Monte Carlo & Event Generators

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CTEQSS-09

This Lecture: Monte Carlo Methods

UA1 Purple Cool-Aid

Missing Energy Events are Confirmed

1) Events are real

2) Not conventional physics
$W \to \tau$, $Q\bar{Q}$, $Z^0 g$
$\hookrightarrow \nu\nu$

3) Not $X \to Z^0 +$ jet(s)
$\hookrightarrow \nu\nu$

4) Not $Z^0 \to X_1 X_2$
$\hookrightarrow$ ν's or stable
$\hookrightarrow$ jet(s)

(ORIGINAL TRANSPARENCY FROM 1986 UA1 'DISCOVERY' OF SUSY

ASPEN CONF, 1986

The Altarelli Cocktail *really* Ellis, Kleiss, Stirling

Table 1. Predicted rates for processes giving large missing transverse energy events passing all event selection cuts.

Data

Process	Events (total)	Events with $L_\tau < 0$	Events with $L_\tau < 0$ and $E_T^{jet} < 40$ GeV
$W \to e\nu$ $W \to \mu\nu$ $W \to \tau\nu \to$ leptons	3.6	2.0	1.4
$W \to \tau\nu$ $\to \nu\bar{\nu}$ + hadrons	36.7	8.0	7.1
$W \to c\bar{s}$	<0.1	<0.1	<0.1
$Z^0 \to \tau^+\tau^-$	0.5	0.1	0.1
$Z^0 \to \nu\bar{\nu}$ (3 neutrino species)	7.4	7.1	5.6
$Z^0 \to c\bar{c}$ and $b\bar{b}$	<0.1	<0.1	<0.1
$c\bar{c}$ and $b\bar{b}$ (direct production)	0.2	0.2	0.2
Jet fluctuations (fake missing energy)	3.8	3.4	3.4
	56	24	
TOTAL	52.2	**20.8** $\pm$ 5.1 $\pm$ 1.0	**17.8** $\pm$ 3.7 $\pm$ 1.0

The Lesson

- Many "negligible" sources of background summed up to explain the data
- The mixing of Standard Model cocktails has become an important component of analyzing collider data
  - relies on a combination of physics tools and measurements
  - event generators are indispensable in this process
- These lectures are focussed on preparing you to do the same at the energy frontier

How much does the $t\bar{t}$ cross section change from TeV to LHC?

- $10\times$
- $100\times$
- $500\times$

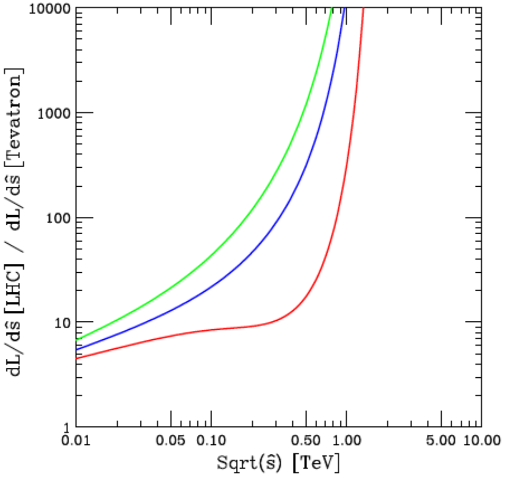
[Kidonakis]

How much does the $t\bar{t}$ cross section change from TeV to LHC?

- $10\times$
- $100\times$ ✓
- $500\times$

[Kidonakis]

$q\bar{q} \to t\bar{t}$ vs $gg \to t\bar{t}$


10000
1000
100
10
1
0.01
0.05
0.10
0.50
1.00
5.00
10.00
Sqrt(ŝ) [TeV]
dL/dŝ [LHC] / dL/dŝ [Tevatron]

How much does the $\widetilde{\chi}^+\widetilde{\chi}^-(m_\chi = 200\ \text{GeV})$ cross section change from TeV to LHC?

- $10\times$
- $100\times$
- $500\times$

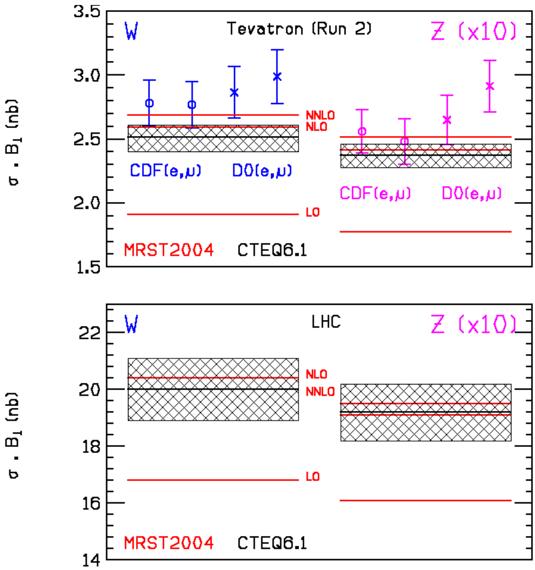
[Pythia]

How much does the $\widetilde{\chi}^{+}\widetilde{\chi}^{-}(m_{\chi} = 200\ \text{GeV})$ cross section change from TeV to LHC?

- $10\times$ ✓
- $100\times$
- $500\times$

[Pythia]

No corresponding gg process at LO


W
Tevatron (Run 2)
Z (x10)
NNLO
NLO
CDF(e,μ)
D0(e,μ)
LO
MRST2004
CTEQ6.1
σ . B$_l$ (nb)
3.5
3.0
2.5
2.0
1.5
W
LHC
Z (x10)
NLO
NNLO
LO
MRST2004
CTEQ6.1
σ . B$_l$ (nb)
22
20
18
16
14

How much does the $Wjjjj$ cross section change from TeV to LHC?

- $10\times$
- $100\times$
- $500\times$

[`MadEvent` parton level, $p_T, k_T > 20$ GeV]

How much does the $Wjjjj$ cross section change from TeV to LHC?

- $10\times$
- $100\times$
- $500\times$ ✓

[`MadEvent` parton level, $p_T, k_T > 20$ GeV]

Many new topologies, lots of phase space

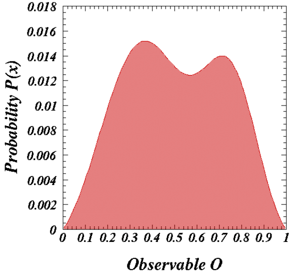
W+4 partons			
TEVATRON		LHC	
Graph	Cross Sect(fb)	Graph	Cross Sect(pb)
Sum	**1035.004**	**Sum**	**577.948**
ug_e+vedggg	112.250	gu_e+vedggg	89.815
gux_e-vexdxggg	112.040	ug_e+vedggg	89.603
uux_e-vexudxgg	112.010	gd_e-vexuggg	45.522
uux_e+veuxdgg	111.900	dg_e-vexuggg	45.342
dux_e-vexddxgg	46.423	uu_e+veudgg	34.174
udx_e+veuuxgg	46.388	dxg_e+veuxggg	15.346
dux_e-vexuuxgg	46.349	gdx_e+veuxggg	15.341
udx_e+veddxgg	46.330	uxg_e-vexdxggg	10.868
gdx_e+veuxggg	40.234	gux_e-vexdxggg	10.866
dg_e-vexuggg	40.122	gg_e+veuxdgg	9.920
udx_e+vegggg	30.906	gg_e+vescxgg	9.907
dux_e-vexgggg	30.867	gg_e-vexsxcgg	9.907
ddx_e-vexudxgg	15.189	gg_e-vexudxgg	9.842
ddx_e+veuxdgg	15.171	du_e+veddgg	8.903
...	...	...	...

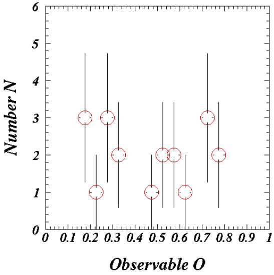
First Steps

- LHC phenomenology begins with re-orienting our Standard Model compass
  - recalibrating our Standard Model tools
- Understanding of the Standard Model relies on Event Generators

Event Generators

Predict multiparticle event configurations in HEP experiments





- P(x) $\Rightarrow$ N performed using Monte Carlo methods
  - Estimate physical quantities (the total cross section)
  - Sample quantities (generate events) one at a time
- Relies on ability to generate (pseudo) random numbers

Monte Carlo

Monte Carlo

Monte Carlo

What is it?

Numerical method for estimating integrals based on “random” evaluations of the integrand

Why do we use it?

- Large dimension of integration variables
- Limits of integration are complicated
- Integrand is a convolution of several functions

Integration & Sampling in HEP

- cross section estimation
- confidence intervals
- systematic uncertainties
- nuisance parameters
- $\cdots$

Nomenclature

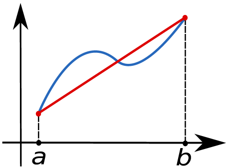
- Some people use *Monte Carlo* to refer to event generators, because they exploit Monte Carlo methods.
- NLO calculations often use the same methods.
- I will try to use *Monte Carlo* as a method, not a program.

Classical Methods

Trapezoidal Rule

$$I \simeq \tfrac{1}{2}(b-a)(f(b)-f(a))$$

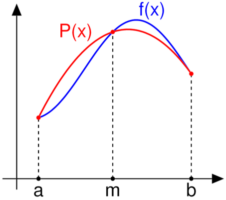
linear fit



Simpson's Rule

$$\begin{aligned} I &\simeq c_b f(b) + c_m f(m) + c_a f(a) \\ &\simeq \frac{1}{3} h(f(b) + 4f(m) + f(a)) \end{aligned}$$

quadratic fit



Limitations

- Well-suited for $d = 1, 2$
- In HEP, typically have to integrate over $d = 100$
- Error estimate: $N^{-k/d}$ scales with d
- Monte Carlo methods independent of d

Monte Carlo Basics

Mean Value Theorem for Integration

$$
\begin{aligned}
I &= \int_{x_1}^{x_2} dx\ f(x) = (x_2 - x_1)\langle f(x)\rangle \quad \left\{\langle \mathcal{O}\rangle = \int dx \frac{d\mathcal{O}}{dx}\right\} \\
&\simeq I_N = (x_2 - x_1)\frac{1}{N}\sum_{i=1}^{N} f(x_i) \\
&\simeq I_N \pm (x_2 - x_1)\sqrt{\frac{(\langle f^2\rangle - \langle f\rangle^2)}{N}}
\end{aligned}
$$

- Randomly select N values of x_i, evaluate $f(x_i)$, and average
- Simple, cheap

Non-uniform sampling can be more efficient:

$$\int_{x_1}^{x_2} dx\ p(x) = 1 \Rightarrow I = \int_{x_1}^{x_2} \{dx\ p(x)\}\, \frac{f(x)}{p(x)}$$

$$I = \left\langle \frac{f(x)}{p(x)} \right\rangle \pm \frac{1}{\sqrt{N}} \sqrt{\left(\left\langle \frac{f(x)^2}{p(x)^2} \right\rangle - \left\langle \frac{f(x)}{p(x)} \right\rangle^2 \right)}$$

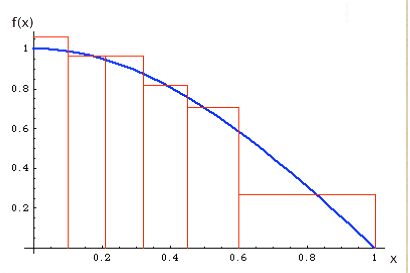
Sample according to $p(x)$ and make f/p as flat as possible (reduce variance)

$$\text{if } f(x) \sim \frac{1}{x} \Rightarrow \text{sample according to } \frac{dx}{x} = d\ln(x)$$

Adaptive Sampling

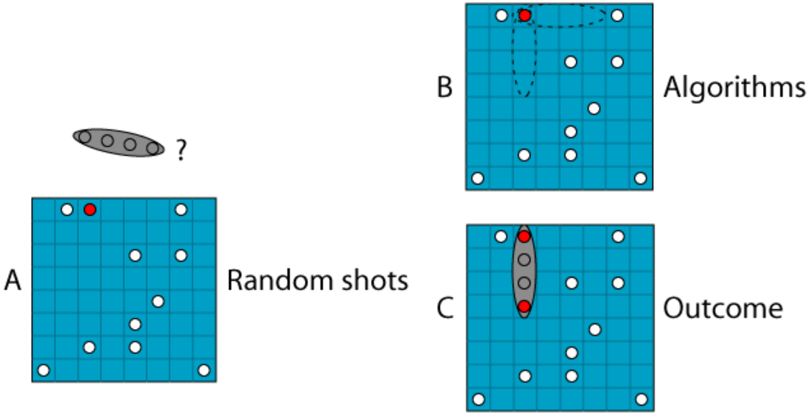
- Importance sampling: choose x_N based on prior knowledge of I_{N-1}
- VEGAS is an adaptive integrator that adjusts step functions to mimic integrand

Vegas

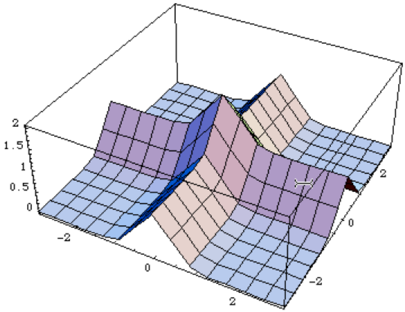


- VEGAS is trying to find $p(x)$ (from previous example) numerically
- Over 30 years old, but still the primary engine in HEP

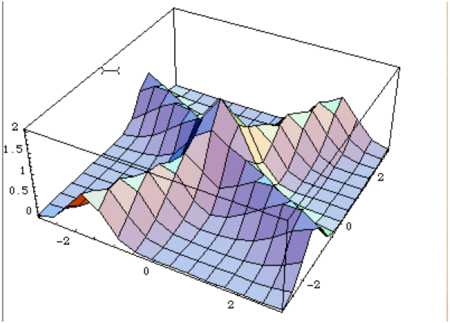
Battleship



`Vegas` in Many Dimensions



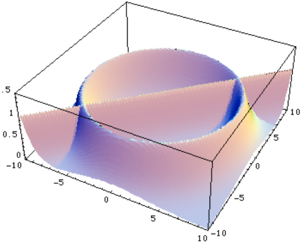
(e) Vegas likes this function: it is aligned with the axes



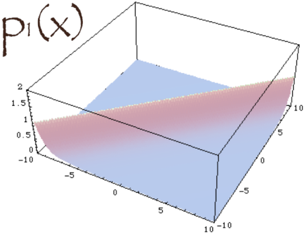
(f) Vegas dislikes this function: but a transformation will align it with the axes

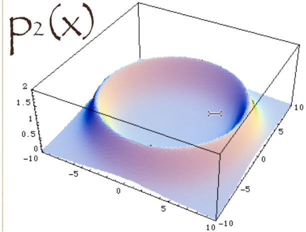
Need to input some information about the behavior of the integrand.
For physical processes, can guess "bad" behavior.

