aaacckk!!

- The Sudakov factor includes the exponentiation of

$$\mathcal{S}(\theta)_{\text{LDLA}} = 1 - \frac{2\alpha_S}{3\pi}\ln^2(\theta^2/4)$$

leading double log approximation

I dropped the power of 2 in the exponentiated form

$$\frac{d\sigma}{d\tau dy dp_T^2} = \left(\frac{d\sigma}{d\tau dy}\right)_B \left[\frac{4\alpha_S}{3\pi}\frac{1}{p_T^2}\ln\left(Q^2/p_T^2\right) e^{-\frac{2\alpha_S}{3\pi}\ln^2(p_T^2/Q^2)}\right]$$

please put that in...the web will be correct...

- calculated the Drell-Yan process for photons, W's, and Z's to the NLO…by cheating multiple times
- noted the leading bad behavior as the unabsorbed new scale (p_T) tends to zero
- found a way to add up an infinite sum of gluons in a particular set of approximations, called Resummation.
- patched up our cross sections for this feature

immediate plans…

1. **take stock of the summation ansatz**
   - note some shortcomings
2. **make some improvements to "resum" all orders**
   - work in b space
   - recover the lost terms from approximations

dare we say "naïve" resummation?

- **not a fair characterization…but there are deficiencies**
  - on the one hand, an infinite summation of glue
  - on the other hand
    - a number of approximations
    - only soft emission…no potential for a hard gluon beyond the order-alpha
    - vector momentum conservation not really taken into account
- **but, the idea: definitely worth pursuing…**

▷ improve by going to impact parameter space:

use the identity: force momentum conservation

$$\delta^2\left(\sum_{i=1}^{n}\vec{k}_{Ti}-\vec{p}_T\right) = \frac{1}{(2\pi)^2}\int d^2b e^{-i\vec{b}\cdot\vec{p}_T}\prod_{i=1}^{n}e^{i\vec{b}\cdot\vec{k}_{Ti}}$$

following from before, one can imagine

$$\frac{d\sigma_n}{dp_T^2}\sim\int d^2k_{T1}\ldots d^2k_{Tn}f(k_{T1})\ldots f(k_{Tn})\delta^2\left(\vec{p}_T+\vec{k}_{T1}+\ldots+\vec{k}_{Tn}\right)$$

where each of the $f(k_{Ti}) = \frac{\alpha}{2\pi}\ln\left(Q^2/k_{Ti}^2\right)$

$$\frac{1}{\sigma_0}\frac{d\sigma}{dp_T^2} = \frac{1}{4\pi^2}\sum_{n=0}^{\infty}\frac{1}{n!}\int d^2b e^{i\vec{b}\cdot\vec{p}_T}\left[\int d^2k_T e^{i\vec{b}\cdot\vec{k}_T}f(k_T)\right]^n$$

▷ a manageable form:

from the Bessel Function identity: $$\frac{i^{-n}}{2\pi}\int_0^{2\pi} e^{i(x\cos\theta+n\theta)}d\theta = J_n(\theta)$$

can get

$$\frac{1}{4\pi^2}\int_0^{\infty} d^2b e^{i\vec{b}\cdot\vec{p}_T} f(b) = \frac{1}{2}\int_0^{\infty} d^2b J_0(p_T b) f(b)$$

so, *which is amenable to computation*

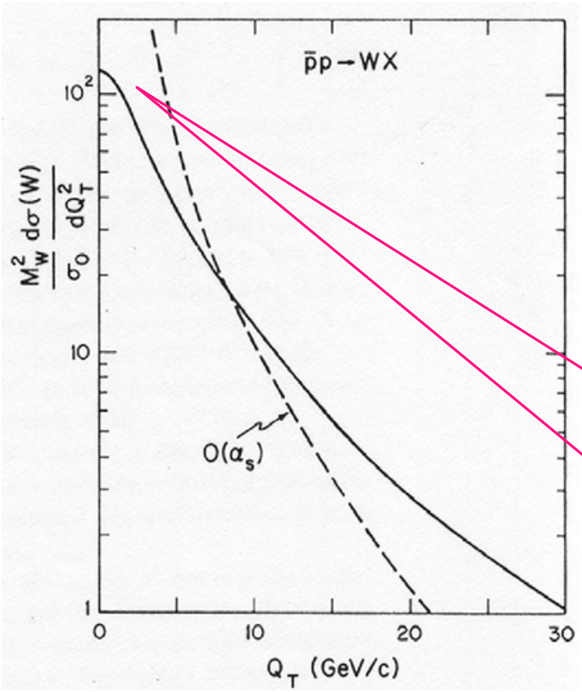
$$\frac{1}{\sigma_0}\frac{d\sigma}{dp_T^2} = \frac{1}{4\pi}\int e^{\Delta(b)} J_0(p_T b) d^2b$$

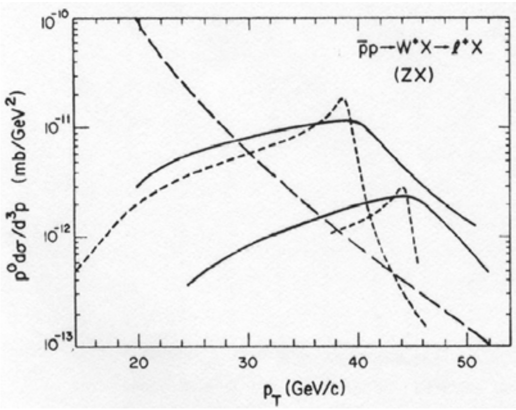
where
$$\Delta(b) = \int f(k_T) e^{i\vec{k}_T\cdot\vec{b}} d^2k_T$$
$$= \pi\int f(k_T)\left[J_0(p_T b) - 1\right] d\vec{k}^2$$

▷ names associated with this approach:

Yu, Dokshitzer, Dyakonov, Troyan, (DDT) Parisi, Petronzio, Curci, Greco, Srivastava

plus, anticipation of W/Z in series by Halzen, Martin, Scott, Tuite, circa 1982





“Identification of *W* Bosons in $p\bar{p}$ Collisions”, Halzen, Martin, Scott, PRD 25 754 (1982)

see? resummation tames the bad behavior at low p_T!

