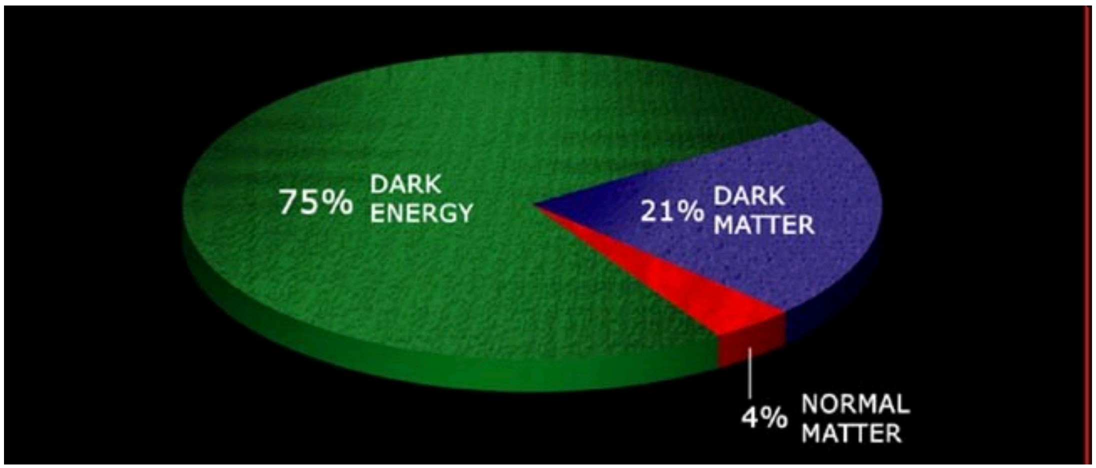
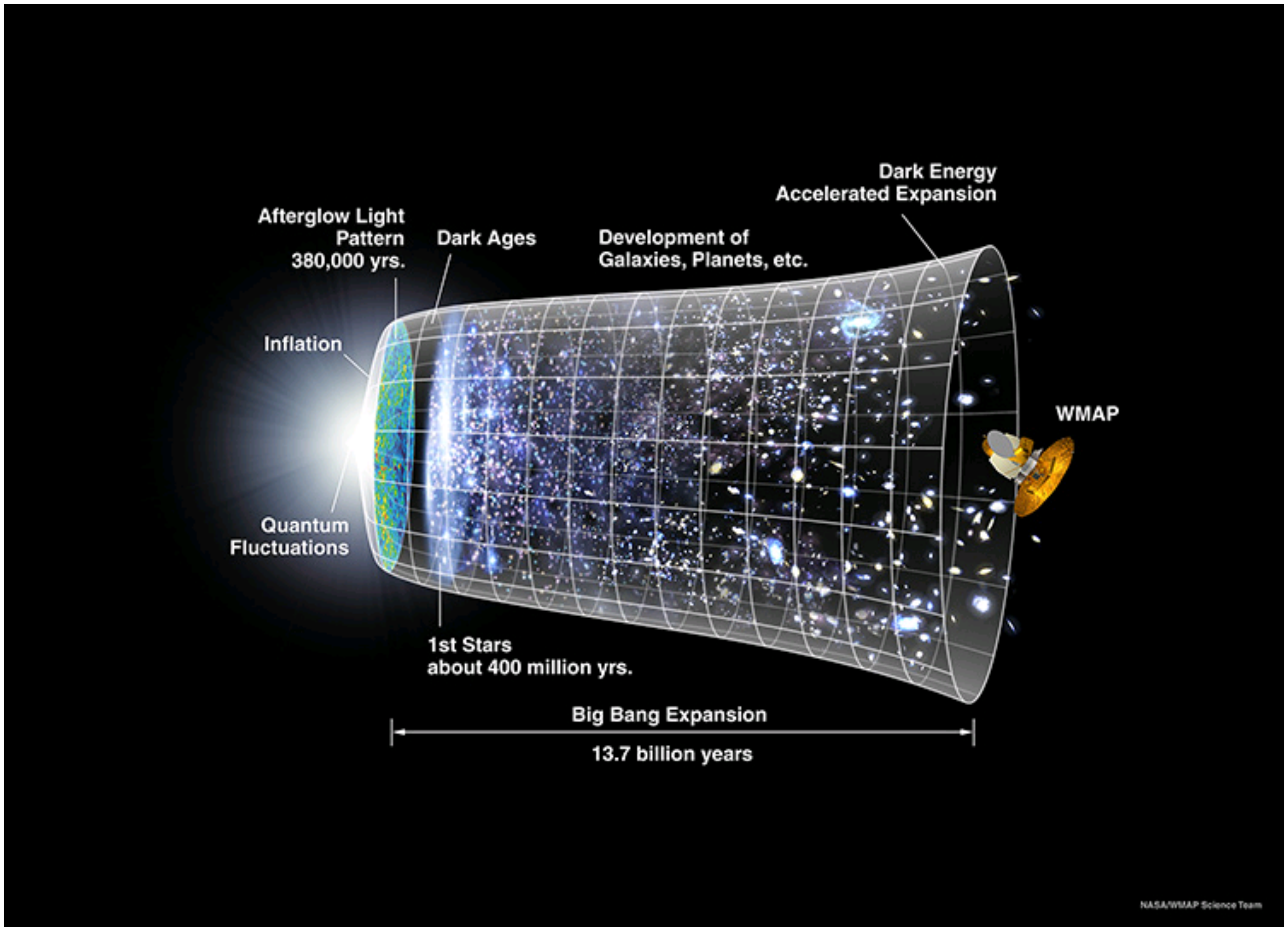
Particle Astrophysics



