Introduction to the Parton Model and Pertrubative QCD

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III. Factorization and Evolution

1. Factorization in DIS
2. DIS at one loop
3. Evolution
4. Factorization in hadron-hadron scattering

1. Factorization in DIS

- Challenge: use AF in observables
  (cross sections, also some amplitudes)
  that are not infrared safe

- Possible if: σ has a short-distance subprocess.
  Separate $IR\ Safe$ from IR: this is factorization

- IR Safe part (short-distance) is calculable in pQCD

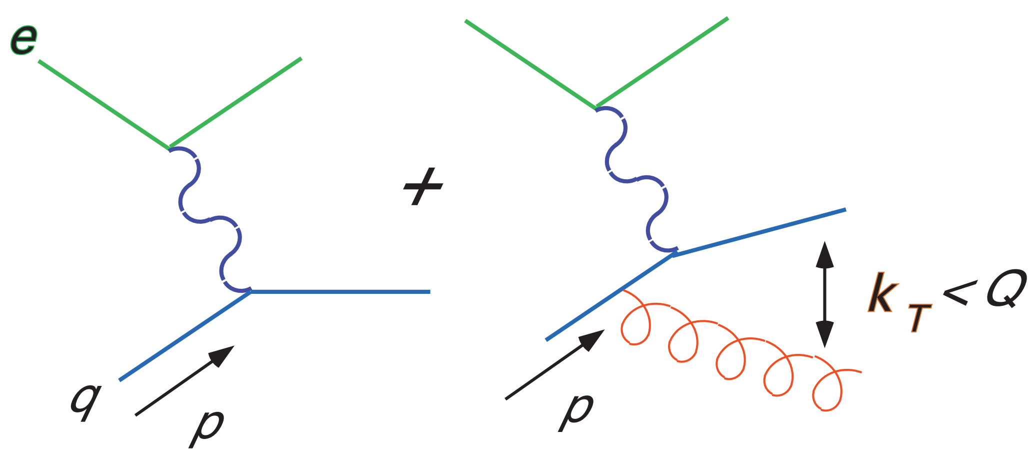
- Infrared part – example: parton distribution –
  measureable and universal

- Infrared safety – insensitive to soft gluon emission
  collinear rearrangements

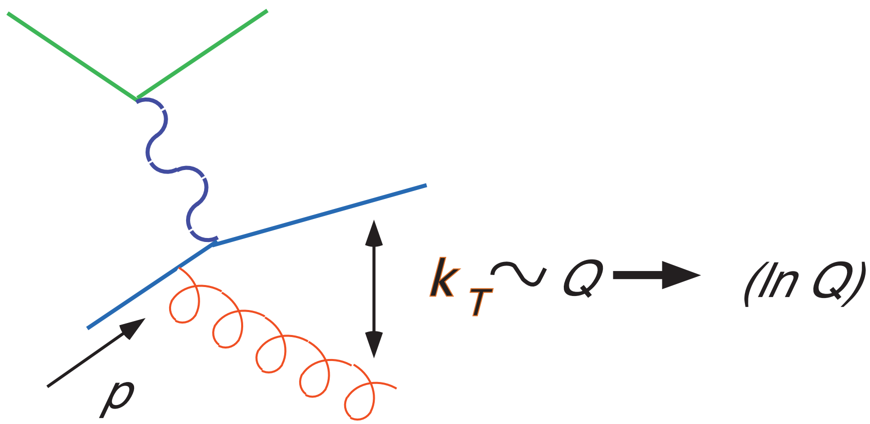
- For DIS, will find a result ...
- Just like Parton Model except in Parton Model the infrared safe part is $\sigma_{\mathrm{Born}} \Rightarrow f(x)$ normalized uniquely

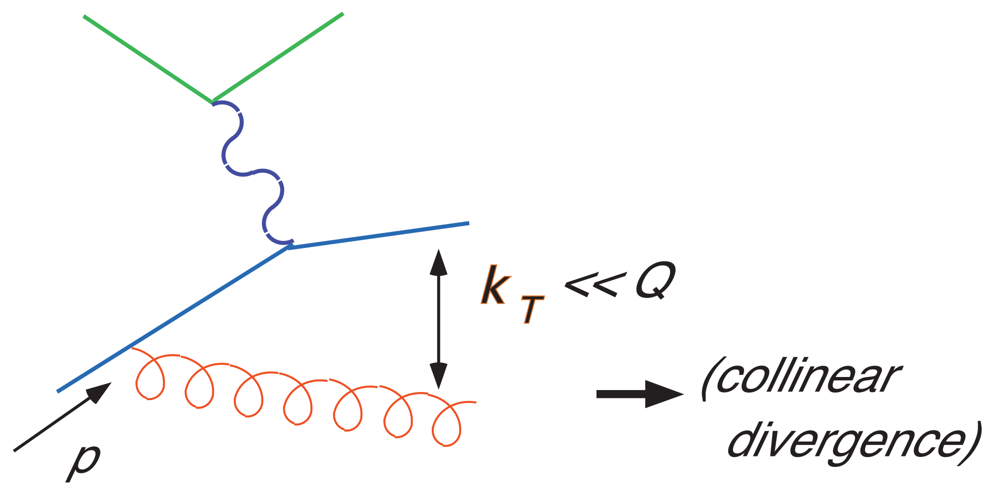
- In pQCD must define parton distributions more carefully: the factorization scheme

- Basic observation: virtual states are not truly frozen. Some states fluctuate on scale $1/Q$ ...



