Distances

How do we measure distances between two objects if the universe is expanding?

For massless particles: $$-dt^2 + a^2(t)dx^2 = 0$$

So, in a time dt, photons travel a comoving distance

$$dx = \frac{dt}{a(t)}$$

So, light leaving a source at t_1 and arriving at t_2 travels

$$x = \int_{t_1}^{t_2} \frac{dt}{a(t)} = \int_{a_1}^{a_2} \frac{da}{a^2(t)H(t)}$$

Distances

The *redshift* of an observed object is related to the scale factor when its light was emitted:

$$1 + z \equiv \frac{1}{a}$$

So the comoving distance out to an object at redshift *z* is:

$$\chi(z) = \int_0^z \frac{dz'}{H(z')}$$

From this comoving distance, *luminosity distance* ($F=L/(4\pi d_L^2)$) and *Angular diameter distance* ($\theta=r/d_A$) can be derived.

Distances

For example, the luminosity distance in a flat universe is:

$$d_L(z) = (1+z)\chi(z)$$

The a

Observed fluxes, angular sizes depend on the expansion history of the universe

$$d_A(z) = \frac{1}{H_0\sqrt{|\Omega_k|}(1+z)} S(H_0\sqrt{|\Omega_k|}\chi)$$

Curvature Density

=sin (closed)
=sinh (open)

Observed brightness depends on how fast the universe was expanding in the past

Deceleration

↓

Fast Expansion in the past

↓

Balloon doubles (*z=1* till today) rapidly

↓

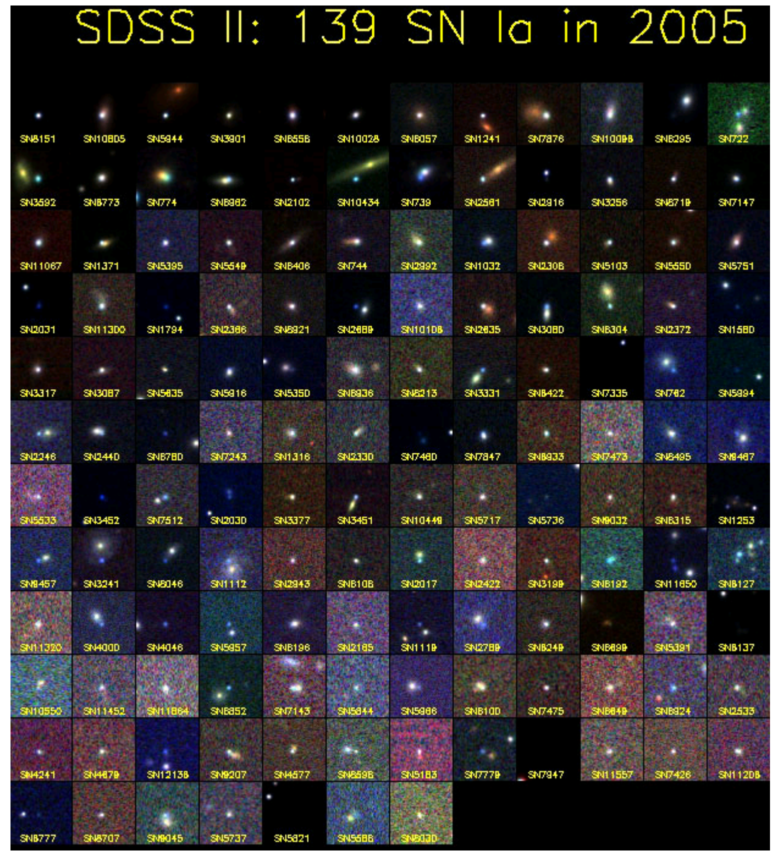
Light travels short distance

↓

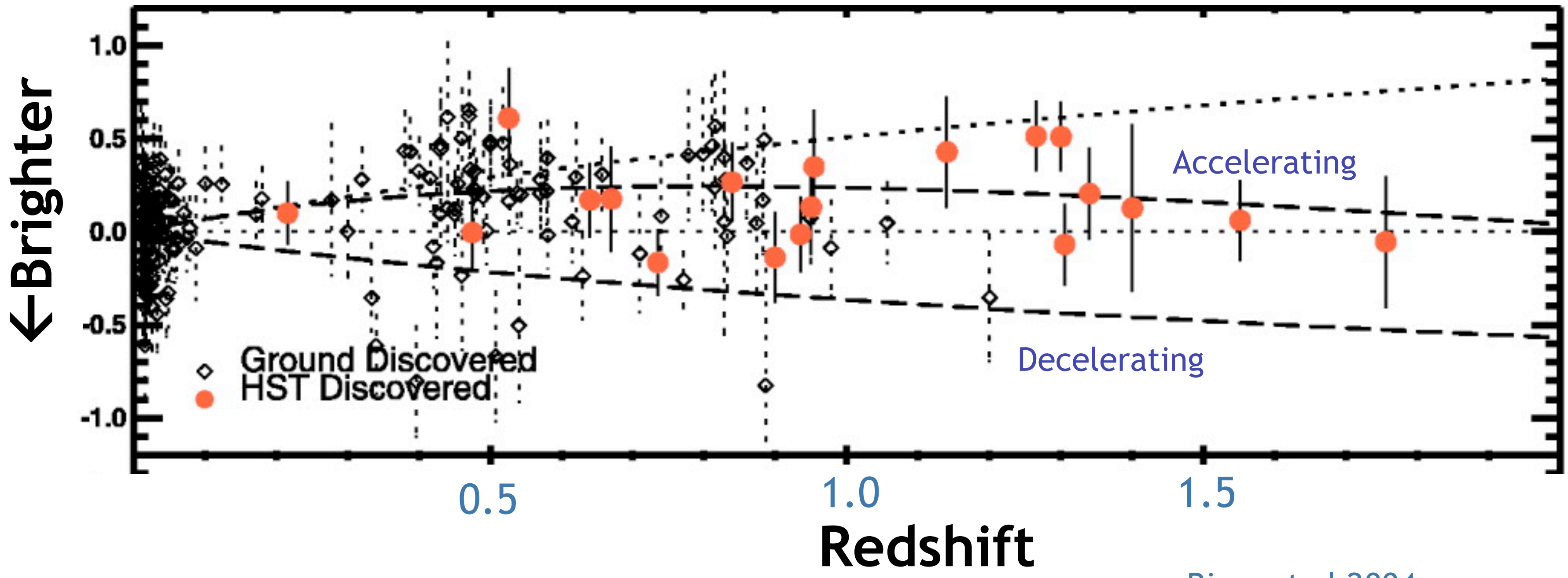
Bright Objects

Type Ia Supernovae are *Standard Candles*

Use these to infer distances and learn about the expansion rate in the past

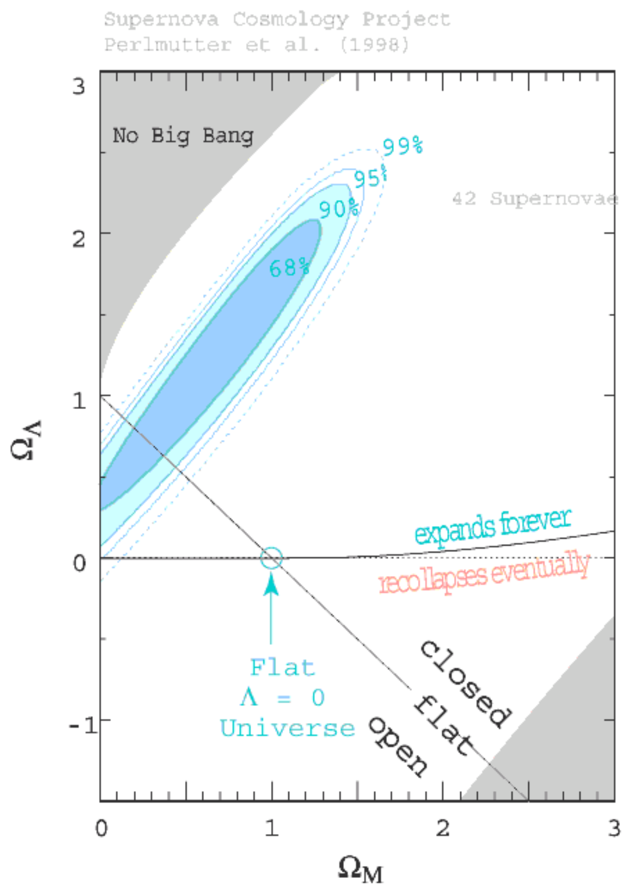


The Universe is Accelerating



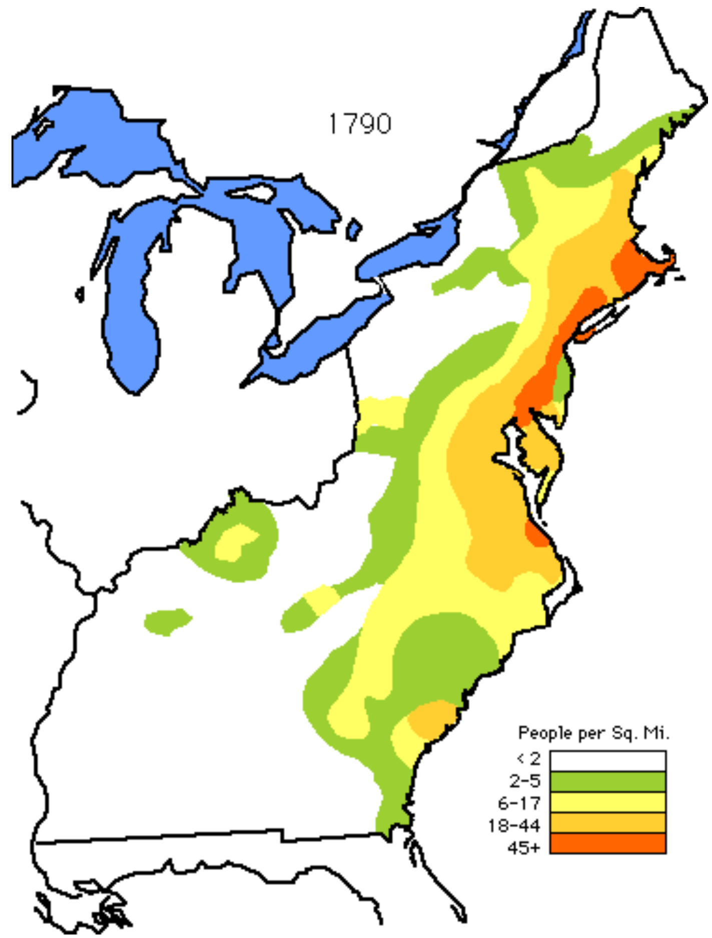
Riess et al 2004

Acceleration Requires Dark Energy



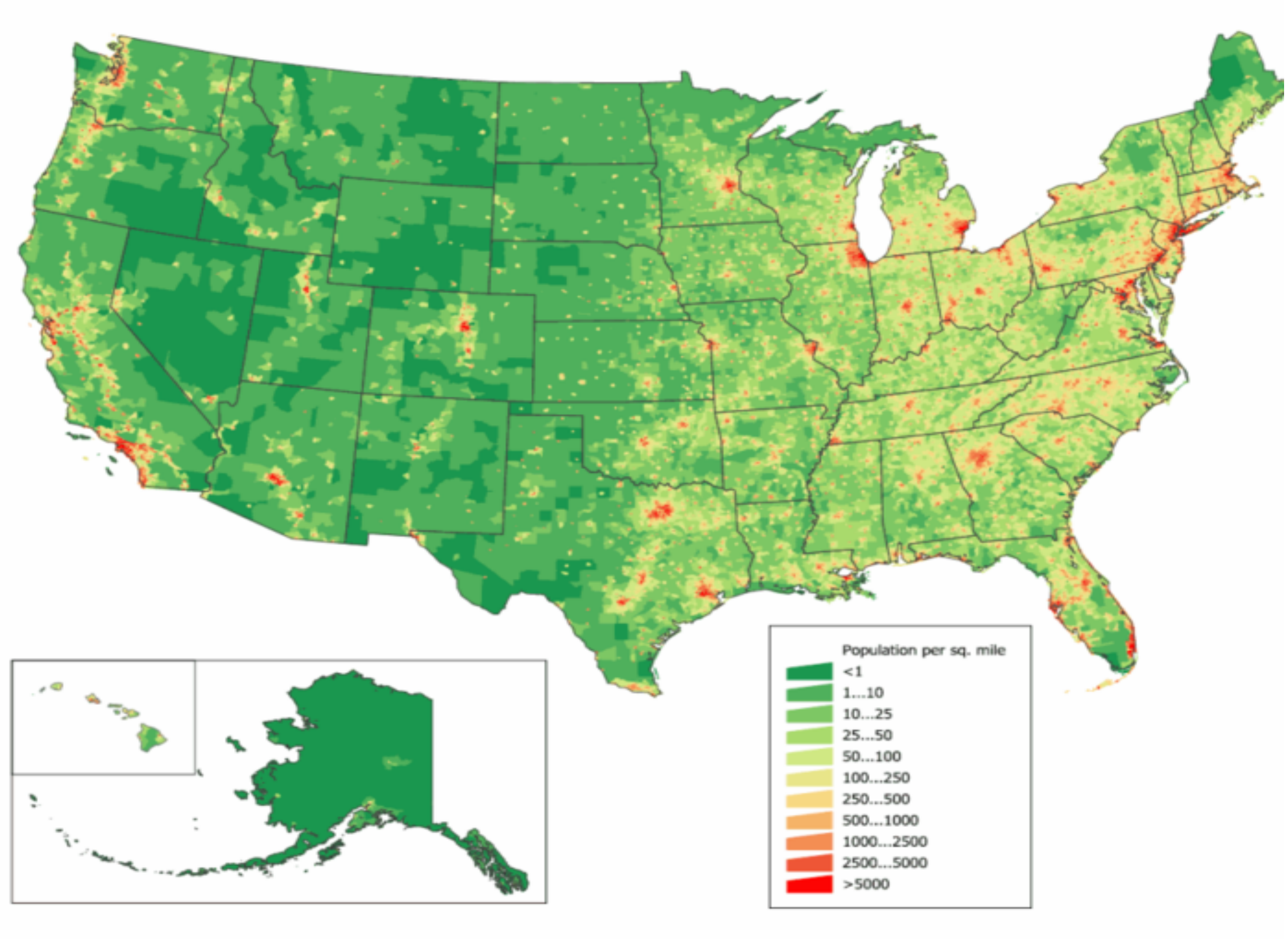
Consider the United States in 1790

- Over-densities of order 50
- Concentrated in East
- Vast Voids with low density