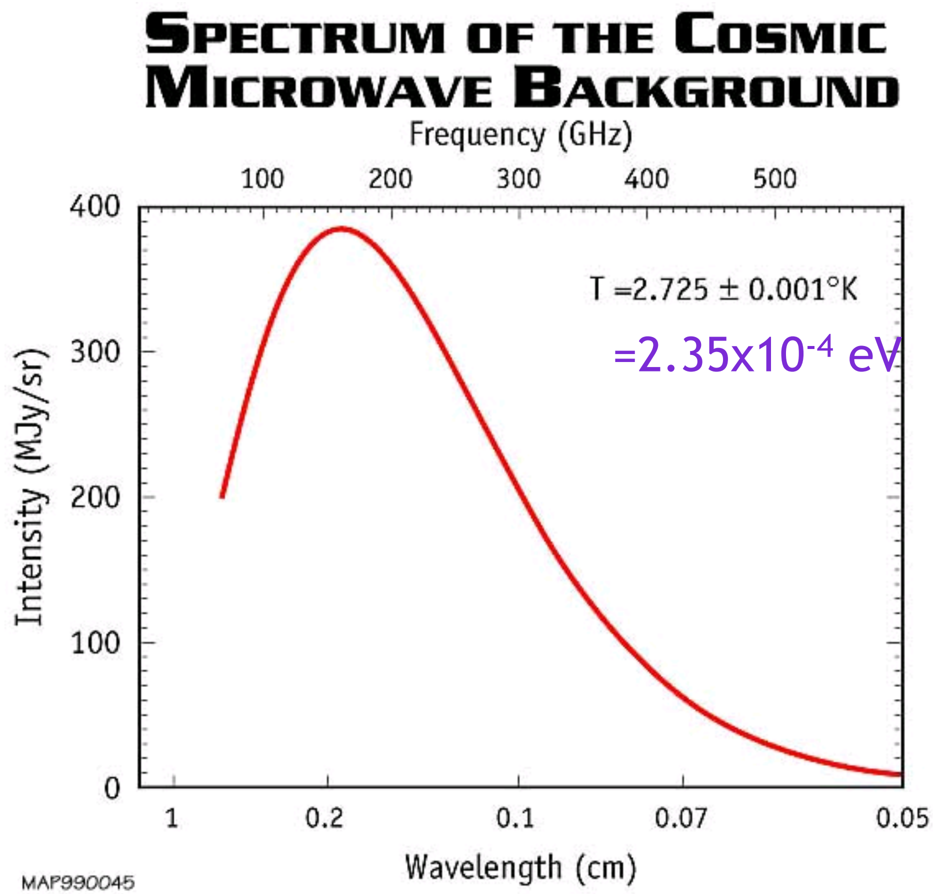
- Production (Early Universe) *Accelerator*
- Signatures (Large Scale Structure & CMB) *Detector*

Neutrinos and Dark Matter were produced in the early universe



Starting Point: Cosmic Photons

The temperature of the cosmic microwave background (CMB) has been measured extremely well. Turn this into a measurement of the energy density.



Photon Energy Density

Energy density of a gas of bosons in equilibrium:

$$\rho = 2\int \frac{d^3 p}{(2\pi)^3} E \frac{1}{e^{E/T} - 1}$$

$\hbar = c = k_B = 1$

Spin states

Sum over phase space

Bose-Einstein distribution

For massless particles, *E=p,* so

$$\rho = \frac{8\pi}{(2\pi)^3}\int_0^\infty \frac{dp\,p^3}{e^{p/T} - 1}$$

$$= \frac{T^4}{\pi^2}\int_0^\infty \frac{dx\,x^3}{e^x - 1}$$

$\pi^4/15$

$$\Rightarrow \rho_\gamma = \frac{\pi^2}{15} T^4$$

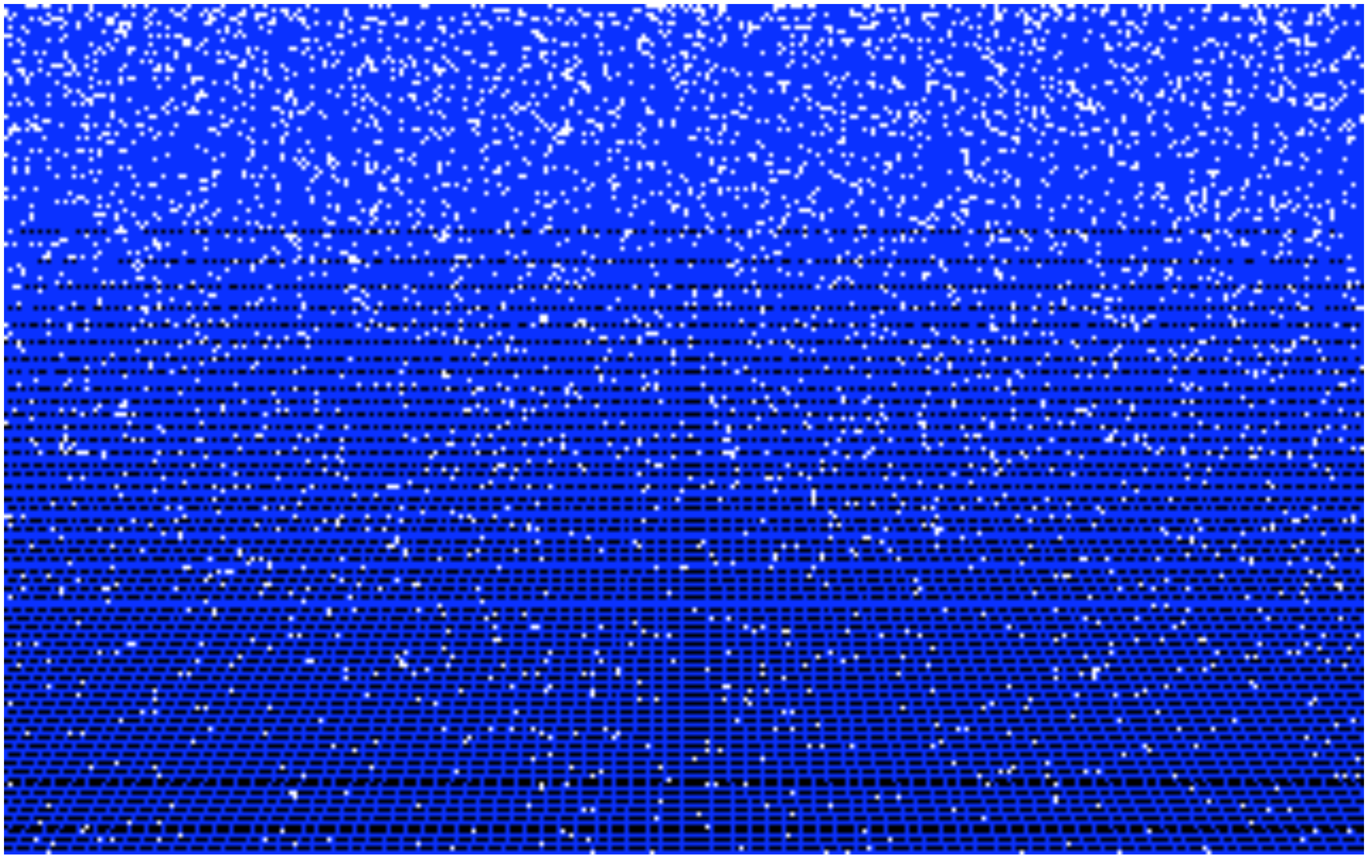
Similar Calculation: Number Density

$$n_\gamma = \frac{2\zeta(3)}{\pi^2} T^3$$

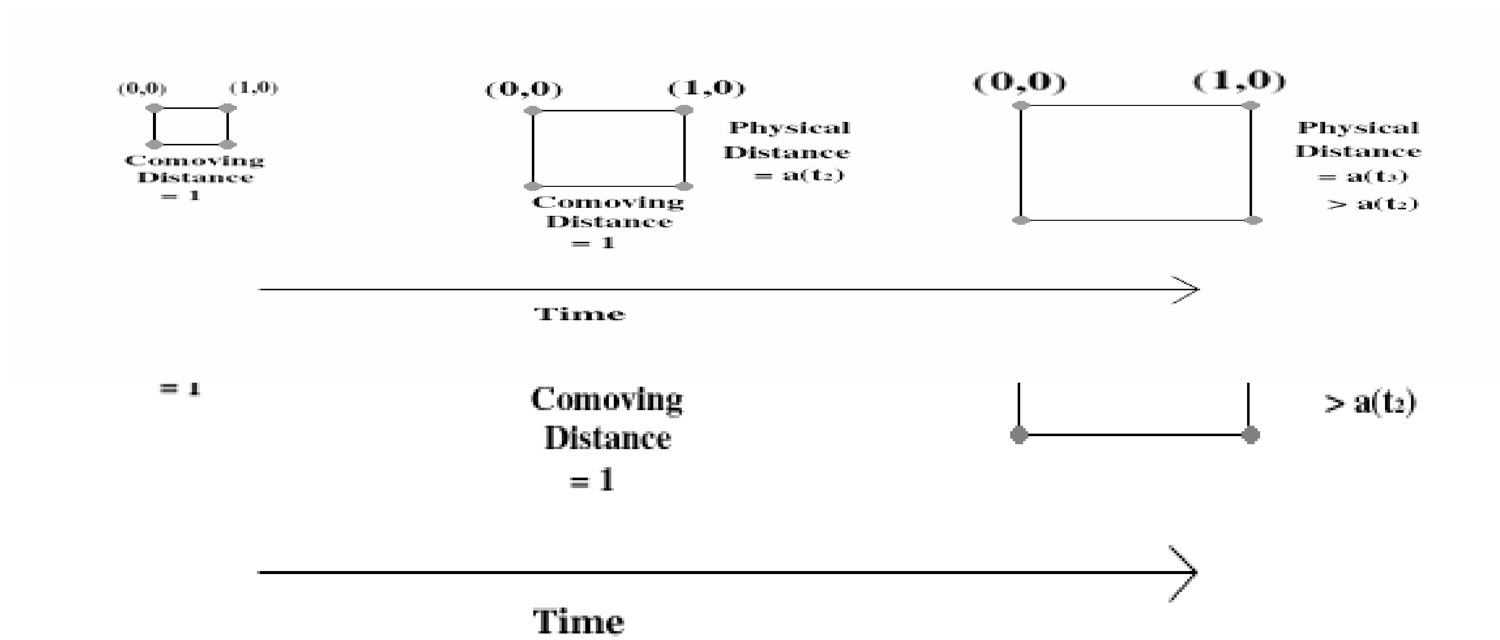
What were the number/energy density in the early universe?

To answer this, we must understand the expansion of the universe, and how this expansion affected its components

The Universe is Expanding



Scale Factor *a* quantifies expansion



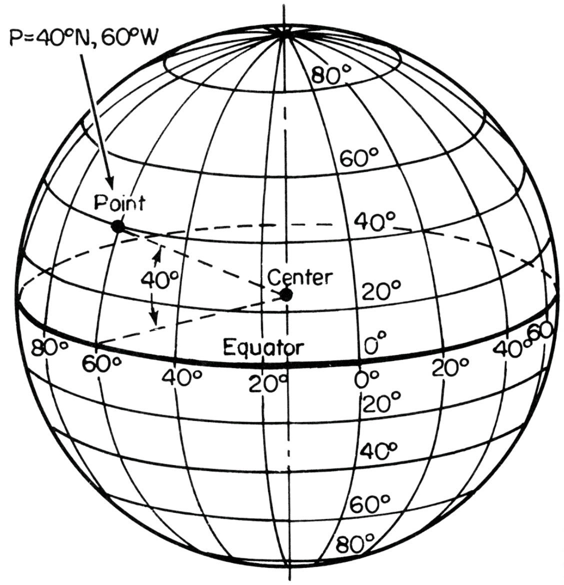
Comoving Coordinates/Distances

The coordinate differences on the grid are called *comoving distances.* They are the equivalent of longitude & latitude.

