Introduction to the Parton Model and Pertrubative QCD

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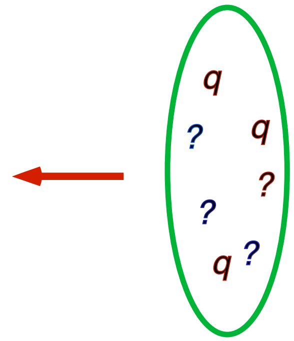
II. From the Parton Model to QCD

1. Color and QCD
2. Field Theory Essentials
3. Infrared Safety



1. From Color to QCD

- Enter the Gluon
- If $\phi_{q/H}(x)$ = probability to find q with momentum xp,
- then,

$$M_q = \sum_q \int_0^1 dx\; x\; \phi_{q/H}(x) \;=\; \text{total fraction of momentum carried by quarks.}$$

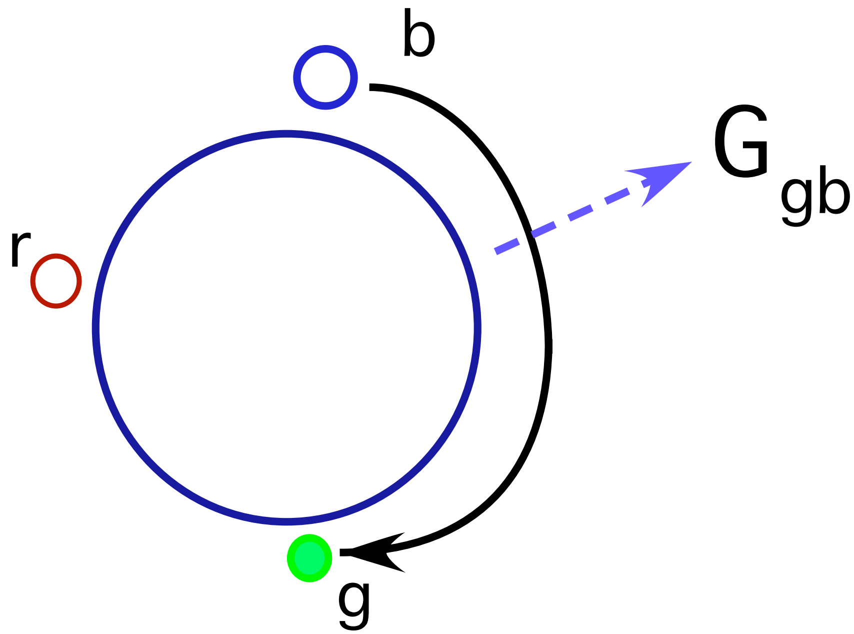
- Experiment gave

$$M_q \sim 1/2$$

- What else? Quanta of force field that holds H together?
- 'Gluons' – but what are they?

- **Where color comes from.**
- **Quark model problem:**
  - $s_q = 1/2 \Rightarrow$ **fermion** $\Rightarrow$ **antisymmetric wave function, but**
  - (uud) **state symmetric in spin/isospin combination for nucleons and**
  - **Expect the lowest-lying** $\psi(\vec{x}_m, \vec{x}'_u, \vec{x}_d)$ **to be symmetric**
  - **So where is the antisymmetry?**
- **Solution: Han Nambu, Greenberg, 1968: Color**
- b, g, r, **a new quantum number.**
- **Here's the antisymmetry:** $\epsilon_{ijk}\psi(\vec{x}_u, \vec{x}'_u, \vec{x}_d)$, **(i,j,k)= (b,g,r)**

- **Quantum Chromodynamics: Dynamics of Color**
- **A globe with no north pole**



- **Position on 'hyperglobe' $\leftrightarrow$ phase of wave function**

  (Yang & Mills, 1954)
- **We can change the globe's axes at different points in space-time, and 'local rotation' $\leftrightarrow$ emission of a gluon.**
- **QCD: gluons coupled to the color of quarks**

  (Gross & Wilczek; Weinberg; Fritzsch, Gell-Mann, Leutwyler, 1973)

