Introduction to Deep Inelastic Scattering (DIS)

Stephen Magill
Argonne National Laboratory
2009 CTEQ Summer School
Madison, WI
Inelastic Scattering
– Probing the Structure of Matter

Pointlike? < $10^{-18}$ m

~few $10^{-10}$ m

1-15 x $10^{-15}$ m

~ $10^{-15}$ m

1898

~ 1992-2007
Quark-Parton Model (QPM) of DIS

Feynman’s QPM explanation of DIS: the nucleon is made up of point-like, spin-1/2, non-interacting constituents – the quarks as partons. DIS is the *incoherent* sum of elastic scattering from these quarks.

Furthermore, the probability $f(x)$ for the quark $f$ to carry a fraction $x$ of the nucleon momentum is an intrinsic property of the nucleon and is **process independent**.

We now know that QCD describes quark interactions with the addition of another “parton” - the gluon (QCD-improved QPM).

- Nucleons are just a “beam of partons” (*incoherent*).
- The $f(x)$s, the “beam parameters”, could be measured in some other process (**process independent**).
Quarks and Gluons as Partons

\[ u(x) : \text{up quark distribution} \]
\[ \bar{u}(x) : \text{up anti-quark distribution} \]
etc. \((d,s,c,b,t)\) and
\[ g(x) : \text{gluon (spin-1)} \]

Momentum has to add up to 1 (“momentum sum rule”):
\[ \int x[u(x)+\bar{u}(x)+d(x)+\bar{d}(x)+s(x)+\bar{s}(x)+\ldots+g(x)]dx = 1 \]

Quantum numbers of the nucleon have to be right:
So for a proton:
\[ \int [u(x)-\bar{u}(x)]dx = 2 \quad \# \text{ } u_{\text{val}} \]
\[ \int [d(x)-\bar{d}(x)]dx = 1 \quad \# \text{ } d_{\text{val}} \]
\[ \int [s(x)-\bar{s}(x)+\ldots]dx = 0 \quad \text{“sea” quark contribution} \]
DIS Kinematics – Scattering Variables

proton in “∞” momentum frame

No transverse momentum

x = fractional longitudinal momentum carried by the struck parton

0 ≤ x ≤ 1

y = fractional energy transfer

0 ≤ y ≤ 1

√s = ep cms energy

Q^2 = -q^2 = 4-momentum transfer squared = sxy
(or virtuality of the “photon”)
**DIS Kinematics – Experimental Variables**

\[ Q^2 = -q^2 = -(k-k')^2 = 2E_eE'_e(1+\cos\theta_e) \]

\[ x = Q^2/2p\cdot q = \frac{E_e}{E_p} \frac{E'_e(1+\cos\theta_e)}{2E_e-E'_e(1-\cos\theta_e)} \]

\( E_e', \theta_e \): electron method

\( E_h, \gamma_h \): Jacquet-Blondel method (energy, angle of struck quark)

\( \theta_e, \gamma_h \): Double-Angle method (angles of scattered electron, struck quark)
Kinematics of DIS Experiments

HERA collider: H1 and ZEUS
1992 – 2007

Fixed target: SLAC, FNAL and CERN completed ~10-20 years ago
Event Topology at the HERA Collider

\[ y = 0.1 \]

\[ \text{key} = 27.5 \text{ GeV} \]

\[ Q^2 = 1000 \text{ GeV}^2 \]

\[ Q^2 = 100 \text{ GeV}^2 \]

\[ Q^2 = 50 \text{ GeV}^2 \]

\[ Q^2 = 10 \text{ GeV}^2 \]

\[ Q^2 = 1 \text{ GeV}^2 \]

\[ y = 0.1 \]

\[ \text{struck quark} \]

\[ \text{scattered electron} \]

\[ \text{key} = 27.5 \text{ GeV} \]
The structure functions of the proton are:

\[ F_2(x, Q^2) = x \sum q e_q^2 (q(x, Q^2) + \bar{q}(x, Q^2)) \]
- the sum of the quark and anti-quark densities

\[ xF_3(x, Q^2) = x \sum q e_q^2 (q(x, Q^2) - \bar{q}(x, Q^2)) \]
- the difference of the quark and anti-quark densities, small for \( Q^2 \ll M_Z^2 \)

\[ F_L(x, Q^2) \sim F_2 - xg(x, Q^2) \]
- the longitudinal structure function which vanishes at LO in QCD and is damped by \( y^2 \) in the cross section
HERA and PDFs: a rough guide

NC DIS:
\[ \frac{d^2 \sigma}{dx dQ^2} \frac{Q^4 x}{2 \pi a^2 Y_+} = \bar{F}_2^+ + \frac{Y_+}{Y_-} x F_3^+ - \frac{y^2}{Y_+} P_L^+ \]

Final States:
(Jets, Charm, ...)