To get a physical distance *dl* from a set of coordinate differences (*dϑ,dφ*), use the *metric.*

$$dl^2 = R^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

$$g_{\theta\theta} = R^2 \quad ; \quad g_{\phi\phi} = R^2\sin^2\theta$$



Friedmann-Robertson-Walker metric

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2\right)$$

In a flat universe (our universe) *k=0,* and the metric reduces to

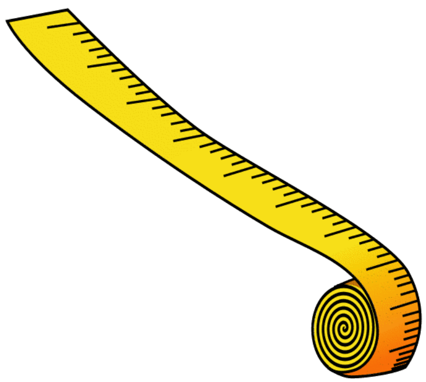
$$ds^2 = -dt^2 + a^2(t)dx^i dx^i$$

or

$$g_{00} = -1 \qquad g_{ij} = \delta_{ij} a^2(t)$$

Comoving Coordinates/Distances

Since we set $a_0=1$, the comoving distance between 2 objects is the physical distance one would get today if an infinitely long tape measure was placed between the objects.



A tape measure placed between the same 2 objects early on would find a physical distance of ($a(t)$ x comoving distance)

The evolution of (*a,ρ*) is determined by Einstein's Equations

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Ricci Tensor

Ricci Scalar

Newton's Constant

Energy Momentum Tensor

General Relativity in 1 Slide

Metric inverse

$$g^{\mu\alpha} g_{\alpha\nu} = \delta^{\mu}{}_{\nu}$$

Raise/lower indices with metric/inverse

$$T^{\alpha}{}_{\beta} = g^{\alpha\mu} T_{\mu\beta}$$

Christoffel Symbol

$$\Gamma^{\alpha}{}_{\mu\nu} = \frac{1}{2} g^{\alpha\beta} \left[\frac{\partial g_{\beta\mu}}{\partial x^{\nu}} + \frac{\partial g_{\beta\nu}}{\partial x^{\mu}} - \frac{\partial g_{\mu\nu}}{\partial x^{\beta}} \right]$$

Ricci Tensor

$$R_{\mu\nu} = \Gamma^{\alpha}{}_{\mu\nu,\alpha} - \Gamma^{\alpha}{}_{\mu\alpha,\nu} + \Gamma^{\alpha}{}_{\beta\alpha} \Gamma^{\beta}{}_{\mu\nu} - \Gamma^{\alpha}{}_{\beta\nu} \Gamma^{\beta}{}_{\mu\alpha}$$

Ricci Scalar

$$R = g^{\mu\nu} R_{\mu\nu}$$

Example: Christoffel Symbol

$$\Gamma^0{}_{ij} = \frac{1}{2} g^{0\alpha}\left[g_{\alpha i,j} + g_{\alpha j,i} - g_{ij,\alpha}\right]$$

$$= \frac{1}{2} g^{00}\left[g_{\cancel{0}i,j} + g_{\cancel{0}j,i} - g_{ij,0}\right]$$

But the metric has no spatial dependence, so …

And $g^{00}=-1$ and $g_{ij}=\delta_{ij}a^2$, so

$$\Gamma^0{}_{ij} = \delta_{ij}\frac{1}{2}\frac{d}{dt}a^2 = \delta_{ij}\dot{a}a$$

Time-Time Component of Einstein Equations

$$R_{00} + \frac{1}{2} R = 8\pi G T_{00} = 8\pi G \rho$$

Straightforward calculations lead to:

$$R_{00} = -3\frac{\ddot{a}}{a} \qquad R = 6\left[\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2\right]$$

Plug in to get the Friedmann equation:

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv H^2 = \frac{8\pi G}{3}\rho$$

Notation

The expansion rate today is called the *Hubble Constant*

$H_0 = 100h$ km sec^{-1} Mpc^{-1}

1 Mpc =*3.1x10^{24}* cm
h=0.73+/-0.03

Cosmologists measure densities in units of the *critical density,*

$\rho_{cr} = 3H_0^2/(8\pi G) = 1.88\, h^2 \times 10^{-29}$ g cm^{-3}

The total density in the universe today is equal to the critical density

The baryon density, e.g., is then written as

$\Omega_b = (\rho_b / \rho_{cr})_0$

Others: $\Omega_m, \Omega_{de}, \Omega_v, \Omega_k$

Space-Space Component of Einstein Equations

$$\frac{\ddot{a}}{a} + \frac{1}{2}\left(\frac{\dot{a}}{a}\right)^2 = -4\pi G P$$

where *P* is the diagonal space-space component of the energy momentum tensor.