2. Field Theory Essentials

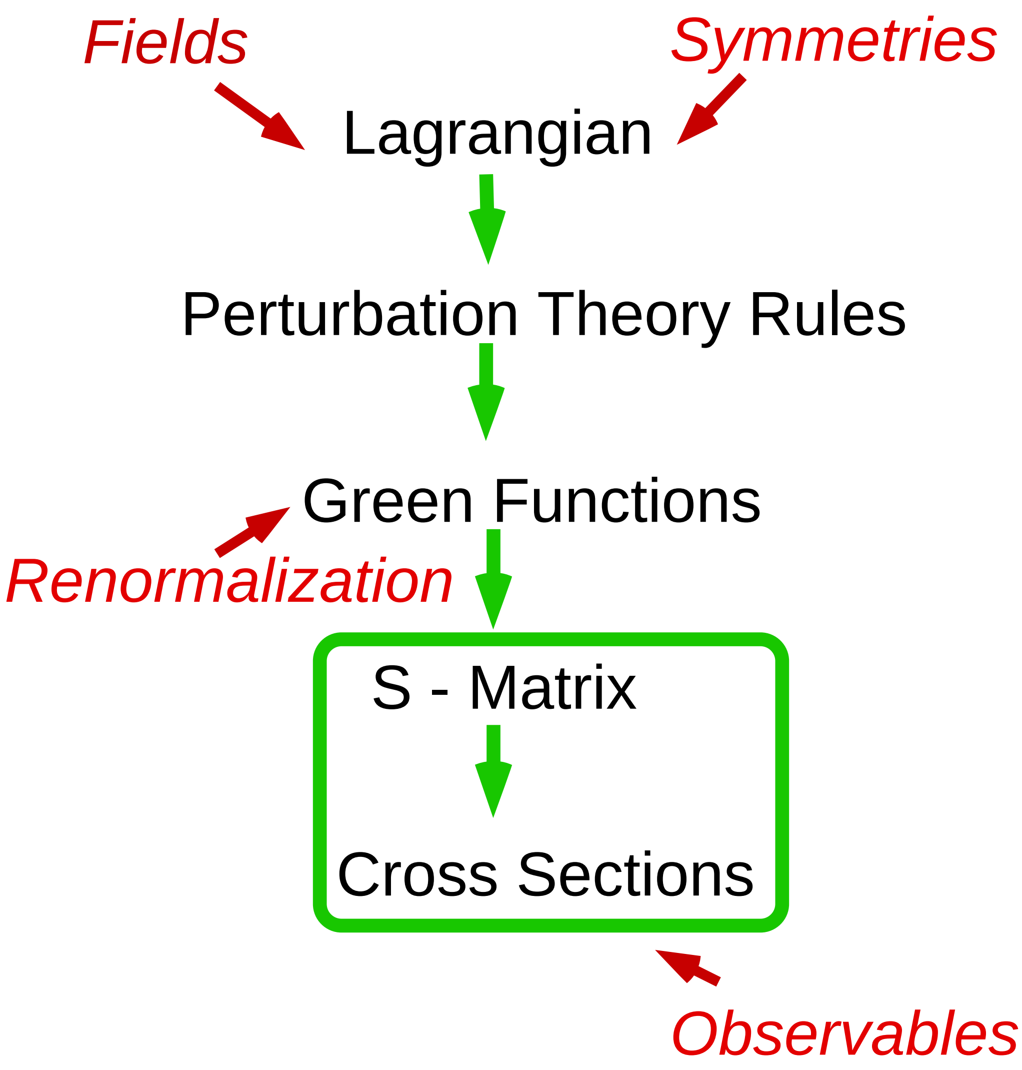
- **Fields and Lagrange Density for QCD**
- $q_f(x)$, $f = u, d, c, s, t, b$: **Dirac fermions (like electron) but extra** $(i, j, k) = (b, g, r)$ **quantum number.**
- $A^\mu_a(x)$ **Vector field (like photon) but with extra** $a \sim (g\bar{b}\ldots)$ **quantum no. (octet).**
- $\mathcal{L}$ **specifies quark-gluon, gluon-gluon propagators and interactions.**

$$\mathcal{L} = \sum_f \bar{q}_f \left(\left[i\partial_\mu - g A_{\mu a} T_a \right] \gamma^\mu - m_f \right) q_f - \frac{1}{4} \left(\partial_\mu A_{\nu a} - \partial_\nu A_{\mu a} \right)^2$$

$$-\frac{g}{2} \left(\partial_\mu A_{\nu a} - \partial_\nu A_{\mu a} \right) C_{abc} A^\mu_b A^\nu_c$$

$$-\frac{g^2}{4} C_{abc} A^\mu_b A^\nu_c C_{ade} A_{\mu d} A_{\nu e}$$

From a Lagrange density to observables, the pattern:

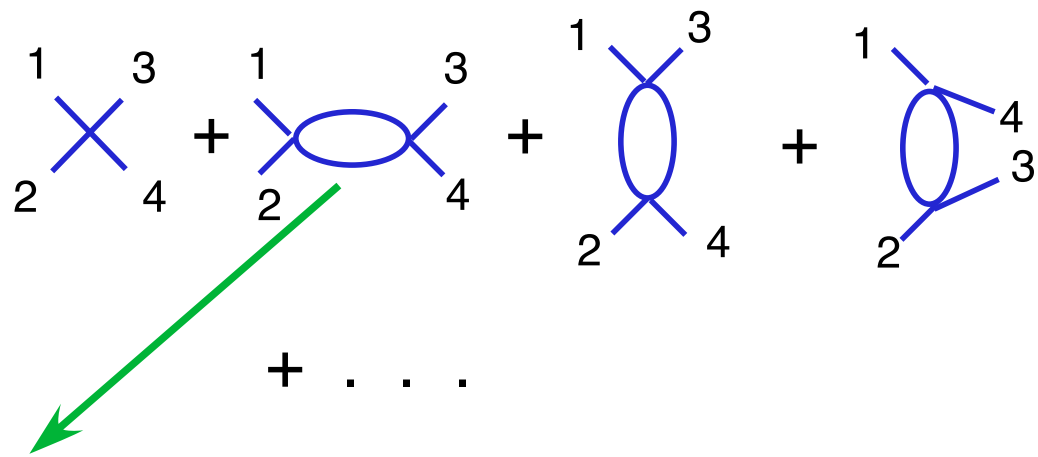


- **UV Divergences (toward renormalization & the renormalization group)**
- **Use as an example**

$$\mathcal{L}_{\phi^4} = \frac{1}{2}\left((\partial_\mu \phi)^2 - m^2\phi^2\right) - \frac{\lambda}{4!}\phi^4$$