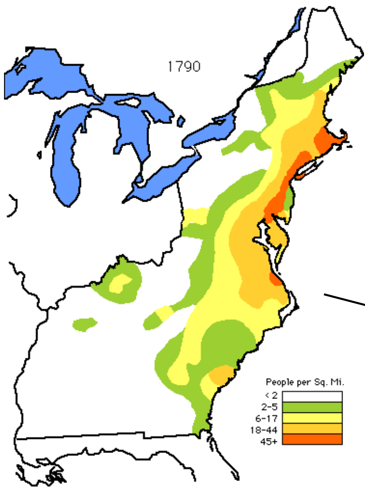


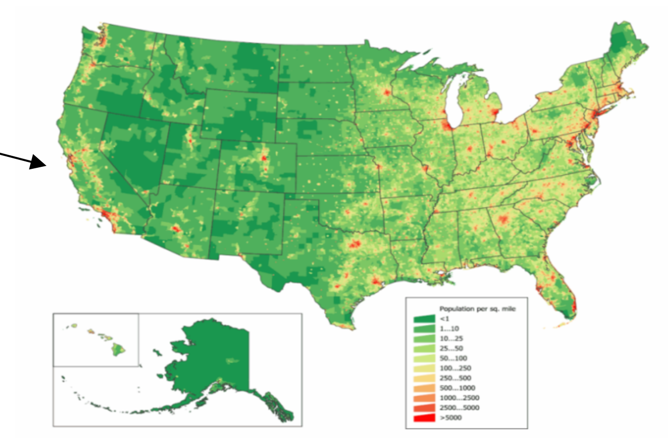
Consider the United States Today

- Over-densities of order 10,000
- Concentration in coasts
- Traces of *primordial* density (Boston-Washington; East > West)
- Vast Voids



The story of this evolution is the story of the United States

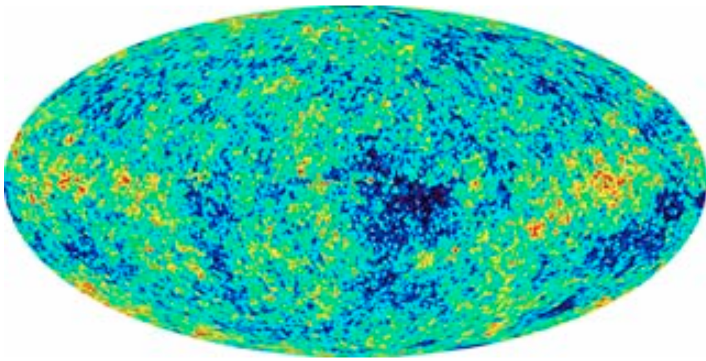




When we understand the evolution from one map to another, we can understand:

- the sociological, economic, and political ***forces*** acting on the US
- the people, or the ***constituents****, of the US*

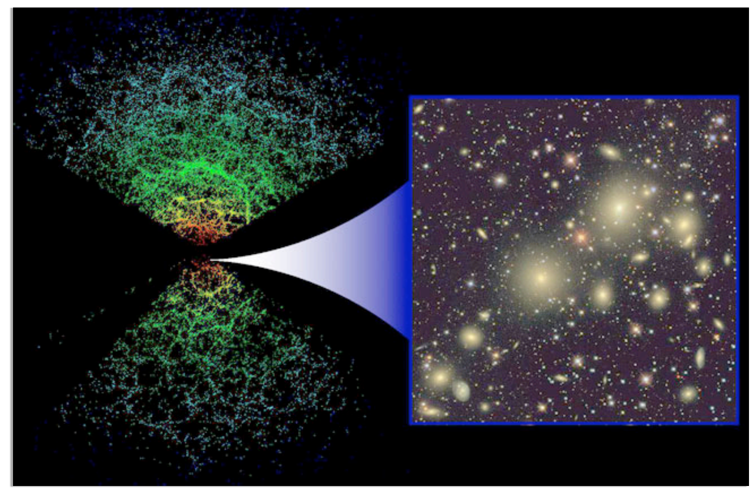
Less parochially, we rely on cosmic maps



WMAP

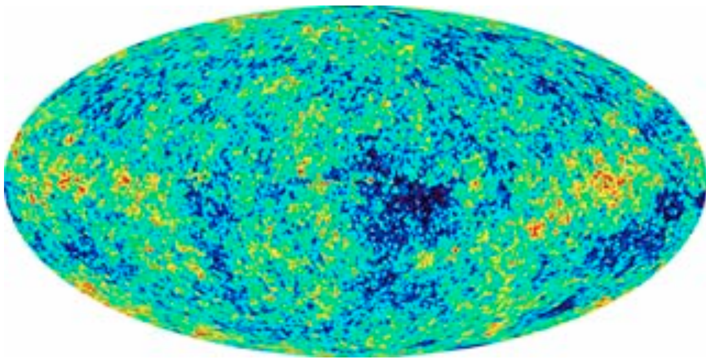
This map of the Cosmic Microwave Background (CMB) shows that the photon/baryon distribution was smooth to one part in 10,000 at *t=400,000* years.

Today, there are huge overdensities: the density in this room is 10^{30} larger than in an average spot in the Universe



Sloan Digital Sky Survey

Less parochially, we rely on cosmic maps



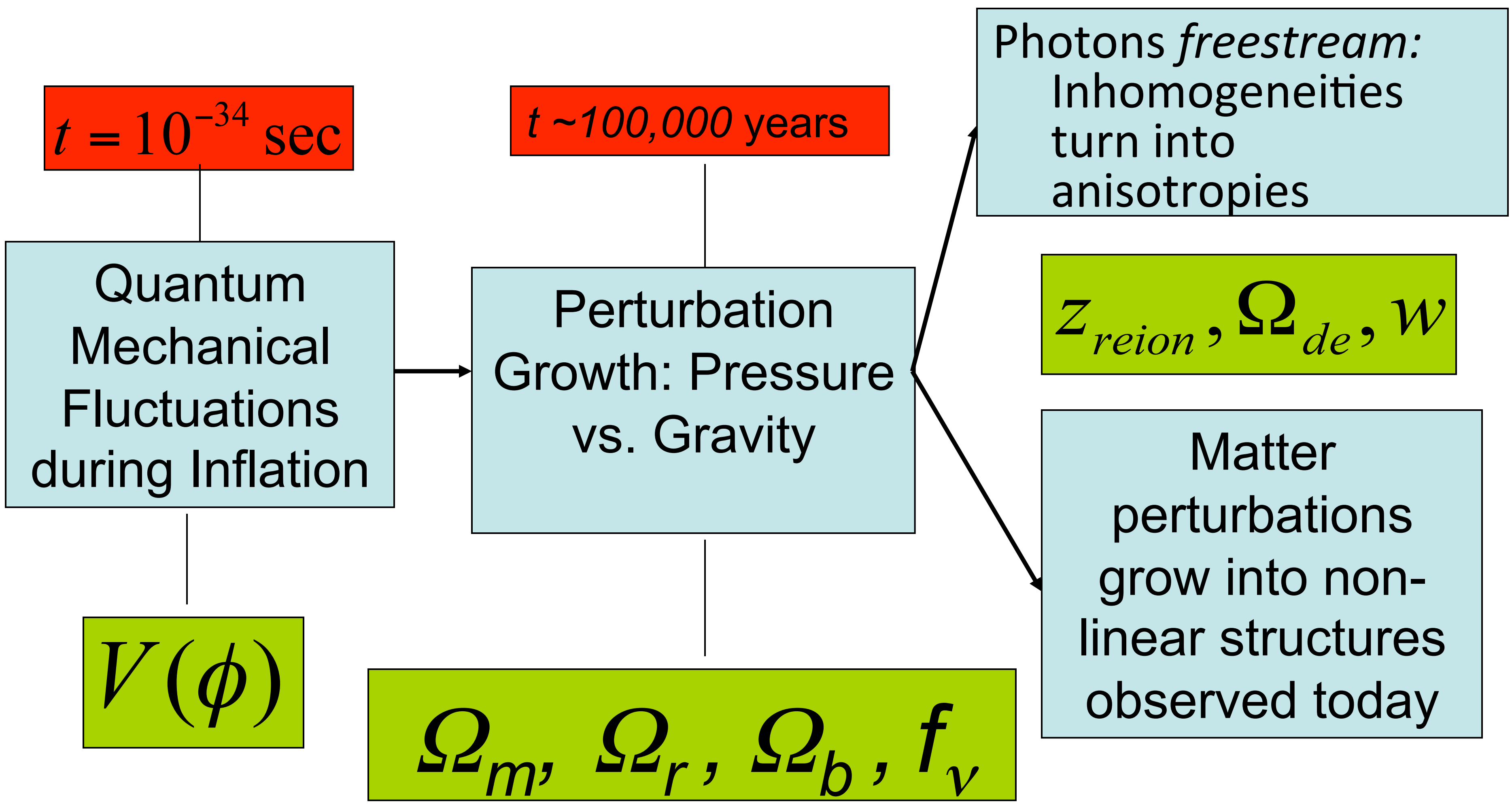
This map of the Cosmic Microwave Background (CMB) shows that the photon/baryon distribution was smooth to one part in 10,000 at *t=400,000* years.

Modern Cosmology quantitatively explains this evolution: Gravitational Instability

Today, there are huge overdensities: the density in this room is 10^{30} larger than in an average spot in the Universe

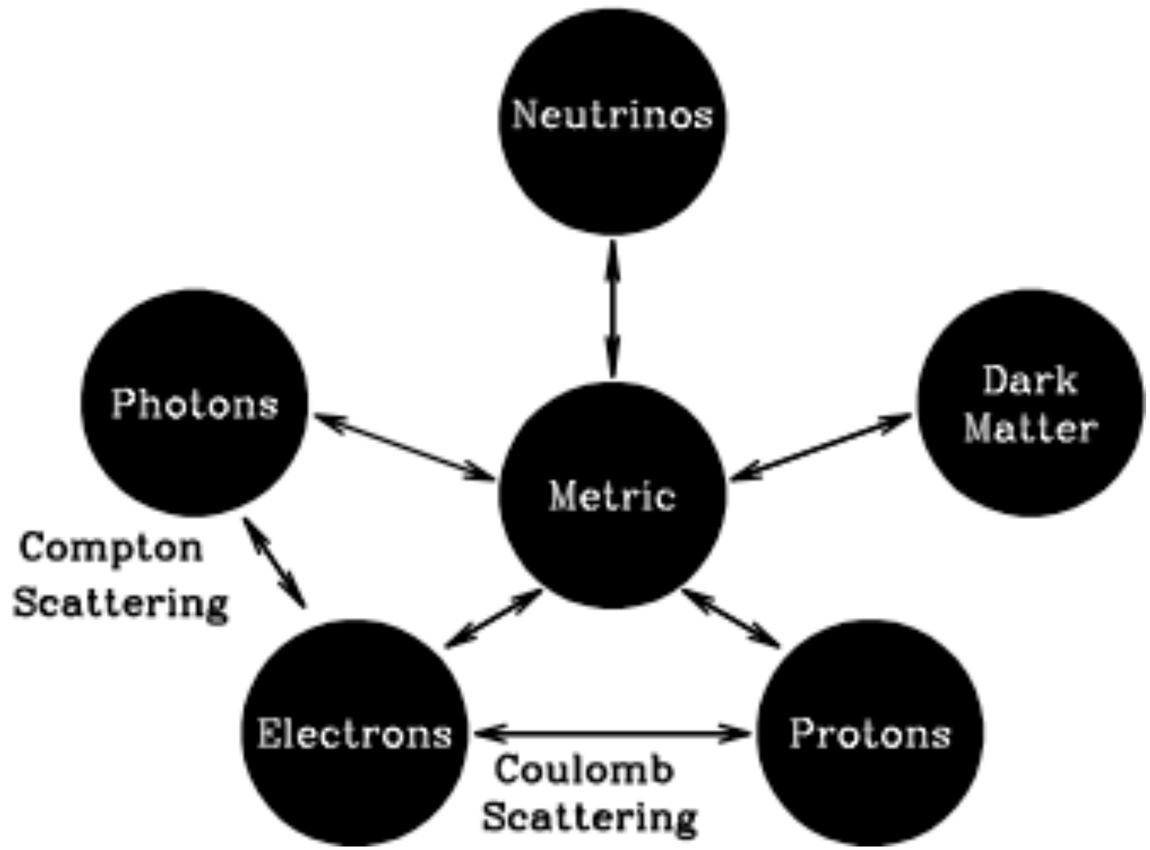
Sloan Digital Sky Survey

Coherent picture of formation of structure in the universe

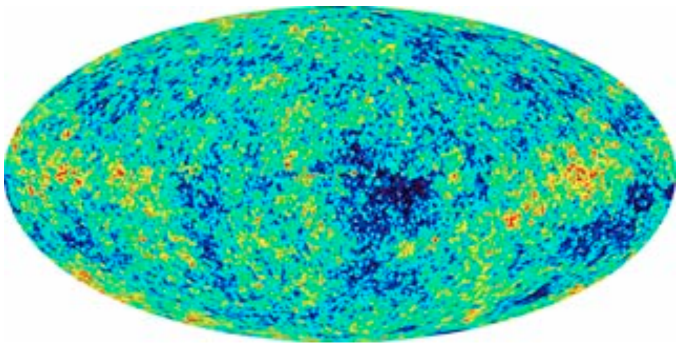


To see how perturbations evolve, need to solve coupled differential equations

Since perturbations are small, work in Fourier space: every Fourier mode evolves independently