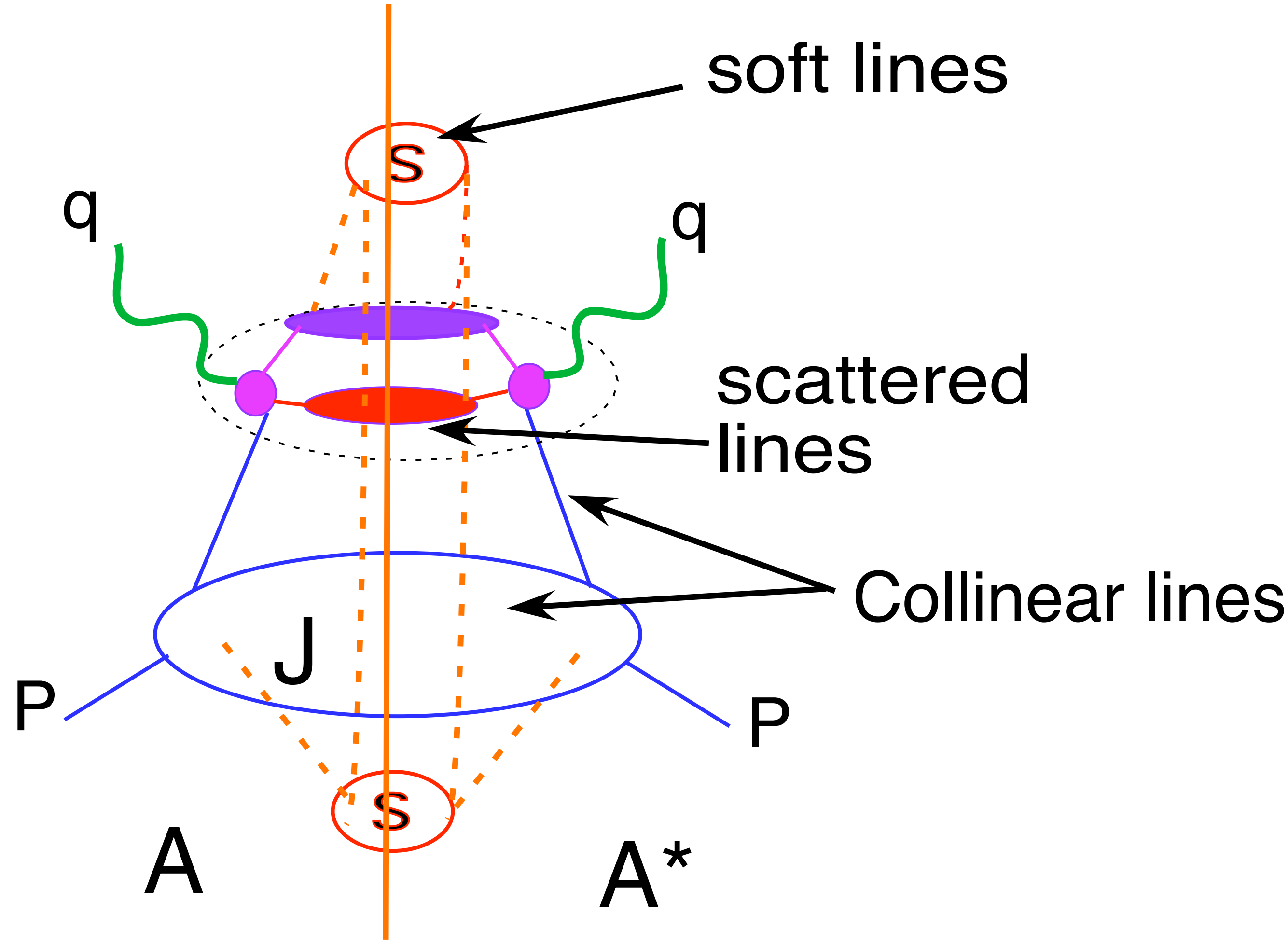
Short-lived states $\Rightarrow \ln(Q)$





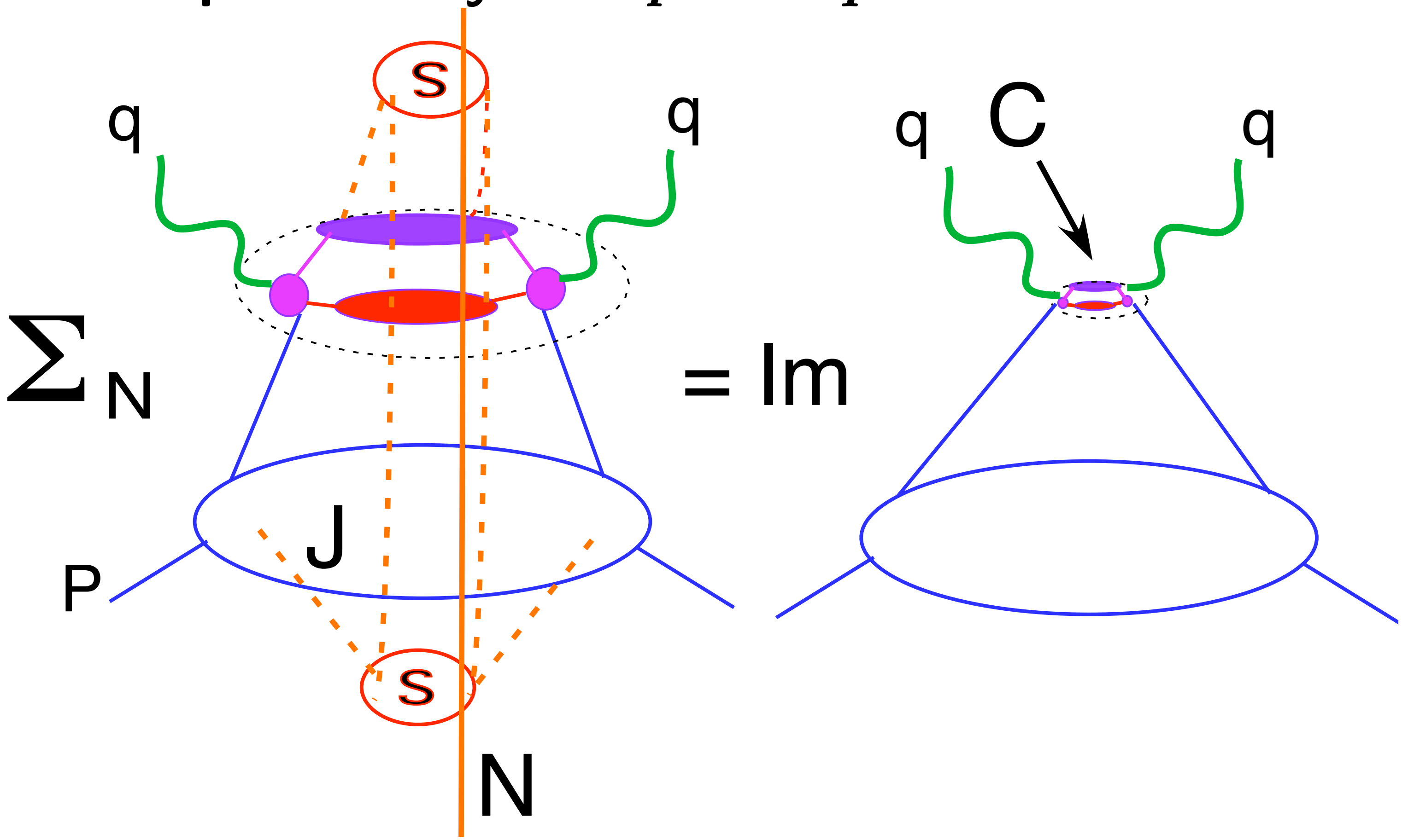
Longer-lived states $\Rightarrow$ Collinear Singularity (IR)

- **Generalization: all sources of long-distance behavior from "physical processes" made of on-shell particles**