- **The "four-point Green function":**

M(s,t) =



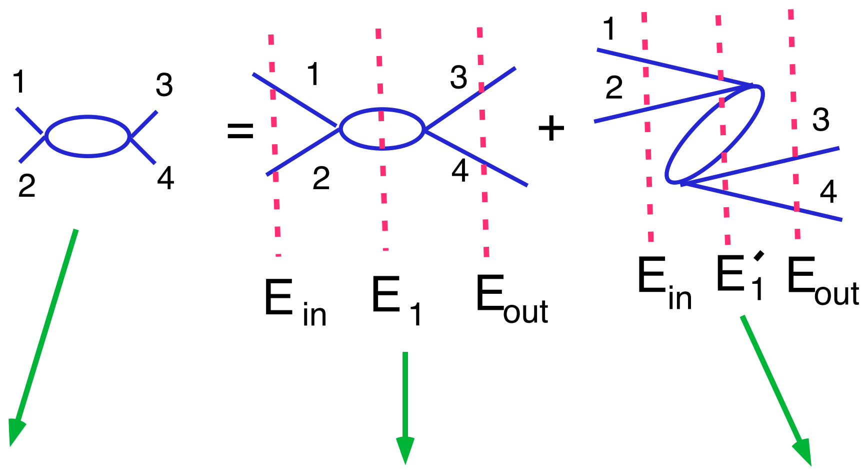
\+ . . .

$$\int^\infty \frac{d^4k}{(k^2 - m^2)((p_1 + p_2 - k)^2 - m^2)} \sim \int^\infty \frac{d^4k}{(k^2)^2} \Rightarrow \infty$$

Interpretation: The UV divergence is due entirely states of high ‘energy deficit’,

$$E_{\text{in}} - E_{\text{state S}} = p_1^0 + p_2^0 - \sum_{i\,\in S} \sqrt{\vec{k}_i^2 - m^2}$$

Made explicit in Time-ordered Perturbation Theory:

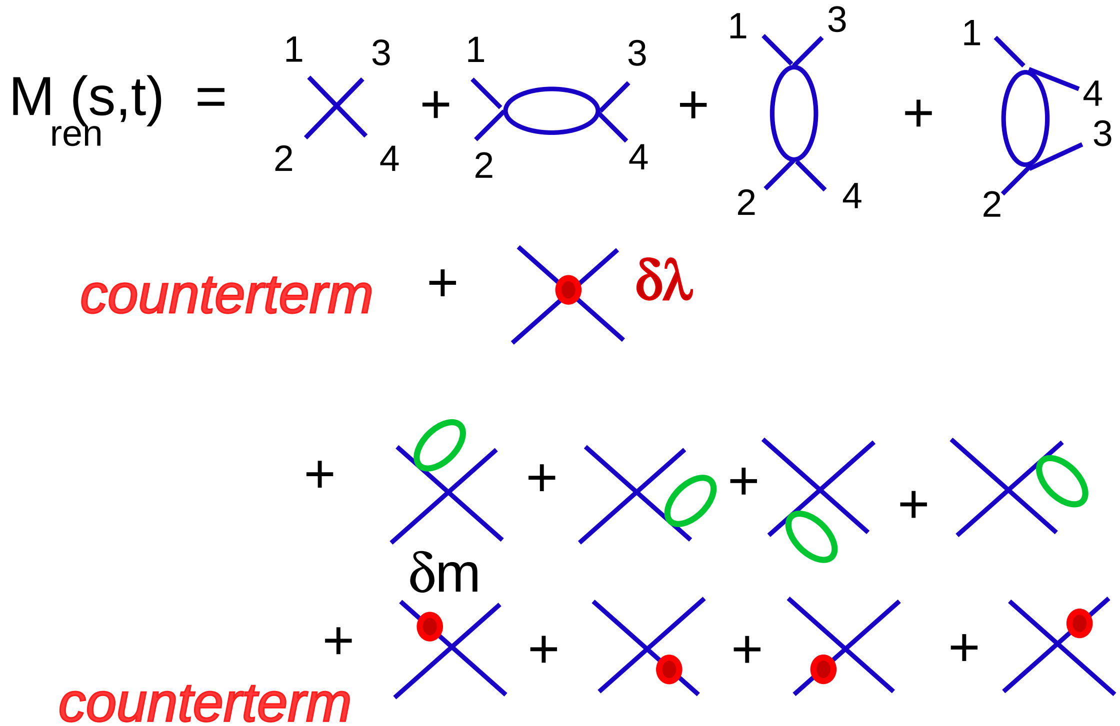


$$\int^{\infty} \frac{d^4k}{(k^2 - m^2)((p_1 + p_2 - k)^2 - m^2)} = \sum_{states} \left[\frac{1}{E_{\text{in}} - E_1} + \frac{1}{E_{\text{in}} - E_1'} \right]$$

Analogy to uncertainty principle $\Delta E \rightarrow \infty \Leftrightarrow \Delta t \rightarrow 0.$