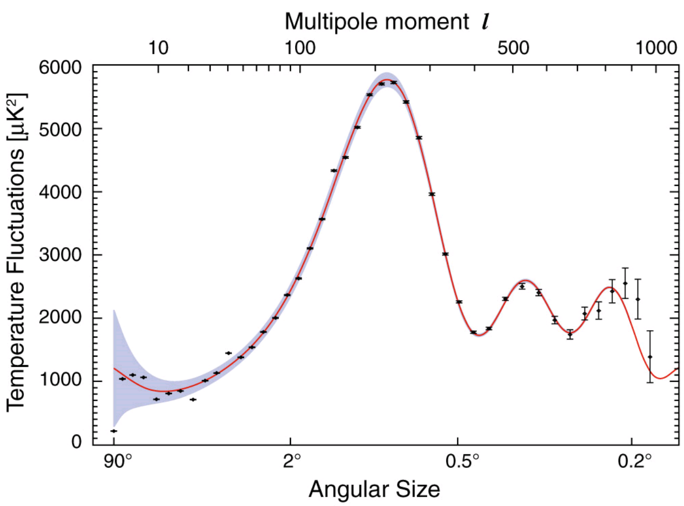
Anisotropies in the CMB



$$T(\theta,\varphi) = \sum_{lm} a_{lm} Y_{lm}(\theta,\varphi)$$

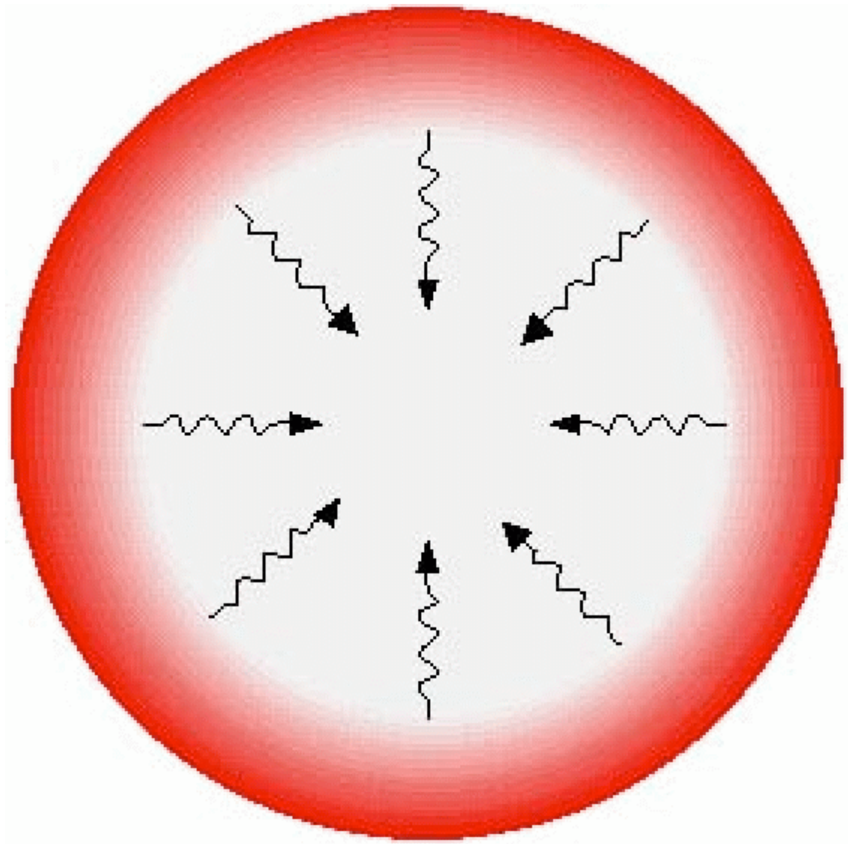
Power on scale l:

$$C_l = \frac{1}{2l+1} \sum_m |a_{lm}|^2$$



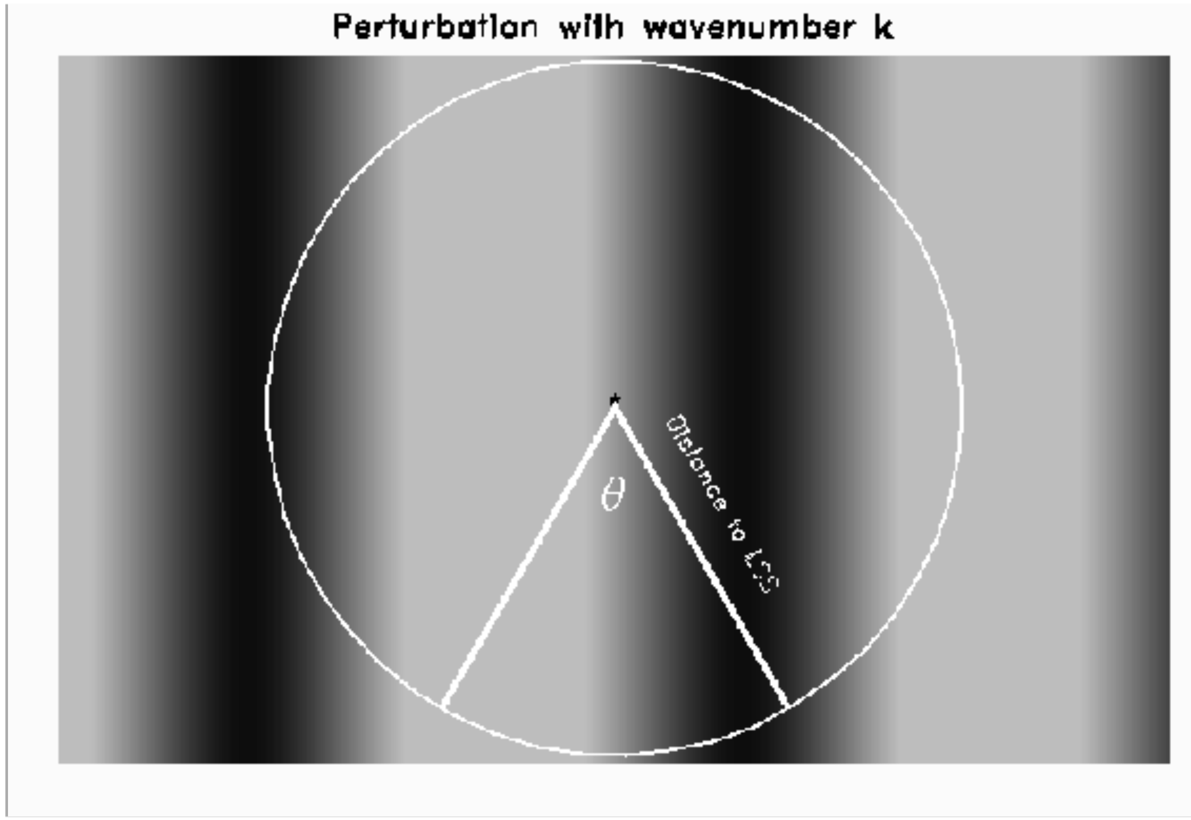
We see photons today from last scattering surface when the universe was just 400,000 years old

The temperature of the CMB is very nearly the same in all directions with small differences of a few parts in a hundred thousand.



How do inhomogeneities at last scattering show up as anisotropies today?

Perturbation w/ wavelength k^{-1} shows up as anisotropy on angular scale $\theta \sim k^{-1}/D_* \sim l^{-1}$



Before recombination, electrons and photons are tightly coupled: equations reduce to

Temperature perturbation
$$\frac{\partial^2 T}{\partial t^2} - c_s^{\,2}\nabla^2 T = F[\Phi]$$

Very similar to …

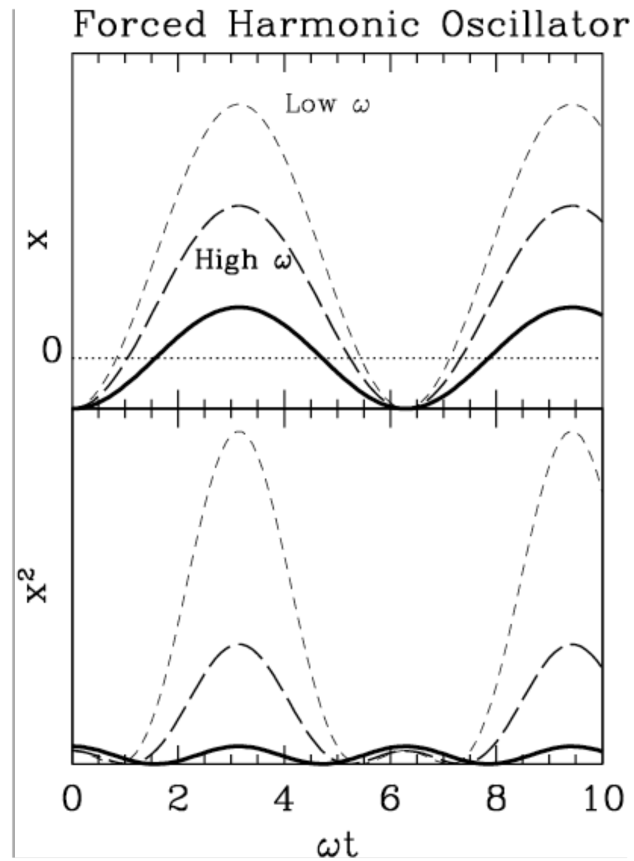
Displacement of a string
$$\frac{\partial^2 y}{\partial t^2} - c_s^2 \frac{\partial^2 y}{\partial x^2} = F$$

Forced Harmonic Oscillator

$$\ddot{x} + \omega^2 x = F$$

with $\omega = kc_s = \dfrac{k}{\sqrt{3(1+3\rho_b/4\rho_\gamma)}}$

Peaks at: $\displaystyle\int_0^{t_*} dt\,\omega_n \equiv k_n r_s = n\pi$



Peaks in Anisotropy Spectrum

Expect peaks at: $$l_n = \frac{n\pi D_{A,*}}{r_{s,*}}$$

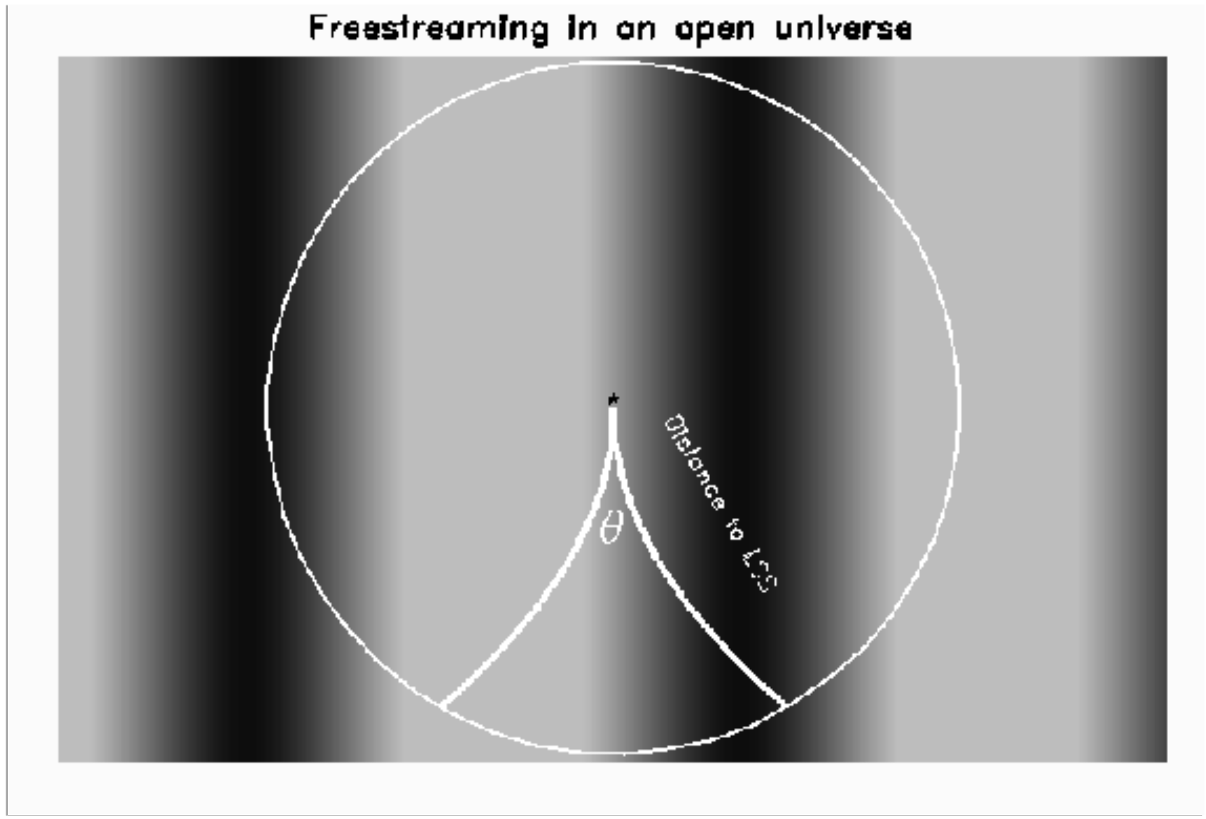
Measure positions of peaks (→ l_n)