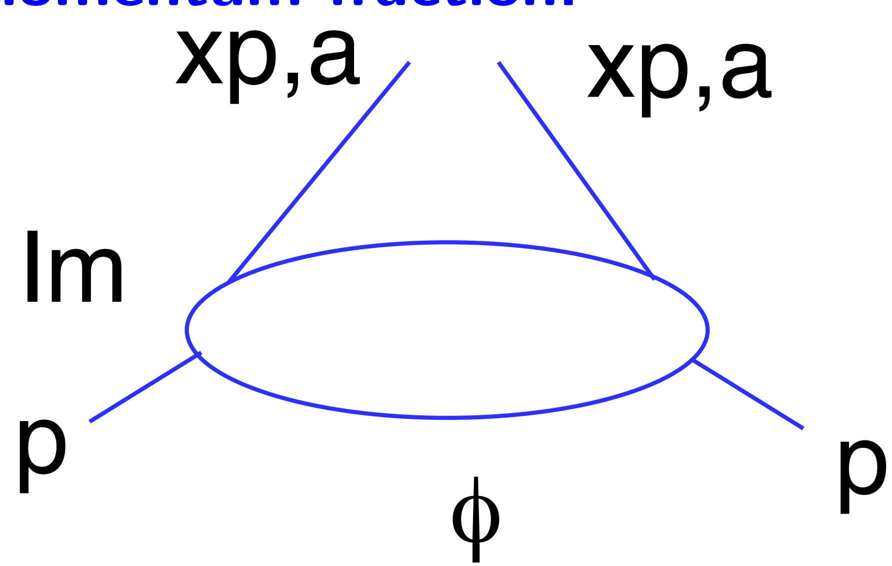
- **The story: h splits into collinear partons, then one of them scatters, producing jets that recede at speed of light, connected only by "infinite wavelength soft" quanta.**

- **Use of the optical theorem. No physical processes in the final state remain, and it collapses to a "short-distance" function C, that depends only on xp and q:**



- **Final-state interactions now suppressed by powers of Q "higher-twist".**

- The partons on each side of the $C(p, q)$ must have the same flavor and momentum fraction.



- Definition of parton distribution generates all the same long-distance behavior left in in the original diagrams (quark case) after the sum over hadronic final states:

$$\phi_{a/h}(x, \mu) = \sum_{\text{spins } \sigma} \int \frac{dy^-}{2\pi} e^{-ixp^+y^-} \langle p, \sigma | \bar{q}(y^-) \gamma^+ q(0) | p, \sigma \rangle$$

- This matrix element requires renormalization: thus the 'μ'.

The result: factorized DIS

$$F_2^{\gamma q}(x, Q^2) = \int_x^1 d\xi \; C_2^{\gamma q}\left(\frac{x}{\xi}, \frac{Q}{\mu}, \frac{\mu_F}{\mu}, \alpha_s(\mu)\right) \times \phi_{q/q}(\xi, \mu_F, \alpha_s(\mu))$$

$$\equiv C_2^{\gamma q}\left(\frac{x}{\xi}, \frac{Q}{\mu}, \frac{\mu_F}{\mu}, \alpha_s(\mu)\right) \otimes \phi_{q/q}(\xi, \mu_F, \alpha_s(\mu))$$

- $\phi_{q/q}$ has $\ln(\mu_F/\Lambda_{\mathrm{QCD}})$ ...
- C has $\ln(Q/\mu)$, $\ln(\mu_F/\mu)$
- Often pick $\mu = \mu_F$ and often pick $\mu_F = Q$. So often see:

$$F_2^{\gamma q}(x, Q^2) = C_2^{\gamma q}\left(\frac{x}{\xi}, \alpha_s(Q)\right) \otimes \phi_{q/q}(\xi, Q^2)$$

2. DIS at one loop

- **But we still need to specify what we** *really* **mean by factorization:** *scheme* **as well as** *scale*.

- **For this, compute** $F_2^{\gamma q}(x, Q)$.

- **Keep** $\mu = \mu_F$ **for simplicity.**

- **"Compute quark-photon scattering"** – *What does this mean?*

  Must use an *IR-regulated* **theory**

  Extract the *IR Safe part* **then take away the regularization**

- **Let's see how it works . . .**

- **At** *zeroth order – no interactions:*

  $$C^{\gamma q_f (0)} = Q_f^2 \, \delta(1 - x/\xi)$$

  (Born cross section; parton model)

  $$\phi^{(0)}_{q_f/q_{f'}}(\xi) = \delta_{f f'} \, \delta(1 - \xi)$$

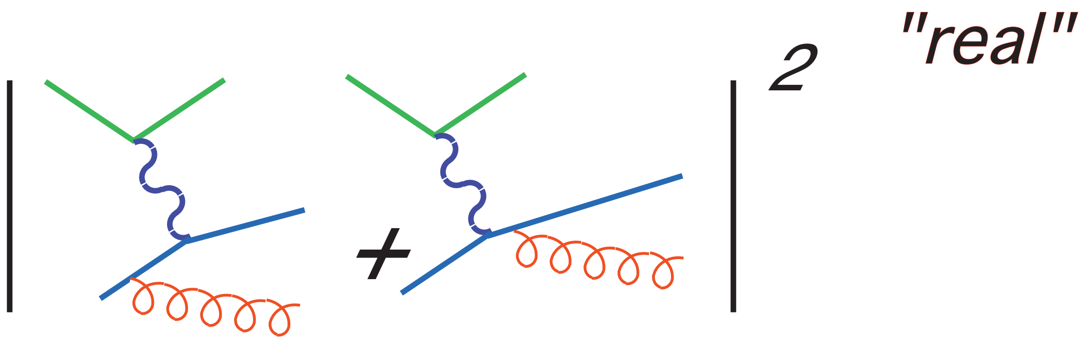
  (at zeroth order, momentum fraction conserved)