- **This suggests: UV divergences are 'local' and can be absorbed into the local Lagrange density. Renormalization.**
- **For our full 4-point Green function, two new "counterterms":**

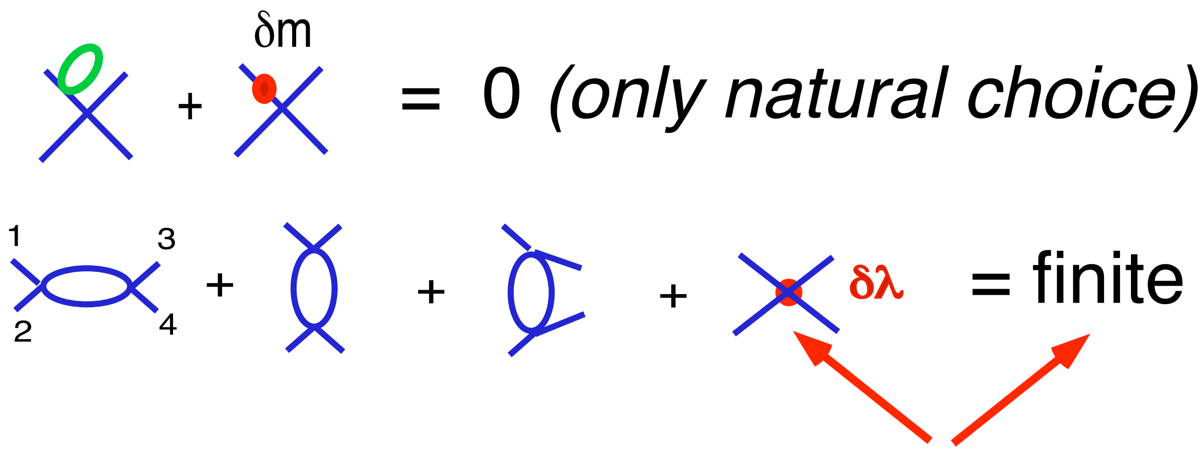
The renormalized 4-point function:



- **The combination is supposed to be finite.**

- **How to choose them? This is the renormalization "scheme"**

Renormalization:



But what should we choose for these?

A B C D

- **For example: define A+B+C b cutting off $\int d^4k$ at $k^2 = \Lambda^2$ (regularization). Then**

$$A + B + C = a \ln \frac{\Lambda^2}{s} + b(s, t, u, m^2)$$

- Now <u>choose</u>:

$$D = -\ a \ln \frac{\Lambda^2}{\mu^2}$$

so that

$$A + B + C + D = a \ln \frac{\mu^2}{s} + b(s, t, u, m^2)$$

independent of Λ.

- Criterion for choosing μ is a "renormalization scheme":
MOM scheme: $\mu = s_0$, some point in momentum space.
MS scheme: same μ for all diagrams, momenta

- But the value of μ is still arbitrary. $\mu =$ renormalization scale.

- Modern view (Wilson) We hide our ignorance of the true high-E behavior.

- All current theories are "effective" theories with the same low-energy behavior as the true theory.

- μ-dependence is the price we pay for working with an effective theory: The Renormalization Group
- As μ changes, mass m and coupling g have to change: $m = m(\mu)\ g = g(\mu)$ "*renormalized*" but ...
- Physical quantities can't depend on μ:

$$\mu \frac{d}{d\mu} \sigma \left(\frac{s_{ij}}{\mu^2}, \frac{m^2}{\mu^2}, g(\mu), \mu \right) = 0$$

- The 'group' is just the set of all changes in μ.
- 'RG' equation (Mass dimension $[\sigma] = d_\sigma$):

$$\left(\mu \frac{\partial}{\partial \mu} + \mu \frac{\partial g}{\partial \mu} \frac{\partial}{\partial g} + \mu \frac{\partial m}{\partial \mu} \frac{\partial}{\partial m} + d_\sigma \right) \sigma \left(\frac{s_{ij}}{\mu^2}, \frac{m^2}{\mu^2}, g(\mu), \mu \right) = 0$$

$$\text{The beta function :} \quad \beta(g) \equiv \mu \frac{\partial g(\mu)}{\partial \mu}$$

- **The Running coupling**
- **Consider <u>any</u> σ ($m = 0$, $d_\sigma = 0$):**

$$\mu \frac{d\sigma}{d\mu} = 0 \quad \rightarrow \quad \mu \frac{\partial \sigma}{\partial \mu} = -\beta(g) \frac{\partial \sigma}{\partial g} \tag{1}$$

- **in PT:**

$$\sigma = g^2(\mu)\sigma^{(1)} + g^4(\mu)\left[\sigma^{(2)}\left(\frac{s_{ij}}{s_{kl}}\right) + \tau^{(2)} \ln \frac{s_{12}}{\mu^2}\right] + \ldots \tag{2}$$

- **(2) in (1) $\rightarrow$**

$$g^4 \tau^{(2)} = 2g\sigma^{(1)}\beta(g) + \ldots$$

$$\beta(g) = \frac{g^3}{2}\frac{\tau^{(2)}}{\sigma^{(1)}} + \mathcal{O}(g^5) \equiv -\frac{g^3}{16\pi^2}\beta_0 + \mathcal{O}(g^5)$$

