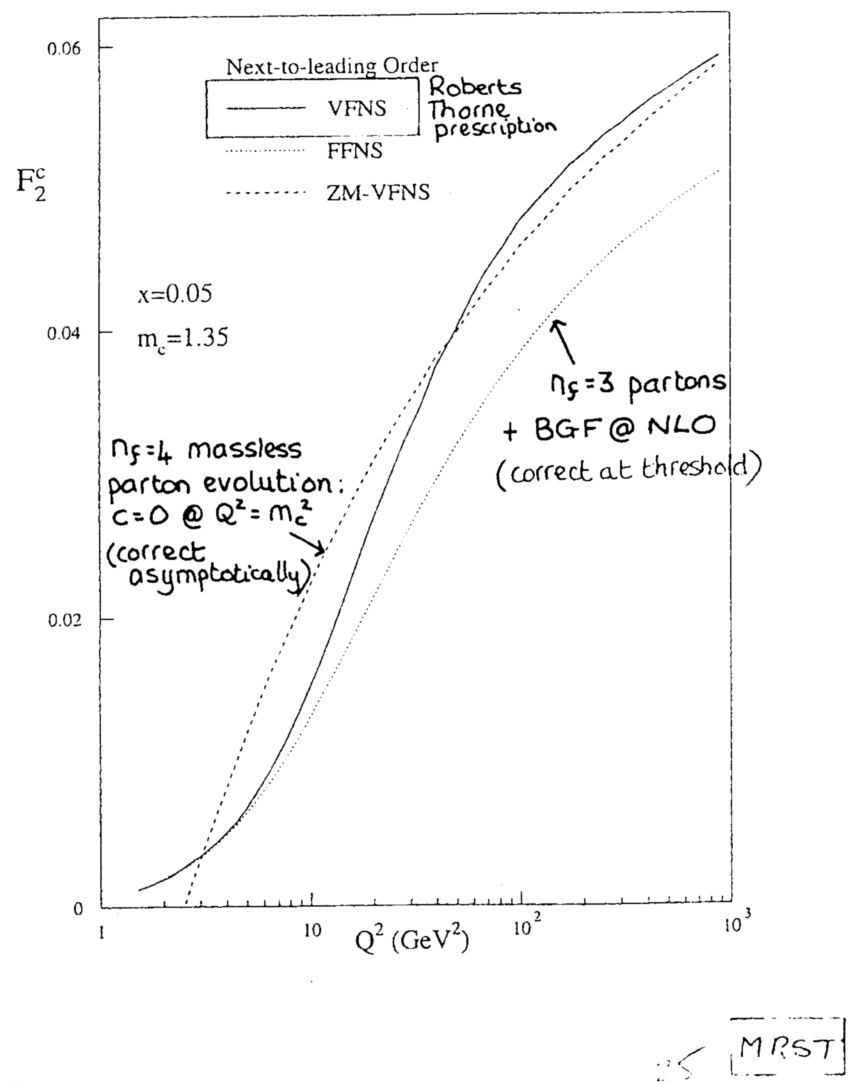
- $\boxed{\bar{u}, \bar{d}}$ — at v. small $x \leftarrow F_2^{\mathrm{HERA}}$

— at medium/high x :
Drell Yan $pN \to \mu\mu X \longrightarrow \bar{u} + \bar{d}$

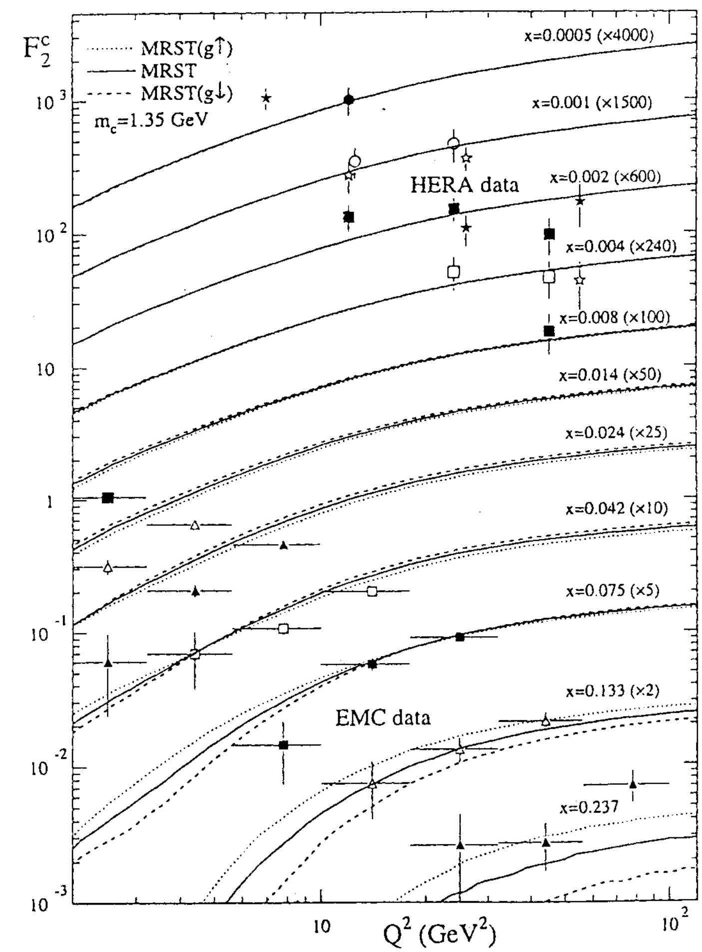
note : GSR : $\int_0^1 (\bar{d} - \bar{u})\,dx \simeq 0.1$ NMC

pp/pn DY : $\bar{d}(x) \neq \bar{u}(x)$ NA51 E866

$$|P\rangle = |p\rangle + |\pi^0 n\rangle + |\pi^- \Delta^{++}\rangle + \cdots$$



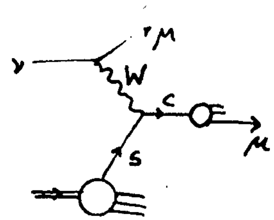
Next-to-leading Order
VFNS
FFNS
ZM-VFNS
Roberts Thorne prescription
F_2^c
x=0.05
m_c=1.35
n_f=4 massless parton evolution: c=0 @ $Q^2 = m_c^2$ (correct asymptotically)
n_f=3 partons + BGF @ NLO (correct at threshold)
Q^2 (GeV^2)
MRST