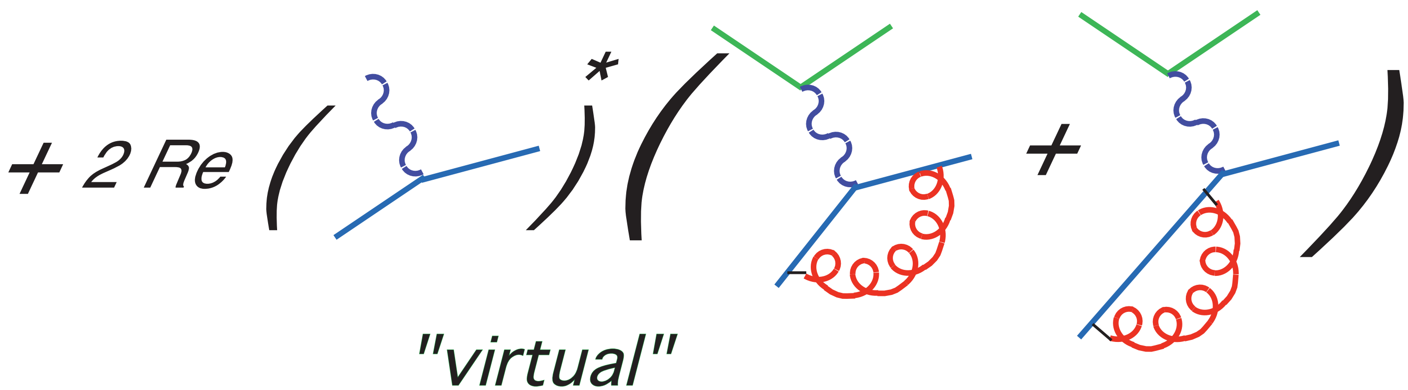
$$
\begin{aligned}
F_2^{\gamma q_f\,(0)}(x,Q^2) &= \int_x^1 d\xi\; C_2^{\gamma q_f\,(0)}\left(\frac{x}{\xi},\frac{Q}{\mu},\frac{\mu_F}{\mu},\alpha_s(\mu)\right) \\
&\quad \times\; \phi^{(0)}_{q_f/q_f}(\xi,\mu_F,\alpha_s(\mu)) \\
&= Q_f^2\;\int_x^1 d\xi\;\delta(1-x/\xi)\;\delta(1-\xi) \\
&= Q_f^2\; x\,\delta(1-x)
\end{aligned}
$$

- **On to one loop ...**

• $F^{\gamma q}$ **at one loop: factorization schemes**

• **Start with** F_2 **for a** *quark***:**





Have to combine final states with different phase space . . .

- **"Plus Distributions":**

$$\int_0^1 dx\, \frac{f(x)}{(1-x)_+} \equiv \int_0^1 dx\, \frac{f(x)-f(1)}{(1-x)}$$

$$\int_0^1 dx\, f(x)\left(\frac{\ln(1-x)}{1-x}\right)_+ \equiv \int_0^1 dx\, \left(\, f(x)-f(1)\,\right)\, \frac{\ln(1-x)}{(1-x)}$$

and so on . . . where

- $f(x)$ **will be parton distributions**
- $f(x)$ **term: real gluon, with momentum fraction** $1-x$
- $f(1)$ **term: virtual, with elastic kinematics**

- **DGLAP "evolution kernel" = "splitting function"**

$$P_{qq}^{(1)}(x) = C_F\, \frac{\alpha_s}{\pi}\, \left[\frac{1+x^2}{1-x}\right]_+$$

- α_s **Expansion:**

$$F_2^{\gamma q}(x, Q^2) = \int_x^1 d\xi \; C_2^{\gamma q}\left(\frac{x}{\xi}, \frac{Q}{\mu}, \frac{\mu_F}{\mu}, \alpha_s(\mu)\right) \times \phi_{q/q}(\xi, \mu_F, \alpha_s(\mu))$$

$$F_2^{\gamma q_f}(x, Q^2) = C_2^{(0)} \; \phi^{(0)} + \frac{\alpha_s}{2\pi} \, C^{(1)} \; \phi^{(0)} + \frac{\alpha_s}{2\pi} \, C^{(0)} \; \phi^{(1)} + \ldots$$

- **And result:**

$$F_2^{\gamma q_f}(x,Q^2) = Q_f^2 \, \{ \, x \, \delta(1-x)$$

$$+ \frac{\alpha_s}{2\pi} C_F \left[\frac{1+x^2}{1-x} \left(\frac{\ln(1-x)}{x} \right) + \frac{1}{4}(9-5x) \right]_+$$

$$+ \frac{\alpha_s}{2\pi} C_F \int_0^{Q^2} \frac{dk_T^2}{k_T^2} \left[\frac{1+x^2}{1-x} \right]_+ \} + \ldots$$

$$F_1^{\gamma q_f}(x,Q^2) = \frac{1}{2x} \left\{ F_2^{\gamma q_f}(x,Q^2) - C_F \, \alpha \, \frac{\alpha_s}{\pi^2} \, 2x \right\}$$

- **Factorization Schemes**

$\overline{\mathrm{MS}}$ **(Corresponds to matrix element above.)**

$$\phi^{(1)}_{q/q}(x,\mu^2) = \frac{\alpha_s}{\pi^2}\, P_{qq}(x)\ \int_0^{\mu^2} \frac{dk_T^2}{k_T^2}$$

With k_T-integral "IR regulated".

Advantage: technical simplicity; not tied to process.

$C^{(1)}(x)_{\overline{\mathrm{MS}}} = (\alpha_s/2\pi)\, P_{qq}(x) \ln(Q^2/\mu^2) + \mu$**-independent**

DIS:

$$\phi_{q/q}(x,\mu^2) = \frac{\alpha_s}{\pi^2}\, F^{\gamma q_f}(x,\mu^2)$$

Absorbs all uncertainties in DIS into a PDF.

Closer to experiment for DIS.

$C^{(1)}(x)_{\overline{DIS}} = (\alpha_s/2\pi)\ P_{qq}(x) \ln(Q^2/\mu^2) + 0$

- **Using the Regulated Theory to Get Parton Distributions for Real Hadrons . . .**

IR-regulated QCD is not $REAL$ **QCD**

BUT it only differs at low momenta

THUS we can use it for IR Safe functions: $C_2^{\gamma q}$**, etc.**

THIS enables us to get PDFs from experiment.

- **Compute $F_2^{\gamma q}$, $F_2^{\gamma G}$ ...**

  Define factorization scheme; find IR Safe C's

  Use factorization in the full theory

$$F_2^{\gamma N} = \sum_{a=q_f,\bar{q}_f,G} C^{\gamma a} \otimes \phi_{a/N}$$

  Measure F_2; then use the known C's to derive $\phi_{a/N}$

  NOW HAVE $\phi_{a/N}(\xi, \mu^2)$ AND CAN USE IT IN ANY OTHER PROCESS THAT FACTORIZES.

- **Multiple flavors and cross sections complicate technicalities; not logic (Global Fits)**

- **Evolution: Q^2-dependence**

- **In general, Q^2/μ^2 dependence still in $C_a\left(x/\xi, Q^2/\mu^2, \alpha_s(\mu)\right)$**

  Choose $\boldsymbol{\mu = Q}$

$$F_2^{\gamma A}(x, Q^2) = \sum_a \int_x^1 d\xi\; C_2^{\gamma a}\left(\frac{x}{\xi}, 1, \alpha_s(Q)\right)\; \phi_{a/A}(\xi, Q^2)$$

  $Q \gg \Lambda_{\rm QCD} \rightarrow$ *compute* $\boldsymbol{C}$*'s in PT*.

$$C_2^{\gamma a}\left(\frac{x}{\xi}, 1, \alpha_s(Q)\right) = \sum_n \left(\frac{\alpha_s(Q)}{\pi}\right)^n C_2^{\gamma a(n)}\left(\frac{x}{\xi}\right)$$

  But still need PDFs at $\mu = Q$: $\phi_{a/A}(\xi, Q^2)$ for different Q's.