- **In QCD:**

$$\beta_0 = 11 - \frac{2n_f}{3}$$

- $-\beta_0 < 0 \rightarrow g$ **decreases as μ increases.**

- **Asymptotic Freedom**: Solution for the QCD coupling

$$\mu \frac{\partial g}{\partial \mu} = -g^3 \frac{\beta_0}{16\pi^2}$$

$$\frac{dg}{g^3} = -\frac{\beta_0}{16\pi^2} \frac{d\mu}{\mu}$$

$$\frac{1}{g^2(\mu_2)} - \frac{1}{g^2(\mu_1)} = -\frac{\beta_0}{16\pi^2} \ln \frac{\mu_2}{\mu_1}$$

$$g^2(\mu_2) = \frac{g^2(\mu_1)}{1 + \frac{\beta_0}{16\pi^2} g^2(\mu_1) \ln \frac{\mu_2}{\mu_1}}$$

- Vanishes for $\mu_2 \longrightarrow \infty$. Equivalently,

$$\alpha_s(\mu_2^2) \equiv \frac{g^2(\mu_2^2)}{4\pi} = \frac{\alpha_s(\mu_1)}{1 + \frac{\beta_0}{4\pi} \alpha_s(\mu_1) \ln \frac{\mu_2}{\mu_1}}$$

- **Dimensional transmutation: $\Lambda_{\rm QCD}$**
  - **Two mass scales appear in**

$$\alpha_s(\mu_2^2) = \frac{\alpha_s(\mu_1)}{1 + \frac{\beta_0}{4\pi}\alpha_s(\mu_1)\ln\frac{\mu_2}{\mu_1}}$$

    but the value of $\alpha_s(\mu_2)$ can't depend on choice of μ_1.
  - **Reduce it to one by defining $\Lambda \equiv \mu_1\, e^{-\beta_0/\alpha_s(\mu_1)}$, independent of μ_1. Then**

$$\alpha_s(\mu_2^2) = \frac{4\pi}{\beta_0 \ln\frac{\mu_2}{\Lambda^2}}$$

- **Asymptotic freedom strongly suggests a relationship to the parton model, in which partons act as if free at short distances. But how to quantify this observation?**

3. Infrared Safety

- To use perturbation theory, would like to choose μ ‘as large as possible to make $\alpha_s(\mu)$ as small as possible.
- But how small is possible?
- A “typical” cross section, , define $Q^2 = s_{12}$ and $x_{ij} = s_{ij}/Q^2$,

$$\sigma\left(\frac{Q^2}{\mu^2}, x_{ij}, \frac{m_i^2}{\mu^2}, \alpha_s(\mu), \mu\right) = \sum_{n=1}^{\infty} a_n\left(\frac{Q^2}{\mu^2}, x_{ij}, \frac{m_i^2}{\mu^2}\right)\alpha_s^n(\mu)$$

with m_i^2 all fixed masses – external, quark, gluon (=0!)

- Generically, the a_n depend logarithmically on their arguments, so a choice of large μ results in large logs of m_i^2/μ^2.

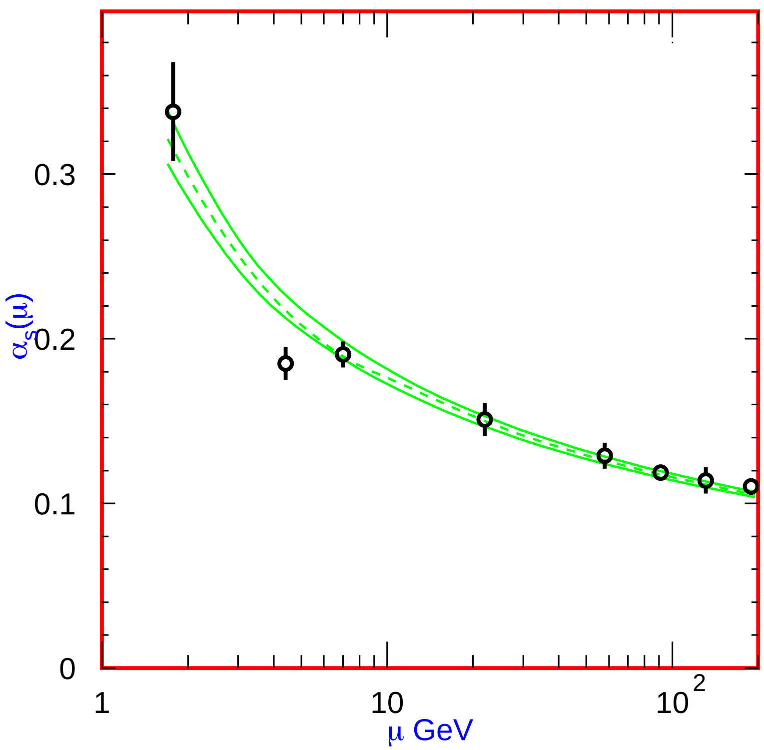
- But if we <u>could</u> find quantities that depend on $m_i's$ only through powers, $(m_i/\mu)^p, p > 0$, the large-μ limit would exist.