Multi-Channel Integration



- Full integrand is horrendous
- Consider as sum of several channels
- $p(x) = \alpha_1 p_1(x) + (1 - \alpha_1) p_2(x)$





Monte Carlo for Sampling

- Up to this point, only considered MC as a numerical integration method
- If function being integrated is a probability density (positive definite), can convert it to a simulation of physical process = an event generator
- Monte Carlo can explore possible histories when there are many degress of freedom
- Events selected with same frequency as in nature

Should Ukraine Attack Afghanistan?

Single Battle Odds

Attacker rolls:	Defender rolls: 2 dice	1 die
3 dice	Att lose 2: 29.26% (2275/7776) Def lose 2: 37.17% (2890/7776) Each lose 1: 33.58% (2611/7776)	Att lose 1: 34.03% (441/1296) Def lose 1: 65.97% (855/1296)
2 dice	Att lose 2: 44.83% (581/1296) Def lose 2: 22.76% (295/1296) Each lose 1: 32.41% (420/1296)	Att lose 1: 42.13% (91/216) Def lose 1: 57.87% (125/216)
1 die	Att lose 1: 74.54% (161/216) Def lose 1: 25.46% (55/216)	Att lose 1: 58.33% (21/36) Def lose 1: 41.67% (15/36)

1. Attacker can roll a die for each army-1 up to 3
2. Defender can roll a die for each army up to 2
3. Defender wins ties

- Often, you don't know the odds as a table
- Maybe you want to test the odds


Iceland
Scandinavia
Ural
Siberia
Northern Europe
Southern Europe
China
India
Siam
Middle East
Egypt


Iceland
Scandinavia
Siberia
Irkutsk
Mongolia
Great Britain
Northern Europe
Southern Europe
Western Europe
China
India
Siam
Middle East
Egypt


land
Iceland
Scandinavia
Great
Britain
Northern
Europe
Southern
Europe
Western
Europe
Egypt
Middle East
India
Siam
China
Mongolia
Afghanistan
Ural
Siberia
Irkutsk
Yak

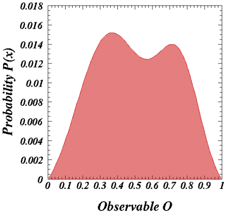

Iceland
Scandinavia
Ural
Siberia
Irkutsk
Great Britain
Northern Europe
Mongolia
Afghanistan
China
Southern Europe
Western Europe
India
Siam
Middle East
Egypt
North Africa
East Africa
INDIAN OCEAN
Indonesia

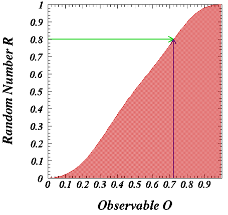

Iceland
Scandinavia
Ural
Siberia
Irkutsk
Great Britain
Northern Europe
Southern Europe
Western Europe
Afghanistan
Mongolia
China
India
Siam
Middle East
Egypt
North Africa
East Africa


Iceland
Scandinavia
Ukraine
Ural
Siberia
Irkutsk
Mongolia
Great Britain
Northern Europe
Southern Europe
Western Europe
Afghanistan
China
India
Siam
Middle East
Egypt
North Africa
East Africa
INDIAN OCEAN

Lesson

- Run more simulations before attacking!





CDF method

Given $f(x) > 0$ over $x_{\text{min}} \leq x \leq x_{\text{max}}$
Prob in $(x + dx, x)$ is $f(x)dx$

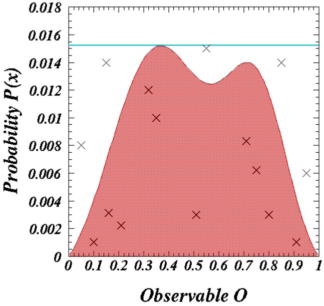
- $\int_{x_{\text{min}}}^{x} f(x)\,\text{dx} = R \int_{x_{\text{min}}}^{x_{\text{max}}} f(x)\,\text{dx}$
- $x = F^{-1}(F(x_{\text{min}}) + R(F(x_{\text{max}}) - F(x_{\text{min}})))$
- assumes $F(x)$, $F^{-1}(x)$ are known
- fraction R of area under $f(x)$ should be to the left of x
- *Realistic $f(x)$ are rarely this nice*

Hit-or-Miss

If $\max[f(x)]$ is known, but not $F^{-1}(x)$, use *hit-or-miss*

1. select $x = x_{\text{min}} + R(x_{\text{max}} - x_{\text{min}})$
2. if $f(x)/f_{\text{max}} \leq$ (new) R, reject x and $\Rightarrow$ 1
3. otherwise, keep x

- Works because probability $f(x)/f_{\text{max}} > R \propto f(x)$
- Acceptable method if $f(x)$ does not fluctuate too wildly
- Often guess at $\max[f(x)]$ and update if a larger estimate is found in a run



$f(x)$ is complicated

Find $g(x)$, with $f(x) \le g(x)$ over x range

- $G(x)$ and its inverse $G^{-1}(x)$ known
- e.g., $\int_{\epsilon}^{z} \mathrm{d}x \frac{1+x^2}{1-x} < \int_{\epsilon}^{z} \mathrm{d}x \frac{2}{1-x} = 2 \ln \left[\frac{1-\epsilon}{1-z} \right]$

1. select an x according to $g(x)$, using Method 1
2. if $f(x)/g(x) \le$ (new) R, reject x and $\Rightarrow$ 1
3. otherwise, keep x

- first step selects x with a probability $g(x)$
- second step retains this choice with probability $f(x)/g(x)$
- total probability to pick a value x is then just the product of the two, i.e. $f(x)\,\mathrm{d}x$

Radioactive Decay

- Know probability $f(t)$ that 'something will happen' (a nucleus decay, a parton branch, a transistor fail) at time t
- *something happens at* t **only** *if it did not happen at* $t' < t$

Equation for nothing $\mathcal{N}(t)$ to happen *up to time t* is ($\mathcal{N}(0) = 1$):

$$-\frac{d\mathcal{N}}{dt} = f(t)\,\mathcal{N}(t) = \mathcal{P}(t)$$

$$\mathcal{N}(t) = \exp\left\{-\int_0^t f(t')\,dt'\right\}$$

$$\mathcal{P}(t) = f(t)\exp\left\{-\int_0^t f(t')\,dt'\right\}$$

- Naive answer $\mathcal{P}(t) = f(t)$ modified by exponential suppression
- In the parton shower, this is the Sudakov form factor

Veto Algorithm

If $F(t)$ and $F^{-1}(t)$ exist:

$$\int_0^t \mathcal{P}(t')\,dt' = \mathcal{N}(0) - \mathcal{N}(t) = 1 - \exp\left\{-\int_0^t f(t')\,dt'\right\} = 1 - R$$

$$F(0) - F(t) = \ln R \quad\Longrightarrow\quad t = F^{-1}(F(0) - \ln R)$$

If not, use **veto algorithm** with a "nice" $g(t)$

1. start with `i = 0` and $t_0 = 0$
2. increment `i` and select $t_i = G^{-1}(G(t_{i-1}) - \ln R)$
3. if $f(t_i)/g(t_i) \leq$ (new) R, $\Rightarrow$ 2
4. otherwise, keep t_i

Unweighted Events

- I have 3 samples of MC events corresponding to different processes.
- Each individual sample has a uniform weight (they have been unweighted).
- How do I select N (uniform weight) events for my cocktail?

Sample	Events	σ (pb)	Weight (pb/evt)	Hit-or-Miss
1	100k	100	10^{-3}	100k
2	300k	60	$.2 \times 10^{-3}$	60k
3	160k	40	$.25 \times 10^{-3}$	40k
Total		200		200k

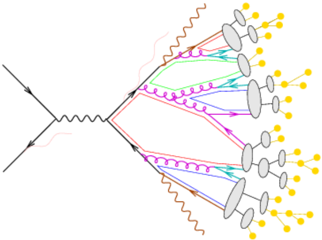
- Select N of these 200k randomly
- Note: the sample with highest weight/evt dominates

MC Overview

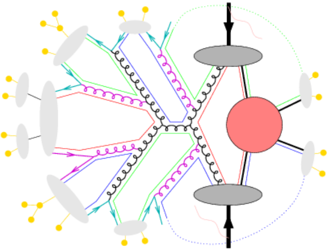
- Use MC to perform integrals and sample distributions
  - Only need a few points to estimate f
  - Each additional point increases accuracy
- Technique generalizes to many dimensions
  - Typical LHC phase space $\sim d^3\vec{p} \times$ 100's particles
  - Error scales as $1/\sqrt{N}$ vs $1/N^{2/d}, 1/N^{4/d}$ (trap,Simp)
- Suitable for complicated integration regions
  - Kinematic cuts or detector cracks
- Can sample distributions where exact solutions cannot be found
- Veto algorithm applied to parton shower

This Lecture: Monte Carlo in Event Generation

Phases of High Energy Collisions

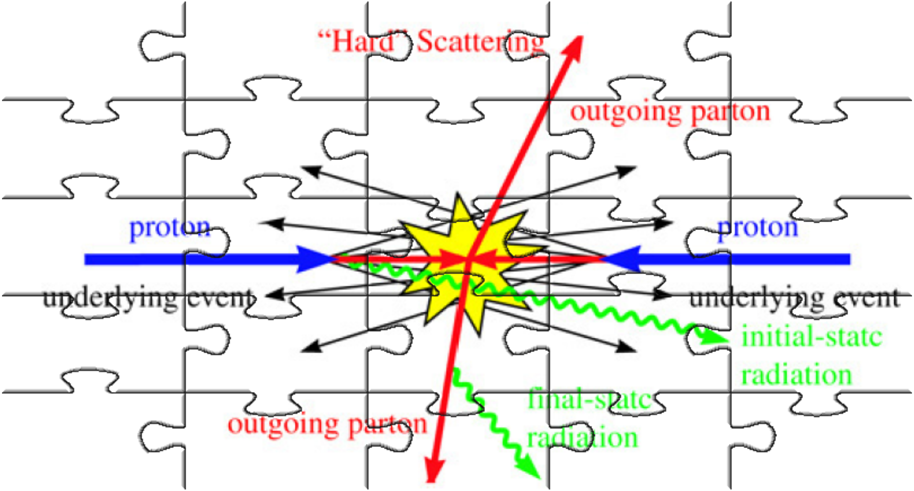


- **hard scattering**
- **initial/final state radiation**
- **partonic decays, $t \to bW$**
- **parton shower evolution**



- **nonperturbative phase**
- **colorless clusters**
- **cluster $\to$ hadrons**
- **hadronic decays**
- **underlying event**

Event Generation is a Puzzle



Hard Scattering center piece first

- Characterizes the rest of the event
- Sets a high energy scale Q
- Fixes a short time scale where partons are free objects
- Allows use of perturbation theory (focus on QCD)
- External partons can be treated as on the mass-shell
    - Valid to $\max[\Lambda, m]/Q$
    - Physics at scales below Q absorbed into parton distribution and fragmentation functions (Factorization Theorem)
- Sets flow of Quantum numbers (Charge, Color)
    - Parton shower and hadronization models use $1/N_C$ expansion
    - Gluon replaced by color-anticolor lines
    - All color flows can be drawn on a piece of paper

Hard Scattering Calculations

- Details of how to calculate in fixed-order perturbation theory have been provided by the other (expert) lecturers
- For the most part, event generators use lowest-order, hard-scattering calculations as their starting point
- When more detailed, tree-level calculations are performed, some care must be taken when adding on parton showers (later)

NOT event generators

- partonic jets: no substructure
- hard, wide-angle emissions only
- colored/fractionally charged states not suitable for detector simulation

Nonetheless, quite useful:

- can guide physics analyses by revealing gross kinematic features
  - Jacobian peak
- can estimate effect of higher-order corrections
- can modify the Lagrangian to implement new models

Towards an Event Generator

HEP Events are approximately modular:

- Events are transformations from $t = -\infty \to t = +\infty$
- Hard Interaction occurs over a short time scale $\Delta t \sim 10^{-2}\text{GeV}^{-1}$
- Perturbation theory ($\alpha_s < \pi$) should work down to time $t = .1 - 1\text{GeV}^{-1}$
- Hadronization on longer time scales
- Particle decays typically on longest time scales

Separation of time scales reduces the complex problem to manageable pieces (modules) which can be treated in series

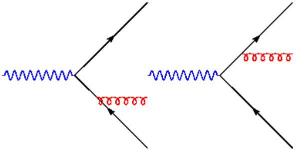
- Previous step sets initial conditions for next one

Next step after hard scatter is the parton shower

Matrix Element to Parton Shower: $\gamma^* \to q\bar{q}g$

Write single gluon emission as:

$$d\sigma(q\bar{q}g) = \sigma_0 \frac{\alpha_s}{2\pi} dz \left\{ \frac{ds_{qg}}{s_{qg}} \left[P_{q\to q}(z) - \frac{s_{qg}}{Q^2} \right] + \frac{ds_{\bar{q}g}}{s_{\bar{q}g}} \left[P_{q\to q}(z) - \frac{s_{\bar{q}g}}{Q^2} \right] \right\}$$



- $\sigma_0 = \sigma(\gamma^* \to q\bar{q})$
- $z = \frac{s_{q\bar{q}}}{Q^2}, P_{q\to q}(z) = \frac{4}{3}\frac{1+z^2}{1-z}$