1984: Collins and Soper, Altarelli, Ellis, and Martinelli had new ideas…

I’ll follow Collins, Soper, and Sterman (“CSS”)…

▷ Here's the idea: remember our exponentiation?

in powers of α_S, in our O(α_S) perturbative calculation,

it went like: $\alpha_S(A)\left[e^{\alpha_S B}\right] \rightarrow \alpha_S(A)\left[1 + \alpha_S B + \alpha_S{}^2 B^2 + ...\right]$

generally, the cross section, in limit of small p_T, would go like:

$$\frac{d\sigma}{dp_T} \propto \frac{1}{p_T^2} \sum_{n=1}^{\infty} \sum_{m=0}^{2n-1} \alpha_S^n \ln^m\left(Q^2 / p_T^2\right)$$

unfold it, with $L \equiv \ln(Q^2 / p_T^2)$:

$$\begin{aligned} \propto \frac{1}{p_T^2}[\quad & \alpha + \quad \alpha L + \\ & \alpha^2 + \quad \alpha^2 L + \quad \alpha^2 L^2 + \quad \alpha^2 L^3 + \\ & \alpha^3 + \quad \alpha^3 L + \quad \alpha^3 L^2 + \quad \alpha^3 L^3 + \quad \alpha^3 L^4 + \quad ...] \end{aligned}$$

▷ **and rearrange this sum, by collecting terms in a particular way…**

$$\propto \frac{1}{p_T^2}[\quad \alpha(L+1)+ \quad \alpha^2(L^3+L^2)+ \quad \alpha^3(L^5+L^4)+\ldots$$

$$+\alpha^2(L+1)+ \quad \alpha^3(L^3+L^2)+\ldots$$

$$+\alpha^3(L+1)+\ldots$$

$$\vdots$$

$$\ldots]$$

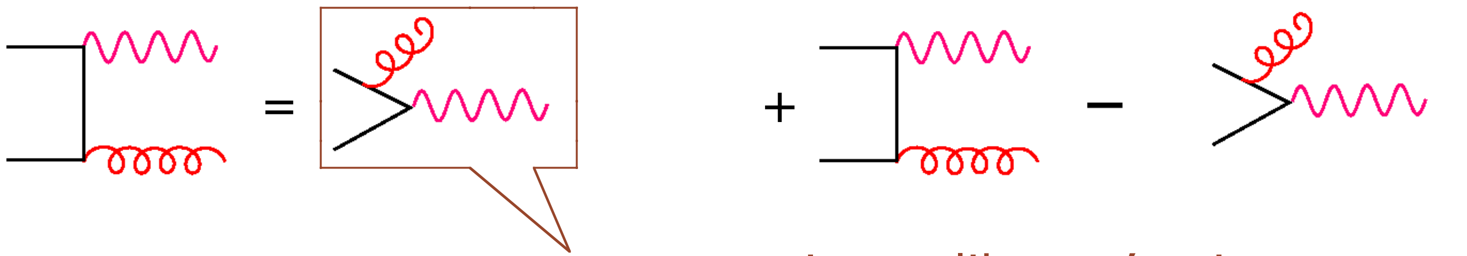
here are our LDLA terms

Collins and Soper took advantage of this expansion and showed that the cross section could be written in the form:

▷ Collins, Soper, and Sterman showed that

$$\frac{d\sigma}{dp_T} \sim \int d^2 b e^{i\vec{p}_T\cdot\vec{b}} W(b,Q,\xi_a,\xi_b) + Y(p_T,Q)$$

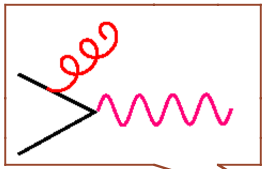
singular as $1/p_T \to 0$ + full perturbative − $1/p_T$ singular terms



exponentiates, like we've just seen

▷ **that is...**

$$\sim \int d^2 b e^{i\vec{p}_T \cdot \vec{b}} W(b, Q, \xi_a, \xi_b)$$



$$W(b, Q, \xi_a, \xi_b) \sim \sum_i \int \frac{d\xi_a}{\xi_a} f_{q_i/A}(\xi_a, Q^2) f_{\bar{q}'_i/B} e^{-S(b)}$$

where Collins and Soper showed that W obeys an evolution equation...so that a renormalization group-inspired form is:

$$S = \int \frac{d\mu^2}{\mu^2} \left[\mathbf{A} \ln(Q^2/\mu^2) + \mathbf{B}\right]$$

with $\mathbf{A} = \sum_j \alpha_S^j A^{(j)}$ $\mathbf{B} = \sum_j \alpha_S^j B^{(j)}$ calculable to specific order (as we'll see)...

▷ I'm intentionally leaving out some complications…

it's very easy to get lost in the technicalities in this business.

the result:

$$\frac{d\sigma(h_1 h_2 \to VX)}{dQ^2\, dQ_T^2 dy} = \frac{1}{(2\pi)^2}\int d^2b\, e^{i\vec{Q}_T\cdot\vec{b}}\widetilde{W}(b,Q,x_1,x_2) + Y(Q_T,Q,x_1,x_2)$$

the strict RG solution:

$$\widetilde{W}(Q,b,x_1,x_2) = e^{-\mathcal{S}(Q,b,C_1,C_2)}\widetilde{W}\left(\frac{C_1}{C_2 b},b,x_1,x_2\right)$$

the Sudadov exponent:

$$\mathcal{S}(Q,b,C_1,C_2) = \int_{C_1^2/b^2}^{C_2^2Q^2}\frac{d\overline{\mu}^2}{\overline{\mu}^2}\left[A\left(\alpha_s(\overline{\mu}),C_1\right)\ln\left(\frac{C_2^2Q^2}{\overline{\mu}^2}\right) + B\left(\alpha_s(\overline{\mu}),C_1,C_2\right)\right]$$

the Q^2 -independent term factorizes:

$$\widetilde{W}\left(\frac{C_1}{C_2 b},b,x_1,x_2\right) = \sum_j e_j^2\, \mathcal{C}_{jh_1}\left(\frac{C_1}{C_2 b},b,x_1\right)\, \mathcal{C}_{jh_2}\left(\frac{C_1}{C_2 b},b,x_2\right)$$

while the C_{jhn} functions are convolutions with pdf's

$$\mathcal{C}_{jh}(Q,b,x) = \sum_a \int_x^1 \frac{d\xi}{\xi}\, C_{ja}\left(\frac{x}{\xi},b,\mu=\frac{C_3}{b},Q\right)\, f_{a/h}\left(\xi,\mu=\frac{C_3}{b}\right)$$

C_1, C_2, C_3 are arbitrary constants with conventional choice for later comparision with fixed order result:

$C_1 = C_3 = 2e^{-\gamma_E} = b_0$ and $C_2 = C_1/b_0 = 1$

▷ suppose, we truncate at the first term:

$$S \quad = \quad \int \frac{d\mu^2}{\mu^2} \left[\alpha A^{(1)} \ln(Q^2/\mu^2) + \alpha B^{(1)} \right] \quad \propto \quad \ln^2(Q^2/\mu^2) \; = L$$

you can see the LDLA coming back…

So, here's the plan:

1. expand *S*, order by order, to a specific order
2. compare with perturbative result, term by term
3. determine *A*'s and *B*'s specified to that particular order

▷ expand the Sudakov exponential...

$$S = \int \frac{d\mu^2}{\mu^2} \left[L(\alpha_S A^{(1)} + \alpha_S^2 A^{(2)} \ldots) + (\alpha_S B^{(1)} + \alpha_S^2 B^{(2)} + \ldots) \right]$$

$$= \underbrace{\int \left[\alpha_S \underbrace{(LA^{(1)} + B^{(1)})}_{X^{(1)}} + \alpha_S^2 (LA^{(2)} + B^{(2)}) + \alpha_S^3 (\ldots) + \ldots \right]}_{\left[\alpha_S X^{(1)} + \alpha_S^2 X^{(2)} + \alpha_S^3 X^{(3)} + \ldots \right]}$$

then...