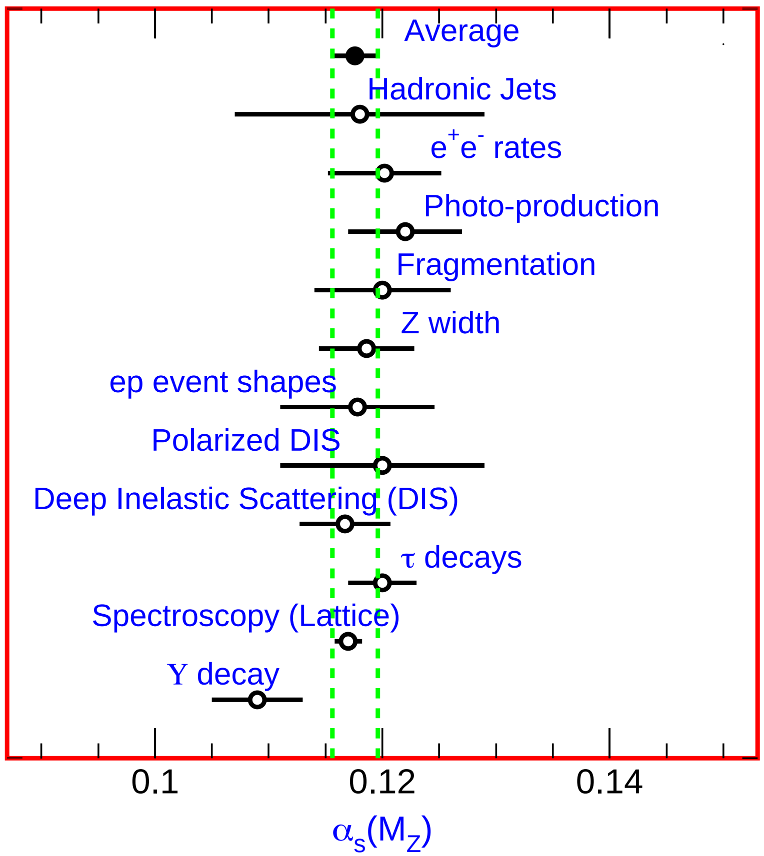
$$\sigma\left(\frac{Q^2}{\mu^2}, x_{ij}, \frac{m_i^2}{\mu^2}, \alpha_s(Q), \mu\right) = \sigma\left(\frac{Q}{\mu}, x_{ij}, \frac{m_i^2}{\mu^2}, \alpha_s(\mu), \mu\right)$$
$$= \sum_{n=1}^{\infty} a_n\left(\frac{Q}{\mu}, x_{ij}\right)\alpha_s^n(\mu) + \mathcal{O}\left(\left[\frac{m_i^2}{\mu^2}\right]^p\right)$$

- Such quantities are called infrared (IR) safe.
- Measure $\sigma \longrightarrow$ solve for α_s. Allows observation of the running coupling.
- Most pQCD is the computation of IR safe quantities.

- **Consistency of $\alpha_s(\mu)$ found as above at various momentum scales**

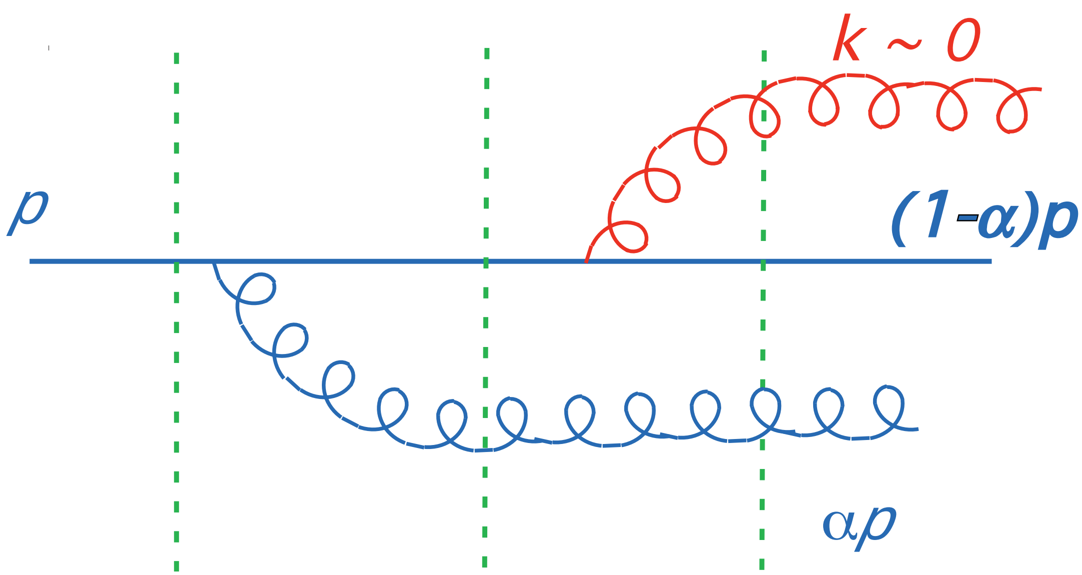
(Particle Data Group)





- **To find IR safe quantities, need to understand where the low-mass logs come from.**

- To analyze diagrams, we generally think of $m \to 0$ limit in m/Q.
- Generic source of IR (soft and collinear) logarithms:



- IR logs come from degenerate states:
  Uncertainty principle $\Delta E \to 0 \Leftrightarrow \Delta t \to \infty$.
- For soft emission and collinear splitting it's "never too late". But these processes don't change the flow of energy . . . The problem is asking for particle content.