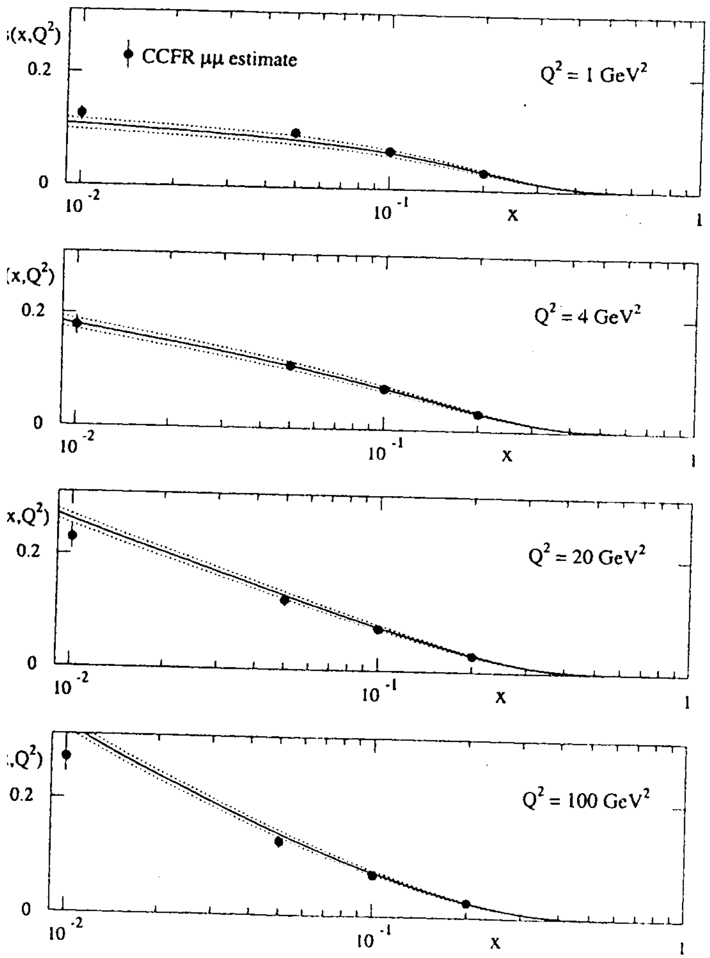


F_2^c
MRST(g↑)
MRST
MRST(g↓)
m_c=1.35 GeV
HERA data
EMC data
x=0.0005 (×4000)
x=0.001 (×1500)
x=0.002 (×600)
x=0.004 (×240)
x=0.008 (×100)
x=0.014 (×50)
x=0.024 (×25)
x=0.042 (×10)
x=0.075 (×5)
x=0.133 (×2)
x=0.237
Q^2 (GeV^2)

determining the
strange quark distribution

$0.5\left(\frac{\bar{u}+\bar{d}}{2}\right)$ @ $Q_0^2 = 1\ \mathrm{GeV}^2$

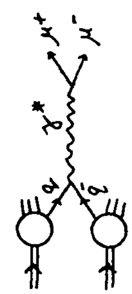


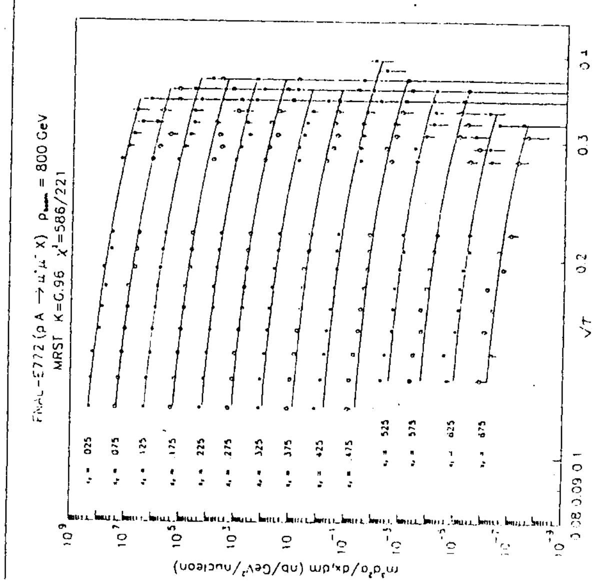


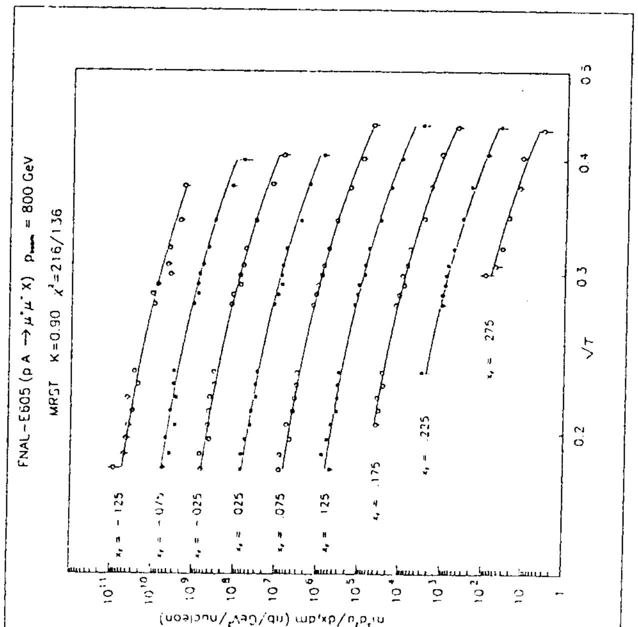
MRST '99

MRST

constraining $\bar{q}(x)$ $(x \lesssim 0.4)$ using — fixed-target Drell-Yan data







high x valence quark
⊗ low x sea quark

Gluon distribution

① small x

$$\frac{\partial F_2}{\partial \log Q^2} \simeq \frac{\alpha_s(Q^2)}{\pi} \sum e_q^2 \, x \int_x^1 \frac{dy}{y} P_{qg}\left(\frac{x}{y}\right) g(y,Q^2)$$