Low Q^2 NC
(\gamma exchange)

High Q^2 NC

Herren\kinematic\ plane

CC

Flavour composition
\[ e^+ : d \quad e^- : u \]

From C. Gwenlan
We will start with the structure function $F_2$:

IF, proton was made of 3 quarks each with 1/3 of proton’s momentum:

$$F_2 = x \sum q e^2_q (q(x) + \bar{q}(x))$$

The partons are **point-like** and **incoherent** - $F_2$ should be independent of $Q^2$.

→ **Bjorken scaling** : $F_2$ has no $Q^2$ dependence.

Does the data support this? →
SLAC-MIT Results (1969)

Partons!

Bjorken Scaling?

νW₂

June 24, 2009

S. R. Magill - 2009 CTEQ
Summer School
Bjorken scaling is ... NOT seen at all x!
QCD – $F_2$ violates Bjorken Scaling

\[ \frac{\partial F_2}{\partial \ln Q^2} \sim \alpha_s x g \]

At low $x$:
Gluon splitting enhances quark density \( \Rightarrow F_2 \) rises with $Q^2$

At high $x$:
Gluon radiation shifts quark to lower $x$ \( \Rightarrow F_2 \) falls with $Q^2$
Proton probe with a photon of virtuality $Q^2$

• Distance scale $r$ at which proton is probed:
  $$r \approx \frac{hc}{Q} = 0.2\text{fm}/Q[\text{GeV}]$$

• Because the virtual photon is absorbed in a time much shorter than the characteristic time of parton-parton interactions (Impulse Approximation), the DIS cross section factorizes as:
  $$\sigma_{\text{DIS}} \sim f_p(x) \otimes \sigma$$

$f_p(x)$: (universal) parton density functions in the proton
$\sigma$: hard scattering partonic cross section $\rightarrow$ pQCD
Higher the resolution (i.e. higher the $Q^2$) more branchings to lower $x$ we “see”.

So what do we expect $F_2$ as a function of $x$ at a fixed $Q^2$ to look like?
Three quarks with $1/3$ of total proton momentum each.

Three quarks with some momentum smearing.

The three quarks radiate partons at low $x$. 
Proton Structure Function $F_2 - Q^2$ Evolution

$Q^2$ dependence quantitatively described by:

Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equations

From R. Yoshida
The evolution of the parton densities with $Q^2$ is given by the DGLAP equations:

$$\frac{\partial f_p}{\partial \ln Q^2} \sim f_p \otimes P$$

First, $P$ represents the four “splitting functions”:

- $P_{qq}(z)$: probability that parton $a$ will radiate a parton $b$ with the fraction $z$ of the original momentum carried by $a$. 
- $P_{gg}(z)$
- $P_{qg}(z)$
- $P_{gq}(z)$

$P_{ba}(z)$: probability that parton $a$ will radiate a parton $b$ with the fraction $z$ of the original momentum carried by $a$. 
So, the DGLAP equations, \( \partial f_p / \partial \ln Q^2 \sim f_p \otimes P \) for quarks and gluons are:

\[
\frac{\partial}{\partial \ln Q^2} \Sigma(x, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \left( [\Sigma \otimes P_{qq}] + [g \otimes 2n_f P_{qg}] \right)
\]

where \( \Sigma(x, Q^2) = \sum_i (q_i(x, Q^2) + \bar{q}_i(x, Q^2)) \) is the quark density summed over all (active) flavors.

And for the gluon:

\[
\frac{\partial}{\partial \ln Q^2} g(x, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \left( [\Sigma \otimes P_{gg}] + [g \otimes P_{gg}] \right)
\]
DGLAP fit (or QCD fit) extracts the parton distributions from measurements.

(Lectures on Friday and Saturday by Pavel Nadolsky)

The Cliffs Notes version:

**Step 1:** parametrize the parton momentum density $f(x)$ at some $Q^2$ ->

$$f(x) = p_1 x + p_2 (1-x) + p_3 (1 + p_4 \sqrt{x} + p_5 x)$$

- $u_v(x)$ u-valence
- $d_v(x)$ d-valence
- $g(x)$ gluon
- $S(x)$ sum of all “sea” (non valence) quarks

**“The original three quarks”**

**Step 2:** find the parameters ($p_1 \rightarrow p_5$) by fitting to DIS (and other) data using the DGLAP equations to evolve $f(x)$ in $Q^2$. 
At $x << 1/3$, quarks and (anti-quarks) are all “sea”.