Easy to calculate (→ $r_{s,*}$)

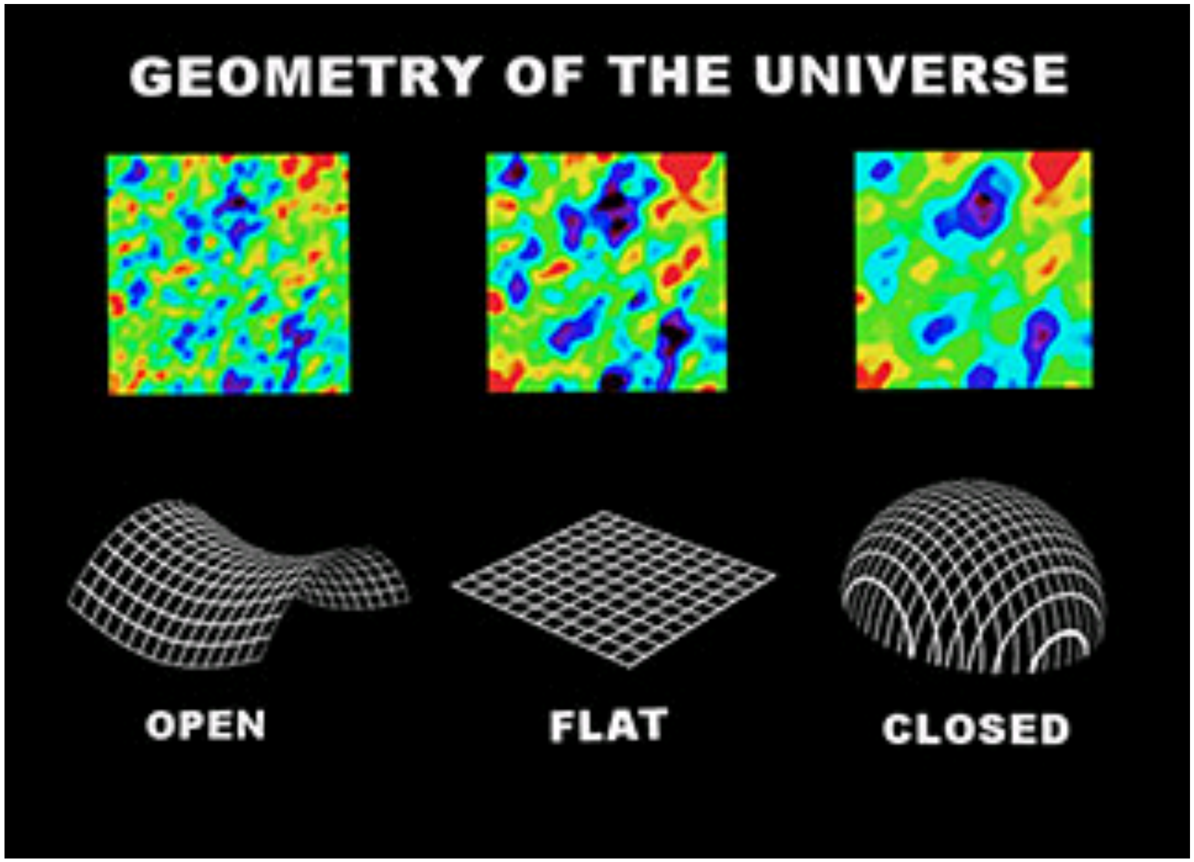
Infer angular diameter distance to the last scattering surface, which depends on geometry.

Open Universe: Light Rays Diverge

- ❑ Same wavelength subtends smaller angle in an open universe
- ❑ Peaks appear on smaller scales in open universe

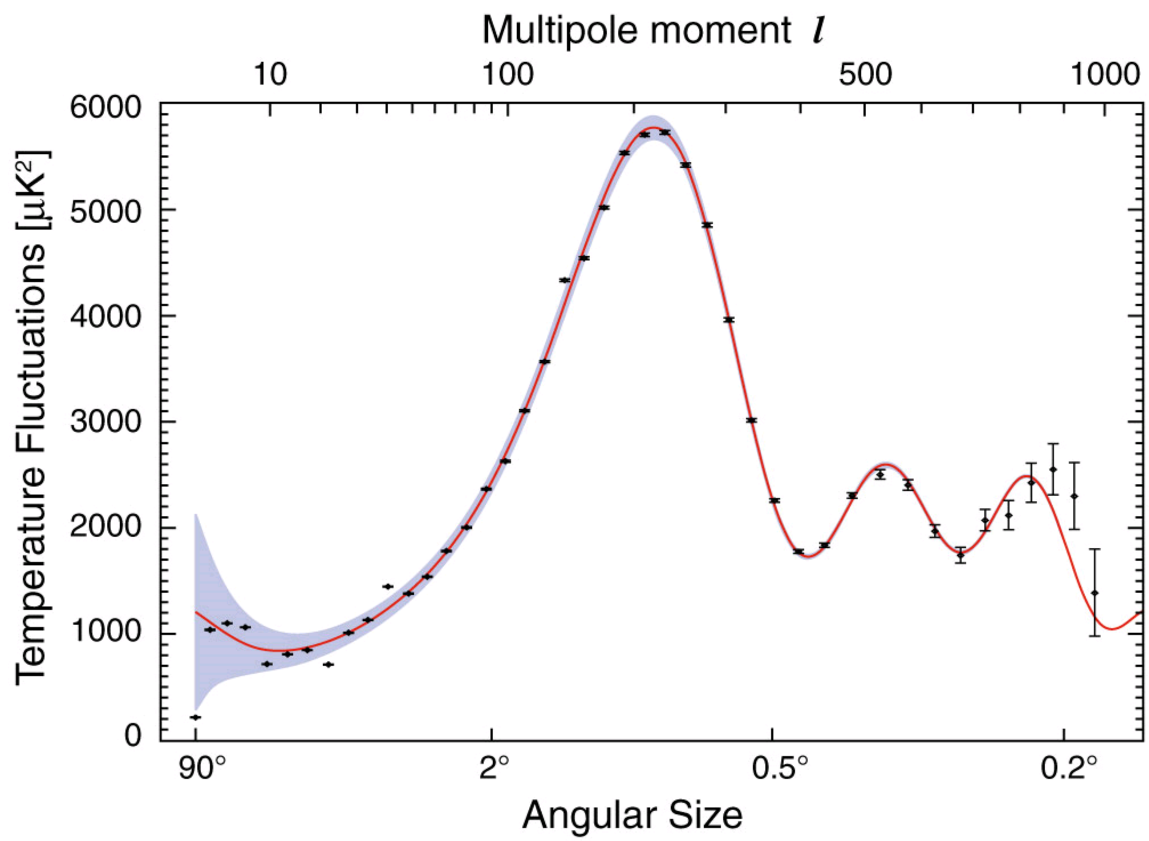


Characteristic Angular Size of Hot/Cold Spots Determines Geometry



Results

As early as 1998, observations favored flat universe



WMAP (2009)

Parameters from WMAP5

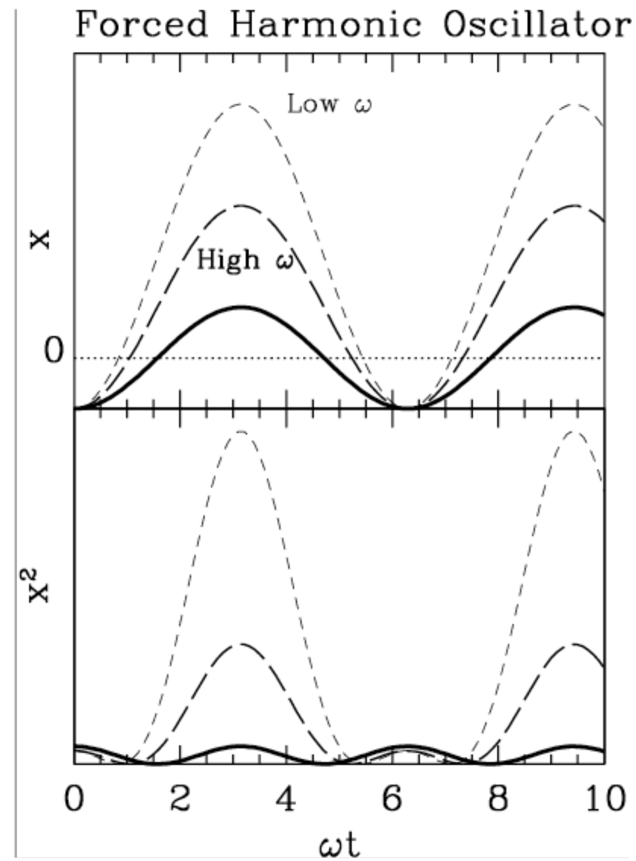
The total energy density in the universe is equal to the critical density

$10^2\Omega_b h^2$	$2.267^{+0.060}_{-0.059}$
$1-n_s$	$0.0095 < 1-n_s < 0.0657$ (95% CL)
C_{220}	5758 ± 42
$d_A(z_*)$	14119^{+187}_{-192} Mpc
h	$0.676^{+0.070}_{-0.068}$
k_{eq}	0.00969 ± 0.00045
ℓ_*	$302.13^{+0.85}_{-0.82}$
Ω_b	$0.0513^{+0.0095}_{-0.0103}$
Ω_c	$0.250^{+0.052}_{-0.056}$
Ω_k	$-0.011^{+0.015}_{-0.014}$
Ω_Λ	$0.710^{+0.053}_{-0.051}$
$\Omega_m h^2$	0.1326 ± 0.0062
Ω_{tot}	$0.99 < \Omega_{tot} < 1.05$ (95% CL)
$r_s(z_d)$	$153.4^{+1.9}_{-2.0}$ Mpc
$r_s(z_d)/D_v(z=0.35)$	$0.1107^{+0.0096}_{-0.0092}$
R	$1.714^{+0.019}_{-0.020}$
A_{SZ}	$1.07^{+0.93}_{-0.67}$
τ	0.087 ± 0.017
θ_*	$0.5958^{+0.0016}_{-0.0017}$ °
z_{dec}	$1087.9^{+1.1}_{-1.2}$
z_{eq}	3177 ± 149
z_*	$1090.59^{+0.89}_{-0.91}$

Baryon density

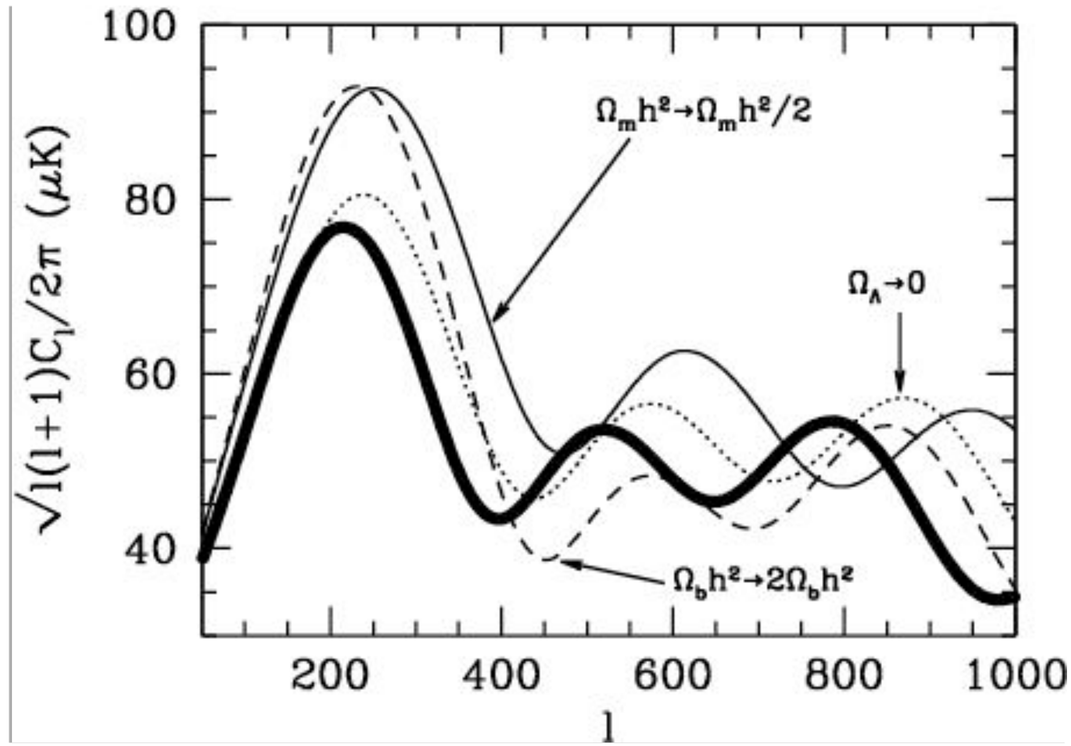
$$\omega = kc_s = \frac{k}{\sqrt{3(1+3\rho_b/4\rho_\gamma)}}$$

As baryon density goes up, frequency goes down. Greater odd/even peak disparity.