$$+ \cdots$$

② medium/high x

as above + hadronic hard scattering cross sections

DIS/photoprod^n

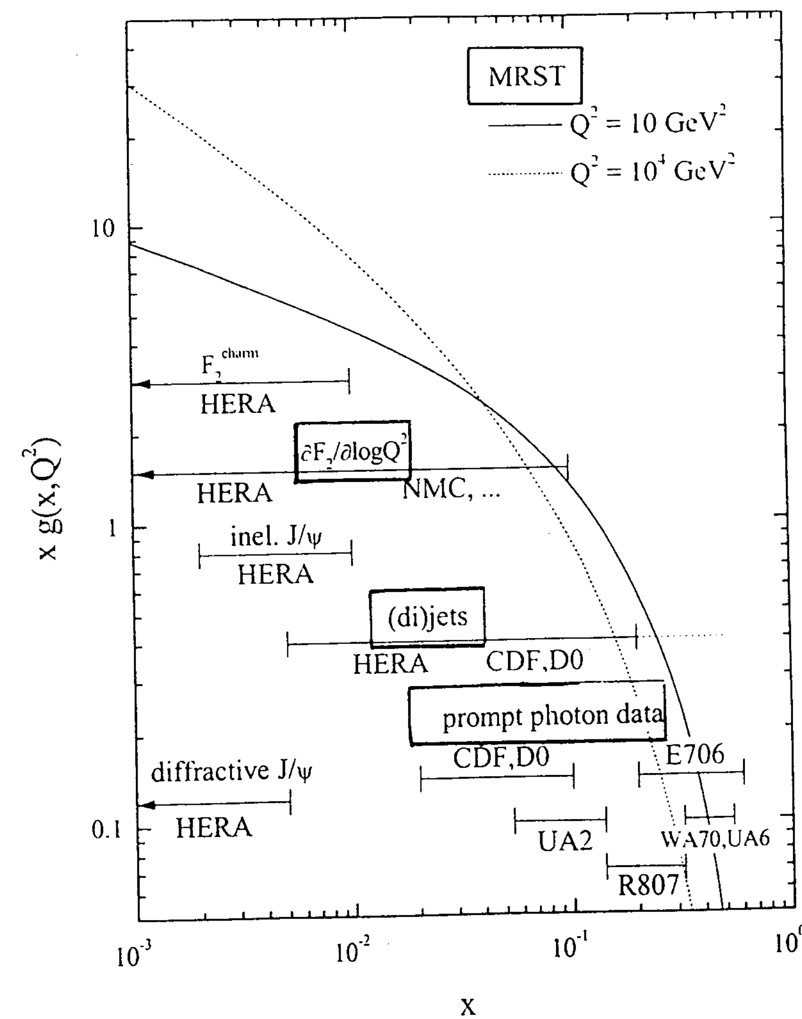
$$\left\{ \begin{array}{ll} \gamma g \to q\bar{q} & \text{jets} \\ \gamma g \to Q\bar{Q} & \text{heavy quarks} \\ \gamma g \to J/\psi\, g & \text{charmonium} \\ \vdots & \vdots \end{array} \right.$$

$pN/p\bar{p}$

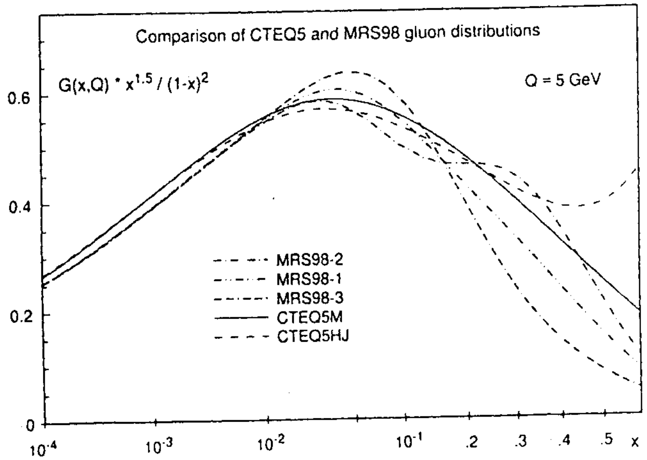
$$\left\{ \begin{array}{ll} qg \to \gamma q & \text{prompt photons} \\ qg \to qg & \text{jets} \\ \vdots & \vdots \end{array} \right.$$

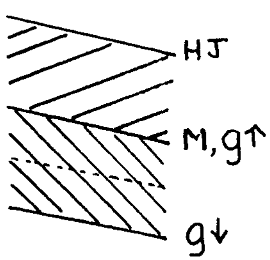
→ fig.

information on the gluon distribution



MRST vs CTEQ5 gluons





CTEQ5

- fit to high E_T Tev. jets
- no prompt γ in fits

MRST

- fit to prompt γ
- needs normalisation ↑ to describe Tev. jet data

$\underline{e}$
J : follows 'enhancement' of highest E_T (CDF) jets

↑ ST ↓ } fit prompt photons with different assumptions on $\langle k_T \rangle$ smearing

— to explore this, we introduce

MRST — 'standard' gluon †

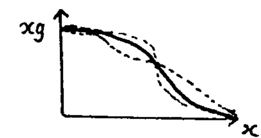
g↑ — 'larger at large x' gluon

g↓ — 'smaller at large x' gluon

then because of

- functional form $[A x^{\lambda}(1+\gamma x+\epsilon\sqrt{x})(1-x)^{\eta}]$
- momentum 'sum rule' $\langle xg \rangle = 1 - \langle x\Sigma q \rangle$
- strong constraint of HERA data at small x

get characteristic behaviour



→ fig.

— see also [J. Huston et al. hep-ph/9801444]

... abandon large p_T photons, jets and study variation allowed by DIS + DY data only

† fit to WA70, E706 } with $\langle k_T \rangle_{int} = \begin{cases} 280 \text{ MeV} \\ 700 \text{ MeV} \end{cases}$

and scale choice $\mu_F = \mu_R = p_T^{\gamma}/2$.

