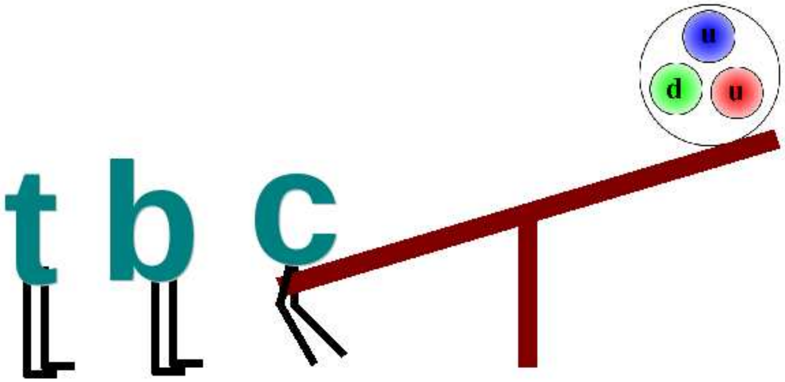
Heavy quarks



Zack Sullivan

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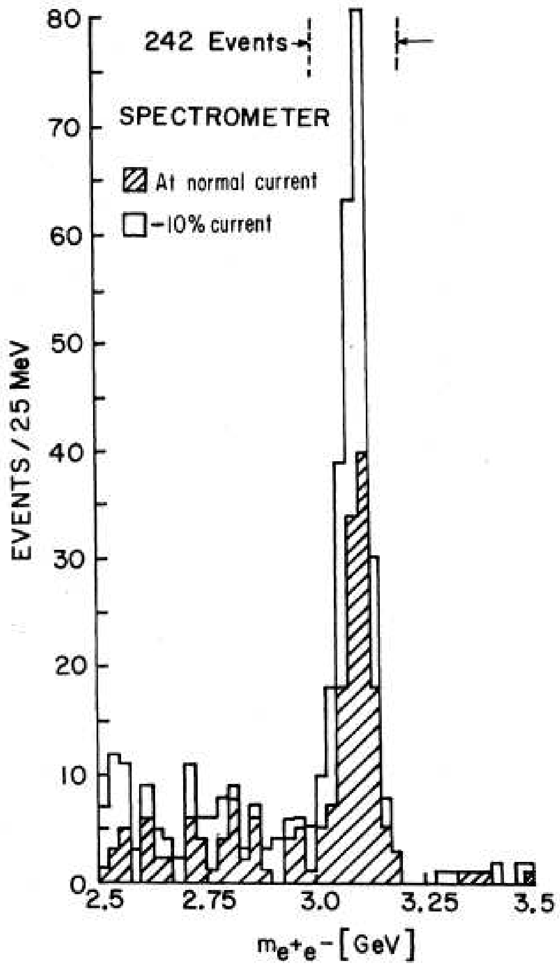
Outline

1. Heavy quark discovery and meaning
2. Heavy quark mass
    - What is it?
    - Why do we care about the top-quark mass in particular?
3. Heavy quark cross sections
    - Top cross section
    - Bottom cross section anomaly
    - (Single-)top cross section (Lecture 2)

A charming discovery



The first heavy quark, charm was discovered in 1974 in $p\bar{p}$ collisions at BNL and e^+e^- at SLAC

The observations were published together:
PRL 33, 1404 (1974); PRL 33, 1406 (1974)

The J/ψ was recognized as a $c\bar{c}$ bound state
$\Rightarrow m_c \sim 1.5$ GeV

The existence of a 4th quark confirmed the Glashow-Iliopoulos-Maiani explanation for why FCNC decays ($s \to d\nu\bar{\nu}$) did not occur.