Since $F_2 = e_q^2 \sum x(q + \bar{q})$, $xS$ is very much like $F_2$

Sea PDF

Fractional uncertainty bands

$Q^2 = 1 \text{ GeV}^2$

ZEUS NLO QCD fit

tot. error ($\alpha_s$ free)
tot. error ($\alpha_s$ fixed)
uncorr. error ($\alpha_s$ fixed)
The gluon pdf is determined from scaling violations, $dF_2/d\ln Q^2$ via the DGLAP equations.

### Gluon PDF

Uncertainties are larger than for quarks.

Scaling violations couple $\alpha_s$ and gluon $g$.

Fractional uncertainty bands

Fit with $\alpha_s$ also a free parameter

**ZEUS NLO QCD fit**

June 24, 2009

S. R. Magill - Summer School
Summarizing so far:

\[ F_2 \sim \sum (q+\bar{q}) \approx S \text{ (sea quarks)} \quad \text{measured directly in NC DIS} \]

Scaling violations

\[ \frac{dF_2}{d\ln Q^2} \sim \alpha_s \cdot g \quad \text{Scaling violations gives gluons (times } \alpha_s \text{). DGLAP equations.} \]

What about valence quarks?

\[ \sum (q-\bar{q}) = u_v + d_v \quad \text{can we determine them separately?} \]

Can we decouple \( \alpha_s \) and \( g \)?
Proton Structure Function $x F_3$

Back to the NC cross section:

$$\frac{d\sigma_{e^+p}^{Q^2}}{dxdQ^2} = \frac{2\pi\alpha^2}{xQ^4} (Y F_2 - y^2 F_L + Y_+ x F_3)$$

$$Y_\pm = (1 \pm (1 - y)^2), \text{ the inelasticity parameter}$$

$$x F_3 = \sum_i (q_i(x,Q^2) - \bar{q}_i(x,Q^2)) \times B_q$$

$$B_q = -2e_q a_q a_e x_z + 4v_q a_q v_e a_e x_z^2$$

$\gamma$-Z interference Z-exchange

$e_q$: electric charge of a quark
$a_q \nu_q$: axial-vector and vector couplings of a quark
$a_e \nu_e$: axial-vector and vector couplings of an electron

$x_z \propto Q^2/(M_Z^2 + Q^2)$

$\rightarrow x F_3$ small if $Q < M_Z$
\[ \sigma_{NC}^{\pm} = F_2(x, Q^2) \mp (Y-/Y+) x F_3(x, Q^2) \]

Note the change of sign from $e^+p$ to $e^-p$.
Recent $x F_3$ (DIS 09) Results from HERA

$\gamma Z$ interference term larger than $Z$ exchange
Neutral and Charged Current Cross-Sections

\[ \frac{d\sigma_{CC}(e^\pm p)}{dx dQ^2} = \frac{G_F^2}{2\pi x} \frac{M_W^2}{M_W^2 + Q^2}^2 \sigma_{CC\pm} \]

\[ \sigma_{CC^+} = x [\bar{u} + \bar{c} + (1 - y)^2(d + s)] \sim d \]

\[ \sigma_{CC^-} = x [u + c + (1 - y)^2(\bar{d} + \bar{s})] \sim u \]

June 24, 2009

S. R. Magill - 2009 CTEQ Summer School
Reduced Charged Current Cross-Section

\[ Q^2 = 280 \text{ GeV}^2 \]

\[ \sigma^{CC\pm} \]

\[ \sigma^{CC^+} \sim d \]

\[ \sigma^{CC^-} \sim u \]
Valence PDFs from QCD Fit

The momenta from valence quarks are producing gluons and sea quarks at low $x$. 