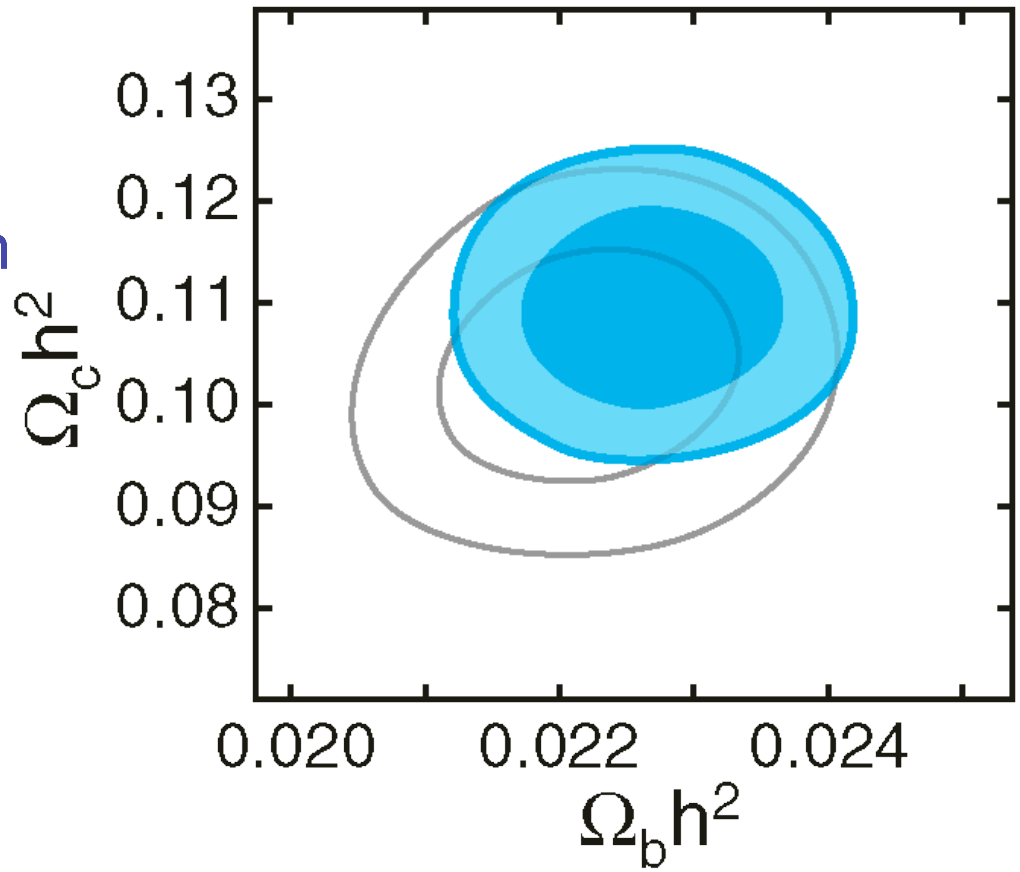
Parameters Redux

- Baryons accentuate odd/even peak disparity
- Less matter implies changing potentials, greater driving force, higher peak amplitudes
- Cosmological constant changes the distance to LSS



Evidence for New Physics

- Total matter density is much greater than baryon density → non-baryonic dark matter
- Total matter density is much less than total density → dark energy



Growth of Structure: Gravitational Instability

Define overdensity:

$$\delta(\vec{x},t) \equiv \frac{\rho(\vec{x},t) - \bar{\rho}(t)}{\bar{\rho}(t)}$$

Fundamental equation governing overdensity in a matter-dominated universe when scales are within horizon:

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\bar{\rho}_m\delta = 0$$

Growth of Structure: Gravitational Instability

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\bar{\rho}_m\delta = 0$$

Example 1: No expansion (H=0,energy density constant)

$$\delta \propto e^{\pm t\sqrt{4\pi G\bar{\rho}_m}}$$

- Two modes: growing and decaying
- Growing mode is exponential (the more matter there is, the stronger is the gravitational force)

Gravitational Instability in an Expanding Universe

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\bar{\rho}_m \delta = 0$$

Example 2: Matter density equal to the critical density in an expanding universe.

The coefficient of the 3rd term is then $3H^2/2$, so

$$\ddot{\delta} + 2H\dot{\delta} - \frac{3}{2}H^2\delta = 0$$

In this universe $a=(t/t_0)^{2/3}$ so $H=2/(3t)$

$$\ddot{\delta} + \frac{4}{3t}\dot{\delta} - \frac{2}{3t^2}\delta = 0$$

Gravitational Instability in an Expanding Universe

$$\ddot{\delta} + \frac{4}{3t}\dot{\delta} - \frac{2}{3t^2}\delta = 0$$

Insert solution of the form: $\delta \sim t^p$

Growing mode: $\delta \sim a$. Dilution due to expansion counters attraction due to overdensity. Result: power law growth instead of exponential growth

Quadratic formula yields:

$$p = \frac{\quad}{6} \pm \frac{\quad}{2}\sqrt{\frac{\quad}{9} + \frac{8}{3}} = \begin{cases} 2/3 \\ -1 \end{cases}$$

Gravitational Potential

Poisson's Equation: $$\nabla^2\Phi = 4\pi G\bar{\rho}\delta$$

In Fourier space, this becomes: $$-\frac{k^2}{a^2}\tilde{\Phi} \propto \frac{\tilde{\delta}}{a^3}$$

So the gravitational potential remains constant! Delicate balance between attraction due to gravitational instability and dilution due to expansion.

Holds only if all the energy is in non-relativistic matter.
Radiation, dark energy or massive neutrinos lead to potential decay.

Matter Power Spectrum

Poisson says: $$k^2\tilde{\Phi} \propto \tilde{\delta}$$

So the power spectrum of matter (which measures the density *squared)* scales as:

$$P_\delta \propto k^4 P_\Phi \propto k^n$$

Valid on large scales which *entered the horizon* at late times when the universe was matter dominated.

Sub-horizon modes oscillate and decay in the radiation-dominated era

Newton's equations - with radiation as the source - reduce to

Here using η as time variable →

$$\ddot{\Phi} + \frac{4}{\eta}\dot{\Phi} + \frac{k^2}{3}\Phi = 0$$

with analytic solution

$$\Phi(\eta) = 3\Phi(0)\frac{\sin(k\eta/\sqrt{3}) - (k\eta/\sqrt{3})\cos(k\eta/\sqrt{3})}{(k\eta/\sqrt{3})^3}$$

Expect less power on small scales

For scales that enter the horizon well before equality,

$$\Phi(\eta_{\rm EQ}) \to \Phi(0)\,\frac{\cos(k\eta_{\rm EQ}/\sqrt{3})}{(k\eta_{\rm EQ}/3)^2}$$

So, we expect the transfer function to fall off as

$$\lim_{k\to\infty}\frac{\Phi_{today}(k)}{\Phi_{initial}(k)} \propto k^{-2}$$

Shape of the Matter Power Spectrum

$$P(k) \propto \begin{cases} k^n & \text{Large scales} \\ k^{n-4}\ln^2(k) & \text{Small scales} \end{cases}$$

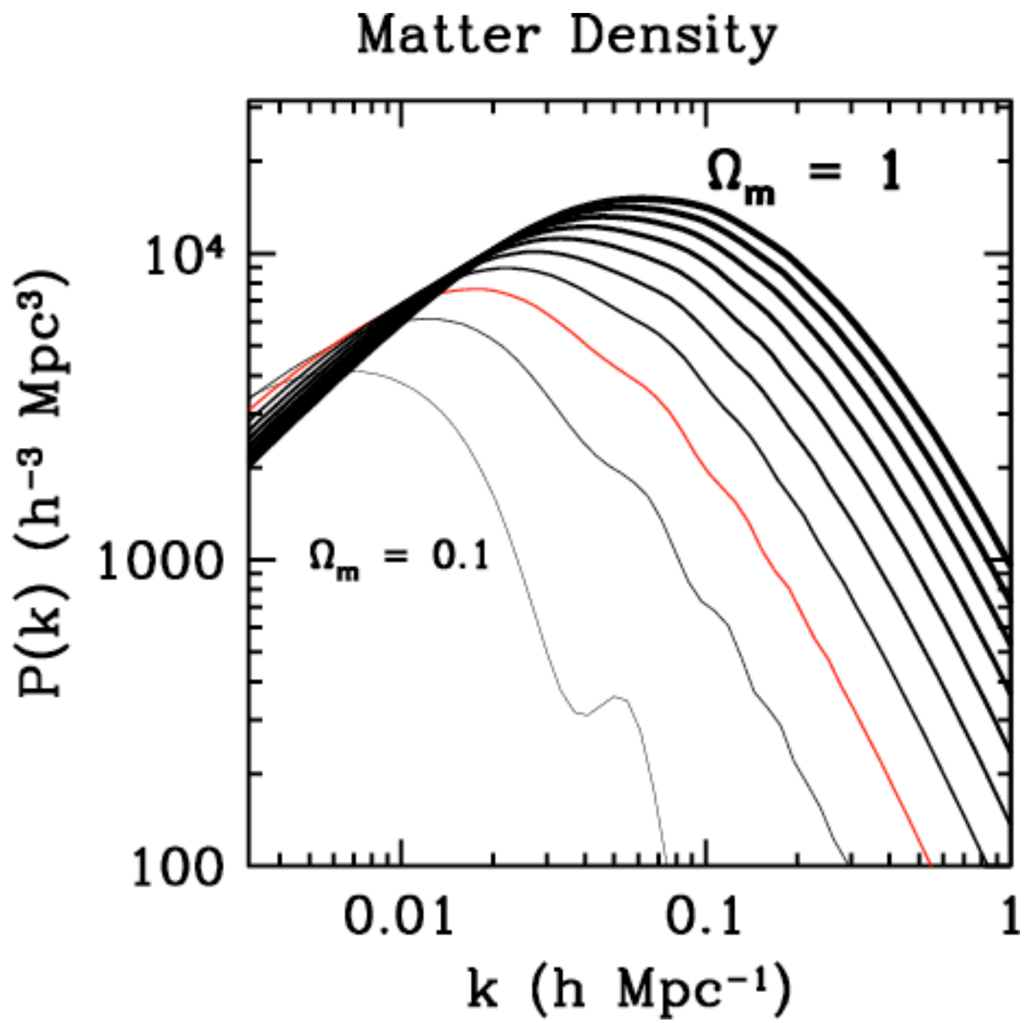
Log since structure grows slightly during radiation era when potential decays

The *turnover* scale is the one that enters the horizon at the epoch of matter-radiation equality:

$$k_{EQ} = 0.073\Omega_m h^2 \mathrm{Mpc}^{-1}$$

Therefore, measuring the shape of the power spectrum will give a precise estimate of Ω_m

Turnover scale sensitive to the matter density



Neutrinos affect large scale structure

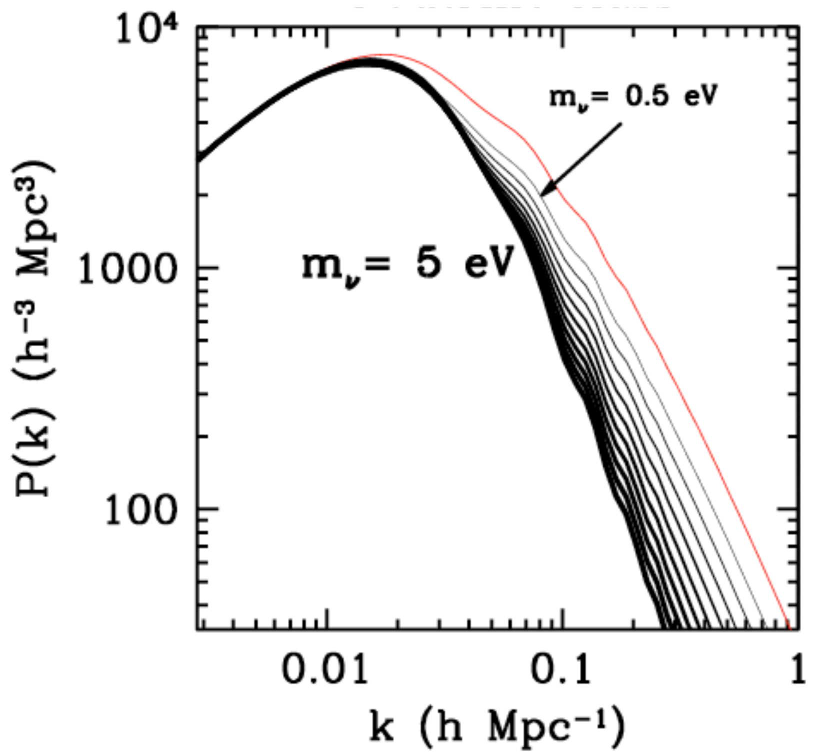
Recall $$\Omega_\nu = 0.02\frac{m_\nu}{1\,\mathrm{eV}}$$

This fraction of the total density does *not* participate in collapse on scales smaller than the freestreaming scale

$$k_{\rm fs}^{-1} \simeq \frac{vt}{a} \simeq \frac{(T/m)H^{-1}}{a}$$

At the relevant time, this scale is 0.02 Mpc^{-1} for a 1eV ν; power on scales smaller than this is suppressed.