- $s_{qg} = 2E_q E_g(1 - \cos\theta_{qg})$
- $s_{qg}, s_{\bar{q}g} \to 0$ when gluon is soft/collinear
- $z \to 1$ when gluon is soft ($E_g = (1-z)E_{\text{mother}}$)
- In soft/collinear limit, independent radiation from q and $\bar{q}$

General Result

- $|\mathcal{M}|^2$ involving $q \to qg$ (or $g \to gg$) strongly enhanced whenever emitted gluon is almost collinear
- Propagator factors (internal lines)

$$\frac{1}{(p_q + p_g)^2} \approx \frac{1}{2E_q E_g (1 - \cos\theta_{qg})} \to \frac{1}{E_q {\color{red}E_g} {\color{blue}\theta_{qg}^2}}$$

- soft $E_g \to 0$ +collinear $\theta_{qg} \to 0$ divergences
- dominant contribution to $|\mathcal{M}|^2$
  - the divergence can overcome smallness of α_s
  - expansion parameter must be redefined

Collinear factorization

- $|\mathcal{M}_{p+1}|^2 d\Phi_{p+1} \approx |\mathcal{M}_p|^2 d\Phi_p \frac{dQ^2}{Q^2}\frac{\alpha_s}{2\pi}P(z)dzd\phi$
- DGLAP kernels:
  $P_{q\to q}(z) = C_F\frac{1+z^2}{1-z},\ \ P_{g\to g}(z) = N_C\frac{(1-z(1-z))^2}{z(1-z)}$
- Note the appearance of $d\ln(Q^2)\alpha_s \sim \frac{d\ln(Q^2)}{\ln(Q^2)}$
- the consideration of successive collinear emissions leads to the parton shower picture

Sudakov Form Factor

Variable $t = \ln(Q^2/\Lambda^2)$, $Q^2 \sim E_q E_g / \theta_{qg}^2$ is like a time-ordering

$$d\mathcal{P}_a = \sum_{b,c} \frac{\alpha_{abc}}{2\pi} P_{a\to bc}(z)\, dt\, dz$$

$$\mathcal{I}_{a\to bc}(t) = \int_{z_-(t)}^{z_+(t)} dz\, \frac{\alpha_{abc}}{2\pi} P_{a\to bc}(z)$$

Probability for no emission in $(t, t+\delta t)$: $1 - \sum_{b,c} \mathcal{I}_{a\to bc}(t)\,\delta t$

Over a longer time period, product of no-emission prob's exponentiates:

$$\mathcal{P}_{\text{no}}(t_0, t) = \exp\left\{ -\int_{t_0}^{t} dt' \sum_{b,c} \mathcal{I}_{a\to bc}(t') \right\} = S_a(t) = \frac{\Delta(t, t_c)}{\Delta(t_0, t_c)}$$

Sudakov FF

$$\mathcal{P}_{\mathsf{no}}(t_0, t) = \exp\left\{ -\int_{t_0}^{t} dt' \sum_{b,c} \mathcal{I}_{a\to bc}(t') \right\} = S_a(t) = \frac{\Delta(t, t_c)}{\Delta(t_0, t_c)}$$

Notation: $S_a(t)$ for `Pythia`, $\Delta(t, t_c)$ for `Herwig`

- The exponentiation of emissions is common to resummation calculations
  - Arises when there are two very different scales in the problem (i.e. the scale of the hard collision vs. the scale of soft/collinear emissions)
- The parton shower includes the probability for many soft and collinear gluons to emitted

Probability that a branching of a occurs at t is:

$$\frac{d\mathcal{P}_a}{dt} = -\frac{d\mathcal{P}_{\mathsf{no}}(t_0,t)}{dt} = \left(\sum_{b,c} \mathcal{I}_{a\to bc}(t)\right) \exp\left\{ -\int_{t_0}^{t} dt' \sum_{b,c} \mathcal{I}_{a\to bc}(t') \right\} .$$

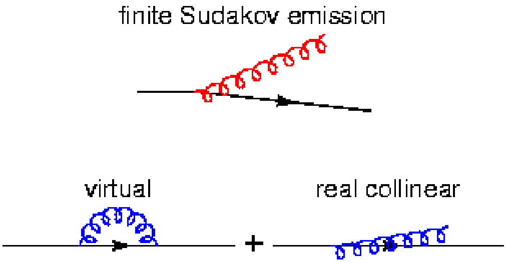
Like Radioactive Decay!

- Can be solved using veto algorithm

$S_a(t) = \mathcal{P}_{\mathsf{no}}(t_0,t)$ is referred to as the Sudakov form factor

- It is the prob. for *nothing* to happen

Diagrammatic Description



- We can only observe emissions (red) above a certain resolution scale (Λ_{QCD}, calorimeter noise?)
- Below resolution scale , singularities (blue) cancel, leaving a finite remnant

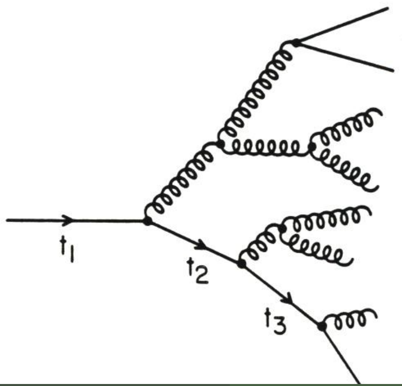
- This cancellation occurs for an infinite tower of possible emissions as long as one considers the leading singularities

Why Leading Log?

- In analytics calculations, the tower is generalizable (NNLL, etc.)
- In parton shower algorithms, a probabilistic interpretation is "easily" implementable for the leading logarithms (LL)
  - LL $\alpha_s \sim \frac{1}{\ln(Q^2)}$
  - LL DGLAP kernels
- Catani and Webber showed that one can get *more* than LL with an appropriate choice of Λ_{QCD} and a K factor

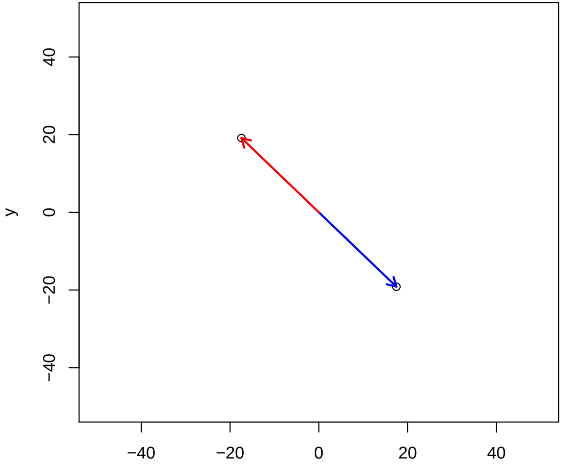
Evolution of the parton shower

- Start parton shower by selecting t from Sudakov FF
- Continue emissions with decreasing t down to the cutoff scale $\sim \Lambda_{\mathrm{QCD}}$

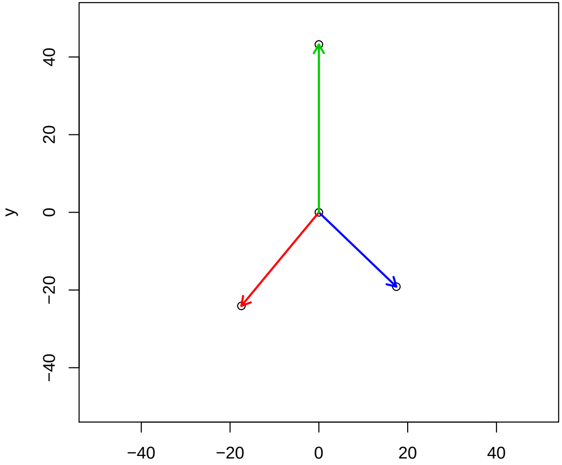


- $t_1 > t_2 > t_3 > t_c$
- (note the ordering)
- $t_c \to \Lambda_{\mathrm{QCD}}$
- Make transition to a model of hadronization at Λ_{QCD}

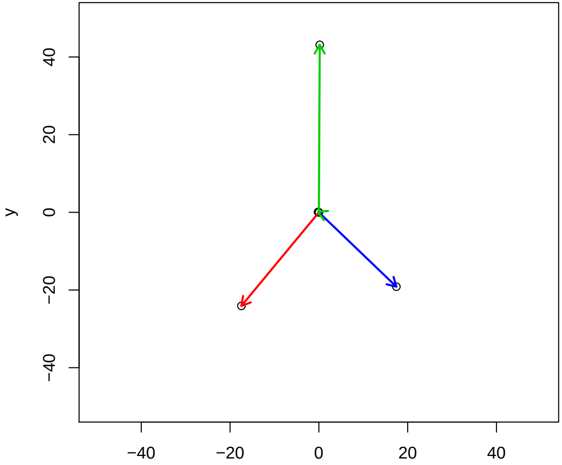
Movie of a Parton Shower



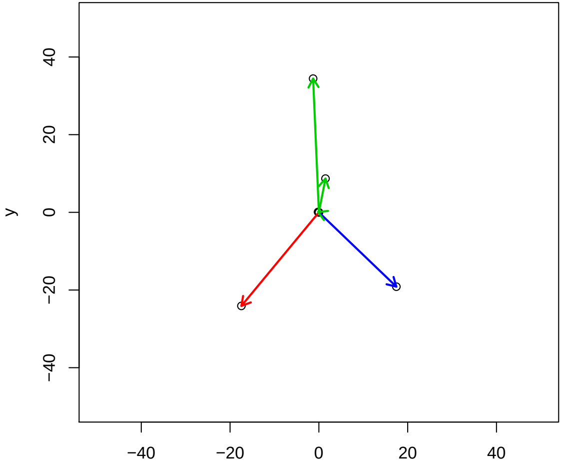
Movie of a Parton Shower



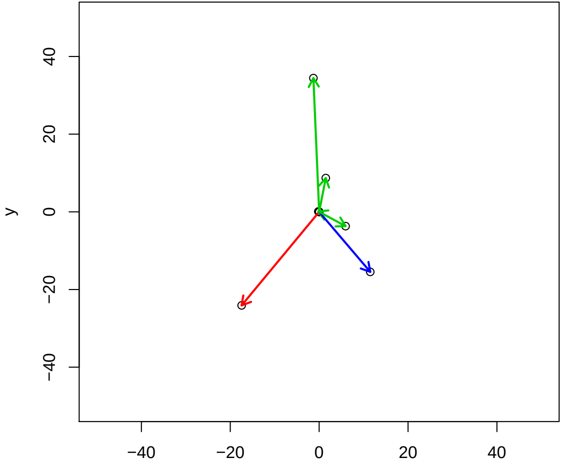
Movie of a Parton Shower



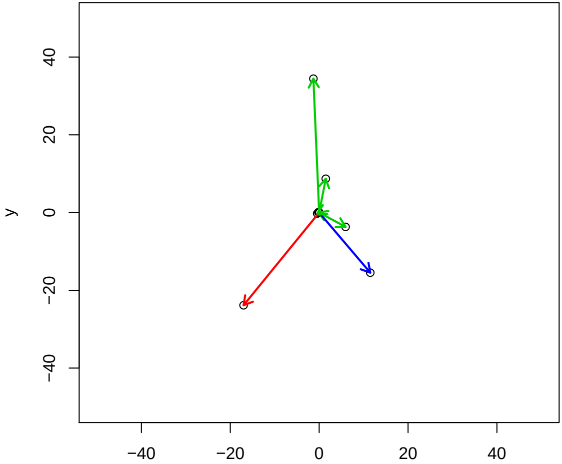
Movie of a Parton Shower



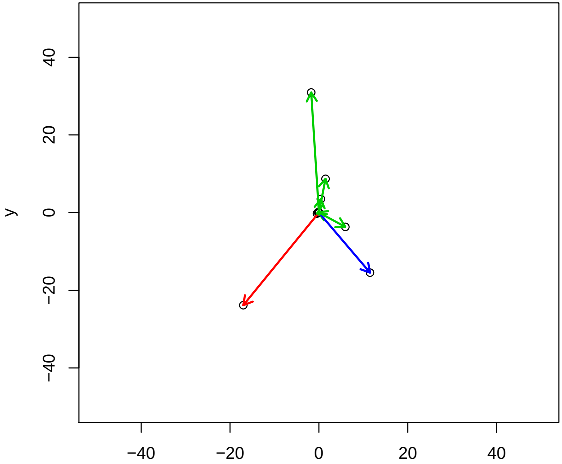
Movie of a Parton Shower



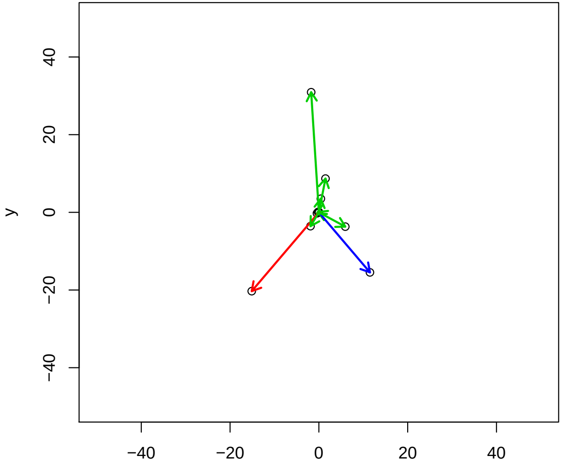
Movie of a Parton Shower



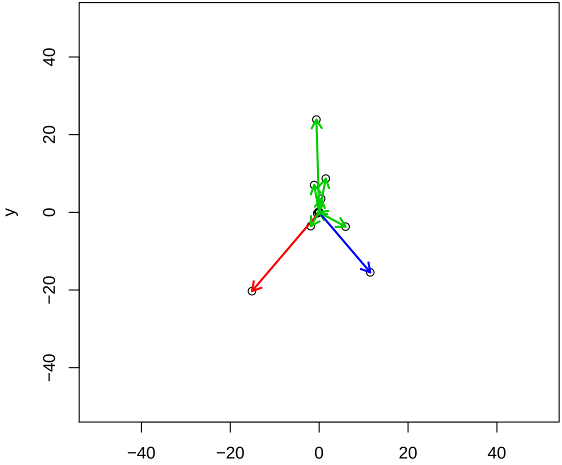
Movie of a Parton Shower



Movie of a Parton Shower



Movie of a Parton Shower

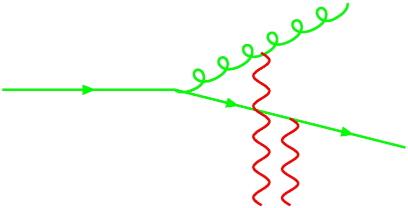


Movie Review

- As this movie demonstrates, the topology generated by the parton shower can be quite complicated
- Such ‘event shapes’ are the forte of the parton shower
    - the bulk of the data cannot be described well by fixed-order calculations
- The total cross section is still given by the hard scattering calculation
    - usually LO
    - experiments will often normalize to data, ignoring higher-order calculations