$$e^{-S} \rightarrow 1 - S + \frac{S^2}{2} - \frac{S^3}{3!} + \ldots$$

(repeating...)

$$
\begin{aligned}
e^{-S} \quad &\rightarrow \quad 1 - S + \frac{S^2}{2} - \frac{S^3}{3!} + \ldots \\
&= \quad 1 - \left[\alpha_S X^{(1)} + \alpha_S^2 X^{(2)} + \alpha_S^3 X^{(3)} + \ldots\right] + \\
&\qquad + \left[\alpha_S X^{(1)} + \alpha_S^2 X^{(2)} + \ldots\right]^2 + \ldots
\end{aligned}
$$

an infinite series made up of grouped infinite series'

the punchline...

$$e^{-S} \to 1 - \left[\alpha_S(LA^{(1)} + B^{(1)}) + \alpha_S^2(LA^{(2)} + B^{(2)}) + \ldots\right]$$

$$+ \Big[\underbrace{\alpha_S(LA^{(1)} + B^{(1)})}_{S_1} + \underbrace{\alpha_S^2(LA^{(2)} + B^{(2)})}_{S_2} + \ldots\Big]^2 - [\ldots]^3 + \ldots$$

...etc.

$$e^{-S} \to 1 - S + \frac{S^2}{2} - \frac{S^3}{3!} + \ldots$$

$$\equiv 1 - (S_1 + S_2 + S_3 + \ldots) + (S_1 + S_2 + S_3 + \ldots)^2 + \ldots$$

the really clever thing here is that S_1 only involves $A^{(1)}$ and $B^{(1)}$ and that each power of S is attached to a power of α

can calculate each S_i perturbatively
....so, match them, term by term with expansion:

▷ term by term...

perturbative calculation		resummed, expanded S
$\alpha_S(1+L)$	which equated, picks out →	$A^{(1)} \quad B^{(1)}$
$\alpha_S^2(L+1)$	ditto →	$A^{(2)} \quad B^{(2)}$

$A^{(1)}$ $B^{(1)}$: sufficient to calculate

$$1 - S_1 + S_1^2 + S_1^3 + \ldots$$

...an infinite set of e^{-S}

$A^{(2)}$ $B^{(2)}$: sufficient to calculate the S_2's which added to the above gives,

$$1 - S_1 + S_1^2 + S_1^3 + \ldots + S_1 S_2 + S_2^2 + S_2^3 + S_2^4 + \ldots$$

...which is a lot more of e^{-S}

resummation is a lot of the total sum

notice, this is greatly improved over the LDLA

$$\frac{d\sigma}{dp_T^2} \propto \frac{1}{p_T^2}\Big[\alpha(L+1)+\alpha^2(L^3+L^2)+\alpha^3(L^5+L^4)+\ldots$$
$$+\alpha^2(L+1)+\alpha^3(L^3+L^2)+\ldots\Big]$$

the top row only uses $A^{(1)}$ and $B^{(1)}$

the second row only uses $A^{(1)}$, $B^{(1)}$, $A^{(2)}$, and $B^{(2)}$

...etc.

these top two "rows" are currently calculated...just numbers:

$$\begin{aligned}
A^{(1)} &= 2C_F\ , \\
A^{(2)} &= 2C_F\Big(N\big(\frac{67}{18}-\frac{\pi^2}{6}\big)-\frac{10}{9}T_R n_f\Big)\ , \\
B^{(1)} &= -3C_F\ , \\
B^{(2)} &= C_F^2\Big(\pi^2-\frac{3}{4}-12\zeta(3)\Big)+C_F N\Big(\frac{11}{9}\pi^2-\frac{193}{12}+6\zeta(3)\Big) \\
&+ C_F T_R n_f\Big(\frac{17}{3}-\frac{4}{9}\pi^2\Big)\ .
\end{aligned}$$

there's a complication

▷ **inherent to the *b*-space formalism is integration over *terra incognita*:**

$$\int_0^\infty db...$$

must integrate over large impact parameter,
small momentum transfer

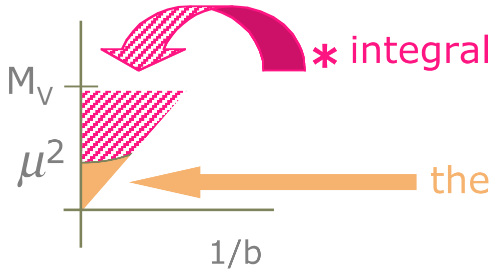
$b > \dfrac{1}{\Lambda_{QCD}}$ defines the long-distance region without a theory

This is handled with a regularization of sorts…and here's how data enter…

▷ bifurcate W:

$$W(b) \rightarrow W(b_*) \quad \text{where} \quad b_* = \frac{b}{\sqrt{1 + b^2 / b_{max}^2}}$$

this cutoff means that W is missing some integral...



the missing contribution to the integral can be reclaimed by measuring it...

$$W(b) \rightarrow W(b_*) e^{-S_{NP}(b)}$$

so-called “non perturbative function” is parameterized in terms of measurables

immediate plans…

1. **test against data**
   - early fit to Drell-Yan data
   - predictions for W/Z
   - global fitting to all Drell-Yan data
2. **conclusions and outlook**
   - pragmatics: M_W and Γ_W
   - fundamentals: what does it mean for QCD?

▷ there have been a variety of parameterizations:

original CSS: $S_{NP}^{CSS}(b) = h_1(b,\xi_a) + h_2(b,\xi_b) + h_3(b)\ln Q^2$

J. Collins and D. Soper, *Nucl.Phys.* **B193** 381 (1981);
erratum: **B213** 545 (1983); J. Collins, D. Soper, and G. Sterman, *Nucl. Phys.* **B250** 199 (1985).

Davies, Webber, and Stirling (DWS): $S_{NP}^{DWS}(b) = b^2\left[g_1 + g_2 \ln(b_{\max}Q)\right]$

C. Davies and W.J. Stirling, Nucl. Phys. B244 337 (1984);
C. Davies, B. Webber, and W.J. Stirling, Nucl. Phys. B256 413 (1985).

Ladinsky and Yuan (LY): $S_{NP}^{LY}(b) = g_1 b\left[b + g_3 \ln(100\xi_a\xi_b)\right] + g_2 b^2 \ln(b_{\max}Q)$

G.A. Ladinsky and C.P. Yuan, Phys. Rev. D50 4239 (1994);
F. Landry, R. Brock, G.A. Ladinsky, and C.P.Yuan, Phys. Rev. D63 013004 (2001).

"Gauss 1": $S_{NP}^{Gauss1}(b) = b^2\left[g_1 + g_1 g_3 \ln(100\xi_a\xi_b) + g_2 \ln(b_{\max}Q)\right]$

F. Landry, "Inclusion of Tevatron Z Data into Global Non-Perturbative QCD Fitting", Ph.D. Thesis, Michigan State University, 2001.
F. Landry, R. Brock, G.A. Ladinsky, and C.P.Yuan, in preparation.