Combine with the Friedmann equation to get:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P)$$

Deceleration unless ρ *+3P* is negative

Evolution of energy density

Combine the Space-Space and Time-Time components of Einstein's Equation to get:

$$\rho(a) = \rho_0 \exp\left\{3\int_a^1 \frac{da'}{a'}\left[1 + w(a')\right]\right\}$$

Energy density today

Equation of state: *w=P/ρ*

Example 1: Non-relativistic matter

$$\rho(a) = \rho_0 \exp\left\{3\int_a^1 \frac{da'}{a'}\left[1 + w(a')\right]\right\}$$

The pressure of non-relativistic matter is very small compared to the energy density (*T<<m*), so *w=0.*

$$\rho_m = \rho_{m,0} a^{-3}$$

Consistent with simple dilution by volume expansion

Example 2: Relativistic particles

$$\rho(a) = \rho_0 \exp\left\{3\int_a^1 \frac{da'}{a'}\left[1 + w(a')\right]\right\}$$

To determine *w*, recall that the pressure is:

$$P = g\int \frac{d^3p}{(2\pi)^3}\frac{p^2}{3E}f(E)$$

For relativistic particles, *E=p*, so *P=ρ/3*, or *w=1/3.*

$$\rho_r = \rho_{r,0}a^{-4}$$

- Volume dilution PLUS wavelength stretching
- *T~1/a*

Example 3: Cosmological Constant

$$\rho(a) = \rho_0 \exp\left\{3\int_a^1 \frac{da'}{a'}\left[1 + w(a')\right]\right\}$$

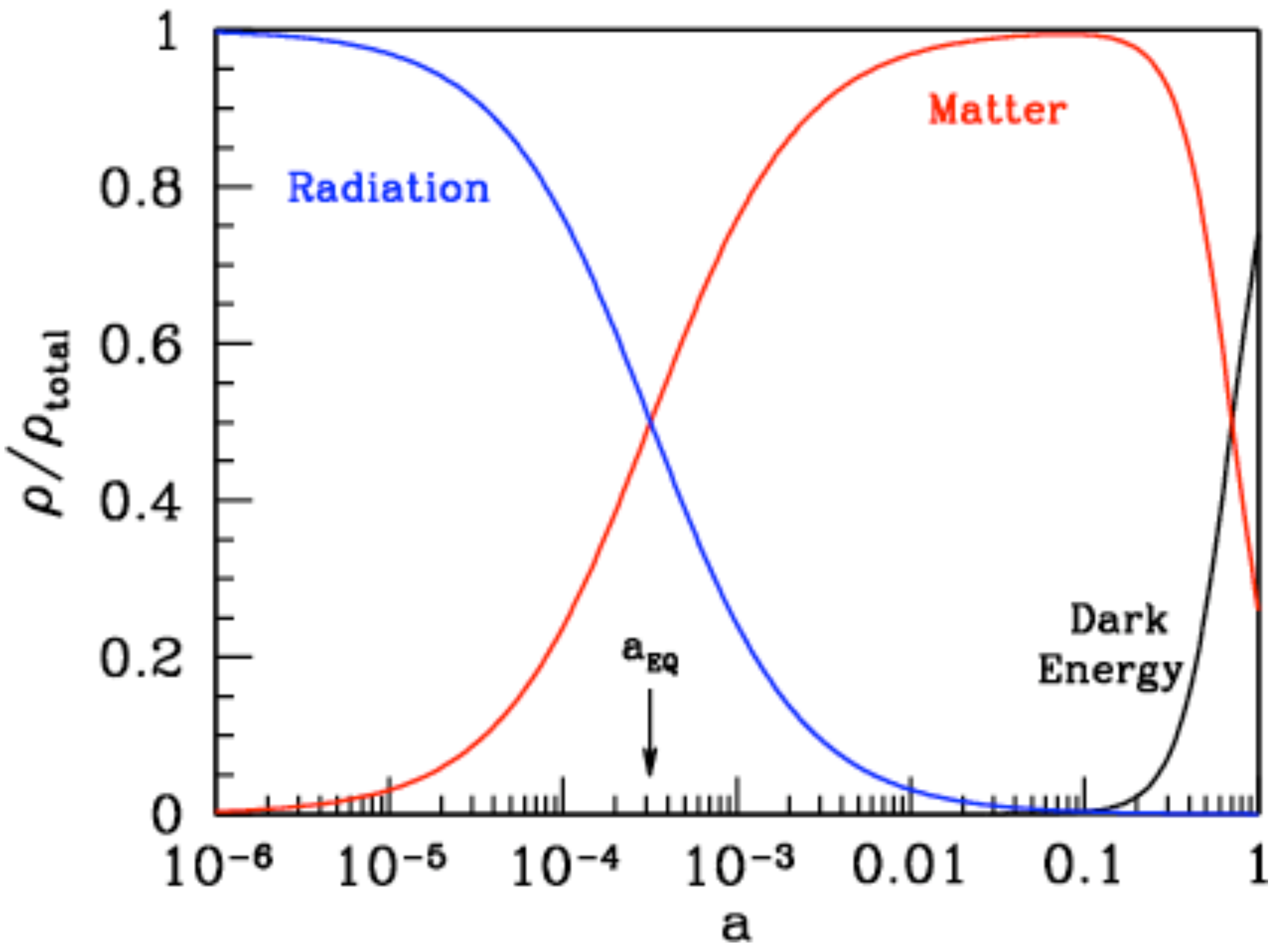
One possibility is that the *dark energy* is a cosmological constant with *w=-1.*

$$\rho_\Lambda = \rho_{\Lambda,0}$$

Empty space contains energy

Thermal History of the Universe

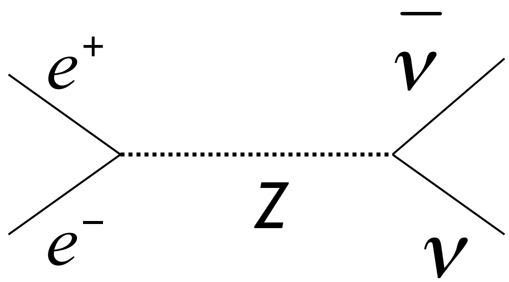
- The equation of state of dark energy is -1 to within 10%
- Structure begins to grow when the universe becomes matter dominated (at a_{EQ})
- Associate a temperature with every *a*: $T=(2.35\times10^{-4}/a)\,\mathrm{eV}$