Color Coherence

Interference effects between emitters are important



Add a soft gluon to a shower of N almost collinear gluons

- incoherent emission: couple to all color

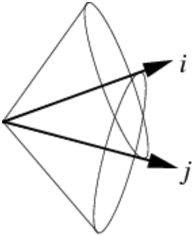
$$|\mathcal{M}_{N+1}|^2 \sim N \times \alpha_s \times N_C$$

- coherent emission: soft (=long wavelength) resolves only overall color charge (that of initial object)

$$|\mathcal{M}_{N+1}|^2 \sim 1 \times \alpha_s \times N_C$$

Color Coherence as Angular Ordering

- Nature chooses coherent emissions
- Choose $Q^2 \to E^2\zeta$
- $\zeta = \dfrac{p_i \cdot p_j}{E_i E_j} = (1 - \cos\theta_{ij}) \sim \theta_{ij}^2/2$



Soft radiation off color lines i, j

$$d\sigma_{n+1} = d\sigma_n \frac{d\omega}{\omega}\frac{d\Omega}{2\pi}\frac{\alpha_s}{2\pi} C_{ij} W_{ij}$$

$$W_{ij} = \frac{1 - \cos\theta_{ij}}{(1 - \cos\theta_{iq})(1 - \cos\theta_{jq})}$$

$$W_{ij} = W^{[i]} + W^{[j]}$$

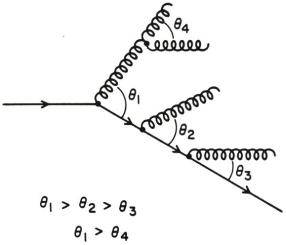
Color Coherence:Derivation

$$W_{ij}^{[i]} = \frac{1}{2}\left(W_{ij} + \frac{1}{1-\cos\theta_{iq}} - \frac{1}{1-\cos\theta_{jq}}\right)$$
$$= \frac{1}{2(1-\cos\theta_{iq})}\left(1 + \frac{\cos\theta_{iq}-\cos\theta_{ij}}{1-\cos\theta_{jq}}\right)$$

Average over azimuthal angle. Choose:

$$\hat{i} = \hat{z} \qquad \hat{j} = \sin\theta_{ij}\hat{x} + \cos\theta_{ij}\hat{z}$$
$$\hat{q} = \sin\theta_{iq}(\cos\phi_{iq}\hat{x} + \sin\phi_{iq}\hat{y}) + \cos\theta_{iq}\hat{z}$$
$$\cos\theta_{jq} = \hat{j}.\hat{q} = \sin\theta_{ij}\sin\theta_{iq}\cos\phi_{iq} + \cos\theta_{ij}\cos\theta_{iq}$$
$$\left\langle \frac{1}{1-\cos\theta_{jq}} \right\rangle = \frac{1}{|\cos\theta_{iq}-\cos\theta_{ij}|}$$
$$\left\langle W_{ij}^{[i]} \right\rangle = \frac{1}{1-\cos\theta_{iq}}\theta(\cos\theta_{iq}-\cos\theta_{ij})$$

θ-Ordering



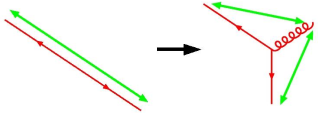
- On average, emissions have decreasing angles w.r.t. emitters
- A strict implementation of this leads to a dead-zone where no radiation occurs ($\Lambda_{\text{QCD}} \sim E_{\text{cut}}\theta_{\text{cut}}$) (`Herwig`)
  - Can be corrected case-by-case, but is complicated

- Decreasing angles can also be enforced with other evolution variables (`Pythia-mass`)
- Another approach is to consider dipole radiation (`Ariadne`, `Pythia-new`)

Generalised Dipoles

- Color charges form dipoles, which beget other dipoles
- $dn_{\text{dipole}} = \alpha_{\text{eff}} \frac{dk_\perp^2}{k_\perp^2} dy = \alpha_{\text{eff}} d\ln(k_\perp^2) dy$
  - $E = k_\perp \cosh(y) \leq \frac{\sqrt{s}}{2}$ ($\sqrt{s}$ is dipole mass)
  - rapidity range $\Delta y \approx ln\left(\frac{s}{k_\perp^2}\right)$

The emission of the first gluon splits the original color dipole into two dipoles which radiate independently



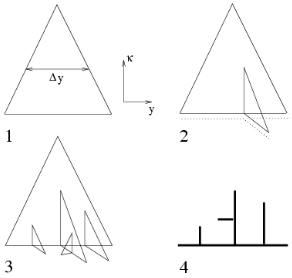
Oragami

- emission of a photon leaves the electromagnetic current unchanged except for small recoil effects
- emission of a gluon changes the current, however:

$$dn(q, g_1, g_2, \bar{q}) = dn(q, g_1, \bar{q})\,[dn(q, g_2, g_1) + dn(g_1, g_2, \bar{q}) - \epsilon]$$

Shower can be traced in origami diagram (triangular phase space):

$\kappa = \ln(k_T^2)$



1. Before emission
2. 1st emission at κ_1
3. After several emissions
4. Bottom view

p_T Ordered Shower

- Retain parton shower evolution
  - $g \to q\bar{q}$ is natural (not so in dipole evolution)
  - easy to generalize to initial state radiation
- Evolution variable $p_T^2 = z(1-z)m^2$
- *Coherence* from choosing dipole frame to determine kinematics
  - Effectively, the boost from the dipole to lab frame "orders" the emissions

Leading Log and Beyond

Neglecting Sudakovs, rate of one emission is:

$$\begin{aligned}\mathcal{P}_{\mathsf{q}\to\mathsf{qg}} &\approx \int \frac{dQ^2}{Q^2}\int dz\,\frac{\alpha_\mathsf{s}}{2\pi}\frac{4}{3}\,\frac{1+z^2}{1-z} \\ &\approx \alpha_\mathsf{s}\ \ln\left(\frac{Q^2_{\mathsf{max}}}{Q^2_{\mathsf{min}}}\right)\ \frac{8}{3}\ \ln\left(\frac{1-z_{\mathsf{min}}}{1-z_{\mathsf{max}}}\right) \sim \alpha_\mathsf{s}\ \ln^2\left(\frac{Q^2_{\mathsf{max}}}{Q^2_{\mathsf{min}}}\right)\end{aligned}$$

Rate for n emissions is of form:

$$\mathcal{P}_{\mathsf{q}\to\mathsf{q}n\mathsf{g}} \sim \left(\mathcal{P}_{\mathsf{q}\to\mathsf{qg}}\right)^n \sim \alpha_\mathsf{s}^n\ \ln^{2n}$$

Next-to-leading log (NLL): include $\alpha_\mathsf{s}^n\ \ln^{2n-1}$

No completely NLL generator, but

- energy-momentum conservation (and "recoil" effects)
- coherence
- scale choice $\alpha_s(p_\perp^2)$
  - absorbs singular terms $\propto \ln z, \ln(1-z)$ in $\mathcal{O}(\alpha_s^2)$ splitting kernels $P_{q\to qg}$ and $P_{g\to gg}$
- ...

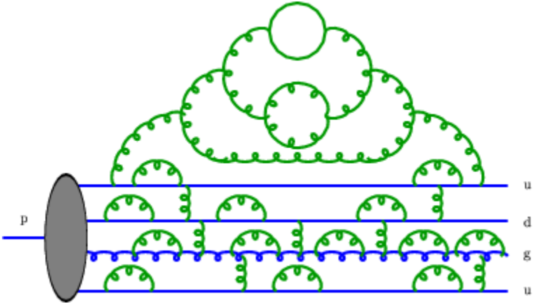
$\Rightarrow$ far better than naive, analytical LL

Initial State Evolution

- So far, have considered final state radiation (FSR)
  - the evolution of the fragmentation functions $D_{h/i}(z, Q^2)$
- The initial state partons of a hard collision can also radiate (ISR)
  - the evolution of the parton distribution functions $f_{i/h}(x, Q^2)$

Parton Distribution Functions

Hadrons are composite, with time-dependent structure:



$f_i(x, Q^2) =$ number density of partons i
at momentum fraction x and probing scale Q^2

$$\frac{df_b(x, Q^2)}{d(\ln Q^2)} = \sum_a \int_x^1 \frac{dz}{z}\, f_a(x', Q^2)\, \frac{\alpha_s}{2\pi}\, P_{a\to bc}\left(z = \frac{x}{x'}\right)$$

Initial-State Shower Basics

- Parton cascades in hadron are continuously born and recombined
- A hard scattering probes fluctuations up to Q^2
- Hard scattering inhibits recombination of the cascade

- Event generation could be addressed by **forwards evolution:** pick a complete partonic set at low Q_0 and evolve, see what happens
- **Inefficient**
  1. have to evolve and check for *all* potential collisions
  2. difficult to steer the production e.g. of a narrow resonance

Backwards evolution

Start at hard interaction and trace what happened "before"
Recast:

$$\frac{df_b(x,Q^2)}{dt} = \sum_a \int_x^1 \frac{dz}{z}\, f_a(x',Q^2)\, \frac{\alpha_\mathsf{s}}{2\pi}\, P_{a\to bc}(z)$$

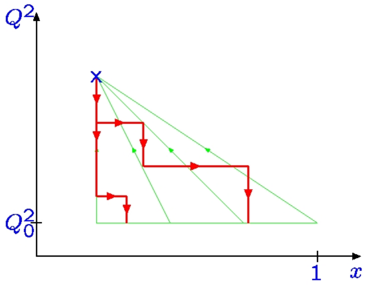
with $t = \ln(Q^2/\Lambda^2)$ and $z = x/x'$
To:

$$\mathrm{d}\mathcal{P}_b = \frac{df_b}{f_b} = |dt| \sum_a \int dz\, \frac{x' f_a(x',t)}{x f_b(x,t)}\, \frac{\alpha_\mathsf{s}}{2\pi}\, P_{a\to bc}(z)$$

- solve for *de*creasing t, i.e. backwards in time
- high Q^2 moving towards lower Q^2
- Sudakov form factor $\exp(-\int \mathrm{d}\mathcal{P}_b)$

Initial State Evolution

- $p_1 \to p_2 + k, p_1^2 = p_2^2 = 0 \Rightarrow k^2 = (p_1 - p_2)^2 = -2p_1 \cdot p_2 < 0$
- Backwards (from hard scatter) evolution of partons with virtualities increasing $\to 0$
- Since backwards, must normalize to the incoming flux of partons



- Hard scattering is characterized by large Q^2, small x
- Valence quarks characterized by large x, small virtualities $Q_0 \sim \Lambda_{\text{QCD}}$

Still *NOT* an Event Generator

- By the end of the parton shower, we have nearly exhausted our ability to apply perturbation theory
  - + Have a description of jet structure
  - + Can ask questions about energy flow and isolation
  - + See if kinematic features survive
- This is still not enough
  - − Don't know response of detector to a soft quark/gluon
  - − Cannot tag a b quark
  - − Can't ask about charged tracks or neutrals
- **Next step is into the Brown Muck**

Parton Shower Summary

- Modern PS models are very sophisticated implementations of perturbative QCD
- Derived from factorization theorems of full gauge theory

- Accelerated electric and color charges radiate
- Parton Shower development encoded in Sudakov FF
- Performed to LL and some sub-LL accuracy with exact kinematics
- Color coherence leads to angular ordering of shower
- Still need hadronization models to connect with data
- Shower evolves virtualities of partons to a low enough values where this connection is possible

This Lecture: Phenomenological Models

Outline

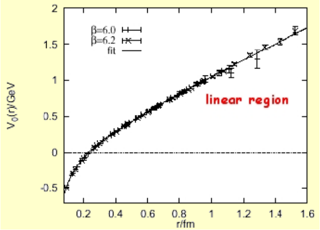
- Hadronization
  - string
  - cluster
- Underlying Event
  - parametrizations
  - multiple-interactions
- The Event Generator Programs
- New Developments

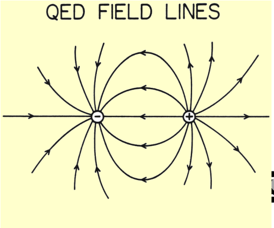
Hadronization

- QCD partons are free only on a **very short time scale**
- **Hadrons** are the physical states of the strong interaction
- Need a description of how partons are confined
- Lacking a theory, we need a model
  - **enough** variables to fit data
  - **few enough** that there is some predictability
  - start related to the end of the parton shower
  - Use **basic** understanding of QCD

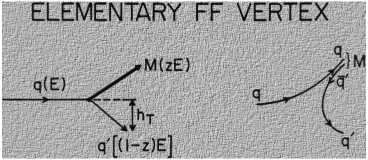
QCD is a confining theory

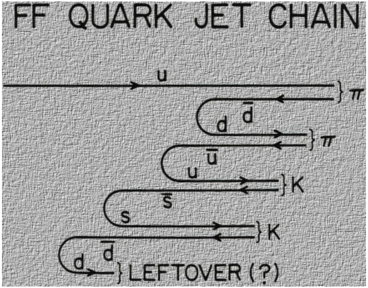
- Linear potential $V_{\text{QCD}}(r) \sim kr$
  - Confirmed by Lattice, Spectroscopy, Regge Trajectories
- Gluons are self-coupling
  - Field lines contract into Flux-tubes
  - Analogy with field behavior inside of superconductors
- Over time, 2 phenomenological models have survived
  - **cluster**
  - **Lund string**
- Not exactly Orthogonal, Exhaustive



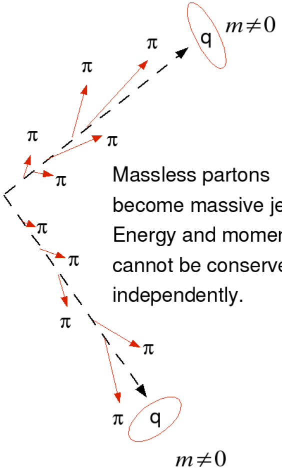


Independent Fragmentation





- FF = Feynman-R. Field
- pure phenomenological model
- imagine $q\bar{q}$ pairs tunnel from the vacuum to dress bare quark
- $f_{q\to h}(z)$ is probability $q \to h$ with fraction z of some E/p variable
- $f_{g\to h}(z)$? $g \to q\bar{q}$?
- Lorentz invariant? (E_q)
- Useful for its time



Massless partons become massive jets. Energy and momentum cannot be conserved independently.