— supplement the 'gluon sets' $g\uparrow\downarrow$ with 'variable α_s sets'

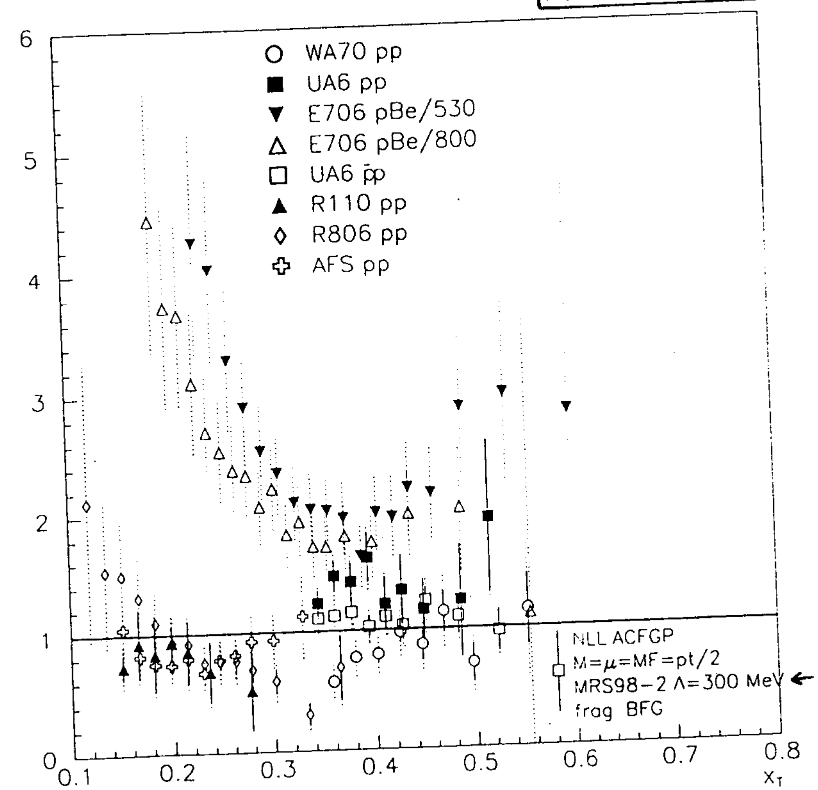
	$\alpha_S(M_Z)$	
MRST	0.1175	← best fit
$\alpha_s\downarrow\downarrow$	0.1125	
$\alpha_s\downarrow$	0.1150	
$\alpha_s\uparrow$	0.1200	
$\alpha_s\uparrow\uparrow$	0.1225	

... to represent a (conservative) 'world average' range

<u>note</u> the <u>implicit</u> α_s dependence of the $f_i(x,Q^2)$ evolved to high Q^2 can compete with the <u>explicit</u> dependence in $\hat{\sigma}$

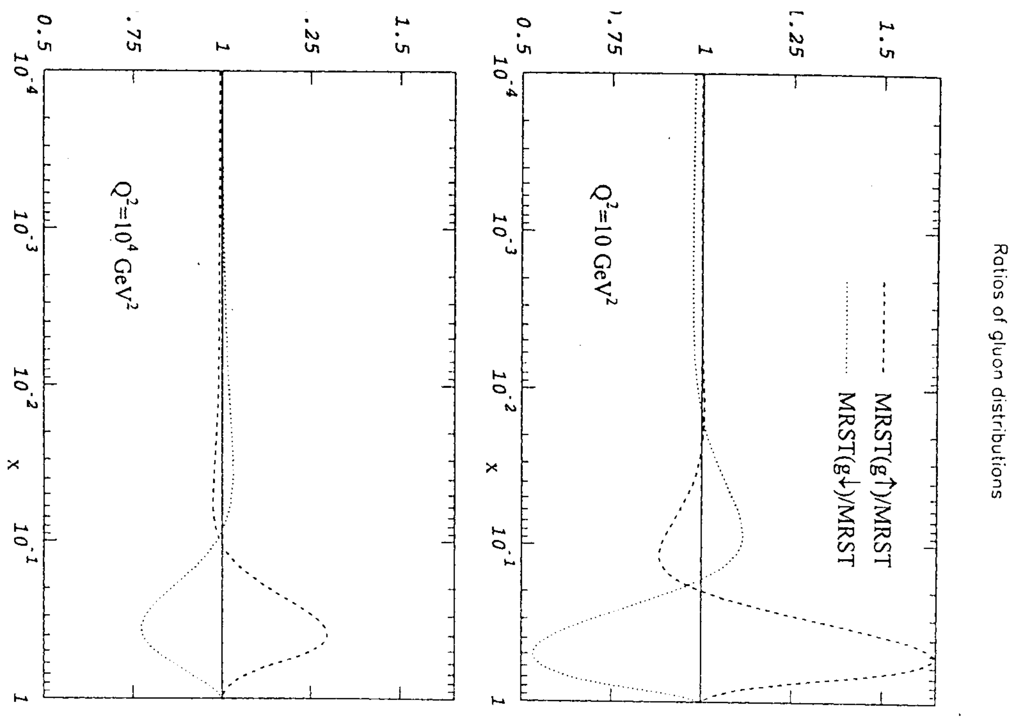
<u>there are a lot of prompt photon data!</u>

Aurenche et al



<u>the issues</u>:

- np 'k_T' smearing
- isolation, fragmenta
- scale dependence
- resummation



J. Huston et al.
hep-ph/9801444

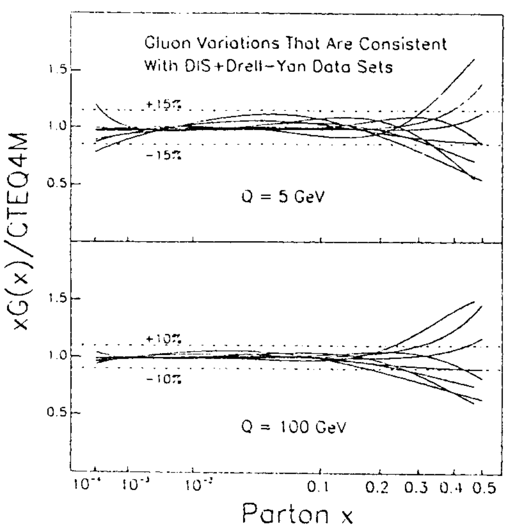


Figure 4: The ratio of gluon distributions compared to CTEQ4M is shown. On top is for $Q=5$ GeV, and on bottom is $Q=100$ GeV. These are the examples that are consistent with DIS+Drell-Yan data sets (see text)