$Q^2 = 1 \text{ GeV}^2$

$xf$

$1$ $10^{-1}$ $10^{-2}$ $0$ $0.2$ $0.4$ $0.6$ $0.8$ $1$

$X$

$zu_x$, $xd_x$

ZEUS-JETS fit

tot. uncert.
uncorr. uncert.
Jet production in DIS (HERA)

Sensitive to $\alpha_s$

$\sigma_{\text{jet}} \sim \alpha_s \cdot f(x)$

Sensitive to gluon
$\sim 10^{-3} < x < \sim 10^{-2}$

Sensitive to quarks
$\sim 10^{-2} < x < \sim 10^{-1}$

Same range as NC and CC

complementary to gluon from $F_2$
Jet Measurements in the Breit Frame

Some definitions:

+\( z \) from IP in proton direction - \textit{Target Region}

-\( z \) from IP in \( \gamma^* \) direction - \textit{Current Region}

\( \gamma^* \) has 4-vector \( q = (0,0,0,-Q) \)

Struck quark in QPM carries away momentum \(-Q/2\)

\begin{align*}
\text{QPM} & \quad \text{No } E_T! \\
\text{LO QCDC (final state)} & \quad \text{LO QCDC (initial state)} \\
\text{LO BGF} & \quad \text{Hard Scale } Q = E_t \\
& \rightarrow \text{High } Q^2 \\
\text{Hard Scale } E_t (>Q) & \rightarrow \text{Low } Q^2
\end{align*}
Gluon distributions

Using only HERA (ZEUS) data including NC, CC and jets

Using HERA (ZEUS) $F_2$ data and FNAL, CERN fixed tgt

June 24, 2009

S. R. Magill - 2009 CTEQ Summer School
Combined PDFs from HERA – DIS Precision

Combine the measured H1 and ZEUS cross sections. Double statistics and take advantage of complementary measurement techniques which result in reduced systematic uncertainties.

Sample of NC $e^+p$ data showing the ZEUS and H1 data and the combined data as a result of the averaging procedure.

Statistical errors shown.
HERAPDF0.2

- **Red**: experimental uncertainties
- **Yellow**: model uncertainties
- **Green**: pdf parametrization uncertainties

**Observations:**
- High-x and valence are mostly affected by the PDF parametrisation
  - The procedure to estimate PDF parametrisation uncertainty addresses the high-x region
- Low-x region interesting to investigate

![H1 and ZEUS Combined PDF Fit](image)
HERAPDF0.2 at $Q^2=2$ GeV$^2$

- At the starting scale gluon is valence like
- $Q_0^2, Q^2_{\text{min}}$ dominate the model uncertainty of gluon and valence PDFs
- PDF parametrisation uncertainty dominates valence PDFs and high $x$ region

H1 and ZEUS Combined PDF Fit

$Q^2 = 2$ GeV$^2$

- $u_{\text{val}}$
- $d_{\text{val}}$
- $U$
- $U_{\text{bar}}$
- Sea
- gluon
- $D$
- $D_{\text{bar}}$
HERAPDF0.2 at $Q^2=10000$ GeV$^2$

- PDF parametrisation uncertainty dominates valence PDFs and high x region
- Impressive precision at the scale relevant to LHC
Gluon Evolution

- Near the starting scale gluon is valence like
  - The model uncertainties are large in low $x$ region
    - Mostly due to $Q_0^2$ variations
  - The PDF param. uncertainty dominates high $x$

- Impressive precision at higher $Q^2$
Understanding DGLAP equations – pdf evolution:

The “incoherence” of the original parton model is preserved. i.e. a parton doesn’t know anything about its neighbor.

The “process independent” partons also survive.

But now parton densities must be “evolved” in $Q^2$.

An example for future analyses →
A parton at $x$ at $Q^2$ is a source of partons at $x' < x$ at $Q'^2 > Q^2$.

In fact, any parton at $x > x'$ at $Q^2$ is a source.

To know the parton density at $x'$, $Q'^2$ it’s necessary (and sufficient) to know the parton density in the range: $x' \leq x \leq 1$ at some lower $Q^2$.

If you know the partons in range $x \leq x \leq 1$ at some $Q^2$, then you know the partons in the range $x' \leq x \leq 1$ for all $Q'^2 > Q^2$.

What does this mean for the LHC?
Fixed target DIS

Tevatron jets

HERA DIS

\( \sim \) safe \( Q^2 \)

"known"
LHC (or hadron-hadron) parton kinematics

\[ \text{rapidity:} \quad y = \frac{1}{2} \ln \left( \frac{E+P_z}{E-P_z} \right) \]

\[ \text{pseudo-rapidity:} \quad \eta = -\ln \tan(\theta/2) \]

\[ \text{angle wrt beam} \]

\[ \text{parton}_1(x_1) + \text{parton}_2(x_2) \rightarrow \text{State with mass } M \]

\[ x_1 = \left( \frac{M}{\sqrt{s}} \right) \exp(y) \quad \quad x_2 = \left( \frac{M}{\sqrt{s}} \right) \exp(-y) \]
So if I want to predict $Z$ or $W$ production cross-section at LHC at some rapidity $y$, say, $-4$:

$$\sigma_{(pp \rightarrow W,Z+X)} \sim q,\bar{q}(x_1,M_{W,Z}^2) \times \bar{q},q(x_2,M_{W,Z}^2) \times \sigma_{(q\bar{q} \rightarrow W,Z)}$$