Neutrino mass suppresses the power spectrum on small scales

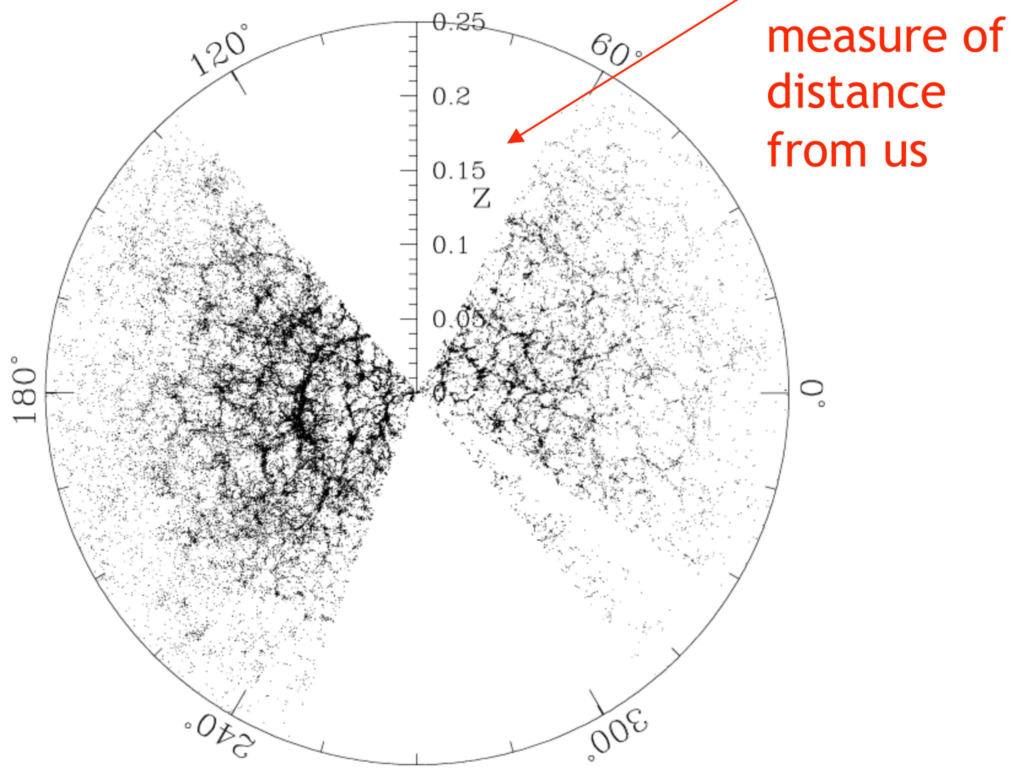


Even for a small neutrino mass, get large impact on structure: power spectrum is excellent probe of neutrino mass

Several Large Galaxy Surveys

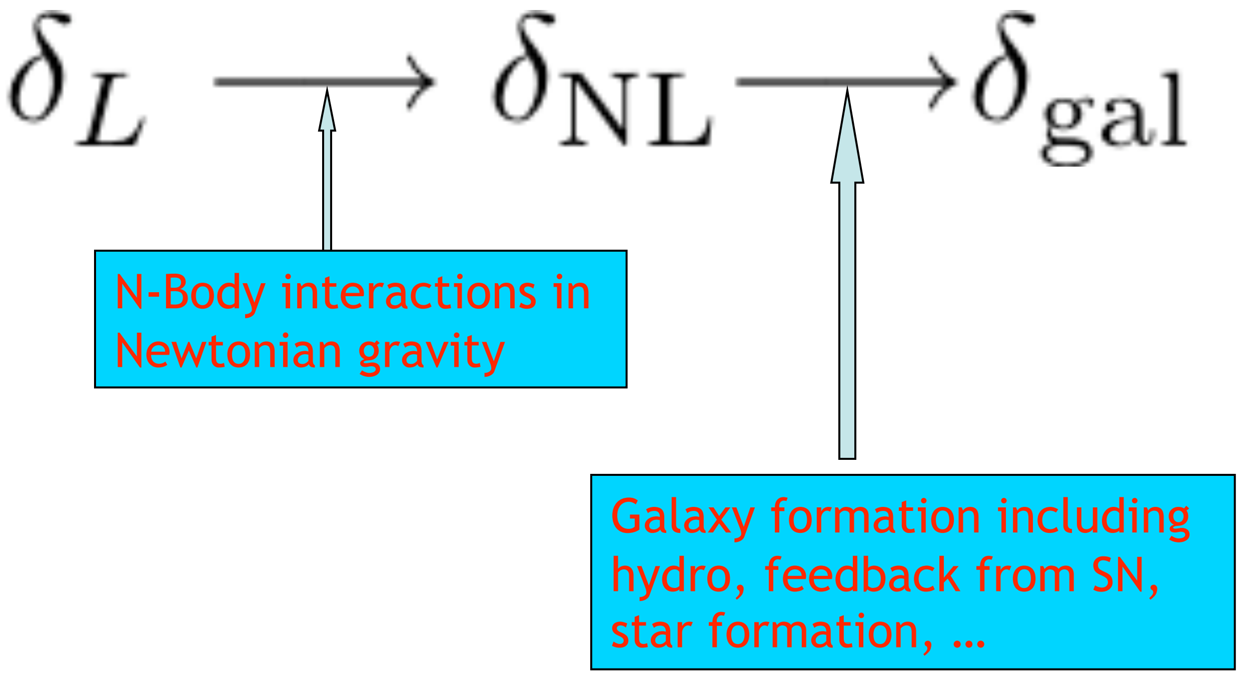
The *Sloan Digital Sky Survey (SDSS)* and the *Two Degree Field (2dF)* both have measured positions and redshifts (which are related to distances) of hundreds of thousands of galaxies

Blanton et al. (2003) (astro-ph/0210215)



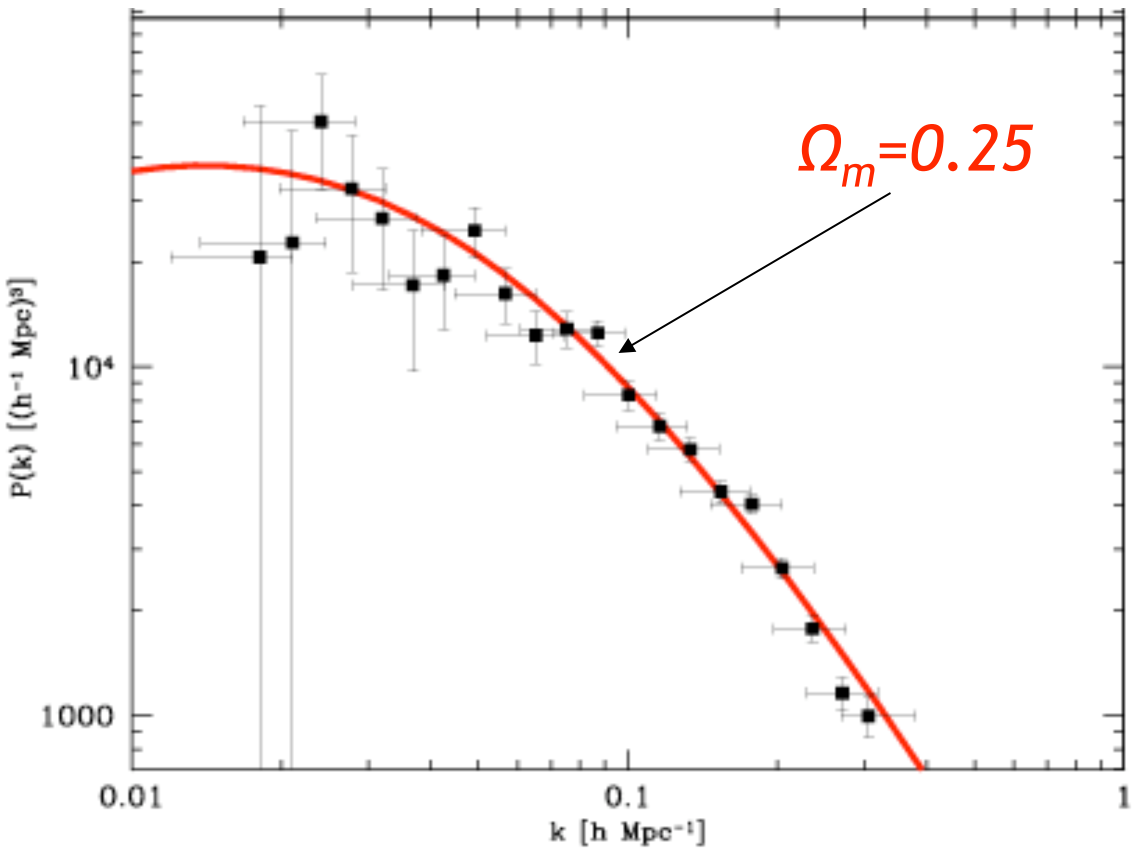
Non-trivial to compare observation to theory

The observables, δ_{gal}, are complicated *functionals* of the easy-to-predict linear matter density field, δ_L.



SDSS Galaxy Power Spectrum

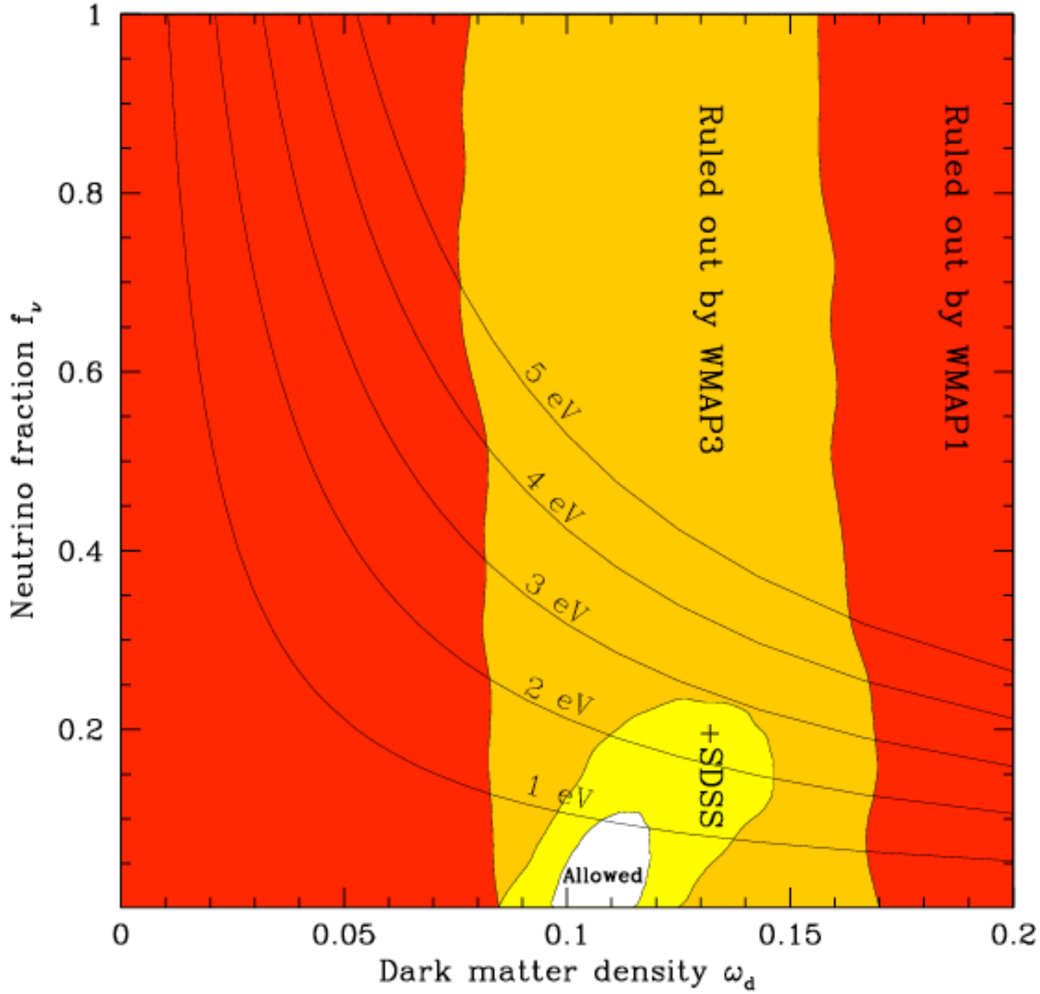
SDSS analysis of >200,000 galaxies



Tegmark et al. 2004

Results

Peaks and troughs in CMB sensitive to matter density: need both CMB and large scale structure to tighten constraints on neutrino mass