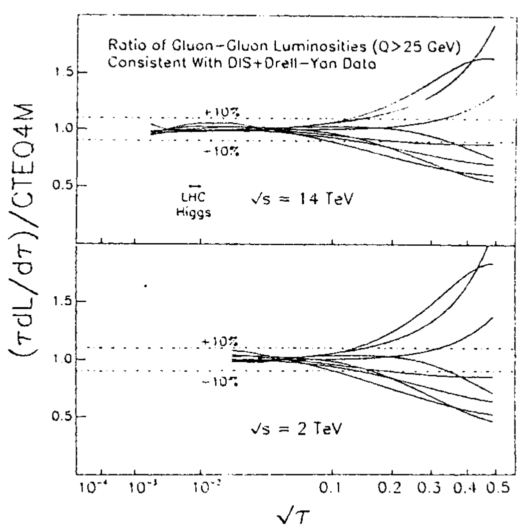
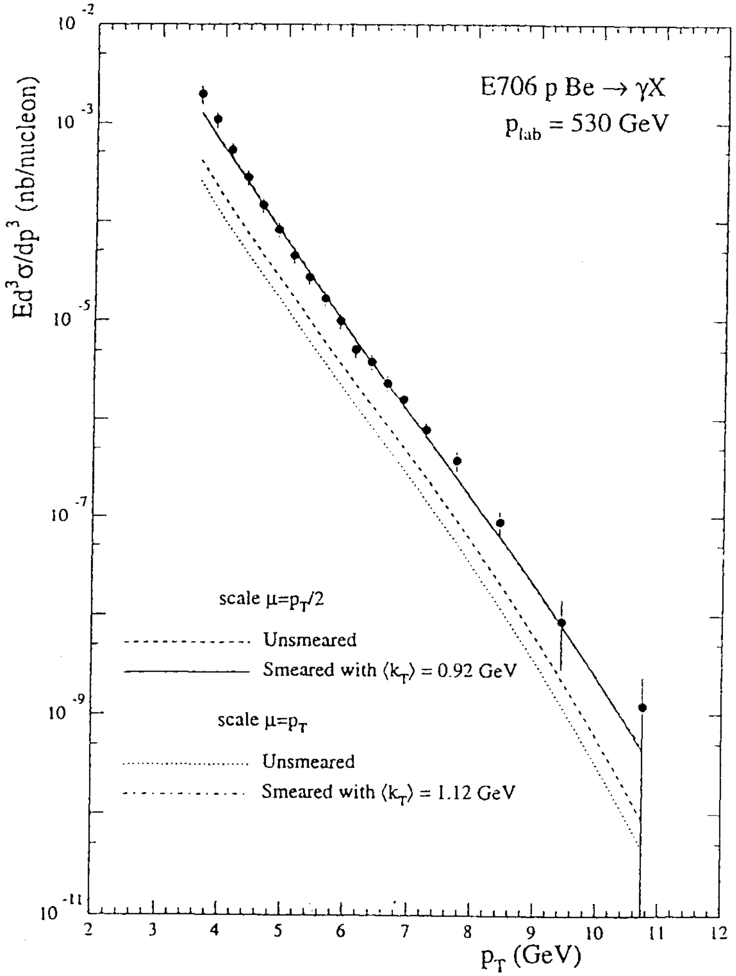
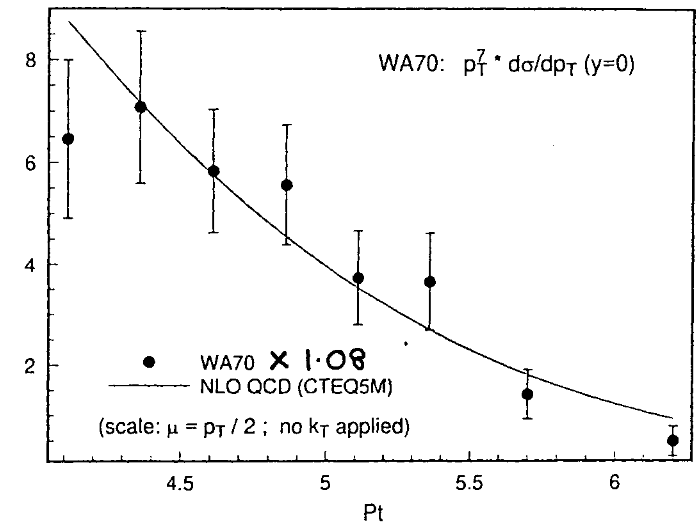
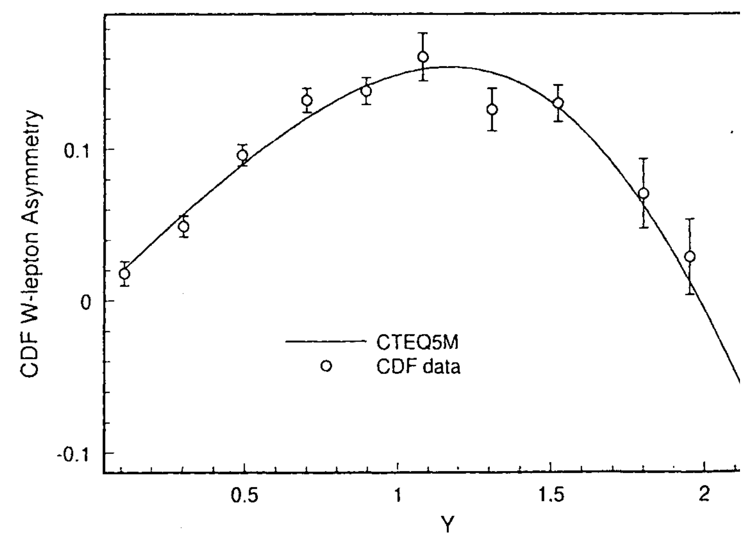


Figure 5: The ratio of integrated gluon-gluon luminosities compared to CTEQ4M is shown as a function of $\sqrt{\tau}$. These are the examples that are consistent with DIS+Drell-Yan data sets.