- **For IR safety, sum over degenerate final states in perturbation theory, and see what the sum is. This requires us to introduce another regularization, this time for IR behavior.**

- **The IR regulated theory is like QCD at short distances, but is better-behaved at long distances.**

  IR-regulated QCD not the same as QCD except for IR safe quantities.

- See how it works for the total $\mathrm{e}^+\mathrm{e}^-$ annihilation cross section to order α_s. Lowest order is $2 \longrightarrow 2, \sigma_2^{(0)} \equiv \sigma_0$, σ_3 starts at order α_s.
  - Gluon mass regularization: $1/k^2 \longrightarrow 1/(k^2 - m_G)^2$

$$\sigma_3^{(m_G)} = \sigma_0 \frac{4}{3}\frac{\alpha_s}{\pi}\left(2\ln^2\frac{Q}{m_g} - 3\ln Q m_g - \frac{\pi^2}{6} + \frac{5}{2}\right)$$

$$\sigma_2^{(m_G)} = \sigma_0\left[1 - \frac{4}{3}\frac{\alpha_s}{\pi}\left(2\ln^2\frac{Q}{m_g} - 3\ln\frac{Q}{m_g} - \frac{\pi^2}{6} + \frac{7}{4}\right)\right]$$

which gives

$$\sigma_{\mathrm{tot}} = \sigma_2^{(m_G)} + \sigma_3^{(m_G)} = \sigma_0\left[1 + \frac{\alpha_s}{\pi}\right]$$

  - Pretty simple! (Cancellation of virtual (σ_2) and real (σ_3) gluon diagrams.)

– **Dimensional regularization: change the area of the sphere from $4\pi R^2$ to $(4\pi)^{(1-\varepsilon)}\frac{\Gamma(1-\varepsilon)}{\Gamma(2(1-\varepsilon))}R^{2-2\varepsilon}$ with $\varepsilon = 2 - D/2$ in D dimensions.**

$$\sigma_3^{(\varepsilon)} = \sigma_0 \frac{4}{3}\frac{\alpha_s}{\pi}\left(\frac{(1-\varepsilon)^2}{(3-2\varepsilon)\Gamma(2-2\varepsilon)}\right)\left(\frac{4\pi\mu^2}{Q^2}\right)^{\varepsilon} \times \left(\frac{1}{\varepsilon^2} - \frac{3}{2\varepsilon} - \frac{\pi^2}{2} + \frac{19}{4}\right)$$

$$\sigma_2^{(\varepsilon)} = \sigma_0 \left[1 - \frac{4}{3}\frac{\alpha_s}{\pi}\left(\frac{(1-\varepsilon)^2}{(3-2\varepsilon)\Gamma(2-2\varepsilon)}\right)\left(\frac{4\pi\mu^2}{Q^2}\right)^{\varepsilon} \times \left(\frac{1}{\varepsilon^2} - \frac{3}{2\varepsilon} - \frac{\pi^2}{2} + 4\right)\right]$$

which gives again

$$\sigma_{\text{tot}} = \sigma_2^{(m_G)} + \sigma_3^{(m_G)} = \sigma_0\left[1 + \frac{\alpha_s}{\pi}\right]$$

- **This illustrates IR Safety: σ_2 and σ_3 depend on regulator, but their sum does not.**

- **Generalized IR safety: sum over all states with the same flow of energy into the final state. Introduce** *IR safe weight* *"$e(\{p_i\})$"*

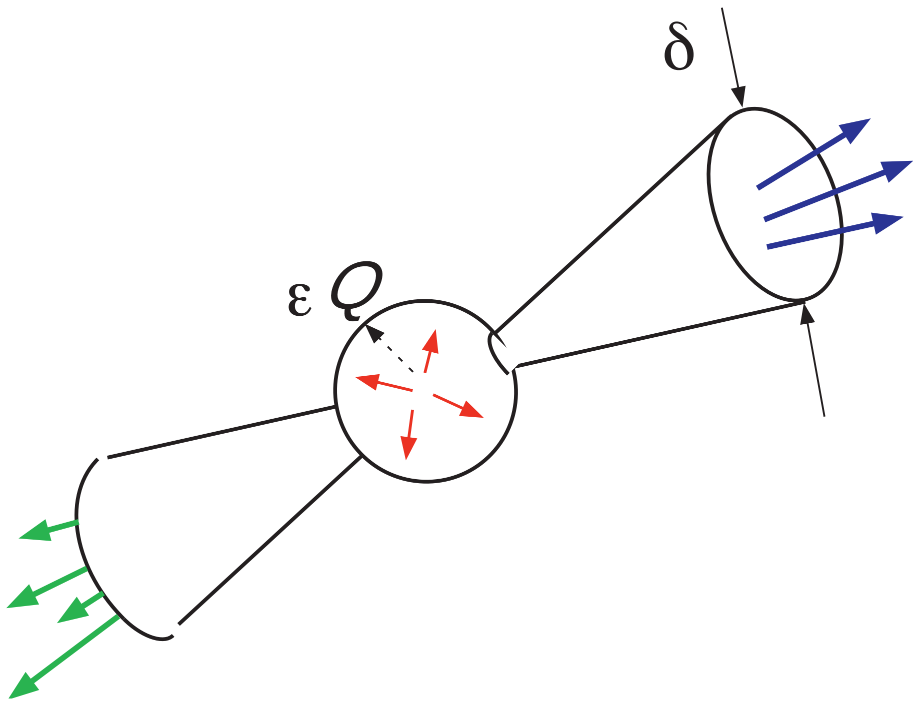
$$\frac{d\sigma}{de} = \sum_n \int_{PS(n)} |M(\{p_i\})|^2 \delta\left(e(\{p_i\}) - w\right)$$