Tegmark, et al. 2006

Final Slide

If you want to get your hands dirty check out …

```
http://camb.info/
```

Meet me at the bar tonight if you have a good idea about …

```
The effect of Dark Matter annihilation
on the spectrum of CMB anisotropies
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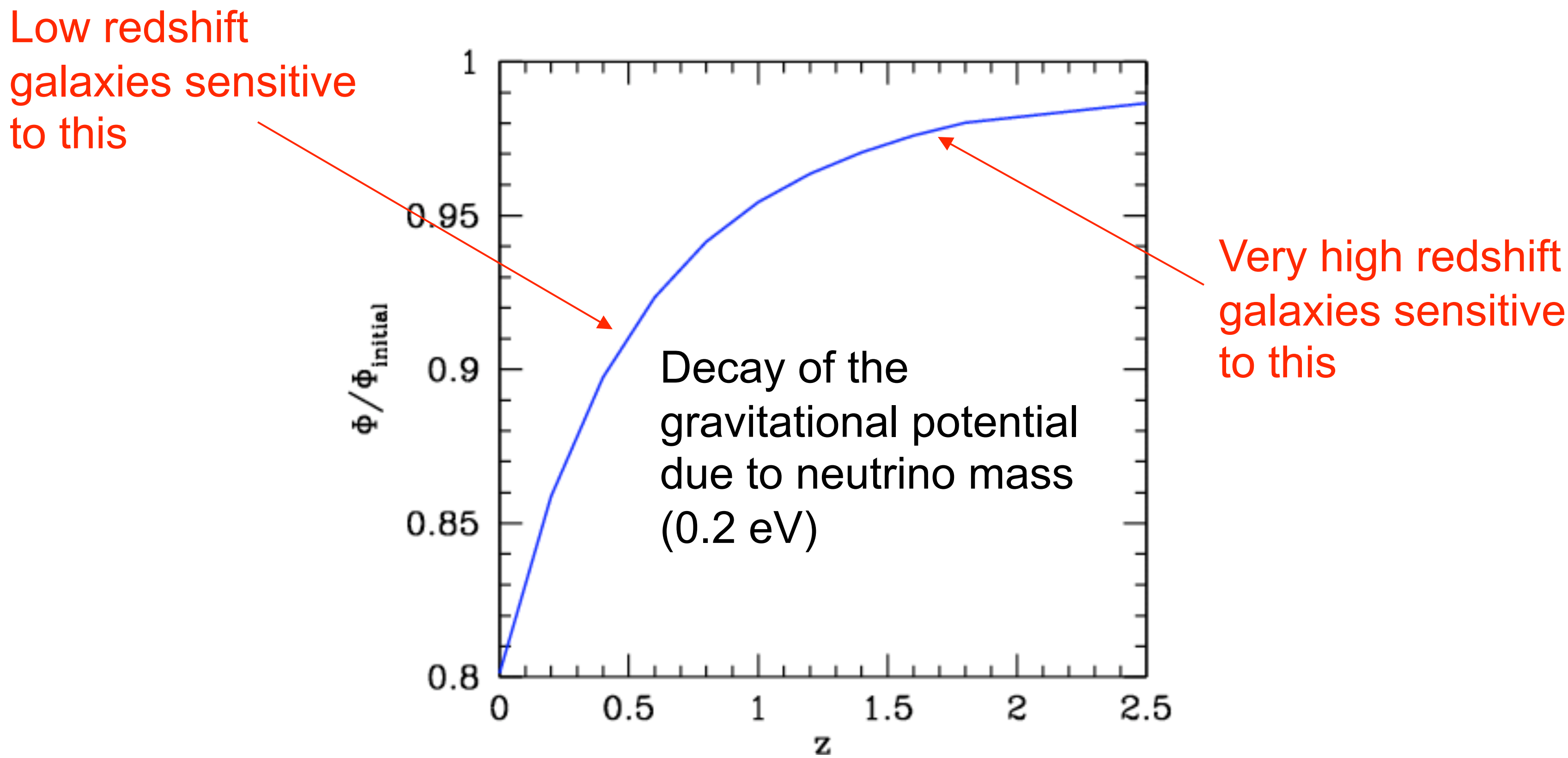
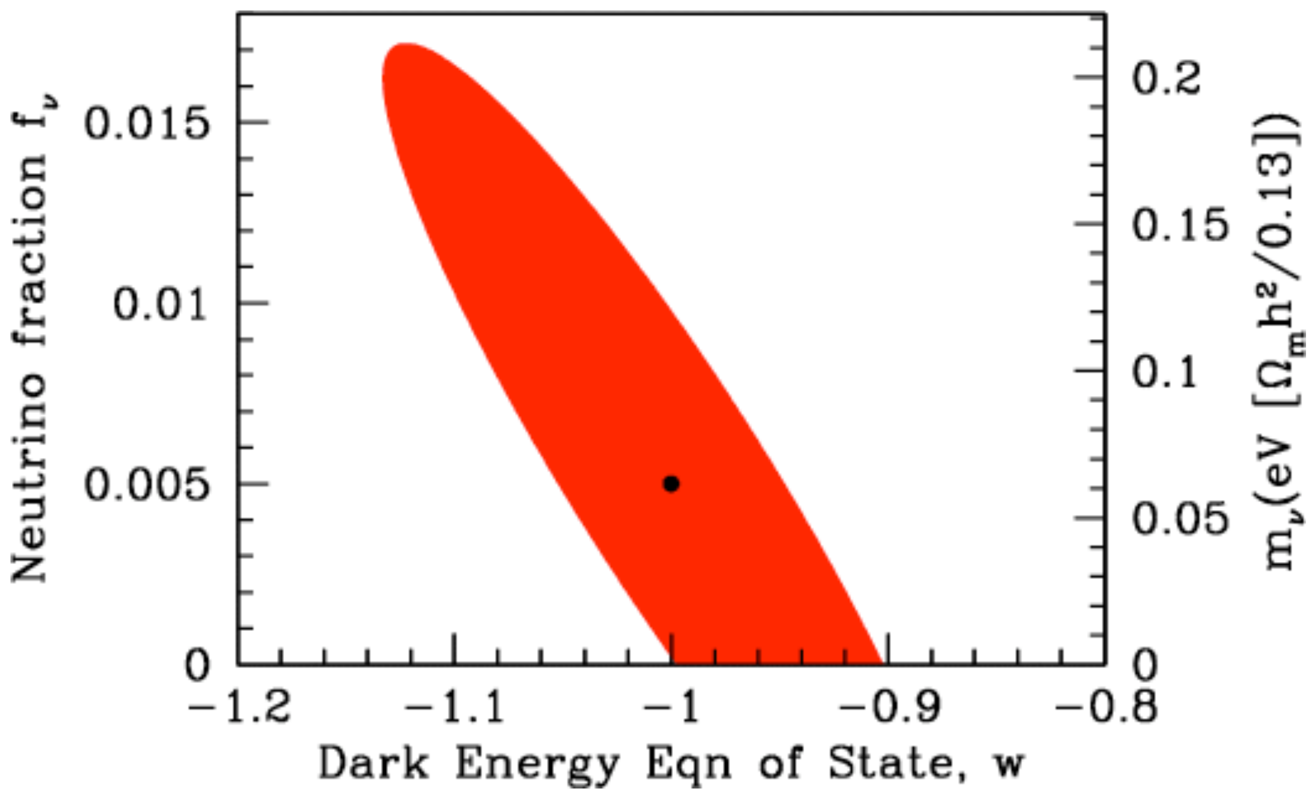
Tomography: Divide Background (Tracer) Galaxies into High and Low Redshift Bins



Even if you're here only to learn about neutrinos, you need to understand dark energy

Projection for deep survey over 1/10 of the sky



Abazjian & Dodelson 2003

Clumping on Scale k

- Dimensionless quantity akin to $l^2 C_l$

$$\Delta^2 \equiv \frac{k^3 P(k)}{2\pi^2}$$

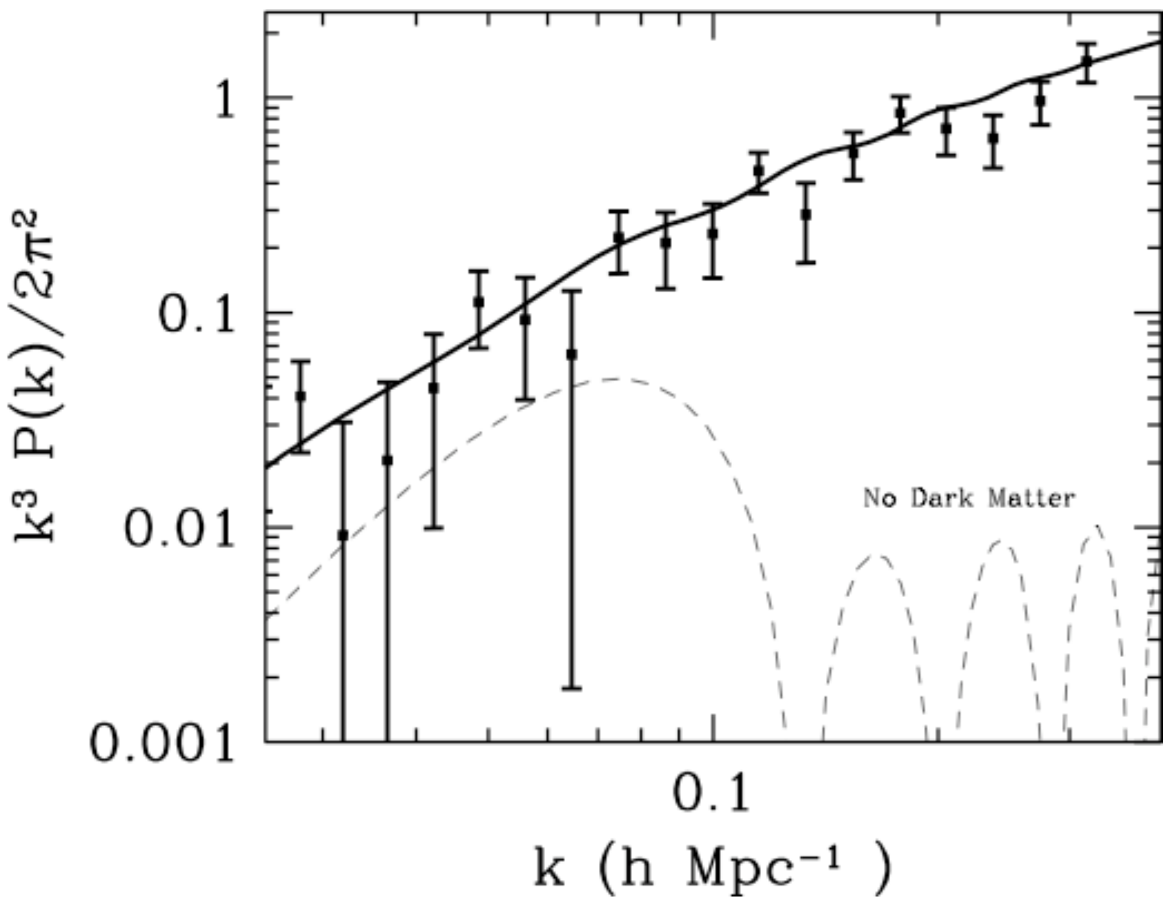
- Variance of density:

$$\left\langle \left(\frac{\delta\rho}{\rho} \right)^2 \right\rangle = \int \frac{dk}{k} \Delta^2(k)$$