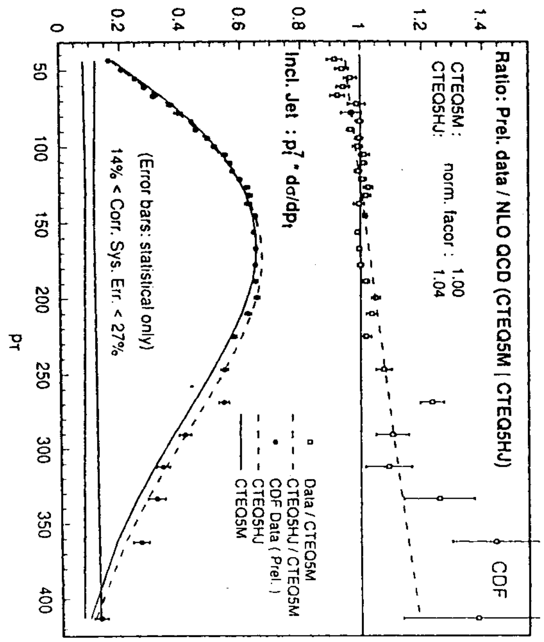

Ed³σ/dp³ (nb/nucleon)
E706 p Be → γX
p_lab = 530 GeV
scale μ=p_T/2
Unsmeared
Smeared with ⟨k_T⟩ = 0.92 GeV
scale μ=p_T
Unsmeared
Smeared with ⟨k_T⟩ = 1.12 GeV
p_T (GeV)

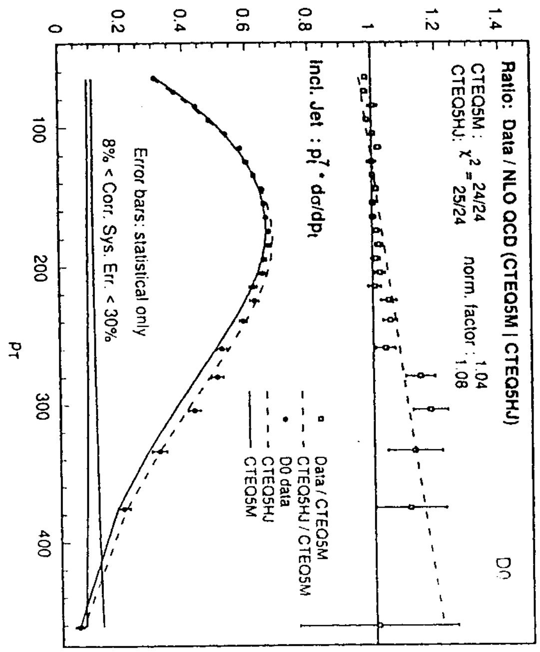
MRST
1998


WA70: p_T^7 * dσ/dp_T (y=0)
WA70 × 1·08
NLO QCD (CTEQ5M)
(scale: μ = p_T / 2 ; no k_T applied)
Pt

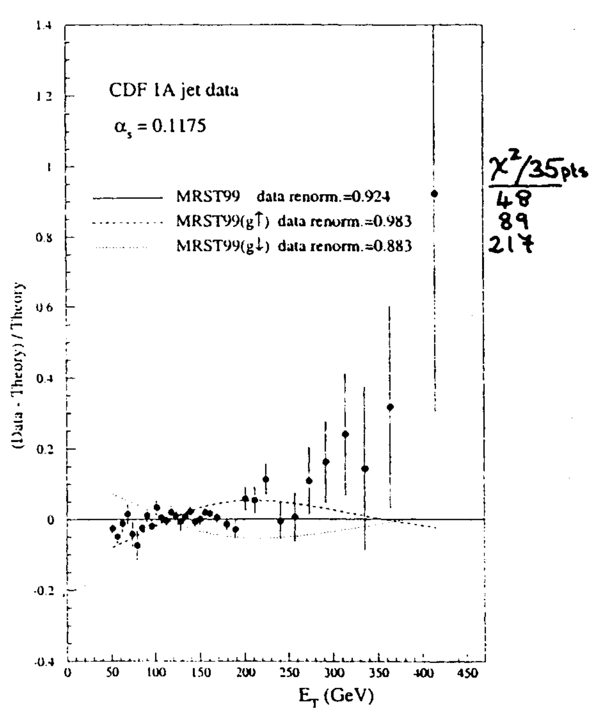

CDF W-lepton Asymmetry
CTEQ5M
CDF data
Y

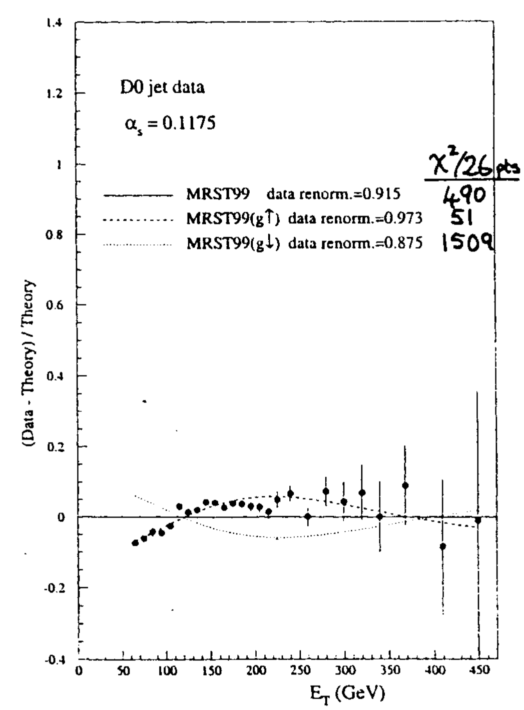
EQ5 description of CDF, DØ jet data





MRST comparison with
Tevatron inclusive jet data



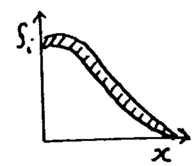


note: statistical errors only

pdf uncertainties

– goal?

$$\left\{ f_i \pm \delta f_i \;,\; \alpha_s \pm \delta\alpha_s \right\}$$



– however ...

- strong correlations between different f_i and different x regions
- influence of parametric forms: crossover points, end points
- many data uncertainties are not "true statistical"
- much of the theoretical input is largely (educated) guesswork
  - scale choices
  - heavy target, deuteron binding corrections
  - "intrinsic k_T"
  - ...