3. Evolution

- **A remarkable consequence of factorization.**

- *Can use* $\phi_{a/A}(x, Q_0^2)$ **to determine**

  $\phi_{a/A}(x, Q^2)$ *and hence* $F_{1,2,3}(x, Q^2)$ *for any* Q

So long at $\alpha_s(Q)$ is still small

- **Illustrate by a 'nonsinglet' distribution**

$$F_a^{\gamma \mathrm{NS}} = F_a^{\gamma p} - F_a^{\gamma n}$$

$$F_2^{\gamma \mathrm{NS}}(x, Q^2) = \int_x^1 d\xi \; C_2^{\gamma \mathrm{NS}}\left(\frac{x}{\xi}, \frac{Q}{\mu}, \alpha_s(\mu)\right) \; \phi_{\mathrm{NS}}(\xi, \mu^2)$$

Gluons, antiquarks cancel

At one loop: $C_2^{\mathrm{NS}} = C_2^{\gamma N}$

- **Basic tool:**

- **'Mellin' Moments and Anomalous Dimensions**

$$\bar{f}(N) = \int_0^1 dx\ x^{N-1}\ f(x)$$

- **Reduces convolution to a product**

$$f(x) = \int_x^1 dy\ g\left(\frac{x}{y}\right)\ h(y) \rightarrow \bar{f}(N) = \bar{g}(N)\ \bar{h}(N+1)$$

- **Moments applied to NS structure function:**

$$\bar{F}_2^{\gamma\mathrm{NS}}(N, Q^2) = \bar{C}_2^{\gamma\mathrm{NS}}\left(N, \frac{Q}{\mu}, \alpha_s(\mu)\right)\, \bar{\phi}_{\mathrm{NS}}(N, \mu^2)$$

(Note $\phi_{\mathrm{NS}}(N, \mu^2) \equiv \int_0^1 d\xi \xi^N f(\xi, \mu^2)$ here.)

- $\bar{F}_2^{\gamma\mathrm{NS}}(N, Q^2)$ is **Physical**

$$\Rightarrow \quad \mu \frac{d}{d\mu}\, \bar{F}_2^{\gamma\mathrm{NS}}(N, Q^2) = 0$$

- ‘Separation of variables’

$$\mu \frac{d}{d\mu} \ln \bar{\phi}_{\rm NS}(N, \mu^2) = -\gamma_{\rm NS}(N, \alpha_s(\mu))$$

$$\gamma_{\rm NS}(N, \alpha_s(\mu)) = \mu \frac{d}{d\mu} \ln \bar{C}_2^{\gamma \rm NS}(N, \alpha_s(\mu))$$

- Because α_s is the only variable held in common.

$$\mu \frac{d}{d\mu} \ln \bar{\phi}_{\rm NS}(N,\mu^2) = -\gamma_{\rm NS}(N,\alpha_s(\mu))$$

$$\gamma_{\rm NS}(N,\alpha_s(\mu)) = \mu \frac{d}{d\mu} \ln \bar{C}_2^{\gamma {\rm NS}}(N,\alpha_s(\mu))$$

- **Only need to know C's $\Rightarrow \gamma_n$ from IR regulated theory!**

$\Downarrow$

Q-DEPENDENCE DETERMINED BY PT

EVOLUTION

THIS WAS HOW WE FOUND OUT QCD IS 'RIGHT'

AND THIS IS HOW QCD PREDICTS PHYSICS AT NEW SCALES

- $\gamma_{\rm NS}$ **at one loop** (5th line is an exercise.)

$$
\begin{aligned}
\gamma_{\rm NS}(N,\alpha_s) &= \mu\frac{d}{d\mu}\ \ln \bar{C}_2^{\gamma \rm NS}(N,\alpha_s(Q)) \\
&= \mu\frac{d}{d\mu}\ \left\{ (\alpha_s/2\pi)\ \bar{P}_{qq}(N) \ln(Q^2/\mu^2) + \mu \text{ indep.} \right\} \\
&= -\frac{\alpha_s}{\pi}\ \int_0^1 dx\ x^{N-1}\ P_{qq}(x) \\
&= -\frac{\alpha_s}{\pi} C_F\ \int_0^1 dx\ \left[\left(x^{N-1} - 1 \right) \frac{1+x^2}{1-x} \right] \\
&= -\frac{\alpha_s}{\pi} C_F\ \left[4\sum_{m=2}^{N} \frac{1}{m} - 2\frac{2}{N(N+1)} + 1 \right] \\
&\equiv -\frac{\alpha_s}{\pi}\ \gamma_{\rm NS}^{(1)}
\end{aligned}
$$

Hint: $(1-x^2)/(1-x) = 1 + x \ldots (1-x^k)/(1-x) = \sum_{i=0}^{k-1} x^k$

- **Solution and scale breaking.**

$$\mu \frac{d}{d\mu}\ \bar{\phi}_{\rm NS}(N,\mu^2) = -\gamma_{\rm NS}(N,\alpha_s(\mu))\ \bar{\phi}_{\rm NS}(N,\mu^2)$$

$$\bar{\phi}_{\rm NS}(N,\mu^2) = \bar{\phi}_{\rm NS}(N,\mu_0^2)\times \exp\left[-\frac{1}{2} \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2}\ \gamma_{\rm NS}(N,\alpha_s(\mu)) \right]$$

$\Downarrow$

$$\bar{\phi}_{\rm NS}(N,Q^2) \ = \bar{\phi}_{\rm NS}(N,Q_0^2)\ \left(\frac{\ln(Q^2/\Lambda^2_{\rm QCD})}{\ln(Q_0^2/\Lambda^2_{\rm QCD})}\right)^{-2\gamma_N^{(1)}/\beta_0}$$

Hint:

$$\alpha_s(Q) = \frac{4\pi}{\beta_0\ \ln(Q^2/\Lambda^2_{\rm QCD})}$$

So also: $\bar{\phi}_{\rm NS}(N,Q^2) \ = \bar{\phi}_{\rm NS}(N,Q_0^2)\ \left(\frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)}\right)^{-2\gamma_N^{(1)}/\beta_0}$

Qualitatively,

$$\bar{\phi}_{\rm NS}(N,Q^2) \;=\; \bar{\phi}_{\rm NS}(N,Q_0^2)\;\left(\frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)}\right)^{-2\gamma_N^{(1)}/\beta_0}$$