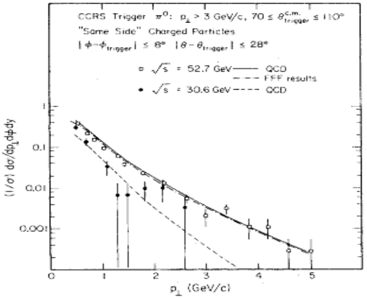
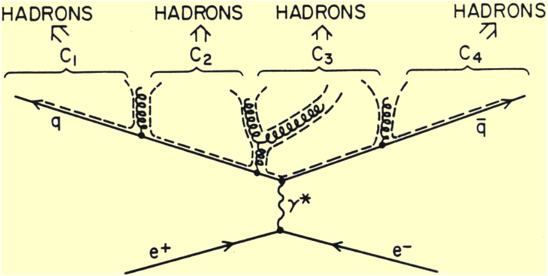


FIG. 15. Toward-side correlation measurements from CCRS collaboration (Ref. 75) together with the predictions of the QCD approach with $\Lambda = 0.4$ GeV/c and the results of the quark–quark black-box model of FFF. Possible background contributions from the fragmentation of the beam and target are *not* included.

Preconfinement

- **Perturbative** evolution of quarks and gluons organizes them into clumps of color-singlet clusters
- In PS, color-singlet pairs end up close in phase space



- Cluster model takes this view to the extreme
- Color connections induce correlations to conserve E, p

Cluster hadronization in a nutshell

- Nonperturbative $g \to q\bar{q}$ splitting ($q = uds$) isotropically Here, $m_g \approx 750\,\text{MeV} > 2m_q$.
- Cluster formation, universal spectrum
- Cluster fission until

$$M^p < M^p_{\text{fiss}} = M^p_{\text{max}} + (m_{q1} + m_{q2})^p$$

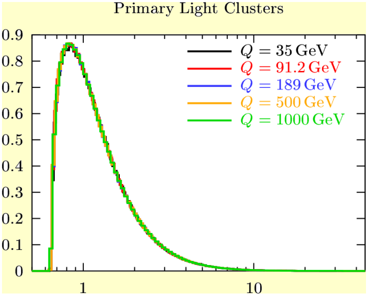
where masses are chosen from

$$M_i = \left[\left(M^P - (m_{qi} + m_{q3})^P\right) r_i + (m_{qi} + m_{q3})^P\right]^{1/P}$$

with additional phase space constraints

- Cluster decay
  - isotropically into pairs of hadrons
  - simple rules for spin, species

Cluster Fission

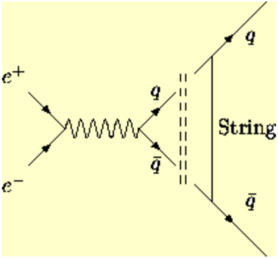


- Mass spectrum of color-singlet pairs asymptotically independent of energy, production mechanism
- Peaked at low mass
- Broad tail at large mass

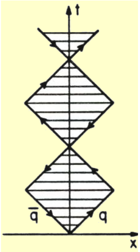
- Small fraction of clusters heavier than typical
  - $\Rightarrow$ Cluster fission (string-like)
- Fission threshold becomes crucial parameter
  - 15% of primary clusters split
  - produces 50% of hadrons

Lund String Model

String=color flux tube is stretched between q and $\bar{q}$



- Classical string will oscillate in space-time
- Endpoints $q, \bar{q}$ exchange momentum with the string



- **Quantum Mechanics:** string energy can be converted to $q\bar{q}$ pairs (tension $\kappa \sim$ 1 GeV/fm)
- dProb/dx/dt = (constant)exp$(-\pi m^2/\kappa)$ **[WKB]**
  - $u : d : s : qq = 1 : 1 : 0.35 : 0.1$
- $dP_n(\{p_j\}; P_{tot}) = \prod^n N_j d^2 p_j \delta({p_j}^2 - {m_j}^2)\delta(\sum^n p_j - P_{tot})\,\exp(-bA)$

String Break-Up

The derivation of the tunnelling probability is the same as Schwinger's for e^+e^- pair production in a static field, but $V(z) = \kappa z$ (QCD potential is linear)

$$\begin{aligned}\Psi(\ell = p_T/\kappa) &= \Psi(0)\exp\left(-\int_0^\ell dz\sqrt{p_T^2 - (\kappa z)^2}\right)\\ &= \Psi(0)\exp\left(-\frac{p_T^2}{\kappa}\int_0^\pi d\theta\sin^2\theta\right)\\ &= \Psi(0)\exp\left(-\frac{\pi p_T^2}{2\kappa}\right)\end{aligned}$$

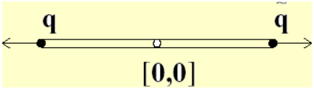
Tunnelling Prob

$$\propto \Psi^*\Psi \Rightarrow \frac{1}{\pi}\exp\left(-\frac{\pi p_T^2}{\kappa}\right)$$

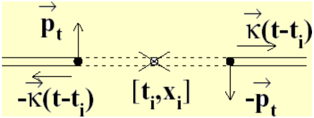
$$p_T^2 \to p_T^2 + m^2$$

Hadron Formation

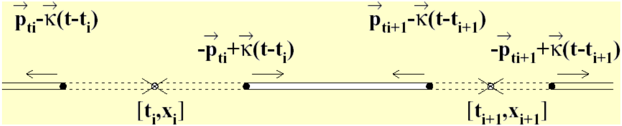
Original String



Tunnelling



Fragmentation



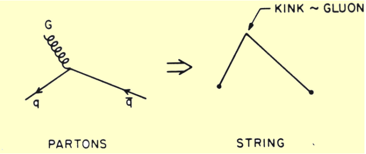
- Adjacent breaks form a hadron
  - $m^2_{had} \propto$ area swept out by string

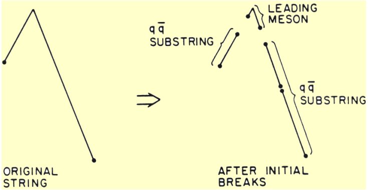
Iterative Solution

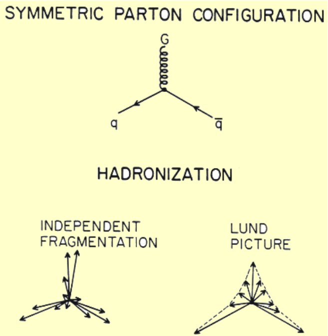
- String breaking and hadron formation can be treated as an iterative process
- Use light-cone coordinates $x^{\pm} = x \pm t$
- Boundary Conditions: $x_0^+ = 2E_0/\kappa, x_{n+1}^- = 2\bar{E}_0/\kappa, x_0^- = x_{n+1}^+ = 0$
  1. select z_i according to $f(z)dz$
     - $f^h(z, p_T) \sim \frac{1}{z}(1-z)^a \exp\left[-\frac{b(m_h^2 + p_T^2)}{z}\right]$
  2. $\Delta x^+ = (x_{i-1}^+ - x_i^+) = z_i x_{i-1}^+$
  3. $\Delta x^- = (x_{i-1}^- - x_i^-) = \frac{-m_i^2}{\kappa^2 \Delta x^+}$
     - mass^2 of hadron $\propto \Delta x^+ \Delta x^-$
  4. Continue until string is consumed

Inclusion of Gluon Radiation

- Perturbative Parton Shower generates gluons
- Gluon = kink on string, i.e. some motion to system
- String effect $\Rightarrow$ particles move in direction of kink







Hadronization Overview

Clusters (`Herwig`)

- perturbation theory can be applied down to low scales if the coherence is treated correctly
- There must be non-perturbative physics, but it should be very simple
- Improving data has meant successively making non-pert phase more string-like

Strings (`Pythia, Ariadne`)

- dynamics of the non-perturbative phase must be treated correctly
- Model includes some non-perturbative aspect of color (interjet) coherence (string effect)
- Improving data has meant successively making non-pert phase more cluster-like

Underlying Event

- Hadrons (protons) are extended objects
- Remnant remains after hard partons scatter
- Need a description of how partonic remnants are confined, similar to the way quarks and gluons from radiation are confined

Historically, Two Approaches

1. Soft parton-parton collisions dominate (parametrize)
2. Semi-Hard parton-parton cross section can be applied even at low p_T

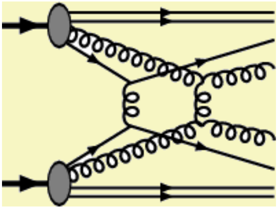
Soft Underlying Event

UA5 Monte Carlo

- hadron-hadron scattering produces two leading clusters and several central ones
- parametrize N_{ch} and sample
- clusters given p_T and y from an *ad hoc* distribution
  - $\dfrac{dN}{dp_T^2} \sim e^{-bp_T}, \dfrac{1}{(p_T + p_0)^n}$
  - $y \sim$ flat with Gaussian tails
- $p_L = m \sinh(y)$

- `Herwig` adds in their cluster model
- UE model is a mechanism for producing the objects used in description of hadronization

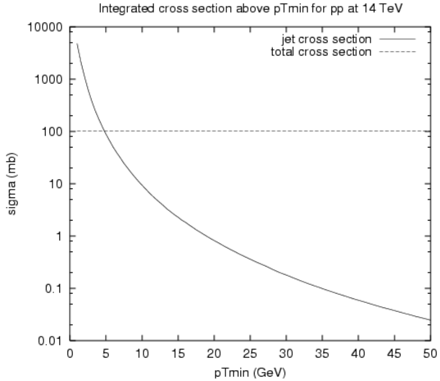
Multiple Interaction Model



- Soft model does not agree well with data
- Multi-interaction dynamics observed by AFS, UA1, CDF
- Implied by the width of the multiplicity distribution in UA5
- forward-backward correlations: UA5
- pedestal effect: UA1, H1, CDF

What are multiple interactions?

QCD $2 \to 2$ interactions dominated by t-channel gluon exchange, so diverges like $\mathrm{d}\sigma/\mathrm{d}p_\perp^2 \approx 1/p_\perp^4$ for $p_\perp \to 0$.



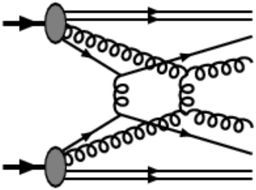
integrate QCD $2 \to 2$
$\mathrm{qq}' \to \mathrm{qq}'$ $\mathrm{q}\overline{\mathrm{q}} \to \mathrm{q}'\overline{\mathrm{q}}'$
$\mathrm{q}\overline{\mathrm{q}} \to \mathrm{gg}$ $\mathrm{qg} \to \mathrm{qg}$ $\mathrm{gg} \to \mathrm{gg}$
$\mathrm{gg} \to \mathrm{q}\overline{\mathrm{q}}$
with CTEQ 5L PDF's

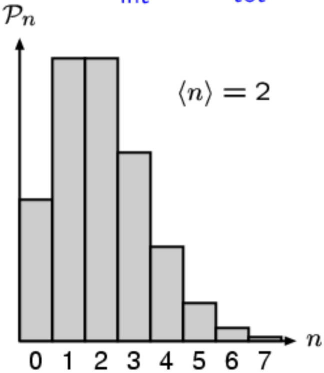
$\sigma_{\text{int}}(p_{\perp\text{min}}) > \sigma_{\text{tot}}$ for $p_{\perp\text{min}} \lesssim 5$ GeV

What does this mean?

Half a solution: many interactions per event

$$\sigma_{\text{tot}} = \sum_{n=0}^{\infty} \sigma_n$$
$$\sigma_{\text{int}} = \sum_{n=0}^{\infty} n\,\sigma_n$$
$$\sigma_{\text{int}} > \sigma_{\text{tot}} \Longleftrightarrow \langle n \rangle > 1$$





If interactions occur independently
then **Poissonian statistics**

$$\mathcal{P}_n = \frac{\langle n \rangle^n}{n!}\, e^{-\langle n \rangle}$$

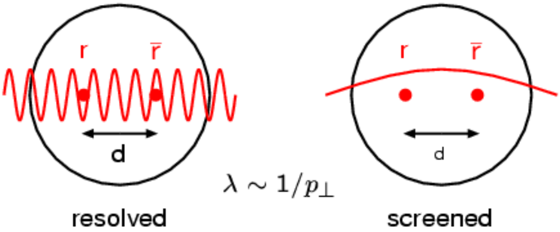
but energy–momentum conservation
$\Rightarrow$ large n suppressed

Other half of solution:
perturbative QCD not valid at small $p_\perp$ since q, g not asymptotic states (confinement!).