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pragmatic approach

$$\cancel{f_i \pm \delta f_i} \longrightarrow \sigma \pm \delta\sigma_{PDF}$$

identify dominant $\{x, Q^2, f_i\}$ contribu to σ, then traceback to global fit constraint

e.g. $\sigma_W : \left\{ x \sim \frac{M_W}{\sqrt{s}} \,,\, Q \sim M_W \,,\, u\bar{d} + d\bar{u} \right\}$ 3

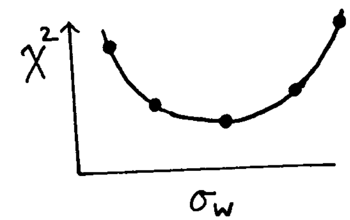
$F_2^{p,n}$ + DGLAP evolution

simple, no new technology, avoids parametrization dependence, expos theoretical uncertainty (if any)

requires separate analysis for each process, ignores correlations between effects (overestimates uncertainty)*

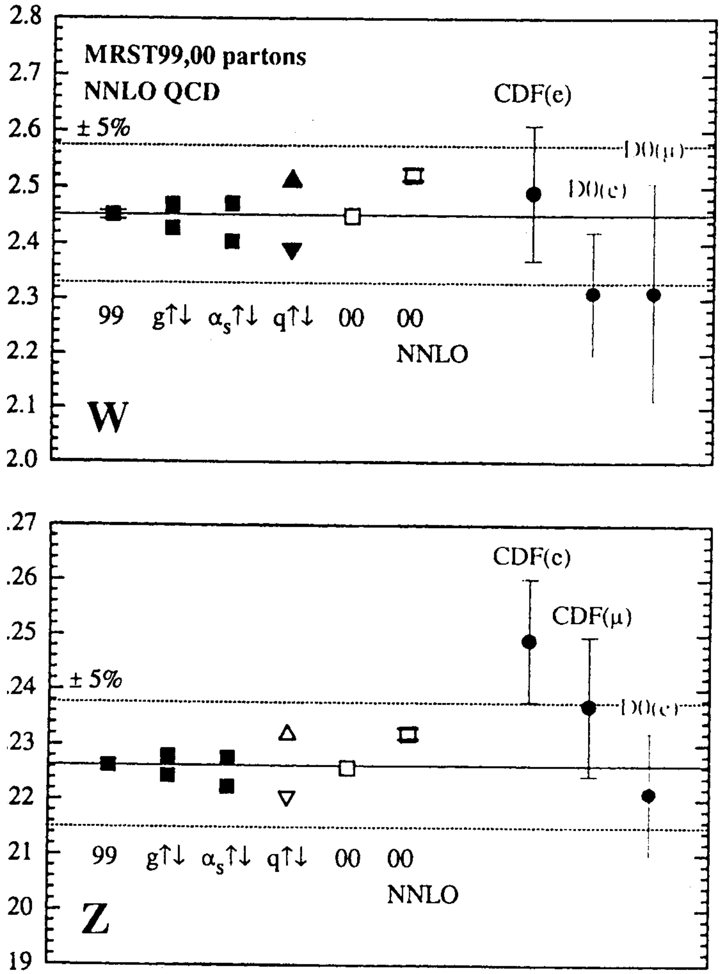
* Lagrange Multiplier Method (CTE(

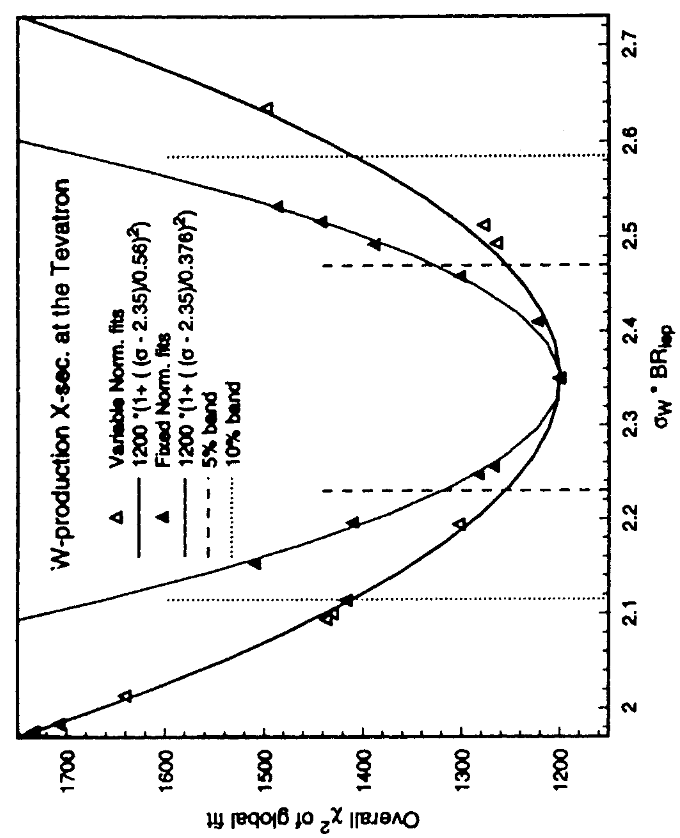
$$\chi^2(\lambda) = \chi^2_{global} + \lambda\sigma_W \Longrightarrow$$



– problem : $\Delta\chi^2 \overset{?}{\rightarrow} \Delta\sigma$

W and Z Cross Sections: Tevatron





"statistical" approaches

(e.g.) <u>Giele Keller Kosower</u>

- ensemble of pdf sets labelled by parameter set $\{\lambda\}$, each with probability $P(\{\lambda\})$, then

$$\mu_O = \sum_{\{\lambda\}} \underbrace{O(\{\lambda\})}_{\text{observable}} P(\{\lambda\}) \, , \quad \sigma_O^2 = \sum_{\{\lambda\}} \left[O(\{\lambda\}) - \mu_O \right]^2 P(\{\lambda\})$$

- in practice, unweighted set of $\mathcal{N}_{pdf}$ pdfs

$$\mu_O = \frac{1}{\mathcal{N}_{pdf}} \sum_{i=1}^{\mathcal{N}_{pdf}} O(\{\lambda\}_i)$$

- can incorporate full information about measurements and their errors correlations and distributions in calculation of $P(\{\lambda\})$

- still in relatively primitive state: limited data sets avoiding 'difficult' uncertainty issues

[problem: α_s comes out too low, because $\underbrace{\alpha_s|_{DIS}}_{\simeq 0.118}$ dominated by CCFR $\underbrace{\nu N}_{\text{heavy target}}$]