— And it loosened the shackles of SU(3)$_{\text{flavor}}$, Gell-Mann's "Eightfold way"

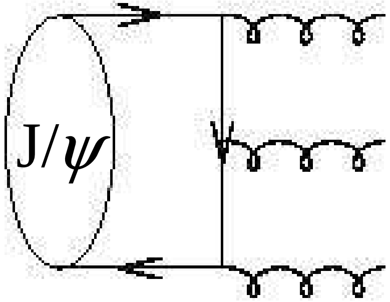
A charming crisis

While the J/ψ was clearly a quark bound state,
it had an extremely narrow width of 88 keV.

This caused a minor crisis in the fledgling QCD. . .

After all how could a strongly interacting state be narrow?
$\Gamma_\rho \sim 150$ MeV, $\Gamma_\omega \sim 8.5$ MeV, $\Gamma_\phi \sim 4.3$ MeV, $\Gamma_{J/\psi} \sim 88$ keV

An explanation was found by Appelquist and Politzer, PRL 34, 43 (75).



Write the width as
$\Gamma(^3S_1 \to 3\text{ gluons}) = |R(0)|^2 |M(q\bar{q} \to ggg)|^2$

Following the model of positronium, solve the Schroedinger Eqn. for $R(r) = \frac{2}{a_0^{3/2}} e^{-r/a_0}$,
where $a_0 = \frac{1}{\alpha_s m_c/2}$.

$|M(q\bar{q} \to ggg)|^2 \sim \alpha_s^3$ — one power for each gluon

$\Rightarrow \Gamma(^3S_1 \to 3\text{ gluons}) \sim 0.2\, \alpha_s^6\, m_c \sim 90\text{ keV}; \alpha_s \approx 0.26$

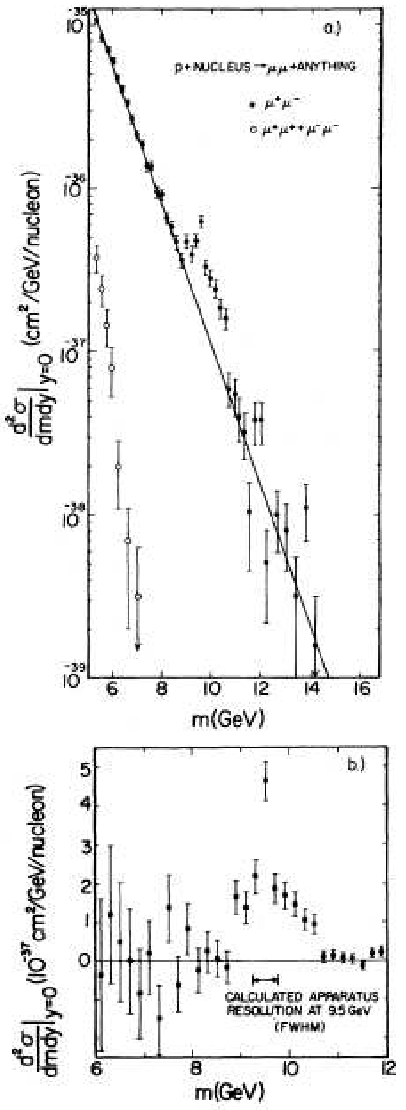
A beautiful discovery

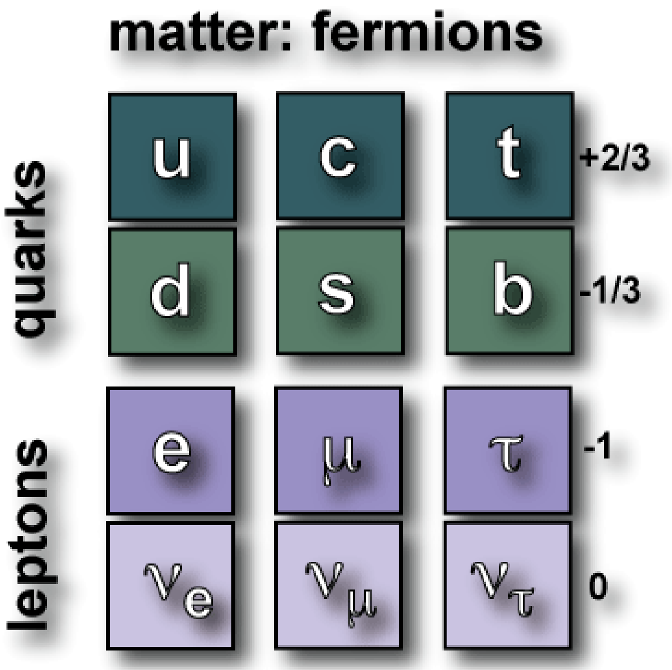
In 1975 the τ was discovered and led to the search for other 3rd-generation particles.