$q,\bar{q}(x_1=10^{-4},Q^2=M_{W,Z}^2) \quad \bar{q},q(x_2=0.3,Q^2=M_{W,Z}^2)$
A test on a standard candle process at LHC

HERA PDFs 0.1 available in LHAPDF

HERAPDF0.2 has factor of 2 smaller uncertainty than 0.1 (more data) at low x
Available soon in LHAPDF
With DIS data up to now:

\[ F_2 \sim \sum (q+\bar{q}) \approx S \text{ (sea quarks)} \quad \text{measured in NC DIS} \]

From scaling violations in \( F_2 \):

\[ dF_2/d\ln Q^2 \sim \alpha_s \cdot g \quad \text{sensitive to gluons (times } \alpha_s) \]

\[ xF_3 \sim \sum (q-\bar{q}) = u_v + d_v \quad \text{valence quarks} \]

Use jet cross sections to decouple \( \alpha_s \) and \( g \)

DGLAP fits with all of the above -> precise predictions at LHC

Now, for the last piece – the longitudinal structure function \( F_L \) to give us direct access to the gluons ->
Longitudinal Structure Function $F_L$

- $F_L$ corresponds to absorption of longitudinally polarized virtual photon.
- Spin 1/2 quarks (with no transverse momentum) cannot absorb a longitudinally polarized boson.

$$F_L = \left( \frac{Q^2}{4\pi^2\alpha} \right) \sigma_L$$

$$F_L = \frac{\alpha_s}{4\pi} x^2 \int_x^1 \frac{dz}{z^3} \left[ \frac{16}{3} F_2 + 8 \sum e_q^2 \left( 1 - \frac{x}{z} \right) zg \right]$$
Analysis strategy

- Direct $F_L$ measurement requires measurement of the reduced cross sections at same $x$ and $Q^2$ but different $y$: 
  $$\sigma_r(x, Q^2, y) = F_2(x, Q^2) - \frac{y^2}{Y_+} \cdot F_L(x, Q^2)$$

- Larger difference in $y$ → better sensitivity to $F_L$ (bigger "lever arm")

- $Q^2 = xys$: different $y$ → different $s$ → different beam energies

- Direct $F_L$ measurement only possible if HERA operates with different proton beam energies

At given $x$ and $Q^2$:
- $F_2$ is the intercept at $y$-axis
- $F_L$ is the negative slope
Measured reduced cross sections

**ZEUS**

Kinematic region:

\[20 \text{ GeV}^2 < Q^2 < 130 \text{ GeV}^2\]
\[5 \cdot 10^{-4} < x < 7 \cdot 10^{-3}\]

- First ZEUS $F_L$ publication available
- Most precise cross section measurement from ZEUS in the kinematic region studied
- Measured cross sections are published and available for fits
- Measured cross sections compared to ZEUS-JETS with and without $F_L$
- Turnover at low $x$ small but visible

D1509 conference, Madrid, Spain. 27 April 2009

Julia Grebenyuk. $F_L$ measurement with ZEUS detector.
Extracted $F_L$ and $F_2$

**ZEUS**

- Most precise $F_2$ measurement from ZEUS at kinematic region studied
- First $F_2$ measurement without assumptions on $F_L$
- Data support a non-zero $F_L$
- Predictions for $F_2$ and $F_L$ are consistent with data
Data support a non-zero $F_L$.

Predictions are consistent with data.

Ratio $R$ is fitted:

$$R = \frac{F_L}{F_2 + F_L}$$

Average $R$ from all data:

$$R = 0.18^{+0.07}_{-0.05}$$
PDF fits with $F_L$ data included

- Measured cross sections for 3 data sets (HER, LER, MER) are included in ZEUS PDF fits
- Data has impact on the low $x$:
  - Steeper rise of gluon at low $x$
  - Sea and gluon uncertainty reduced

→ For more details see talk of A. Cooper-Sarkar
Saturation region

Regge region

BFKL evolution

DGLAP evolution

Non-perturbative

Perturbative

1/x

Q^2 (GeV^2)