with

$$e(\ldots p_i \ldots p_{j-1}, \alpha p_i, p_{j+1} \ldots) = e(\ldots (1+\alpha) p_i \ldots p_{j-1}, p_{j+1} \ldots)$$

- **Neglect long times in the initial state for the moment and see how this works in e^+e^- annihilation: event shapes and jet cross sections.**

- **“Seeing” Quarks and Gluons With Jet Cross Sections**
- **Simplest example: cone jets in $\mathrm{e}^+\mathrm{e}^-$ annihilation**



- **Intuition: eliminating long-time behavior ⇔ recognize the impossibility of resolving collinear splitting/recombination of massless particles**
- **No factors Q/m or $\ln(Q/m)$ Infrared Safety.**

- In this case,

$$\sigma_{2J}(Q,\delta,\epsilon) = \frac{3}{8}\sigma_0(1+\cos^2\theta) \times \left(1 - \frac{4\alpha_s}{\pi}\left[4\ln\delta\ln\epsilon + 3\ln\delta + \frac{\pi^2}{3} + \frac{5}{2}\right]\right)$$

- Perfect for QCD: asymptotic freedom $\rightarrow d\alpha_s(Q)/dQ < 0$.
- No unique jet definition. $\leftrightarrow$ Each event a sum of possible histories.
- Relation to quarks and gluons always approximate but corrections to the approximation computable.

- The general form of a jet cross section:

$$\sigma_{\text{jet}} = \sigma_0 \sum_{n=0}^{\infty} c_n(y_i, N, C_F)\alpha_s^n(Q)$$

- Choices for y_i: δ, Ω_{jet}, T, $y_{\text{cut}}, \ldots$
- δ, cone size; Ω, jet direction
- Shape Variable, e.g. thrust ($T = 1$ for "back-to-back" jets

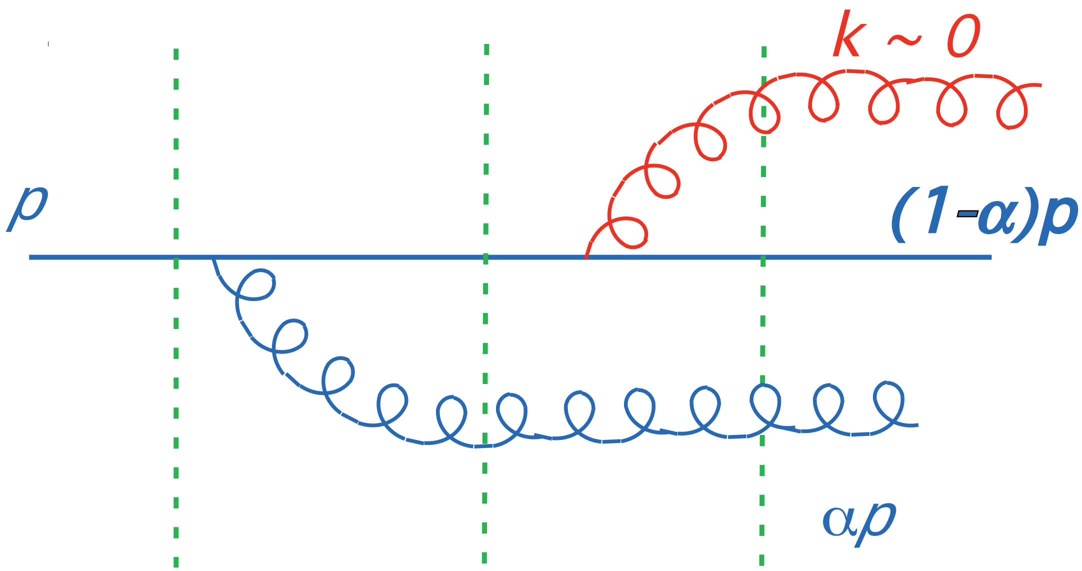
$$T = \frac{1}{s}\max_{\hat{n}} \sum_i |\hat{n} \cdot \vec{p}_i|$$

- y_{cut} Cluster Algorithm: $y_{ij} > y_{\text{cut}}$,

$$y_{ij} = 2\text{min}\left(E_i^2, E_j^2\right)\left(1 - \cos\theta_{ij}\right)$$

Summarize: what makes a cross section infrared safe?

- **Independence of long-time interactions:**



More specifically: should depend on only the flow of energy into the final state. This implies independence of collinear re-arrangements and soft parton emisssion.

But if we prepare one or two particles in the initial state (as in DIS or proton-proton scattering), we will always be sensitive to long time behavior inside these particles. The parton model suggests what to do: factorize. This is the subject of Part III.