In 1977 the Upsilon (a $b\bar{b}$ bound state) was observed at the Fermilab Tevatron. PRL 39, 252 (1977)

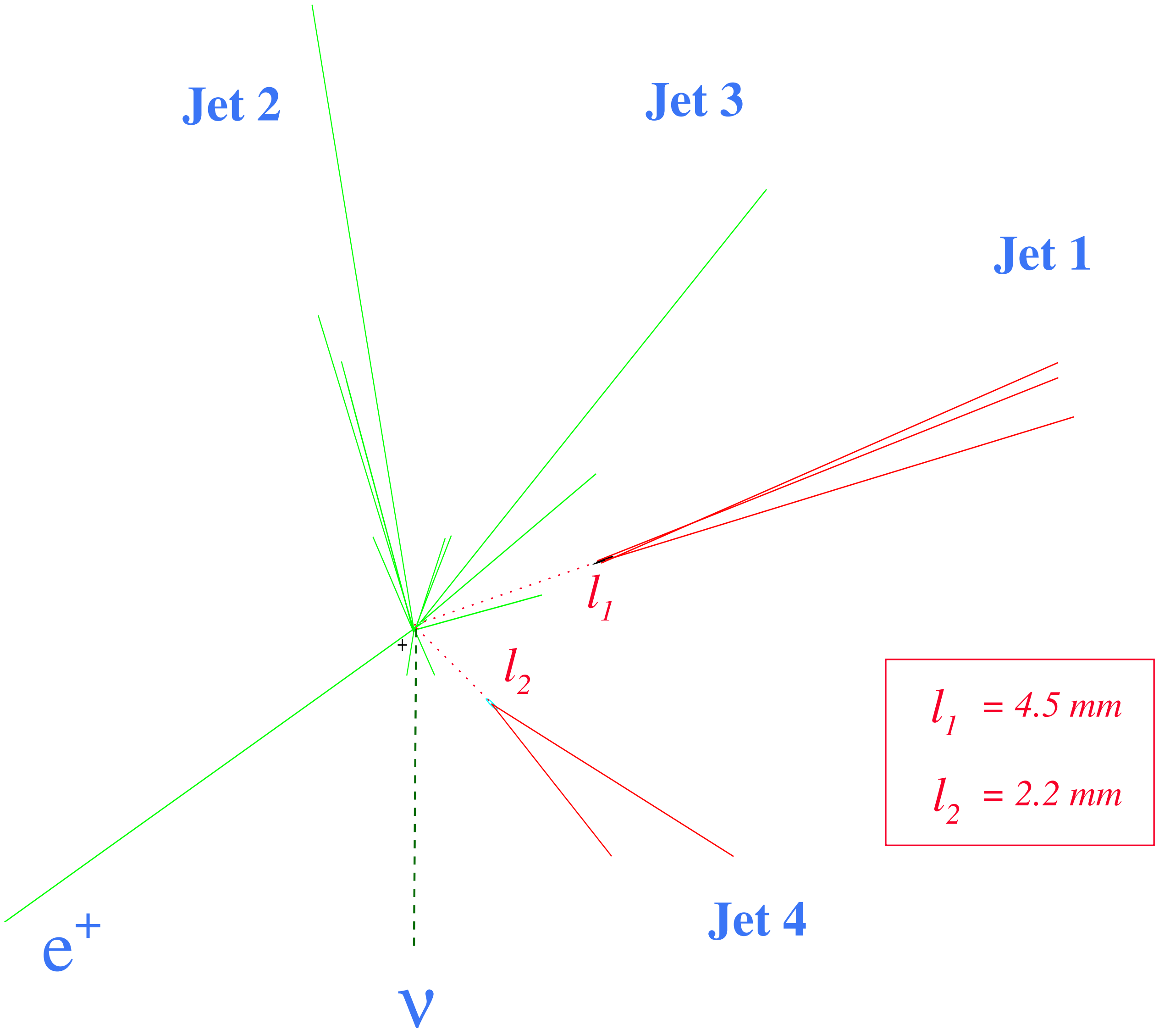
(The Upsilon is also very narrow.)