Neutrinos are produced in the early universe

Alpher, Herman, & Gamow 1953

Assume there are no neutrinos initially when the temperature is much larger than m_e. The rate for producing them via, e.g.,



is of order

$$n\sigma \sim T^3 \frac{\alpha^2 T^2}{m_Z^4} \sim 100\left(\frac{T}{10MeV}\right)^5 \sec^{-1}$$

At those times, electrons and positrons are effectively massless so have the same abundance as photons.

Compare this to the expansion rate

$$H = \sqrt{\frac{8\pi G\rho}{3}} \sim \frac{T^2}{m_{Planck}} \sim 10\left(\frac{T}{10MeV}\right)^2 \sec^{-1}$$

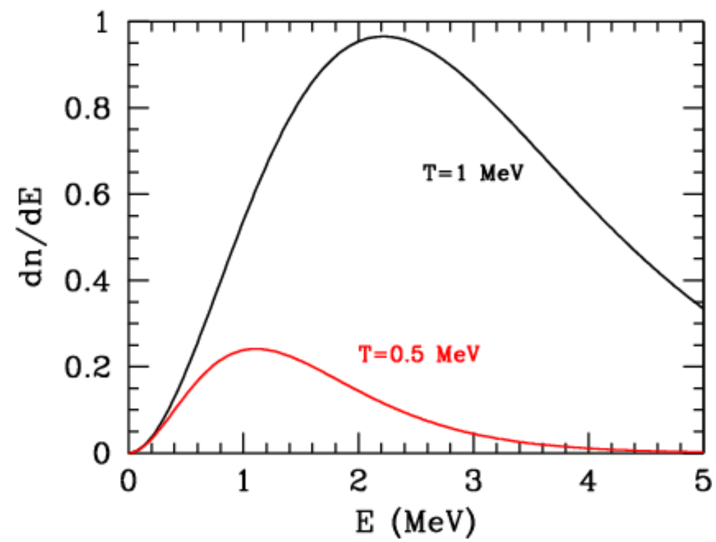
since the universe is radiation dominated at early times

The ratio of the neutrino production rate to the expansion rate is therefore

$$\frac{\Gamma_\nu}{H} \sim \left(\frac{T}{5MeV}\right)^3$$

Above 5 MeV, neutrinos are in equilibrium with the rest of the cosmic plasma

Fermi-Dirac distribution with temperature *T* equal to the electron/photon temperature.



After the neutrinos decouple from the rest of the plasma (*T*<MeV), they still maintain Fermi-Dirac distribution with *T~1/a.*

You might think neutrinos and photons have the same temperature today …

But photons gained energy from electron/positron annihilation when $T \sim m_e$

Use:

- entropy conservation: $sa^3 = constant$
- neutrino temperature does scale as a^{-1}

$$\frac{s}{T_\nu^{\;3}} = const$$

Compute this ratio before and after electron/ positron annihilation

photons — electrons/positrons — neutrinos

Initially:

$$\frac{s}{T_\nu^{\,3}} = \frac{cT^3\left(2 + (7/8)[4+6]\right)}{T^3} = c\,\frac{43}{4}$$

Finally:

$$\frac{s}{T_\nu^{\,3}} = \frac{c\left(2T_\gamma^{\,3} + (7/8)6T_\nu^{\,3}\right)}{T_\nu^{\,3}} = c\left(2[T_\gamma/T_\nu]^3 + 21/4\right)$$

Equate the two to get:

$$\frac{T_\gamma}{T_\nu} = \left(\frac{11}{4}\right)^{1/3}$$

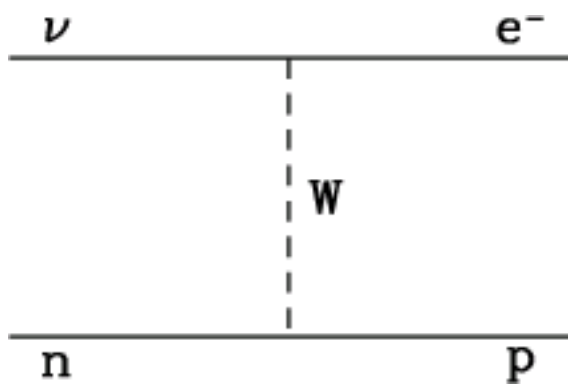
Calibrate off the well-known photon temperature to get the prediction

$$n_{\nu\bar{\nu}} = 115\, N_\nu\, cm^{-3}$$

Number of species of weakly interacting neutrinos

There are ~ a hundred quadrillion cosmic neutrinos (flux of 115x3c = 10^{13} cm^{-2} sec^{-1}) passing through this screen (~10^4 cm^2) every second.

Unfortunately, we can't detect these because the cross-section is too small



$$\sigma \sim 10^{-63}\,cm^2\left(\frac{E_\nu}{10^{-4}\,eV}\right)^2$$

So the detection rate is of order

$$\Gamma \sim 10^{-50}\times 10^{27}\left(\frac{M_{\rm detector}}{1\,{\rm kg}}\right)\times 3\times 10^{7}\,yr^{-1}$$

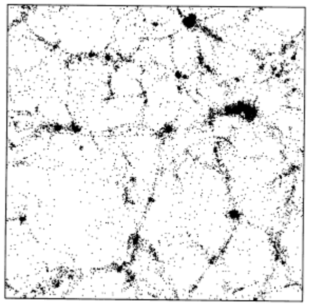
But … these neutrinos do contribute to the energy density of the universe.