- Is 'mild' scale breaking, to be contrasted to

- Case of $\alpha_s \longrightarrow \alpha_0 \neq 0$, get a power Q-dependence:

$$\left(Q^2\right)^{\gamma^{(1)}\frac{\alpha_s}{2\pi}}$$

- $\Rightarrow$ QCD's consistency with the Parton Model (73-74)

- **Inverting the Moments.**

$$\mu \frac{d}{d\mu}\, \bar{\phi}_{\rm NS}(N,\mu^2) = -\gamma_N(\alpha_s(\mu))\, \bar{\phi}_{\rm NS}(N,\mu^2)$$

$$\Downarrow$$

$$\mu \frac{d}{d\mu}\, \bar{\phi}_{\rm NS}(N,\mu^2) = \int_x^1 \frac{d\xi}{\xi}\, P_{\rm NS}(\xi,\alpha_s(\mu))\, \bar{\phi}_{\rm NS}(\xi,\mu^2)$$

Splitting function $\leftrightarrow$ Moments

$$\int_0^1 dx\, x^{N-1}\, P_{qq}(x,\alpha_s) = \gamma_{qq}(N,\alpha_s)$$

- **Singlet (Full) Evolution**

$$\mu \frac{d}{d\mu} \bar{\phi}_{b/A}(N, \mu^2) = \sum_{b=q,\bar{q},G} \int_x^1 \frac{d\xi}{\xi} P_{ab}(\xi, \alpha_s(\mu)) \bar{\phi}_{b/A}(\xi, \mu^2)$$

- **The Physical Context of Evolution**

  – **Parton Model:** $\phi_{a/A}(x)$ **density of parton** a **with momentum fraction** x**, assumed independent of** Q

  – **PQCD:** $\phi_{a/A}(x, \mu)$**: same density, but with transverse momentum** $\leq \mu$

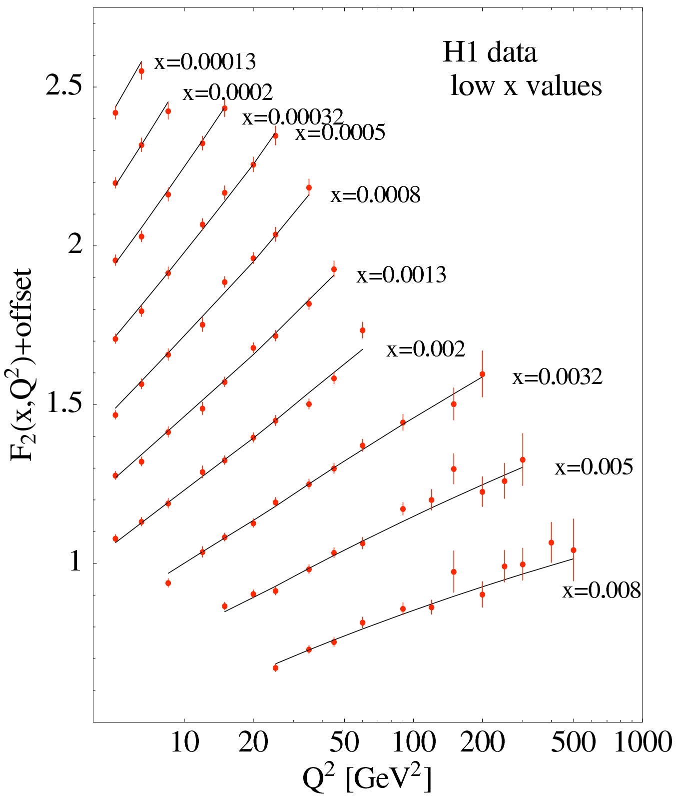
- **If there *were* a maximum transverse momentum Q_0, each $\phi_{a/h}(x, Q_0)$ would freeze for $\mu \geq Q_0$**

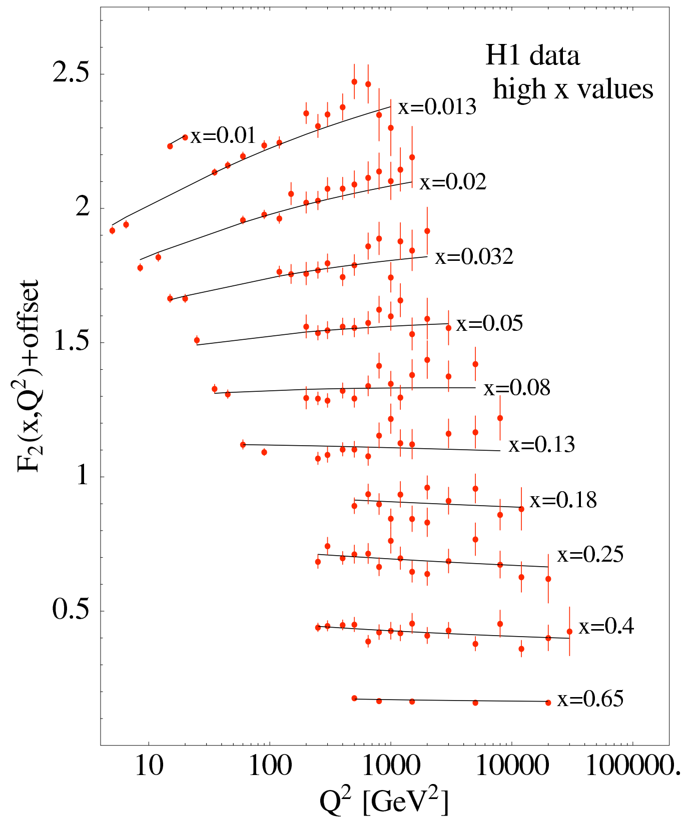
- ***Not so* in renormalized PT**

- **Scale breaking measures the change in the density as maximum transverse momentum increases**

- **Cross sections we compute still depend on our choice of μ through uncomputed "higher orders" in C and evolution**

• Evolution in DIS (with CTEQ6 fits)





4. Factorization in hadron-hadron scattering

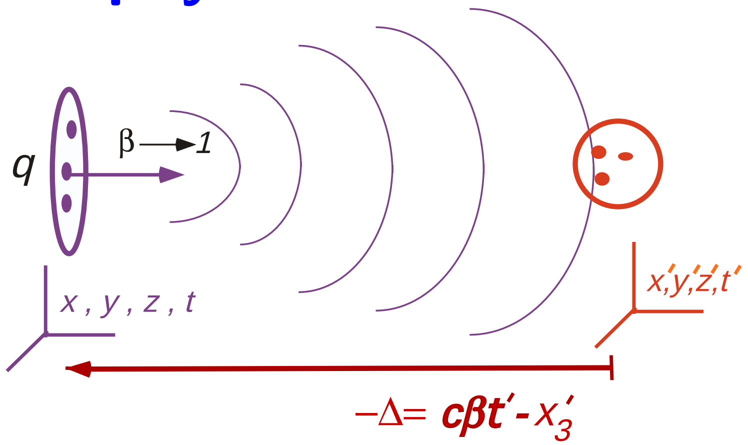
- General relation for hadron-hadron scattering for a hard, inclusive process with momentum transfer M to produce final state $F + X$:

$$d\sigma_{\mathrm{H_1H_2}}(p_1, p_2, M) = \sum_{a,b} \int_0^1 d\xi_a \, d\xi_b d\hat{\sigma}_{ab\to F+X}\,(\xi_a p_1, \xi_b p_2, M, \mu) \times \phi_{a/H_1}(\xi_a, \mu)\, \phi_{b/H_2}(\xi_b, \mu),$$

- "Factorization proofs: justifying the "universality" of the parton distributions.