Naively breakdown at

$$p_{\perp\mathrm{min}} \simeq \frac{\hbar}{r_\mathrm{p}} \approx \frac{0.2\ \mathrm{GeV}\cdot\mathrm{fm}}{0.7\ \mathrm{fm}} \approx 0.3\ \mathrm{GeV} \simeq \Lambda_\mathrm{QCD}$$

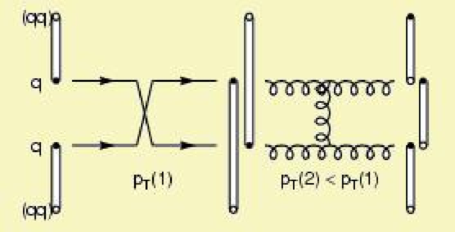
. . . but better replace r_p by (unknown) colour screening length d in hadron



- $\bar{n} = \sigma_{\mathsf{hard}}(p_{\perp\mathsf{min}})/\sigma_{\mathsf{nd}}(s) > 1$
- Not a violation of unitarity! σ_{hard} is inclusive
- On average, $\bar{n}$ semi-hard interactions **in one hard collision**
- Collisions ranked in $x_\perp = 2p_\perp/E_{\mathsf{cm}}$, produced with prob
  $$f(x_\perp) = \frac{1}{\sigma_{\mathsf{nd}}(s)} \frac{d\sigma}{dx_\perp}$$
- The probability that the hardest interaction is at $x_{\perp 1}$:
  $$f(x_{\perp 1}) \exp\left\{ -\int_{x_{\perp 1}}^{1} f(x'_\perp)\, dx'_\perp \right\}$$
  - like radioactive decay

- generate a chain of scatterings $1 > x_{\perp 1} > x_{\perp 2} > \cdots > x_{\perp i}$ using $x_{\perp i} = F^{-1}(F(x_{\perp i-1}) - \ln R_i)$
  - $F(x_\perp) = \int_{x_\perp}^{1} f(x'_\perp)\, dx'_\perp = \frac{1}{\sigma_{\mathsf{nd}}(s)} \int_{sx_\perp^2/4}^{s/4} \frac{d\sigma}{dp_\perp^2}\, dp_\perp^2$

Strings and the UE



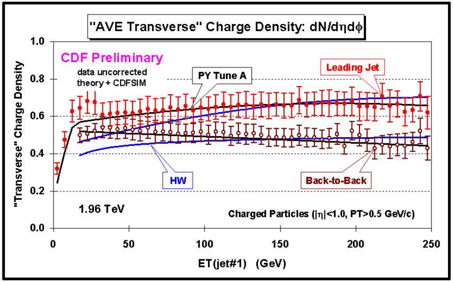
- Each additional interaction adds more color flow
  - Color information encoded in strings
  - The way subsequent interactions color-connect is a parameter of the model
  - Fits prefer a minimization of total string length

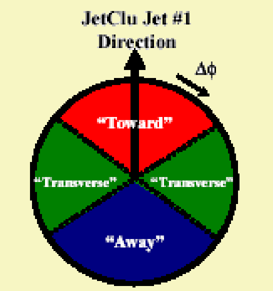
Pythia Options (already outdated!)

```
MSTP(82) :
    (D=1) structure of multiple interactions. For QCD processes, used down to
values below , it also affects the choice of structure for the
one hard/semi-hard interaction.

    = 0 :
        simple two-string model without any hard interactions. Toy model only!
    = 1 :
        multiple interactions assuming the same probability in all events,
with an abrupt cut-off at PARP(81). (With a slow energy dependence given by
PARP(89) and PARP(90).)
    = 2 :
        multiple interactions assuming the same probability in all events,
with a continuous turn-off of the cross section at PARP(82). (With a slow
energy dependence given by PARP(89) and PARP(90).)
    = 3 :
        multiple interactions assuming a varying impact parameter and a
hadronic matter overlap consistent with a Gaussian matter distribution, with
a continuous turn-off of the cross section at PARP(82). (With a slow energy
dependence given by PARP(89) and PARP(90).)
    = 4 :
        multiple interactions assuming a varying impact parameter and a
hadronic matter overlap consistent with a double Gaussian matter distribution
given by PARP(83) and PARP(84), with a continuous turn-off of the cross
section at PARP(82). (With a slow energy dependence given by PARP(89) and
PARP(90).)
```

Pythia at Run2: Underlying Event





PYTHIA 6.206 and CDF Tune A (CTEQ5L)			
Parameter	Default	Tune	Description
PARP(67)	1.0	4.0	Scale factor for ISR
MSTP(82)	1.0	4	Double Gaussian matter distribution
PARP(82)	1.9	2.0	Cutoff (GeV) for MPIs
PARP(83)	0.5	0.5	Warm Core with % of matter
PARP(84)	0.2	0.4	within a given radius
PARP(85)	0.33	0.9	Prob. that two gluons have NNC
PARP(86)	0.66	0.95	gg versus $q\bar{q}$
PARP(89)	1000.0	1800.0	Reference energy (GeV)
PARP(90)	0.16	0.25	Power of Energy scaling for cutoff

Status of UE Tunes

PYTHIA 6.2 Tunes

Use LO α_s with Λ = 192 MeV!

K-factor (Sjöstrand)

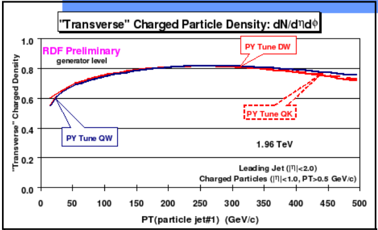
UE Parameters

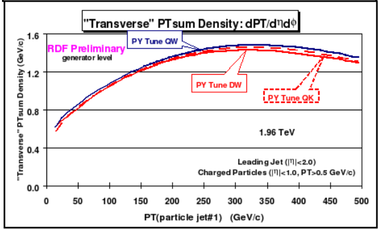
ISR Parameter

Intrinsic KT

Parameter	Tune DW	Tune DWT	ATLAS	Tune QW	Tune QWT	Tune QK	Tune QKT
PDF	CTEQ5L	CTEQ5L	CTEQ5L	CTEQ6.1	CTEQ6.1	CTEQ6.1	CTEQ6.1
MSTP(2)	1	1	1	1	1	1	1
MSTP(33)	0	0	0	0	1	1	1
PARP(31)	1.0	1.0	1.0	1.0	1.0	1.8	1.8
MSTP(81)	1	1	1	1	1	1	1
MSTP(82)	4	4	4	4	4	4	4
PARP(82)	1.9 GeV	1.9409 GeV	1.8 GeV	1.1 GeV	1.1237 GeV	1.9 GeV	1.9409 GeV
PARP(83)	0.5	0.5	0.5	0.5	0.5	0.5	0.5
PARP(84)	0.4	0.4	0.5	0.4	0.4	0.4	0.4
PARP(85)	1.0	1.0	0.33	1.0	1.0	1.0	1.0
PARP(86)	1.0	1.0	0.66	1.0	1.0	1.0	1.0
PARP(89)	1.8 TeV	1.96 TeV	1.0 TeV	1.8 TeV	1.96 TeV	1.8 TeV	1.96 TeV
PARP(90)	0.25	0.16	0.16	0.25	0.16	0.25	0.16
PARP(62)	1.25	1.25	1.0	1.25	1.25	1.25	1.25
PARP(64)	0.2	0.2	1.0	0.2	0.2	0.2	0.2
PARP(67)	2.5	2.5	1.0	2.5	2.5	2.5	2.5
MSTP(91)	1	1	1	1	1	1	1
PARP(91)	2.1	2.1	1.0	2.1	2.1	2.1	2.1
PARP(93)	15.0	15.0	5.0	15.0	15.0	15.0	15.0

PYTHIA 6.2 Tunes



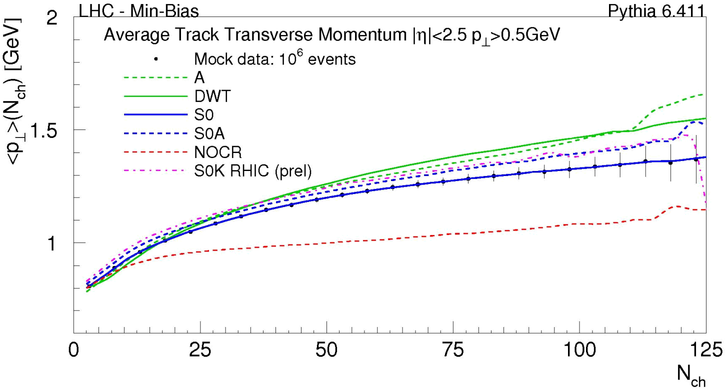


	1.96 TeV		14 TeV	
	P_{T0}(MPI) GeV	σ(MPI) mb	P_{T0}(MPI) GeV	σ(MPI) mb
Tune DW	1.9409	351.7	3.1730	549.2
Tune DWT	1.9409	351.7	2.6091	829.1
ATLAS	2.0	324.5	2.7457	768.0
Tune QW	1.1237	296.5	1.8370	568.7
Tune QK	1.9409	259.5	3.1730	422.0
Tune QKT	1.9409	259.5	2.6091	588.0

- Remember the p_T cut-off, P_{T0}, of the MPI cross section is energy dependent and given by $P_{T0}(E_{cm})$ = PARP(82)×$(E_{cm}/E_0)^{\varepsilon}$ with ε = PARP(90) and E_0 = PARP(89).
- Average charged particle density and PTsum density in the "transverse" region (p_T > 0.5 GeV/c, |η| < 1) versus P_T(jet#1) at 1.96 TeV for PY Tune DW, Tune QW, and Tune QK.

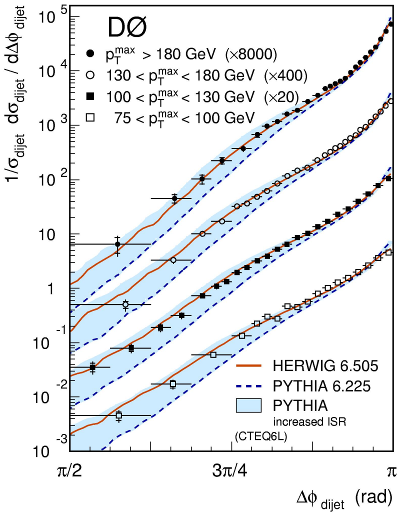
More Detailed Models

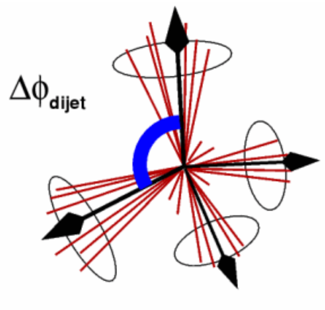
The p_T ordered shower in `Pythia` was developed to have a consistent description of ISR and UE, and to allow for fiddling of the color connections



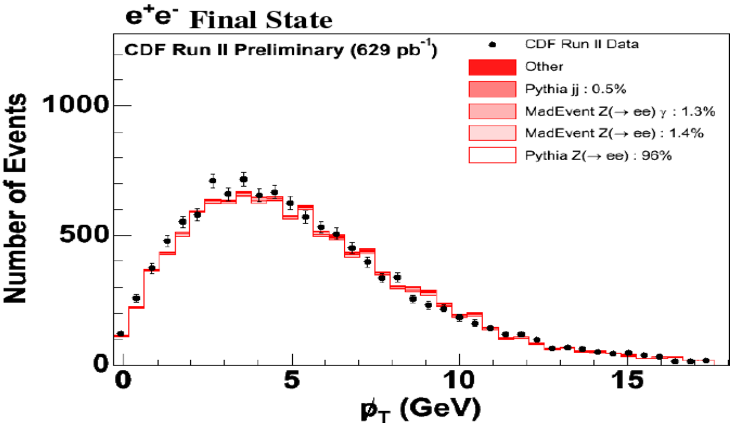
- Tune parameters affect much more than just the charged track properties
- These are full “Event” tunes

DØ Dijet Azimuthal Correlation





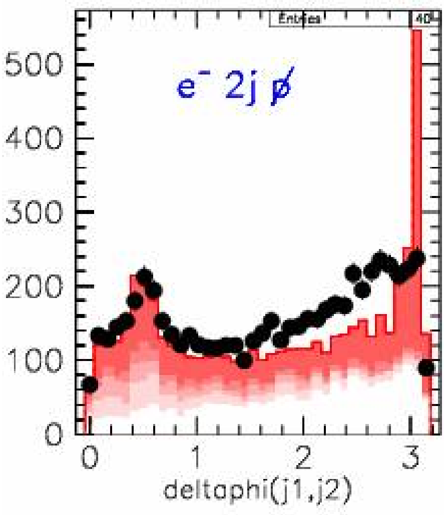
Large Intrinsic k_T



- Even resummation calculations need non-pert. k_T
- Catalysis for "-W"[1] tunes

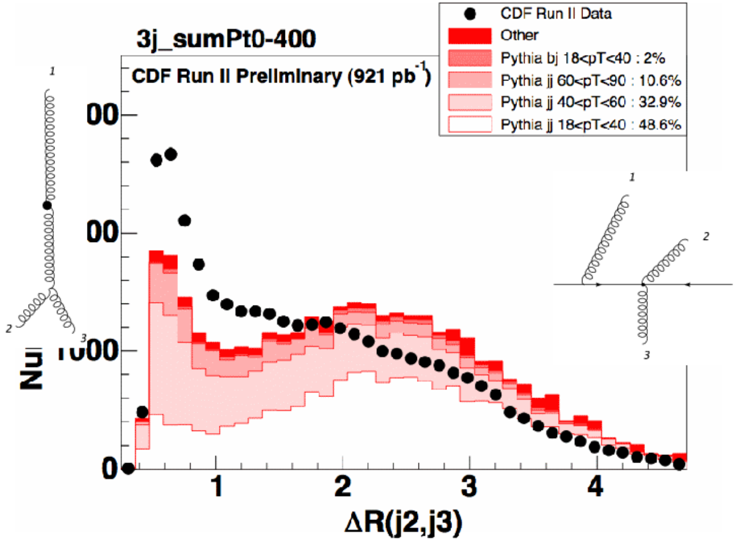
[1] W=Willis Sakumoto

High-p_T is sensitive to UE



Should allow FSR for multiple parton interactions

Tune A gives too much ISR



The `f77` Parton Shower Programs

	`Pythia`	`Herwig`	`Ariadne`
PS Ordering	Mass (θ veto) p_T	Angle	k_T
Hadronization	String	Cluster	String
Underlying Event	Mult. Int	UA6/(Jimmy)	LDCM

Finding them:

- `http://www.thep.lu.se/tf2/staff/torbjorn/Pythia.html`
- `http://hepwww.rl.ac.uk/theory/seymour/herwig/`
- `http://www.thep.lu.se/~leif/ariadne/`

`Fortran` codes

`http://www.ibiblio.org/pub/languages/fortran/ch1-1.html`

- Herwig-f77 frozen, Pythia-f77 evolving: primary tools at Tevatron

Why so many programs?