"q_T resummation": $\tilde{F}^{NP}(q_T) = 1 - e^{-\tilde{a}q_T^2}$ (not in b-space…see below)

R.K. Ellis, Sinisa Veseli, Nucl.Phys. B511 (1998) 649-669
R.K. Ellis, D.A. Ross, S. Veseli, Nucl.Phys. B503 (1997) 309-338

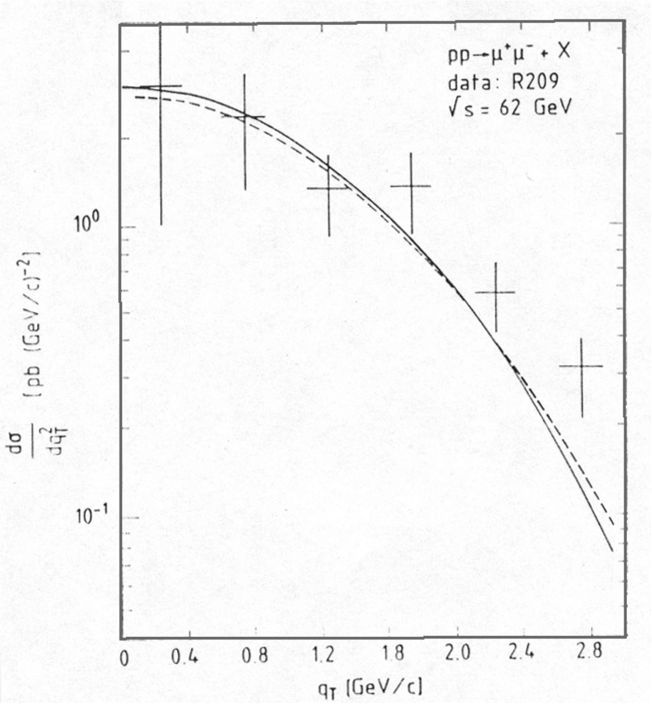
▷ the first attempt in 1984:

from a fit to necessarily sparse data, they found:

g_1 = 0.15 GeV^2

g_2 = 0.4 GeV^2

using b_{max} - (2 GeV)$^{-1}$



They used only the resummed piece…

A whole theory needs the soft and the hard parts:

▷ Remember the original formulation…

$$\frac{d\sigma(h_1 h_2 \to VX)}{dQ^2\, dQ_T^2 dy} = \frac{1}{(2\pi)^2} \int d^2b\, e^{i\vec{Q}_T \cdot \vec{b}} \widetilde{W}(b, Q, x_1, x_2) + \; Y(Q_T, Q, x_1, x_2)$$

singular as
p^2_T, in the limit of zero p^2_T

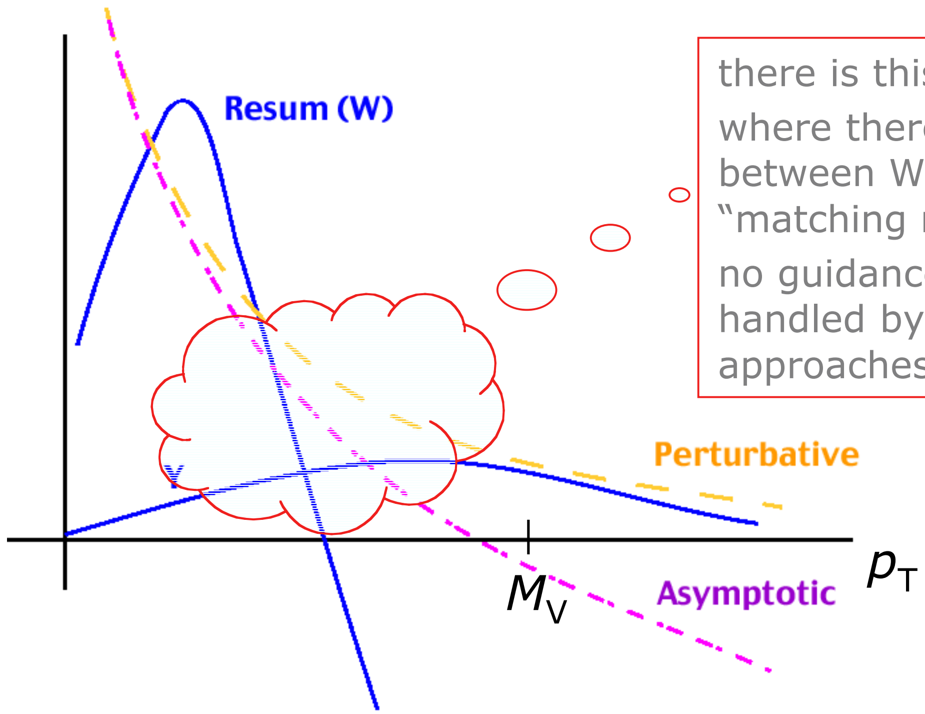
puts back terms left out from the leading $1/p^2_T$
expansion: *less* singular than $1/p^2_T$

one can extract Y at a particular order as = that remaining after the terms singular as $1/p^2_T$ are subtracted at that order.

called the Asymptotic piece

Y = *P* - *A* …at a particular order, finite.

▷ so, the whole cross section comes in pieces:



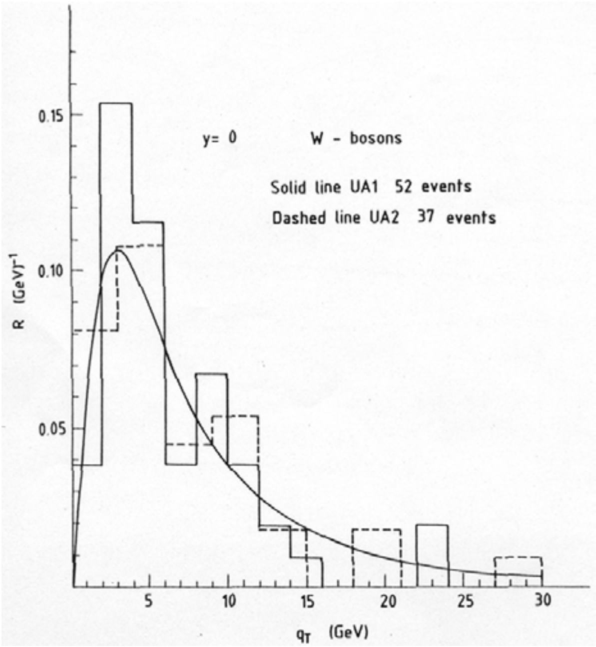
there is this funny region
where there is a discontinuity between W and Y - the so-called "matching region"
no guidance….this clouded region is handled by seat-of-the-pants approaches

- the "answer" is in blue
- negative cross sections are meaningful
- *W* heads south at about *Q*/2
- *A* heads south at about *Q*