The energy density of massive neutrinos is:

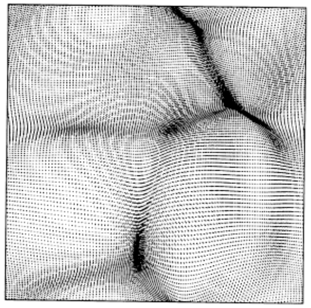
$$\Omega_\nu = \frac{\rho_\nu}{\rho_{cr}} = \frac{n_\nu \sum m_\nu}{\rho_{cr}} = \frac{0.01}{h^2}\left(\frac{\sum m_\nu}{1eV}\right)$$

This could be as large as 10% of the total matter, so affects large scale structure.

Massless Neutrinos

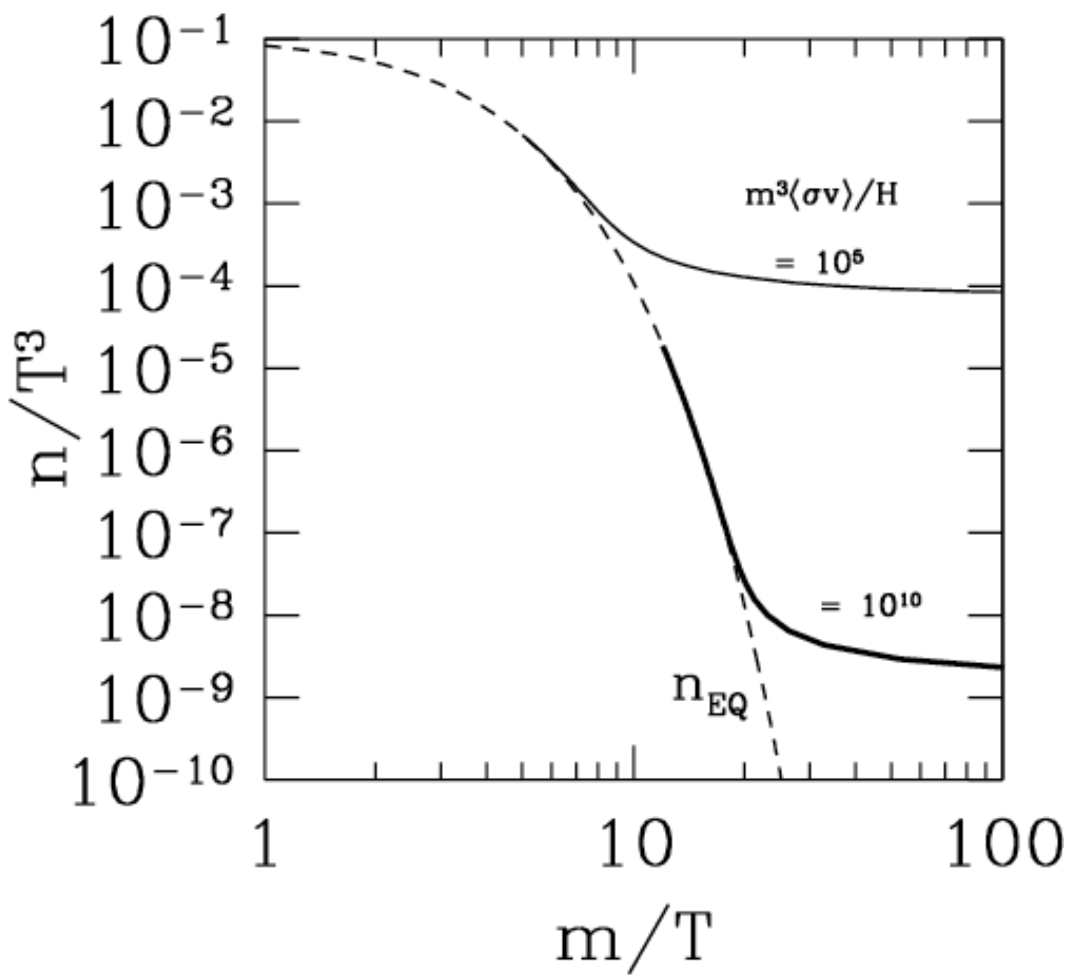


Massive Neutrinos



Weakly interacting stable massive particles (WIMPs) could be the dark matter

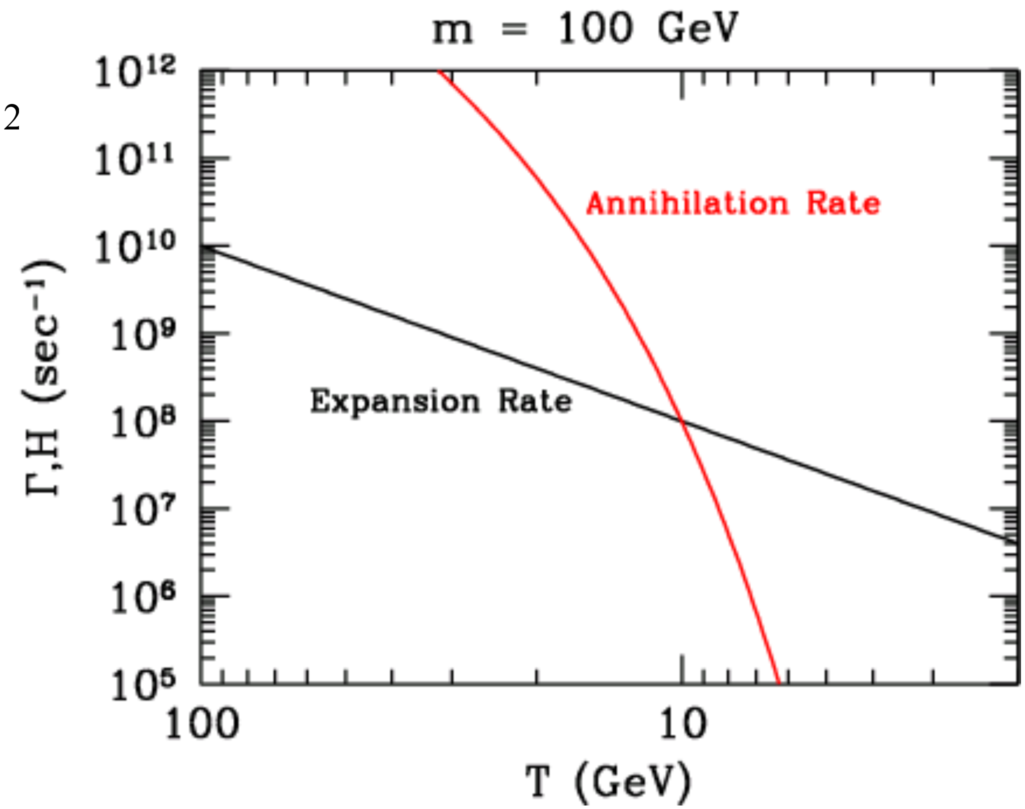
- All particles start with density roughly equal to photon density
- When temperature drops beneath mass, annihilations deplete density …
- until freeze-out



Annihilation rate compared to expansion rate

$$\Gamma \sim n_X \sigma v \sim (mT)^{3/2} e^{-m/T} \sigma \left(\frac{T}{m} \right)^{1/2}$$

$$H \sim \frac{T^2}{m_{Planck}}$$



Freeze-out takes place when the two rates are equal

$$e^{-m/T_{fo}}\sigma m \sim m_{Planck}{}^{-1}$$

After freeze-out, WIMP number scales as photon number density

$$\frac{n_X}{n_\gamma} \sim \left(\frac{m}{T_{fo}}\right)^{3/2} e^{-m/T_{fo}} \sim \frac{1}{m\sigma m_{Planck}}\left(\frac{m}{T_{fo}}\right)^{3/2}$$

Multiply by mass to estimate the contribution to the energy density today