As time allows . . . heuristic arguments for factorization, and a hint of the origin of factorization for fragmentation functions in pQCD.

- **The physical basis: classical fields**



$$\Delta \equiv x'_3 - \beta c t'$$

- **Why a classical picture isn't far-fetched . . .**

  The correspondence principle is the key to to IR divergences.

  An accelerated charge must produce classical radiation,

  and an infinite numbers of soft gluons are required to make a classical field.

Transformation of a scalar field:

$$\phi(x) \;=\; \frac{q}{(x_T^2 + x_3^2)^{1/2}} \;=\; \phi'(x') \;=\; \frac{q}{(x_T^2 + \gamma^2 \Delta^2)^{1/2}}$$

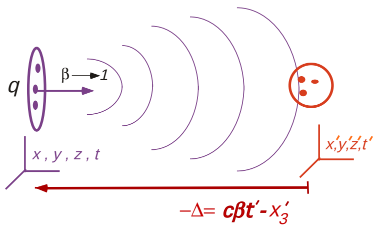
From the Lorentz transformation:

$$x_3 = \gamma(\beta c t' - x_3') \equiv -\gamma \Delta.$$

Closest approach is at $\Delta = 0$, i.e. $t' = \frac{1}{\beta c} x_3'$.

The scalar field transforms "like a ruler": At any fixed $\Delta \neq 0$, the field decreases like $1/\gamma = \sqrt{1-\beta^2}$.

Why? Because when the source sees a distance x_3, the observer sees a much larger distance.



field	x frame	x' frame
scalar	$\frac{q}{\lvert\vec{x}\rvert}$	$\frac{q}{(x_T^2+\gamma^2\Delta^2)^{1/2}}$
gauge (0)	$A^0(x) = \frac{q}{\lvert\vec{x}\rvert}$	$A'^0(x') = \frac{-q\gamma}{(x_T^2+\gamma^2\Delta^2)^{1/2}}$
field strength	$E_3(x) = \frac{q}{\lvert\vec{x}\rvert^2}$	$E'_3(x') = \frac{-q\gamma\Delta}{(x_T^2+\gamma^2\Delta^2)^{3/2}}$
Gauge fields :	$E_3 \sim \gamma^0,$	$E_3 \sim \gamma^{-2}$

- **The "gluon" $\vec{A}$ is enhanced, yet is a total derivative:**

$$A^\mu = q\frac{\partial}{\partial x'_\mu}\ \ln\left(\Delta(t', x'_3)\right) + \mathcal{O}(1-\beta) \sim A^-$$

- **The "large" part of A^μ can be removed by a gauge transformation!**

- The "force" $\vec{\mathrm{E}}$ field of the incident particle does not overlap the "target" until the moment of the scattering.

- "Advanced" effects are corrections to the total derivative:

$$1-\beta \;\sim\; \frac{1}{2}\left[\sqrt{1-\beta^2}\right]^2 \;\sim\; \frac{m^2}{2E^2}$$

- Power-suppressed! These are corrections to factorization.

- At the same time, a gauge transformation also induces a phase on charged fields:

$$q(x) \Rightarrow q(x)\; e^{i\,\ln(\Delta)}$$

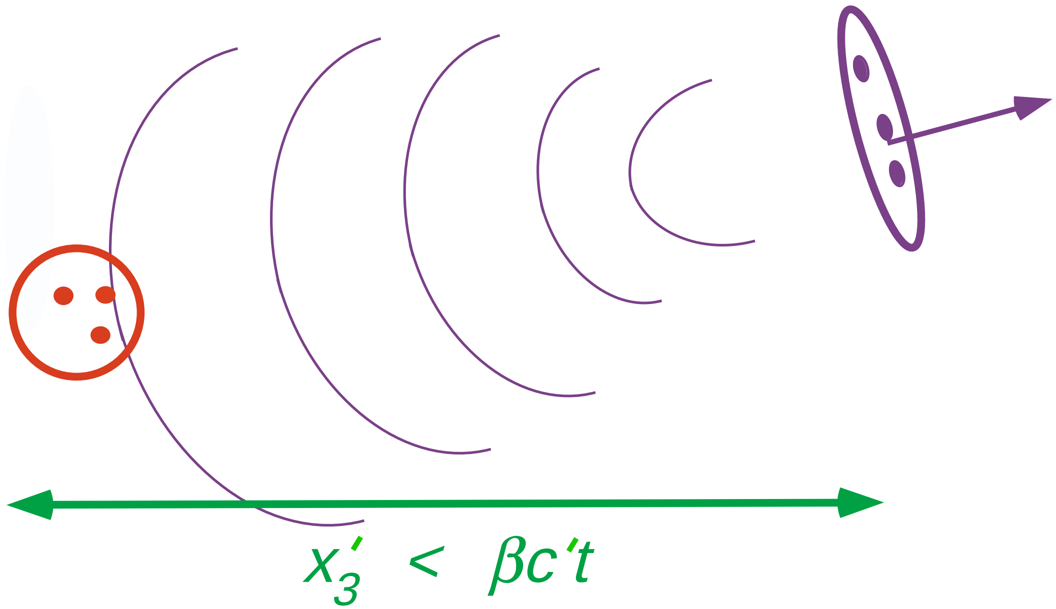
Cancelled if the fields are well-localized $\Leftrightarrow$ σ inclusive

- **Initial-state interactions decouple from hard scattering**
- **Summarized by multiplicative factors: the parton distributions**

  $\Rightarrow$ Cross section for inclusive hard scattering is IR safe, with power-suppressed corrections.