▷ first done, to leading order, by Altarelli, Ellis, Greco, and Martinelli, 1984/5

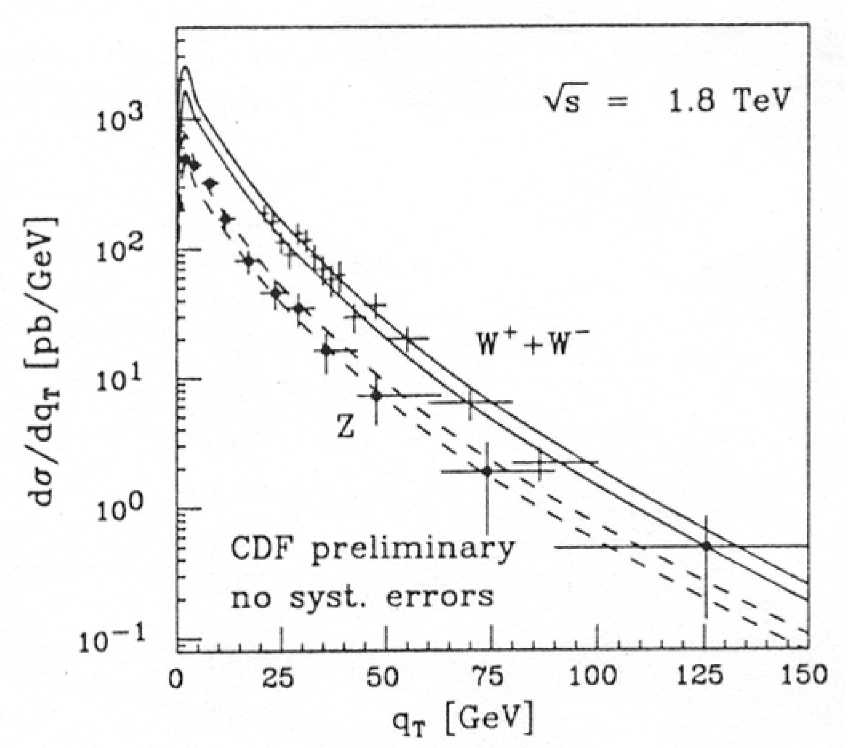
(not strictly CSS, but a variant of the approach)

- prediction for *W/Z* - where the *Y* piece is necessary
- attempted to match the *W* piece to the *Y* piece
- and evaluate the error



▷ Arnold and Kauffman (and Reno) extended to NLO

- explicitly worried about an <u>algorithm</u> for the matching region
- estimated the error
- strict CSS
- used the DWS parameterization and fits



- **Ladinsky and Yuan fit to modern Drell Yan data, 1994**
- **low p_T, low mass Drell Yan data dominated all fits**

included: ISR data
fixed target Tevatron
Run 0 CDF Z's (tiny sample)

has the effect of marginalizing g_2 in 2 parameter form

$$S_{NP}^{DWS}(b) = b^2\left[g_1 + g_2 \ln(b_{\max}Q)\right]$$

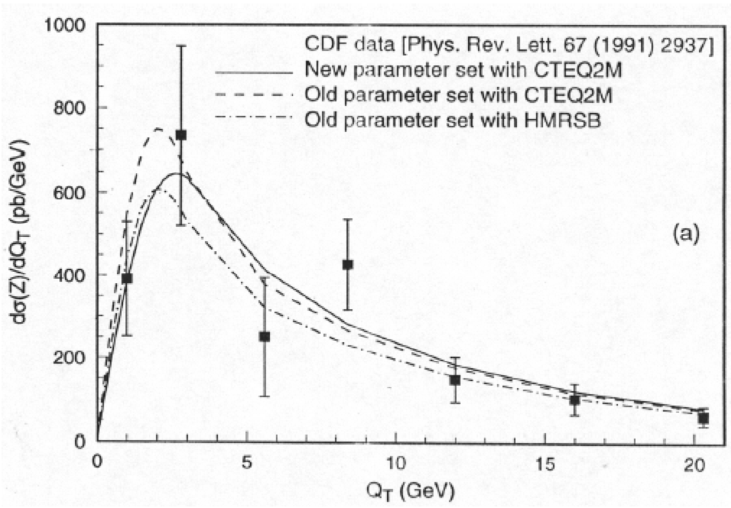
so L-Y modified the form…to include
some τ dependence - a heuristic choice

$$S_{NP}^{LY}(b) = g_1 b\left[b + g_3 \ln(100\xi_a\xi_b)\right] + g_2 b^2 \ln(b_{\max}Q)$$

results:

$g_1 = 0.11^{+0.04}_{-0.03}$ GeV^2 $g_2 = 0.68^{+0.1}_{-0.2}$ GeV^2

$g_3 = -0.60 \pm 0.1$ GeV^{-1}



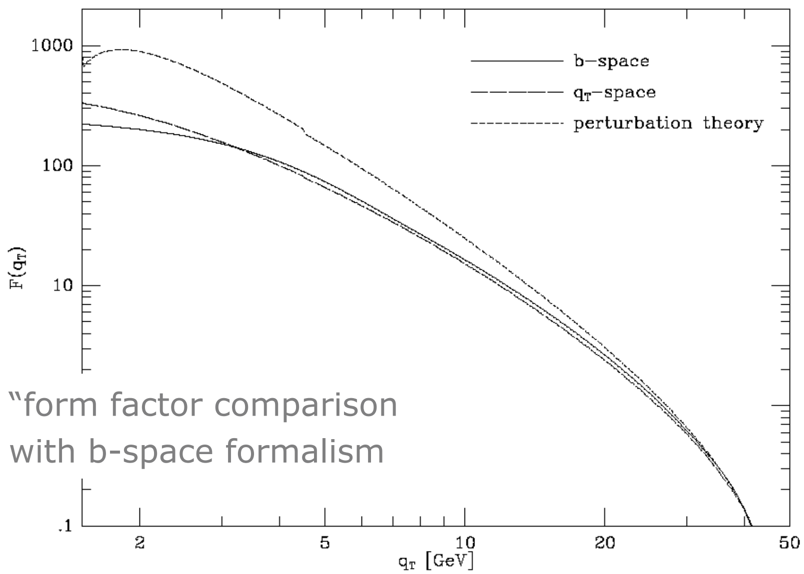
▷ an alternative approach in 1997 by Ellis and Velesi