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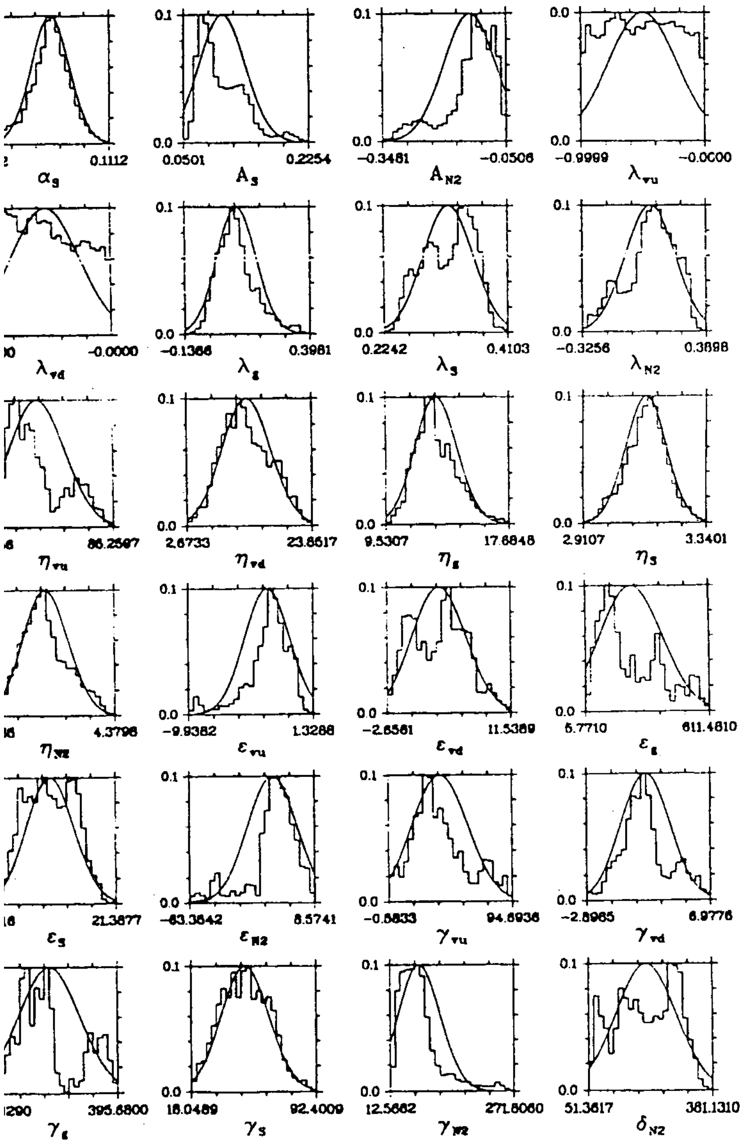
Current fit

1. Draconian measures needed to restart from scratch and re-evaluate each issue
2. Fixed renormalization and factorisation scale
3. Data affected by nuclear binding effects are excluded.
4. Use a MRS-style parametrisation
5. Evolution by Mellin transform method
6. Massless quarks
7. Positivity constraint on F2

At the moment we are using H1 and BCDMS(proton) for our core set. With their full correlation matrix and <u>assuming Gaussian distribution</u> we can calculate the $\chi^2(\{\lambda\})$ and $P(\{\lambda\}) \approx exp(-\chi^2/2)$.

→ generate 50000 unweighted PDFs according to the probability function (an overnight project on a pc).

(note: 532 data points, minimum $\chi^2 = 530$ for 23 parameters)



α_S
A_S
A_{N2}
λ_{vu}
λ_{vd}
λ_g
λ_S
λ_{N2}
η_{vu}
η_{vd}
η_g
η_S
η_{N2}
ε_{vu}
ε_{vd}
ε_g
ε_S
ε_{N2}
γ_{vu}
γ_{vd}
γ_g
γ_S
γ_{N2}
δ_{N2}

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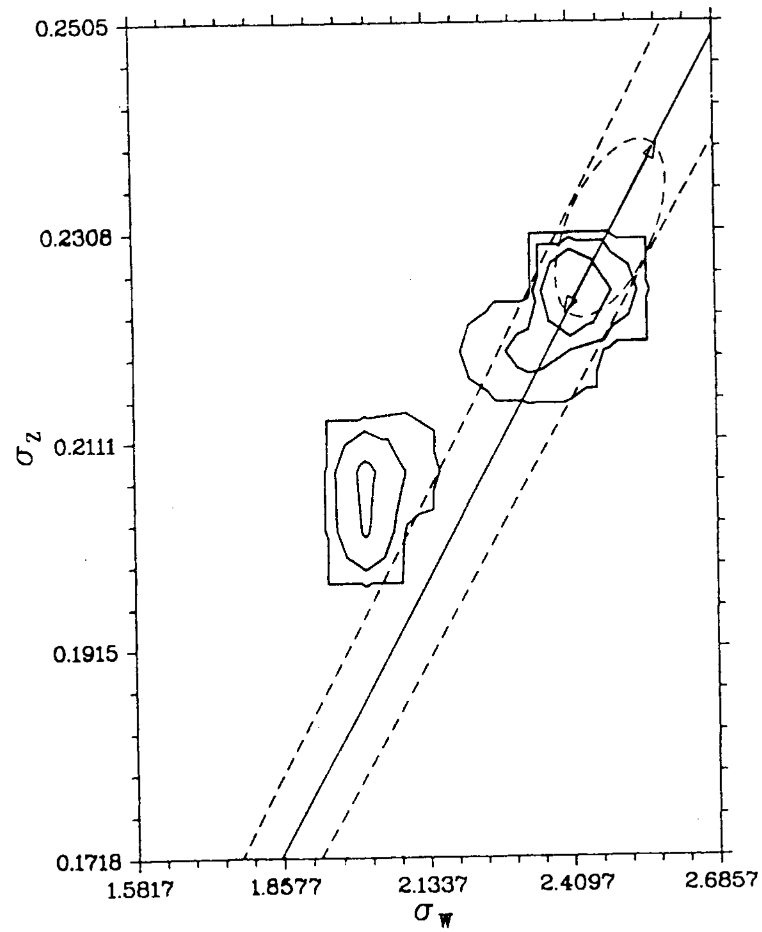

Correlation of σ_W vs σ_Z
σ_Z
σ_W
0.2505
0.2308
0.2111
0.1915
0.1718
1.5817
1.8577
2.1337
2.4097
2.6857