- **But what about cross sections where we observe specific particles in the final state? Single hadrons, dihadron correlations, etc?**

- **Much of the same reasoning holds:**

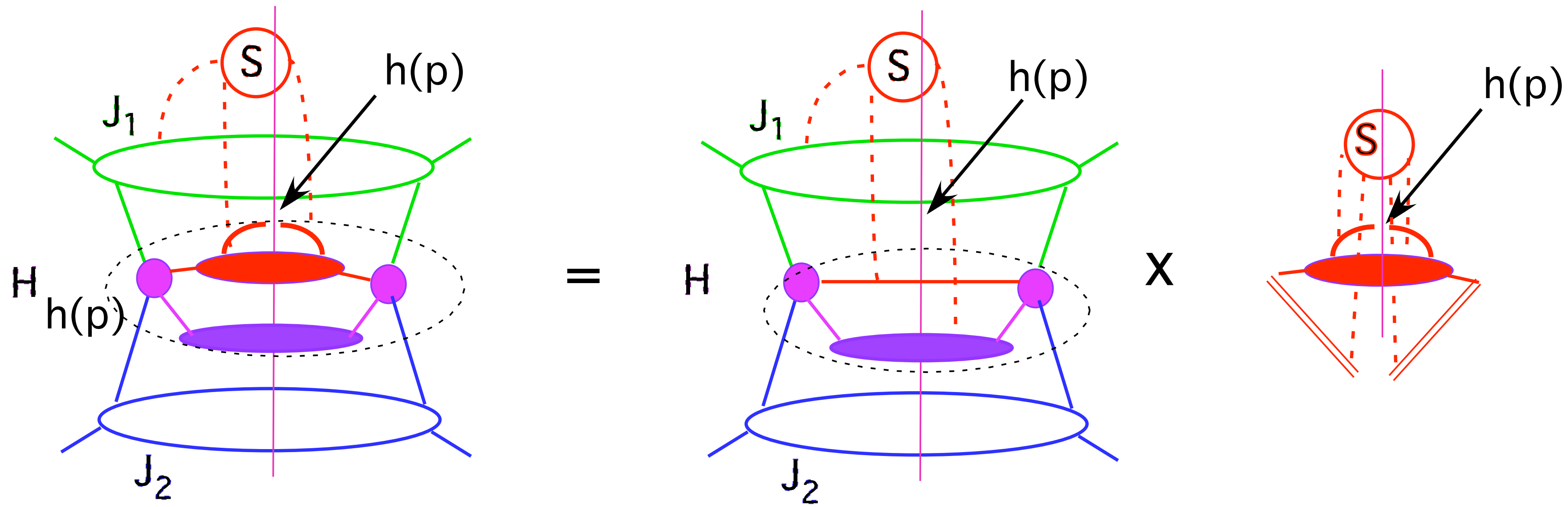


- **For single-particle inclusive . . .**

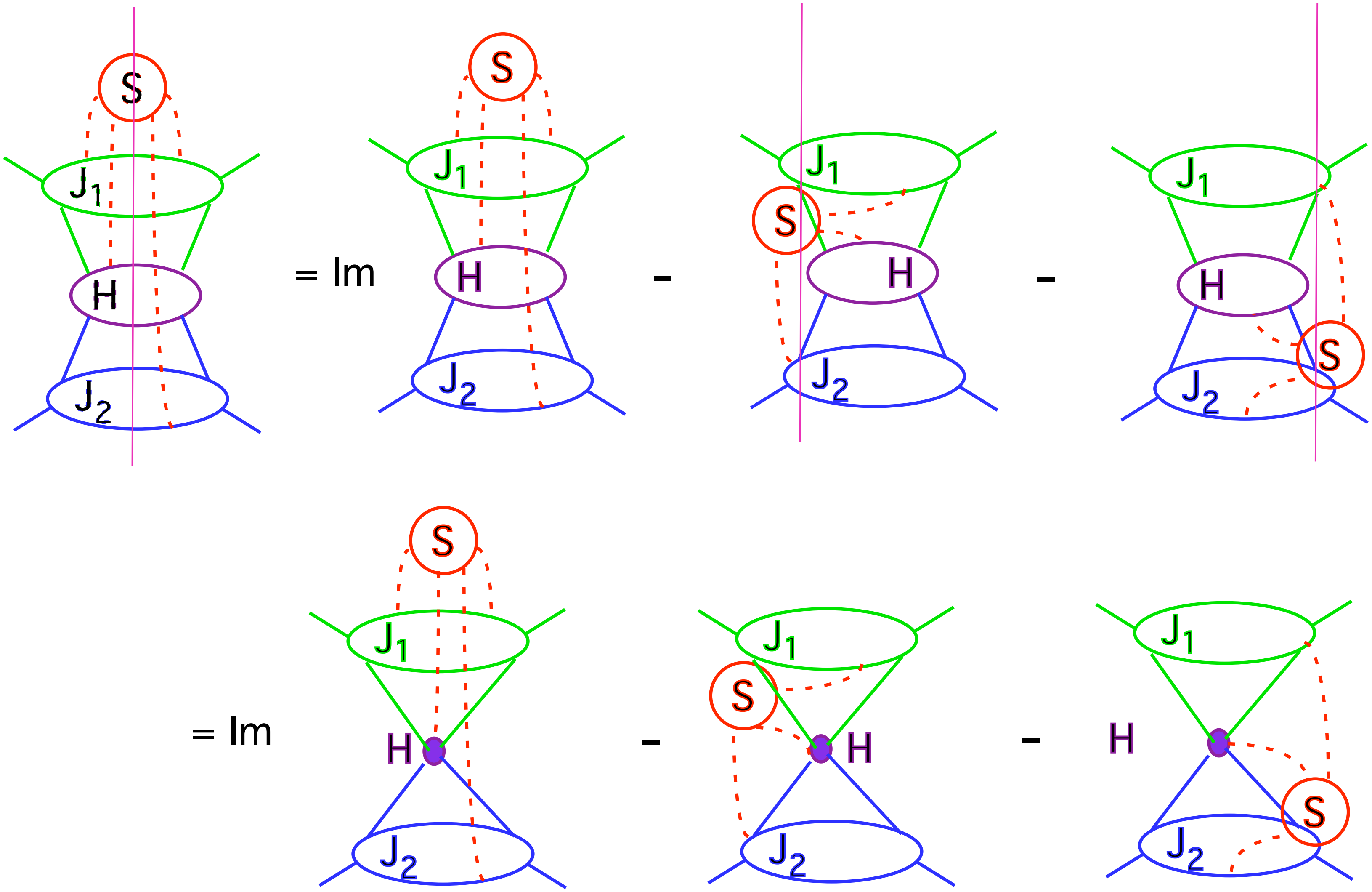
  Interactions after the scattering are too late to affect large momentum transfer, creation of heavy particle, etc.

  The fragmentation of partons to jets is too slow to know details of the hard scattering: factorization of fragmentation functions.

- **How it works in pQCD, with pictures as in DIS:**
- **Separation of soft quanta from fragmenting partons:**



- **The all-orders cancellation of soft singularities that connect initial and final states for single-particle inclusive and other short-distance cross sections in hadron-hadron scattering:**



- **all terms on RHS are power-suppressed**