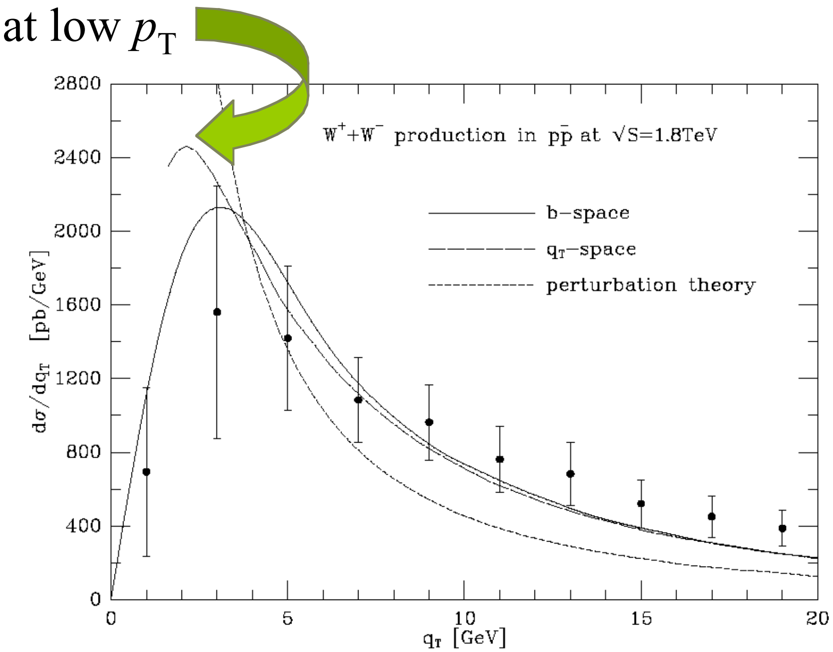
fix normalization and fit for one NP parameter -

- matching is at low p_T
- no *b*-space Fourier transform, so numerically <u>very fast</u>

$\tilde{F}^{NP}(q_T)$ then fills in here

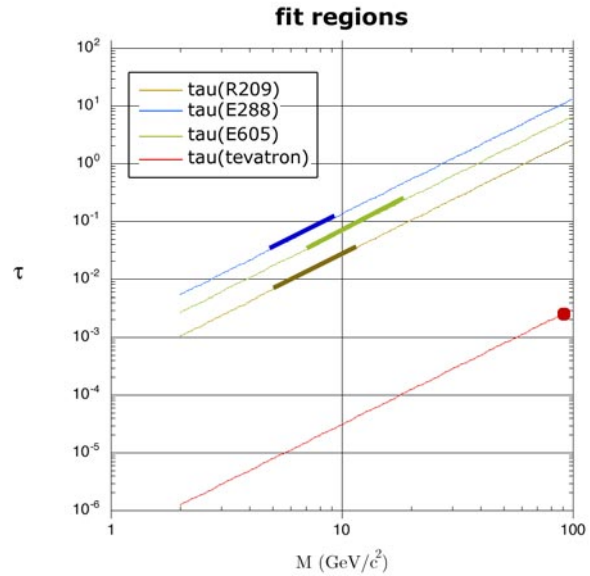
so, matching is at low p_T

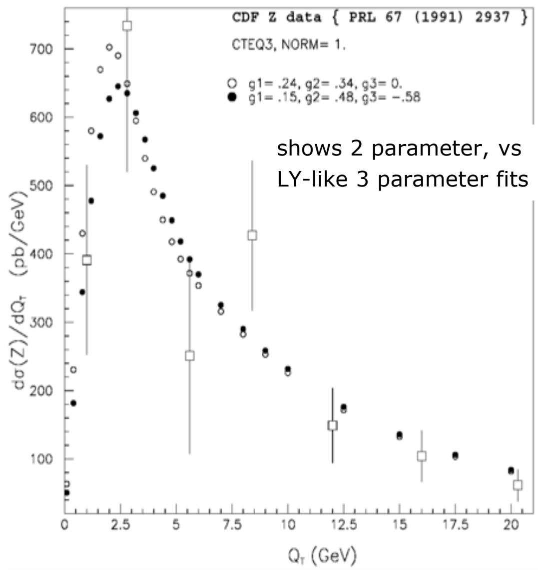




▷ lately, MSU has added to fitting…

- include more modern DY fixed target data
- fixed mistake in neutron parameterization in original LY
- added new pdf's
- predicted Tevatron sensitivity
- found normalization difficulties with some low energy DY data…which matter





results

$g_1 = 0.15^{+0.04}_{-0.03}\ \mathrm{GeV}^2 \quad g_2 = 0.48^{+0.04}_{-0.05}\ \mathrm{GeV}^2$

$g_3 = -0.58 \pm 0.26\ \mathrm{GeV}^{-1}$

but, with the completion of highly precise D0 and CDF Z p_T data, things dramatically changed

▷ Quality collider data made a huge difference

$S_{NP}^{Gauss1}(b) = b^2\left[g_1 + g_1 g_3 \ln(100\xi_a \xi_b) + g_2 \ln(b_{\max} Q)\right]$ was preferred

a Gaussian fit:

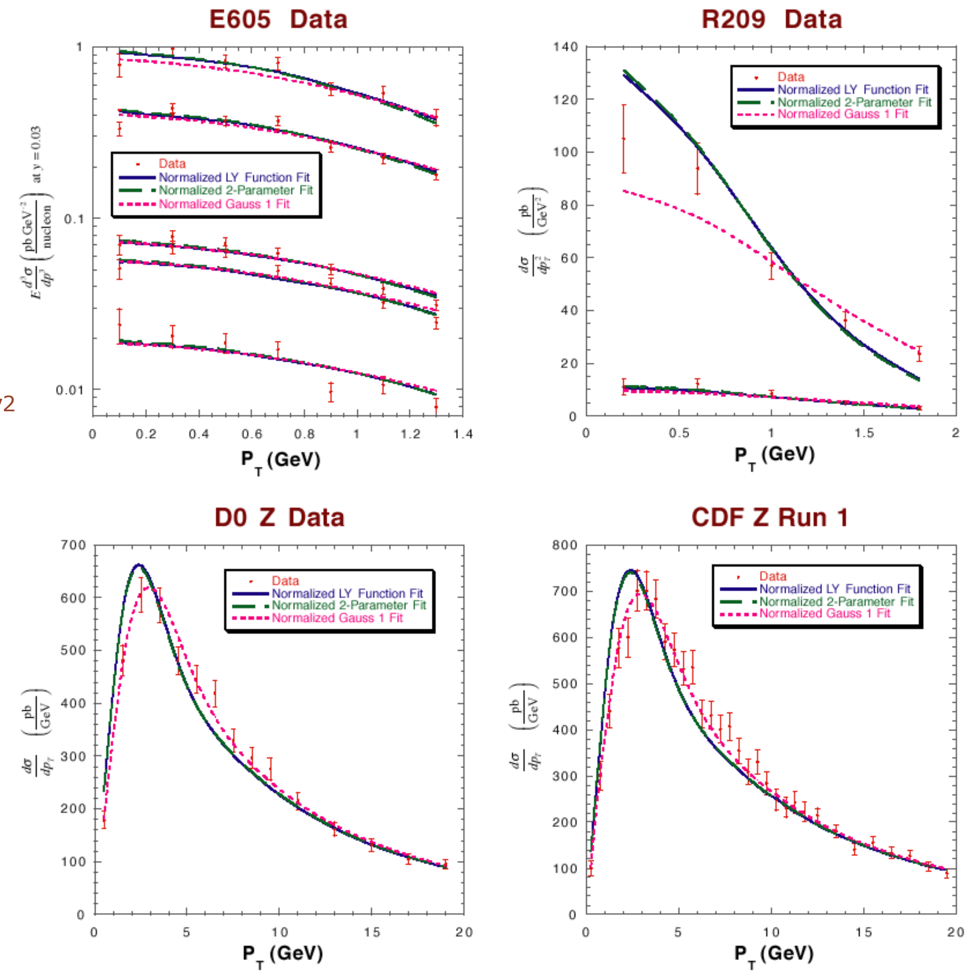
- gives best quality fit
- allowed inclusion of low Q data that were rejected previously
- gave acceptable normalizations (a free parameter in the fits)