- Need to resum large logarithms, because there are two scales in the program
- The large scale is $M_W, M_Z, m_t, \cdots$
- Which small scale? The mass of jets? p_T? $E_0\theta_{qg}$?
- How are they related?

$$m^2 = 2E_iE_j(1-\cos\theta_{ij})$$

$$E_i = zE_0, E_j = (1-z)E_0; 2(1-\cos\theta_{ij}) = 4\sin^2(\theta_{ij}/2) \to \theta_{ij}^2$$

$$q^2_{\text{Py-old}} = m^2 \times \theta(\theta_{\text{old}} - \theta_{\text{new}})$$

$$q^2_{\text{Hw}} = E_0^2\theta_{ij}^2 = \frac{m^2}{z(1-z)}$$

$$q^2_{\text{Ar}} = z(1-z)m^2 = q^2_{\text{Py-new}}$$

The `cpp` programs

- `Pythia` & `Herwig` being rewritten
  - QCD FSR, QCD ISR, particle decays, etc.
  - Improvements to showers, accounting of particle properties, couplings
- `Herwig++` "will be ready for LHC"; `Pythia8` likely same

`Sherpa` is also `C++` event generator in a different framework
Includes some new ideas with and some older models

- overlap with some `Pythia` physics assumptions
  - hadronization is the Lund string model
  - parton shower is virtuality ordered with some modifications
  - underlying event is of the multiple-interaction kind
- "automatic" inclusion of higher-order (tree level) matrix elements

For all new generators, there is a long road of tuning and validation ahead

This Lecture: Special Topics

Improvements

- The parton showers were developed using the soft and collinear approximations
- We would like to control this approximation and make systematic improvements
- How can we include more hard jets in the “hard scattering”?
- Can we include NLO normalization?

How to do Tree Level Calculations

- Read Feynman rules from $i\mathcal{L}_{int}$ from a textbook
- Use Wave Functions from Relativistic QM
  - Propagators (Green functions) for internal lines
- Specify initial and final states
  - Track spins/colors/etc. if desired
- Draw all valid graphs connecting them
  - Tedious, but straight-forward
- Calculate (Matrix Element)^2
  - Evaluate Amplitudes, Add and Square
  - Symbolically Square, Evaluate
  - ALPHA (numerical functional evaluation with no Feynman graphs)
- Integrate over Phase Space

Learn by hand, then automate

Complications:

- $|\mathcal{M}|^2$: Number of graphs grows quickly with number of external partons
- $d\Phi_n$: Efficiency decreases with number of internal lines

Programs:

- `MadEvent, CompHep, Alpgen, Amegic++`
- Differ in methods of attack
- Most rely on `VEGAS` for MC integration

Limitations:

- Fixed number of partons
- No control of large logarithms as $E_g, \theta_{qg}, \theta_{gg} \to 0$

New Matrix Element Programs

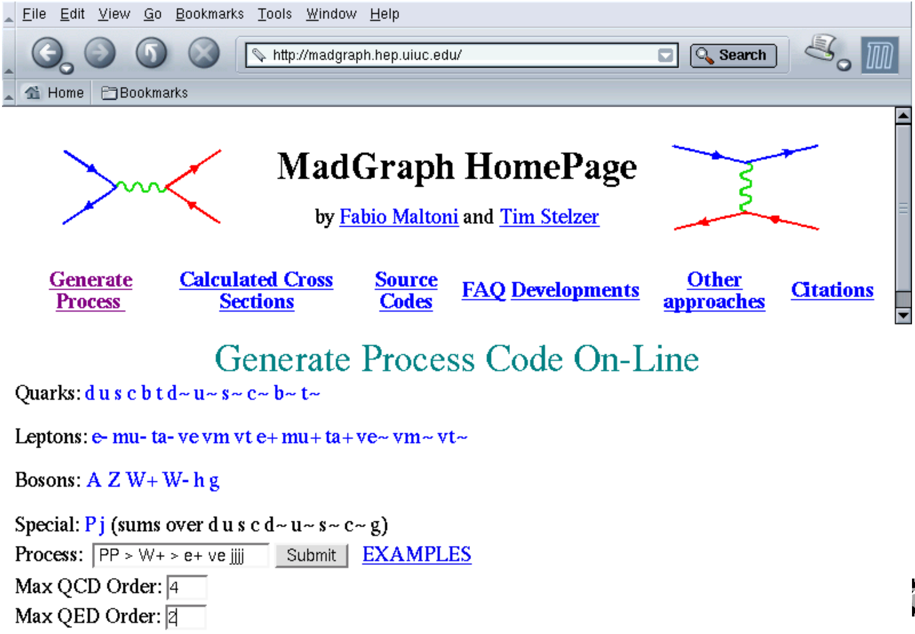
Automatically calculate code needed for a given HEP process and generate events

List of those actively supporting hadron colliders

- `Alpgen@ http://m.home.cern.ch/m/mlm/www/alpgen/`
- `CompHep@ http://theory.sinp.msu.ru/comphep`
- `Grace@ http://atlas.kek.jp/physics/nlo-wg/grappa.html`
- `MadEvent@ http://madgraph.hep.uiuc.edu/index.html`
- `Sherpa/Amegic++@ http://141.30.17.181/`

Advantages and disadvantages of each

An impressive improvement from several years ago


File Edit View Go Bookmarks Tools Window Help
http://madgraph.hep.uiuc.edu/
Search
Home
Bookmarks
MadGraph HomePage
by Fabio Maltoni and Tim Stelzer
Generate Process
Calculated Cross Sections
Source Codes
FAQ
Developments
Other approaches
Citations
Generate Process Code On-Line
Quarks: d u s c b t d~ u~ s~ c~ b~ t~
Leptons: e- mu- ta- ve vm vt e+ mu+ ta+ ve~ vm~ vt~
Bosons: A Z W+ W- h g
Special: P j (sums over d u s c d~ u~ s~ c~ g)
Process: PP > W+ > e+ ve jjjj
Submit
EXAMPLES
Max QCD Order: 4
Max QED Order: 2

Interfacing with PS Tools: Les Houches Accord

Initialization

```
 INTEGER MAXPUP
 PARAMETER (MAXPUP=100)
 INTEGER IDBMUP,PDFGUP,PDFSUP,IDWTUP,NPRUP,LPRUP
 DOUBLE PRECISION EBMUP,XSECUP,XERRUP,XMAXUP
 COMMON/HEPRUP/IDBMUP(2),EBMUP(2),PDFGUP(2),PDFSUP(2),IDWTUP,
&NPRUP,XSECUP(MAXPUP),XERRUP(MAXPUP),XMAXUP(MAXPUP),LPRUP(MAXPUP)
```

`IDBMUP`: incoming beam particles (PDG codes, $\mathrm{p} = 2212$, $\overline{\mathrm{p}} = -2212$)

`EBMUP`: incoming beam energies (GeV)

`PDFGUP`, `PDFSUP`: PDFLIB parton distributions (not used by PYTHIA)

`IDWTUP`: weighting strategy

- = `1`: PYTHIA mixes and unweights events, according to known $\mathrm{d}\sigma_{\mathrm{max}}$
- = `2`: PYTHIA mixes and unweights events, according to known σ_{tot}
- = **3**: unit-weight events, given by user, always to be kept
- = `4`: weighted events, given by user, always to be kept
- = `-1`, `-2`, `-3`, `-4`: also allow negative $\mathrm{d}\sigma$

`NPRUP`: number of separate user processes

`XSECUP(i)`: σ_{tot} for each user process

Sufficiently Describe the Hard Scattering

The event

```
 INTEGER MAXNUP
 PARAMETER (MAXNUP=500)
 INTEGER NUP,IDPRUP,IDUP,ISTUP,MOTHUP,ICOLUP
 DOUBLE PRECISION XWGTUP,SCALUP,AQEDUP,AQCDUP,PUP,VTIMUP,SPINUP
 COMMON/HEPEUP/NUP,IDPRUP,XWGTUP,SCALUP,AQEDUP,AQCDUP,
&IDUP(MAXNUP),ISTUP(MAXNUP),MOTHUP(2,MAXNUP),ICOLUP(2,MAXNUP),
&PUP(5,MAXNUP),VTIMUP(MAXNUP),SPINUP(MAXNUP)
```

`IDPRUP`: identity of current process

`XWGTUP`: event weight (meaning depends on `IDWTUP` weighting strategy)

`SCALUP`: scale Q of parton distributions etc.

`AQEDUP`: α_{em} used in event

`AQCDUP`: α_{s} used in event

`NUP`: number of particles in event

`IDUP(i)`: PDG identity code for particle `i`

`ISTUP(i)`: status code ($-1 =$ incoming parton, $1 =$ final-state parton, $2 =$ intermediate resonance with preserved m)

`MOTHUP(i,j)`: position of one or two mothers

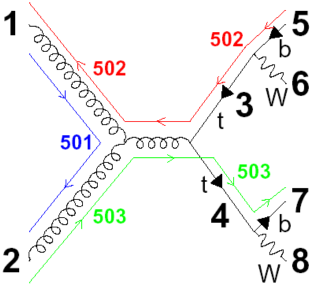
Cartoons

Examples of colour flows and indices

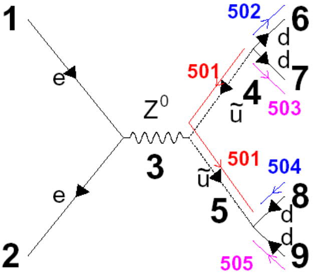
ICOLUP(j,i): colour and anticolour indices

= colour line tags, in the $N_C \to \infty$ limit, starting e.g. with number 501.

Example 1: hadronic $t\bar{t}$ production



Example 2: baryon number violation



user-process BNV not (yet) implemented in PYTHIA

Event Generators for Many Hard Partons

- Want to use these matrix-element tools with parton showers
- Each topology (e.g. $W + 0, 1, 2, 3, 4$ partons) has no soft/collinear approximation
- How do I rigorously add a parton shower to each topology with no double counting of hard emissions?

Solution (`CKKW`):

1. Make the $|\mathcal{M}|^2$ result "look" like a parton shower down to a reasonable cutoff scale ($k_T^{\text{cut}}/Q_{\text{hard}} \sim .1$)
2. Add on ordinary parton shower below k_T^{cut}

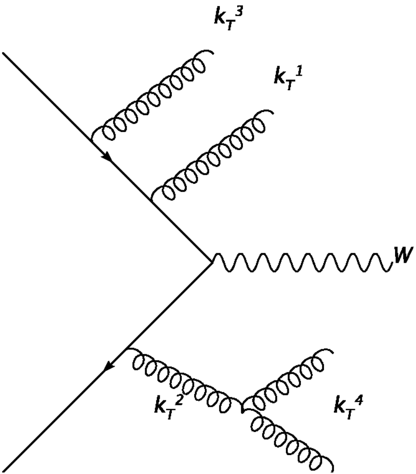
Review of Matching

Pseudo-Shower Method

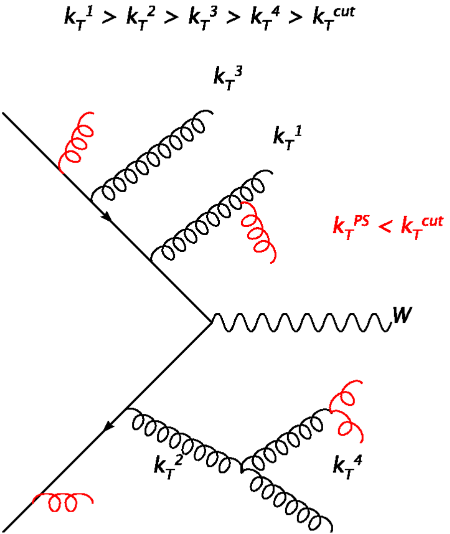
1. Generate $W + N$ parton events, applying a cut ${p_T}^2_{\text{cut}}$ on shower p_T^2 (p_T^2 for ISR, $z(1-z)m^2$ for FSR)
2. Form a p_T^2-ordered parton shower history
3. Reweight with $\alpha_s(p_T^2)$ for each emission
4. Add parton shower and keep if no emission harder than $p^2_{T\,\text{cut}}$: (save the first event with full topology)
5. Remove softest of N partons, fix up kinematics, add parton shower and keep if no emission harder than $p^2_{T\,\text{softest}}$
6. Go to 4 until no partons remain, or an emission is too hard
7. If not rejected, use the saved event

ISR Parton Shower–Matrix Element Movie

$$k_T^1 > k_T^2 > k_T^3 > k_T^4 > k_T^{cut}$$



ISR Parton Shower–Matrix Element Movie



ISR Parton Shower–Matrix Element Movie

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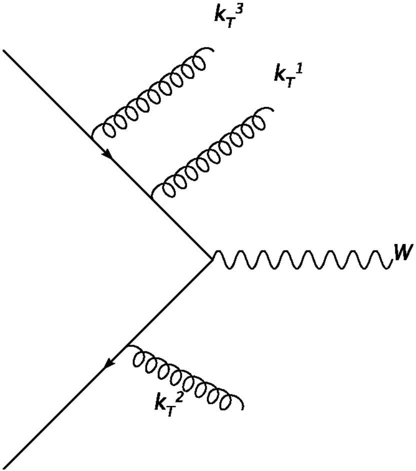
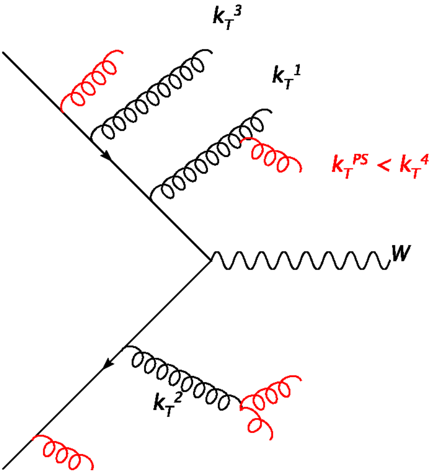