$$n_X \sim \frac{n_\gamma}{m\sigma m_{Planck}}\left(\frac{m}{T_{fo}}\right)^{3/2}$$

$$\Omega_x \sim \frac{T^3}{\rho_{cr}\sigma m_{Planck}}\left(\frac{m}{T_{fo}}\right)^{3/2}$$

Plug in numbers

$$\Omega_x = 0.3h^{-2}\left(\frac{m}{10T_{fo}}\right)^{3/2}\frac{10^{-37}\,cm^2}{\sigma}$$

Mild mass dependence, but mostly depends on cross-section only.

WIMP Miracle

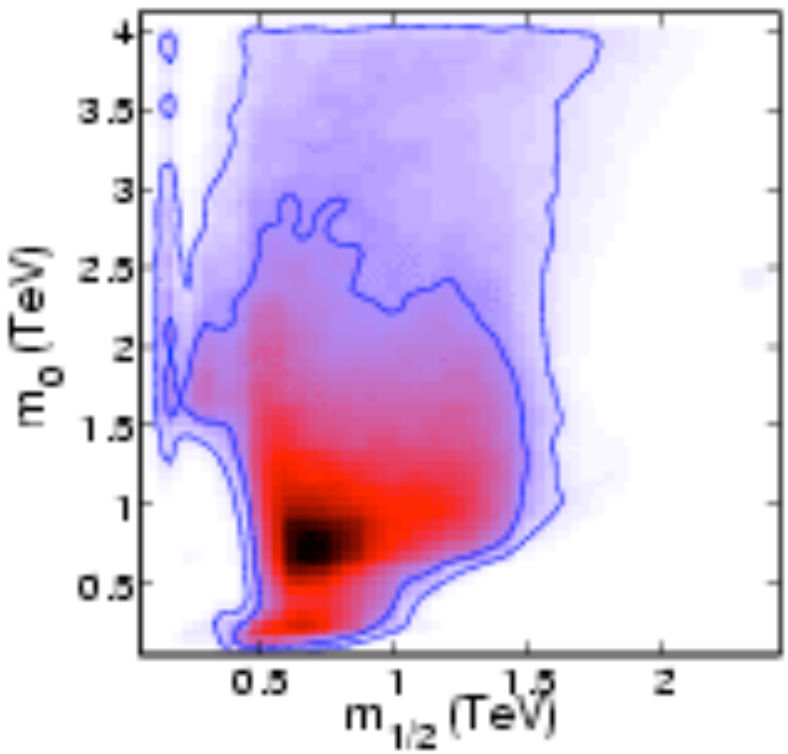
Need a cross section of order 10^{-37} cm^2

$$\sigma \sim \frac{\alpha^2}{m^2} \sim \frac{10^{-4}}{(100GeV)^2}\left(\frac{100GeV}{m}\right)^2$$

$$\sim \frac{10^{-4}(2\times 10^{-14}\,GeV cm)^2}{(100GeV)^2}\left(\frac{100GeV}{m}\right)^2$$

$$\sim 4\times 10^{-36}\left(\frac{100GeV}{m}\right)^2 cm^2$$

Easy to get correct dark matter abundance in supersymmetric models



De Austri, Trotta, & Roszkowski 2006

Final Slide

If you want to get your hands dirty check out …

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http://www.physto.se/~edsjo/darksusy/
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Meet me at the bar tonight if you have a good idea about …

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Cosmic Dark Matter and the LHC
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