results

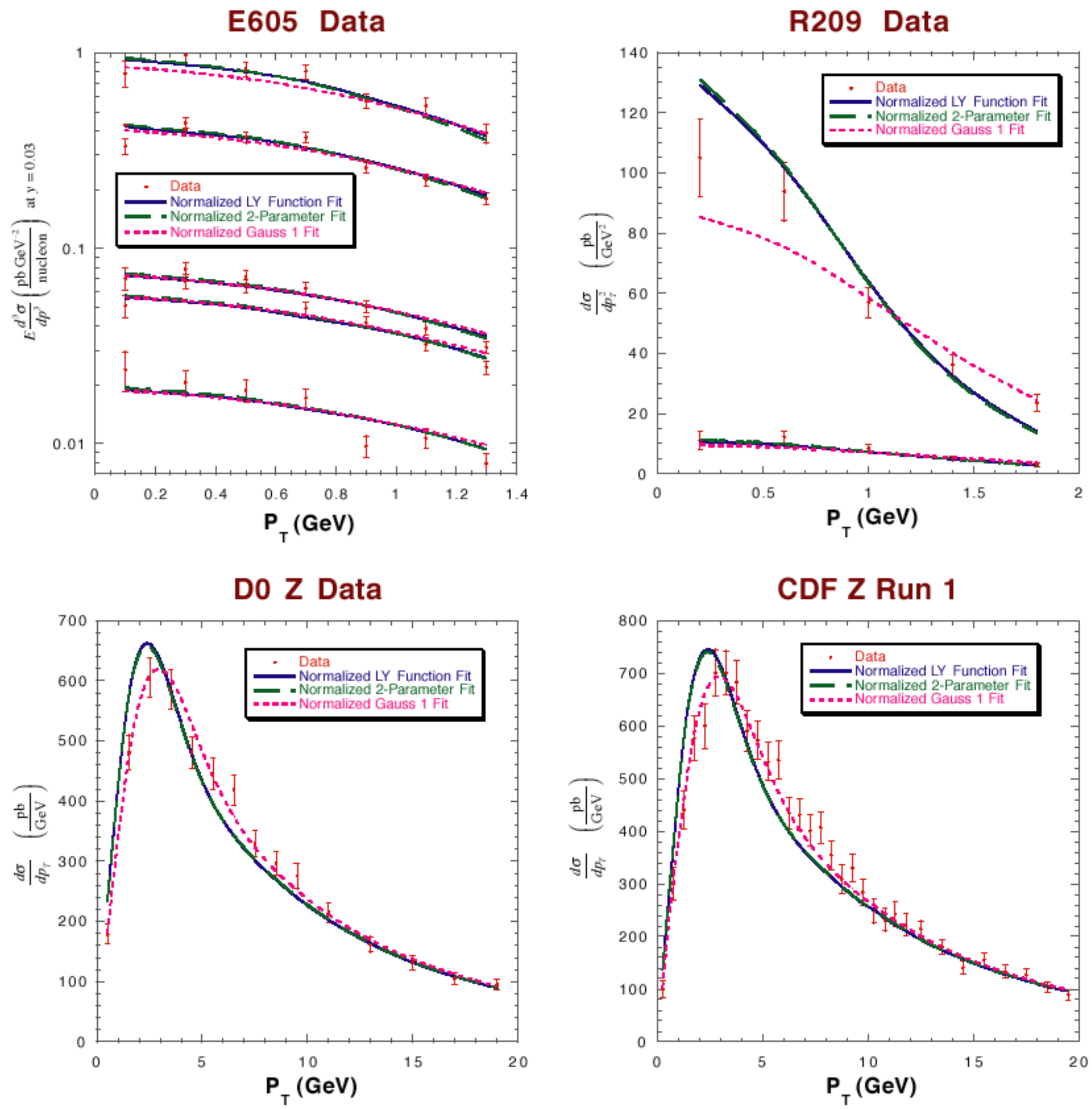
g_1 = 0.21±0.01 GeV^2 g_2 = 0.68±0.02 GeV^2

$g_3 = -0.60^{+0.05}_{-0.04}$ GeV^{-1}

- better precision in g_2
- χ^2/dof = 1.48



better view


E605 Data
R209 Data
D0 Z Data
CDF Z Run 1
Data
Normalized LY Function Fit
Normalized 2-Parameter Fit
Normalized Gauss 1 Fit
P_T (GeV)
at y = 0.03

▷ A fundamental test of QCD

if the theory is right, it is a universal description of 2 scale problems

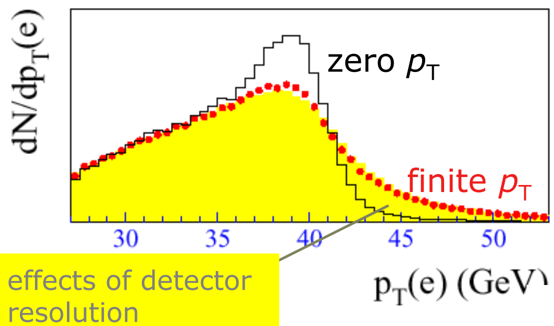
- then, appropriate as a description of p_T for all sorts of reactions: *W/Z*, 2γ, *h*, etc.
- the NP functions should apply to all

if the theory is right, is there any physical picture for the NP piece?