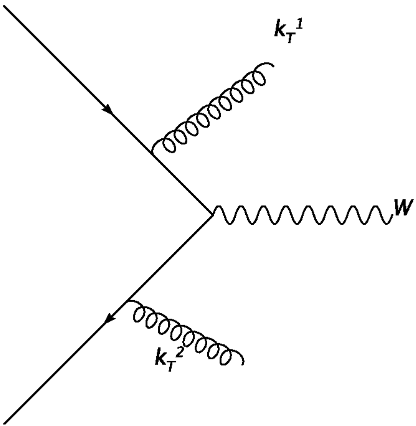
ISR Parton Shower–Matrix Element Movie

$$k_T^1 > k_T^2 > k_T^3 > k_T^4 > k_T^{cut}$$



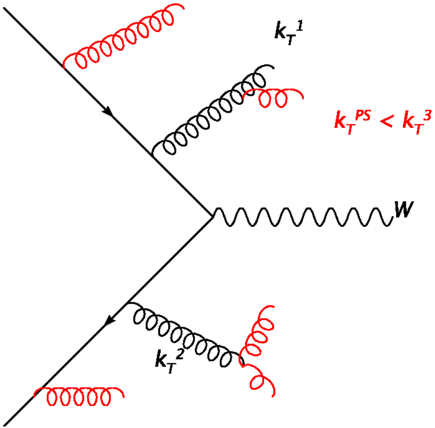
ISR Parton Shower–Matrix Element Movie

$$k_T^1 > k_T^2 > k_T^3 > k_T^4 > k_T^{cut}$$



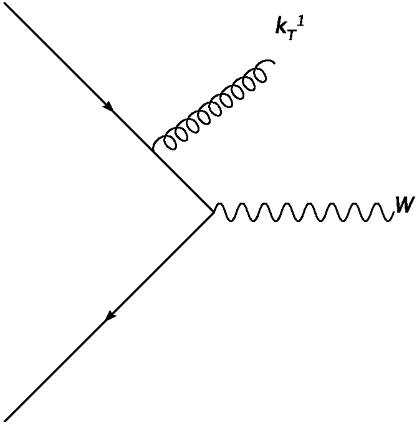
ISR Parton Shower–Matrix Element Movie

$$k_T^1 > k_T^2 > k_T^3 > k_T^4 > k_T^{cut}$$



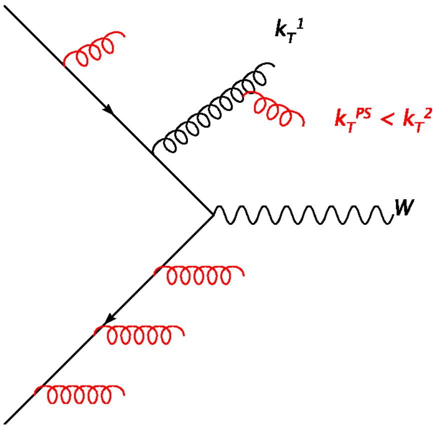
ISR Parton Shower–Matrix Element Movie

$$k_T^{1} > k_T^{2} > k_T^{3} > k_T^{4} > k_T^{cut}$$



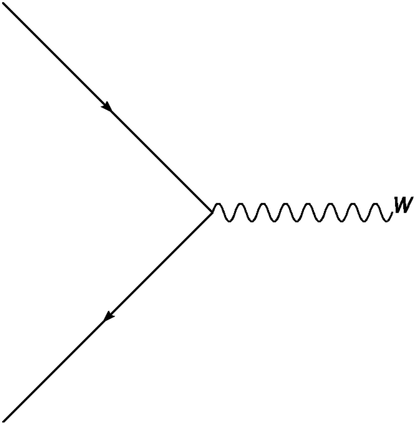
ISR Parton Shower–Matrix Element Movie

$$k_T^1 > k_T^2 > k_T^3 > k_T^4 > k_T^{cut}$$



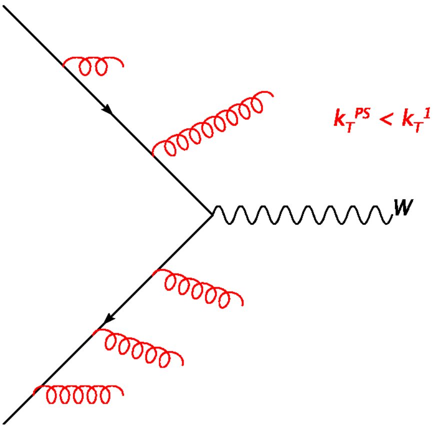
ISR Parton Shower–Matrix Element Movie

$$k_T^{1} > k_T^{2} > k_T^{3} > k_T^{4} > k_T^{cut}$$



ISR Parton Shower–Matrix Element Movie

$$k_T^1 > k_T^2 > k_T^3 > k_T^4 > k_T^{cut}$$



Why it works

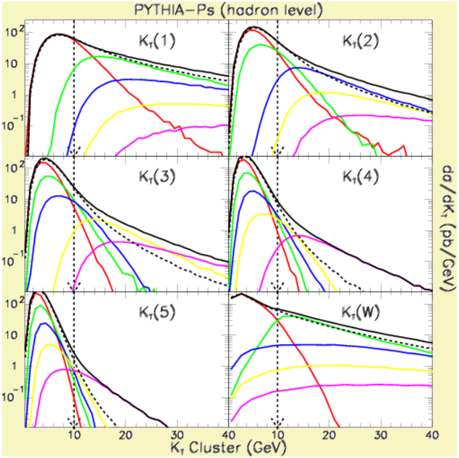
- For each N, PS does not add any jet harder than $p^2_{T\,cut}$
- Can safely add different N samples with no double-counting
  - Apply looser rejection on highest N
- Pseudo-showers assure correct PS limit, while retaining hard emissions
  - Rejection of hard emissions weights by Sudakov probabilities

Why it is necessary

- Suppress unphysical enhancements in tree level calculations from $\alpha_s^n(p_T)\,\ln^{(2n,2n-1)}\left(\frac{Q}{p_T}\right)$
- Account for many topologies in physical observables, e.g.

  $H_T = \sum p_T(\text{hard object})$
- Tames hard emissions from PS

W+0 $\oplus \cdots \oplus$ W+4 hard partons

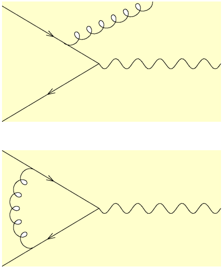


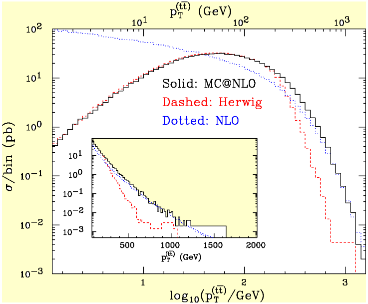
Dashed is `Pythia` with default (ME) correction
Solid is Pseudoshower result

- Other methods for performing such matching are “MLM” and “CKKW”
- There is no attempt to account for individual “K”-factors for different topologies
- Such calculations are currently included in CDF and DØ Standard Model cocktails
- Theoretical uncertainty on such methods is beginning to limit Run2 prospects for extracting top properties

Event Generator At NLO

- NLO Calculations give an improved description of the hard kinematics and cross sections, but are inclusive, i.e. **not** (exclusive) event generators
- Solution (`MC@NLO`): Remove divergences by adding and subtracting the Monte Carlo result for one emission





Toy Parton Shower

Consider a system that can emit a number of quanta (photons) with energy $z_0 < x < x_{\mathsf{max}}(x)$, $x_{\mathsf{max}}(1) = 1$

$$0 \leq Q(z) \leq 1, \quad \lim_{z \to 0} Q(z) = 1,$$

IF the prob. of one emission is $a \frac{Q(x)}{x} dx$
THEN the *Sudakov form factor* is

$$\Delta(x_2, x_1) = \exp\left[-a \int_{x_1}^{x_2} dz \frac{Q(z)}{z}\right],$$

Limit	Sudakov	# of Quanta
a $\ll$ 1	$\Delta \sim 1 - a\frac{Q(x)}{x}dx$	few
a $\gg$ 1	$\Delta \sim 0$	many

Constructing an "Event" Generator

Event $\equiv$ original system + emissions down to scale x_0

Take $Q(x) = 1$
To solve for the shower evolution:

1. Pick $r = \exp\left(-a \int_x^{x_2} dx/x \right) = (x/x_2)^a$
2. Solve $x = x_2 r^{(1/a)}$
3. Calculate remaining energy x_2
4. Continue until $x < x_0$

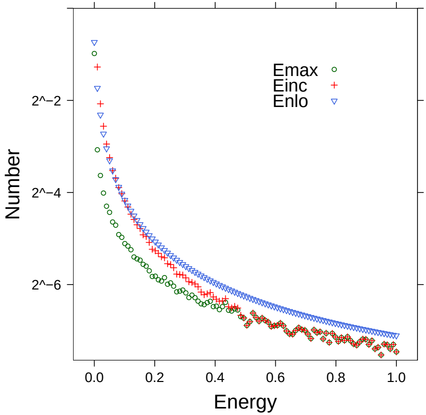
This generates an energy-ordered shower with multiple photon emissions

Example Event Record

Event listing (summary)

I	particle/jet	KS	KF	orig	E
1	e-	1	11	0	1.000
2	nu_e	1	12	0	0.000
3	(e-)	11	11	0	0.296
4	gamma	1	22	3	0.704
5	(e-)	11	11	3	0.285
6	gamma	1	22	3	0.011
7	(e-)	11	11	5	0.283
8	gamma	1	22	5	0.002
9	e-	1	11	7	0.282
10	gamma	1	22	7	0.001
		sum:	-1.00		1.000

Spectra for Toy Model



- Real (NLO) spectrum = $\frac{d\sigma}{dx} = a\frac{R(x)}{x}$
- $R(x) \to Q(x)$ as $x \to 0$
- Here: $R(x) = (1 + x/10)^2$

Enlo = energy at NLO

Einc = summed energy from PS

Emax = max[E] from PS

Parton shower underestimates high energy emissions

PS@NLO

NLO Computation for Toy Model

$$
\begin{aligned}
\left(\frac{d\sigma}{dx}\right)_{\mathrm{B}} &= B\delta(x), \\
\left(\frac{d\sigma}{dx}\right)_{\mathrm{V}} &= a\left(\frac{B}{2\epsilon}+V\right)\delta(x), \\
\left(\frac{d\sigma}{dx}\right)_{\mathrm{R}} &= a\frac{R(x)}{x},
\end{aligned}
$$

$$\lim_{x\to 0} R(x) = B.$$

infrared-safe observable O

$$\langle O\rangle = \lim_{\epsilon\to 0}\int_0^1 dx\, x^{-2\epsilon} O(x)\left[\left(\frac{d\sigma}{dx}\right)_{\mathrm{B}} + \left(\frac{d\sigma}{dx}\right)_{\mathrm{V}} + \left(\frac{d\sigma}{dx}\right)_{\mathrm{R}}\right],$$

Subtraction Method

Write the real contribution as:

$$\langle O \rangle_{\mathsf{R}} = aBO(0) \int_0^1 dx \, \frac{x^{-2\epsilon}}{x} + a \int_0^1 dx \, \frac{O(x)R(x) - BO(0)}{x^{1+2\epsilon}} \; .$$

Set $\epsilon = 0$ in the second term

$$\langle O \rangle_{\mathsf{R}} = -a\frac{B}{2\epsilon}O(0) + a \int_0^1 dx \, \frac{O(x)R(x) - BO(0)}{x} \; .$$

NLO prediction:

$$\langle O \rangle_{\mathsf{sub}} = \int_0^1 dx \left[O(x)\frac{aR(x)}{x} + O(0)\left(B + aV - \frac{aB}{x} \right) \right] .$$

$$\langle O \rangle_{\text{sub}} = \int_0^1 dx \left[O(x) \frac{aR(x)}{x} + O(0) \left(B + aV - \frac{aB}{x} \right) \right]$$

Adding a parton shower makes it difficult to cancel singularities

$$\Delta(x_2, x_1) = \exp \left[-a \int_{x_1}^{x_2} dz \frac{Q(z)}{z} \right]$$ Sudakov

$O(0)$ and $O(x)$ observables *both* contribute to order a:

- $Ba\frac{Q(x)}{x} + a\frac{R(x)}{x}$ (double counting problem)

Showering with full NLO corrections

Modified Subtraction Method (Frixione and Webber: `MC@NLO`)

$$\left(\frac{d\sigma}{dO}\right)_{\text{msub}} = \int_0^1 dx \left[I_{\text{MC}}(O, x_{\text{M}}(x)) \frac{a[R(x) - BQ(x)]}{x} + I_{\text{MC}}(O, 1) \left(B + aV + \frac{aB[Q(x) - 1]}{x} \right) \right]$$

Singular terms cancel among themselves

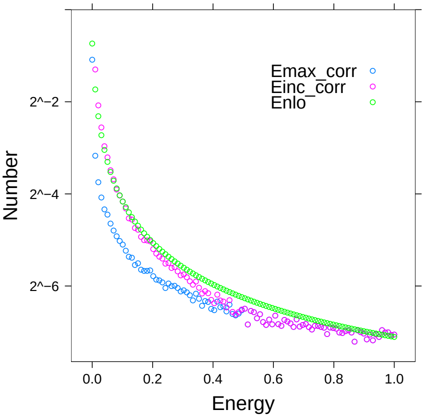
$O(0)$ and $O(x)$ observables still *both* contribute to $O(a)$

They cancel to yield $a\dfrac{R(x)}{x}$

Assignment: read (Soper and Kraemer: `Beowulf + PS`)

Alternative

Matrix Element Correction to Parton Shower



Assume the parton shower samples all of phase space and gives the hardest emission first

For the 1st emission, weight according to $\frac{R(x)}{Q(x)}$

Here: $(1 + x/10)^2 < 2$

Parton shower gets correct limit for large x and includes multiple photon emission

Summary

- Event Generators accumulate our understanding of the Standard Model into one package
- Apply perturbation theory whenever possible
  - hard scattering, parton showering, decays
- Rely on models or parametrizations when present calculational methods fail
  - hadronization, underlying event, beam remnants

Summary (cont)

- Out of the box, they give reliable estimates of the full, complicated structure of HEP events
- Attentive users will find more flexibility & applications
- Understanding the output can lead to a broader understanding of the Standard Model (and physics beyond)
- Many new developments
  (more difficult questions $\Rightarrow$ better tools)