- Onset on nonlinearity: $\Delta^2 > 1$

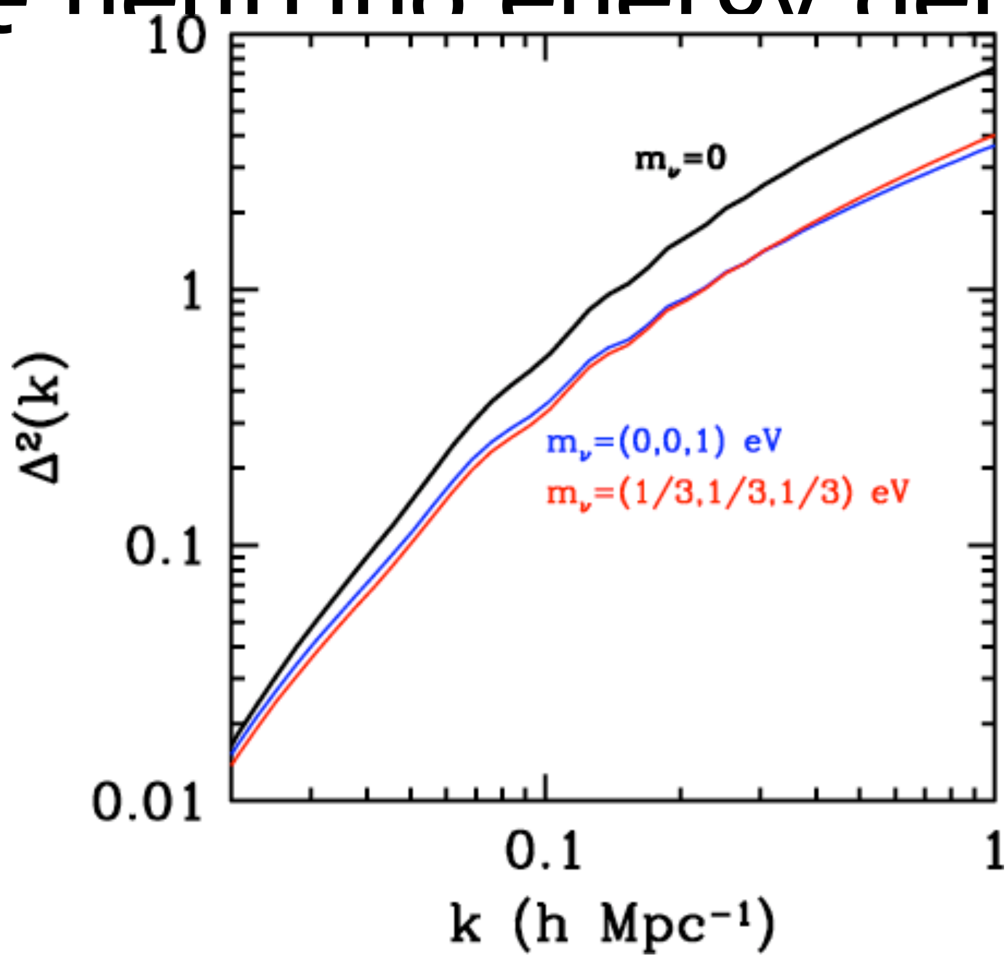
No-Dark-Matter is strongly ruled out

Y-axis shows dimensionless measure of clumpiness; if it stays below one (as it would if there were no DM), no nonlinear structures form in the universe

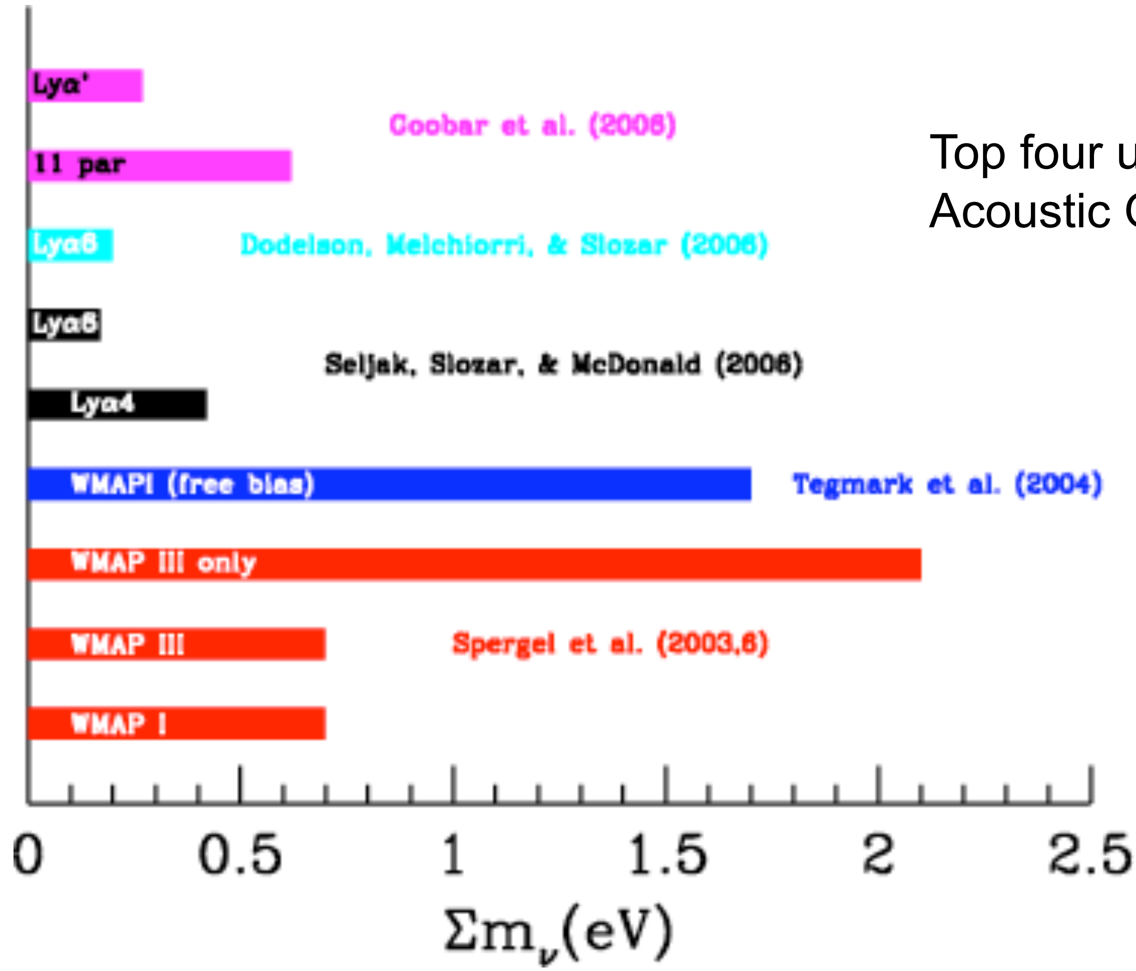


Power spectrum depends only on massive neutrino energy density

Large Scale Structure probes Σm_ν

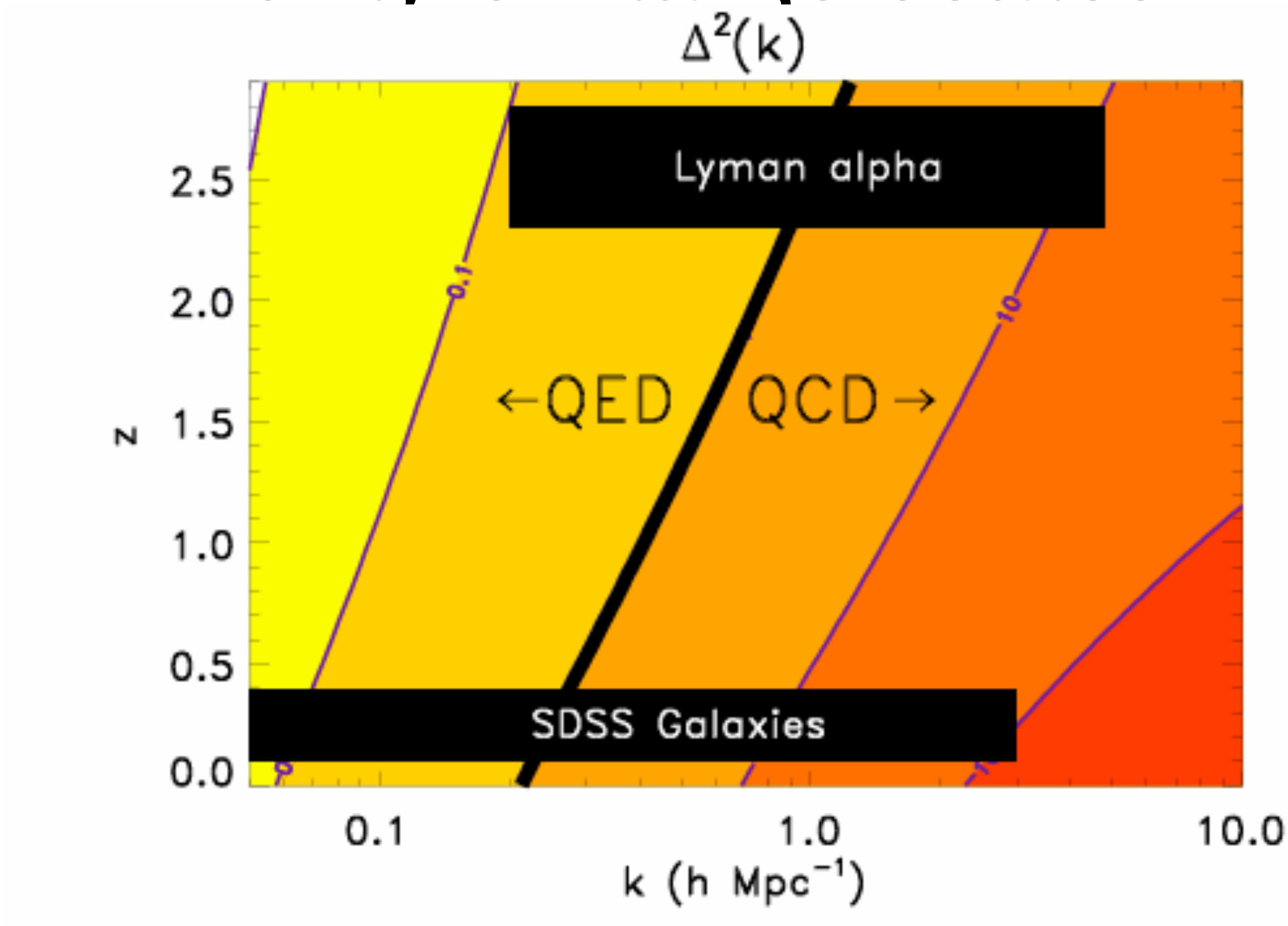


Neutrino Mass constraints are typically sub-eV



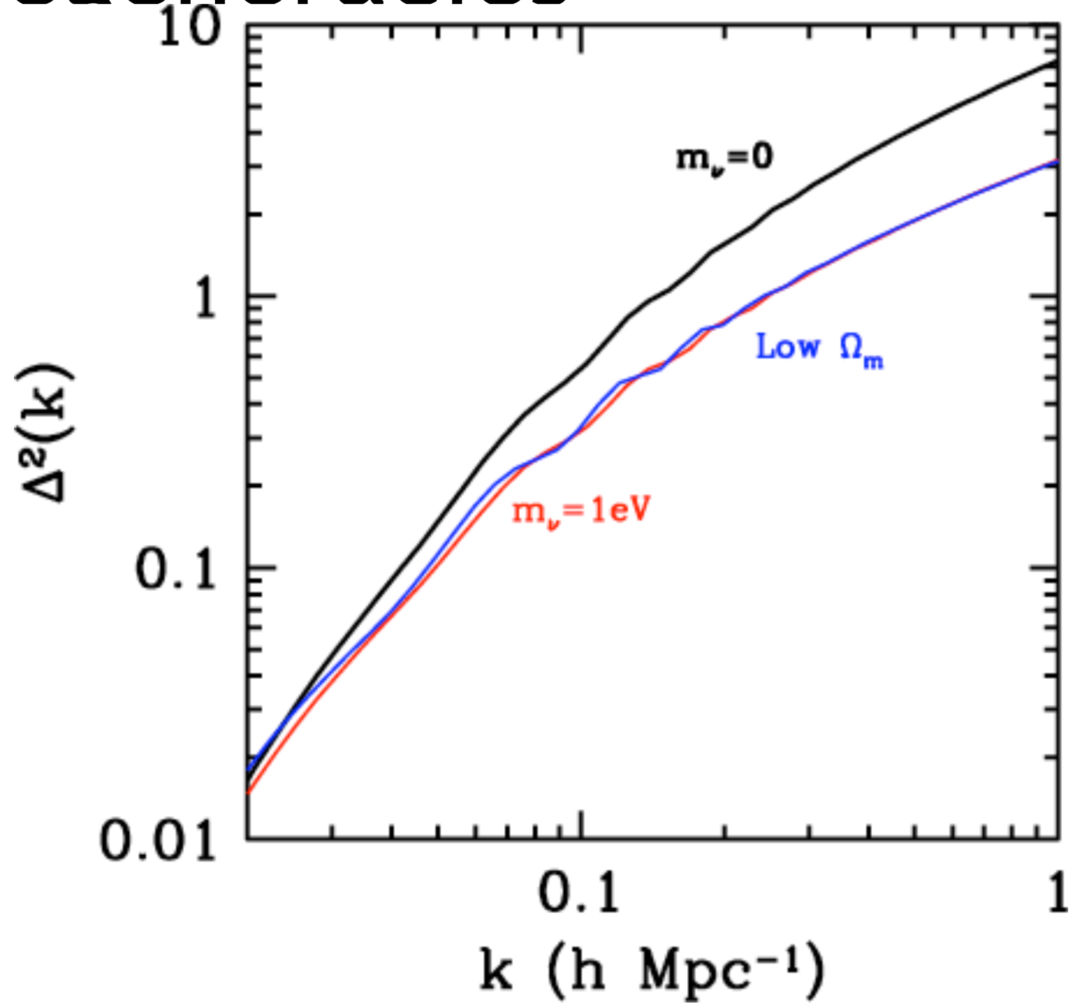
Top four use Baryon Acoustic Oscillations

Comparing to predictions is easy only on large scales

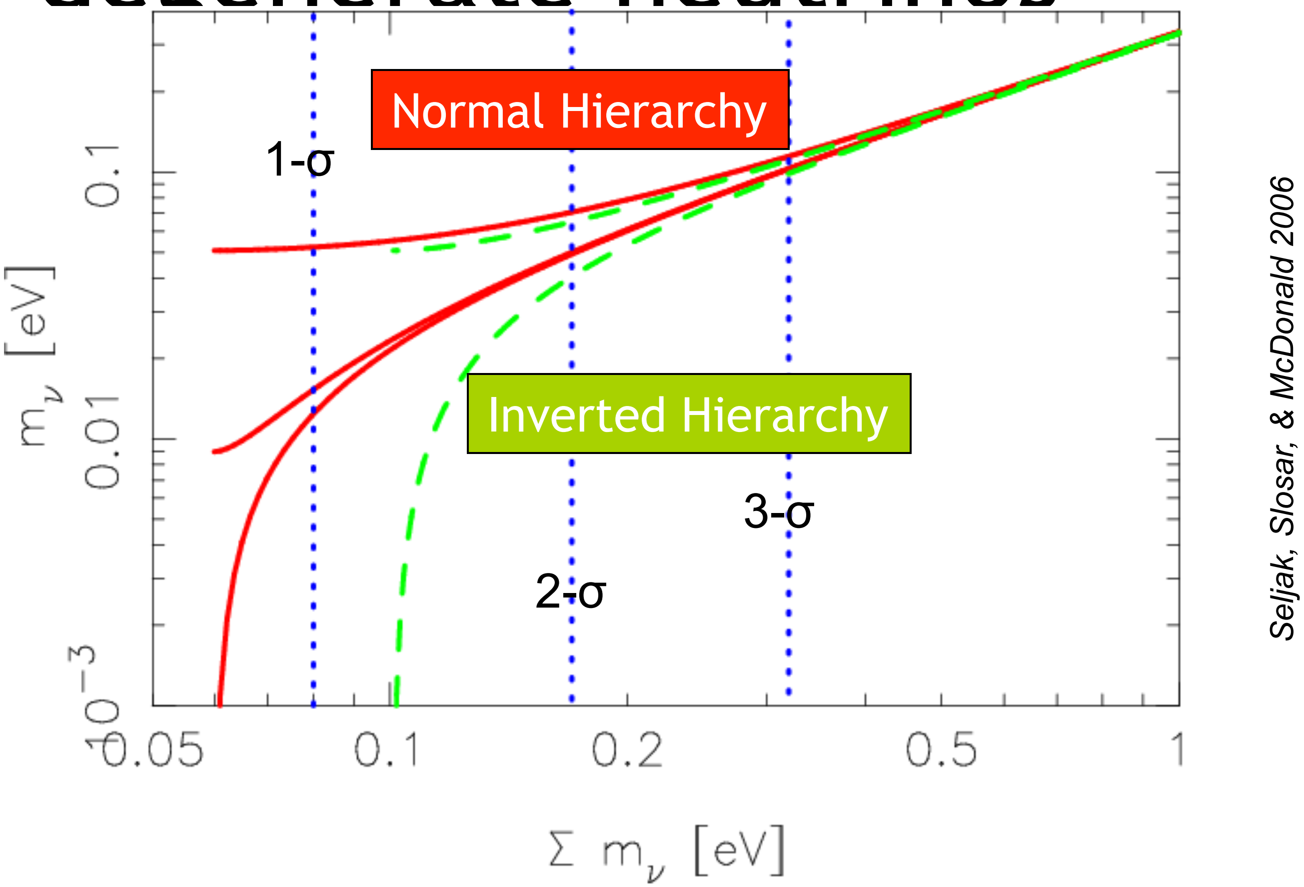


Degeneracies

- ❑ Lowering the matter density suppresses the power spectrum
- ❑ Close to degenerate with non-zero neutrino mass



Most aggressive limit disfavors 3 degenerate neutrinos



Expect Deceleration