▷ It's also important as a description of and predictor for EW physics…

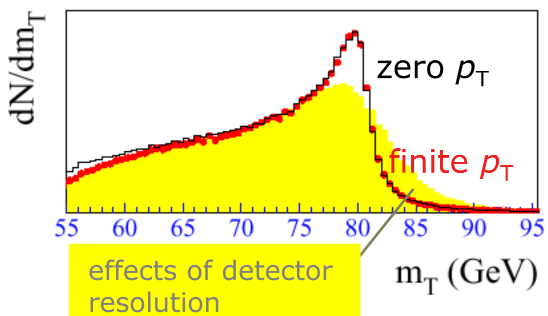
sort of in second order…

▷ Determination of M_W and Γ_W will require p_T modeling

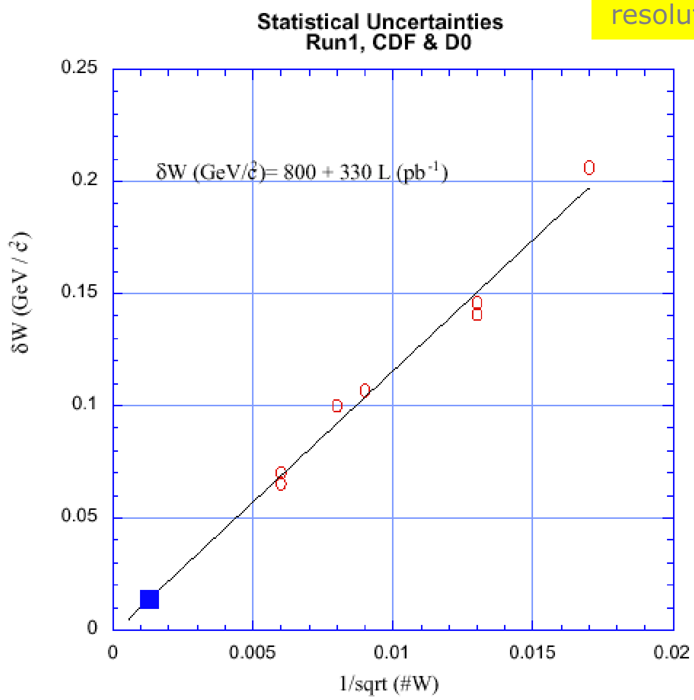


effects of detector resolution

effects of p_T led to use of transverse mass for M_W determination



effects of detector resolution



statistical precision in Run II will be miniscule…placing an enormous burden on control of modeling uncertainties.

pdf and p_T(W) uncertainties will need to be controlled to few-MeV/c^2 equivalent levels

currently:

~10-15 & 5-10 MeV/c^2

▷ that this is tough stuff…

Not much by way of resummation references. Here's what has helped me:

books:

1. *QCD and Collider Physics*, R.K. Ellis, W.J.Stirling, and B.R.Webber, Cambridge, 1996. great book…very complete and very readable
2. *Applications of Perturbative QCD*, R.D.Field, Addison Wesley, 1989. very detailed and complete
3. *Collider Physics*, V.D.Barger and R.J.N.Phillips, Addison Wesley, 1987. everything is here…I give it to all of my students

articles:

1. "Handbook of Perturbative QCD", Sterman et al., **RMP 67**, 157 (1995), (http://www.phys.psu.edu/~cteq/handbook/handbook.pdf) thorough and readable…probably needs an update? George?
2. "*W* and *Z* Production at Next-to-Leading Order: From Large q_T to Small", P.B. Arnold and R.P Kauffman, Nucl.Phys. **B349**, 381 (1991). nice pedagogical "introduction" to CSS - the only pedagogical exposition of CSS!

▷ Computer codes:

Legacy (Ladinsky and Yuan)

resummation code, p_T distributions for W/Z/h/ production

Resbos (Balazs)

lepton generator, from legacy grids
http://www.pa.msu.edu/~balazs/ResBos/

Legacy, faster and interactive (P. Nadolsky)
http://ht11.pa.msu.edu/wwwlegacy/

q_T Resummation, (Ellis and Veseli)

for example, see:

http://www-theory.fnal.gov/people/ellis/Talks/LHC.ps.gz

Resummation is an ingeneous, very technical description of how to

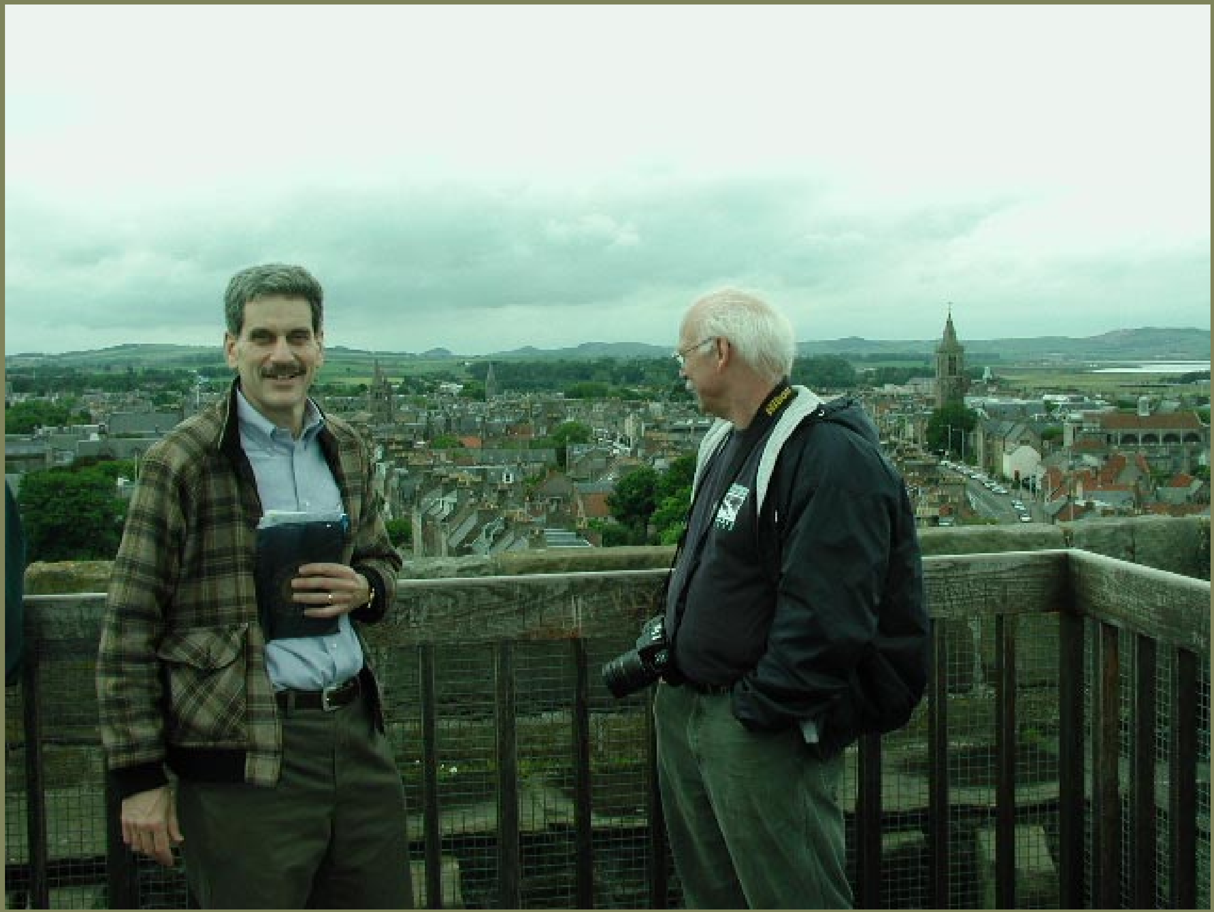
- have your cake
  - (account for a large fraction of an otherwise infinite sum of gluons)
- and eat it too

  (yet expend calculational energy only toward perturbative results)

Run II data will further enhance the test of the universal nature of this description

- even more precise *Z* data
- two photon data will begin to be a player

thanks for your attention…



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