Once the bottom quark was found it was clear that a sixth quark was needed to complete the family structure.





"This is the top quark."



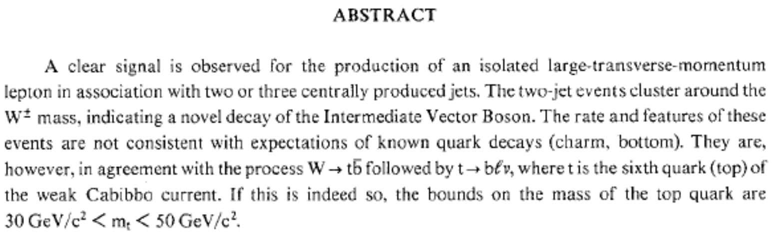
$l_1 = 4.5\ mm$

$l_2 = 2.2\ mm$

$$\mathbf{M^{Fit}_{top} = 170 \pm 10\ GeV/c^2}$$

24 September, 1992
run #40758, event #44414

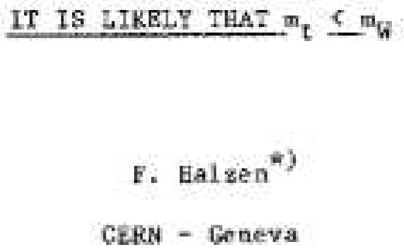
Of course the top had been found before. . .



ABSTRACT

A clear signal is observed for the production of an isolated large-transverse-momentum lepton in association with two or three centrally produced jets. The two-jet events cluster around the $W^\pm$ mass, indicating a novel decay of the Intermediate Vector Boson. The rate and features of these events are not consistent with expectations of known quark decays (charm, bottom). They are, however, in agreement with the process $W \to t\bar{b}$ followed by $t \to b\ell\nu$, where t is the sixth quark (top) of the weak Cabibbo current. If this is indeed so, the bounds on the mass of the top quark are $30\ \mathrm{GeV}/c^2 < m_t < 50\ \mathrm{GeV}/c^2$.

UA1, Phys. Lett. B 147, 493 (1984)

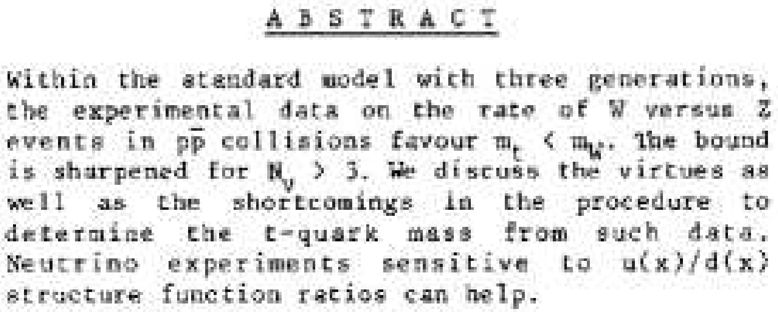


IT IS LIKELY THAT $m_t < m_W$

F. Halzen[*)]

CERN - Geneva

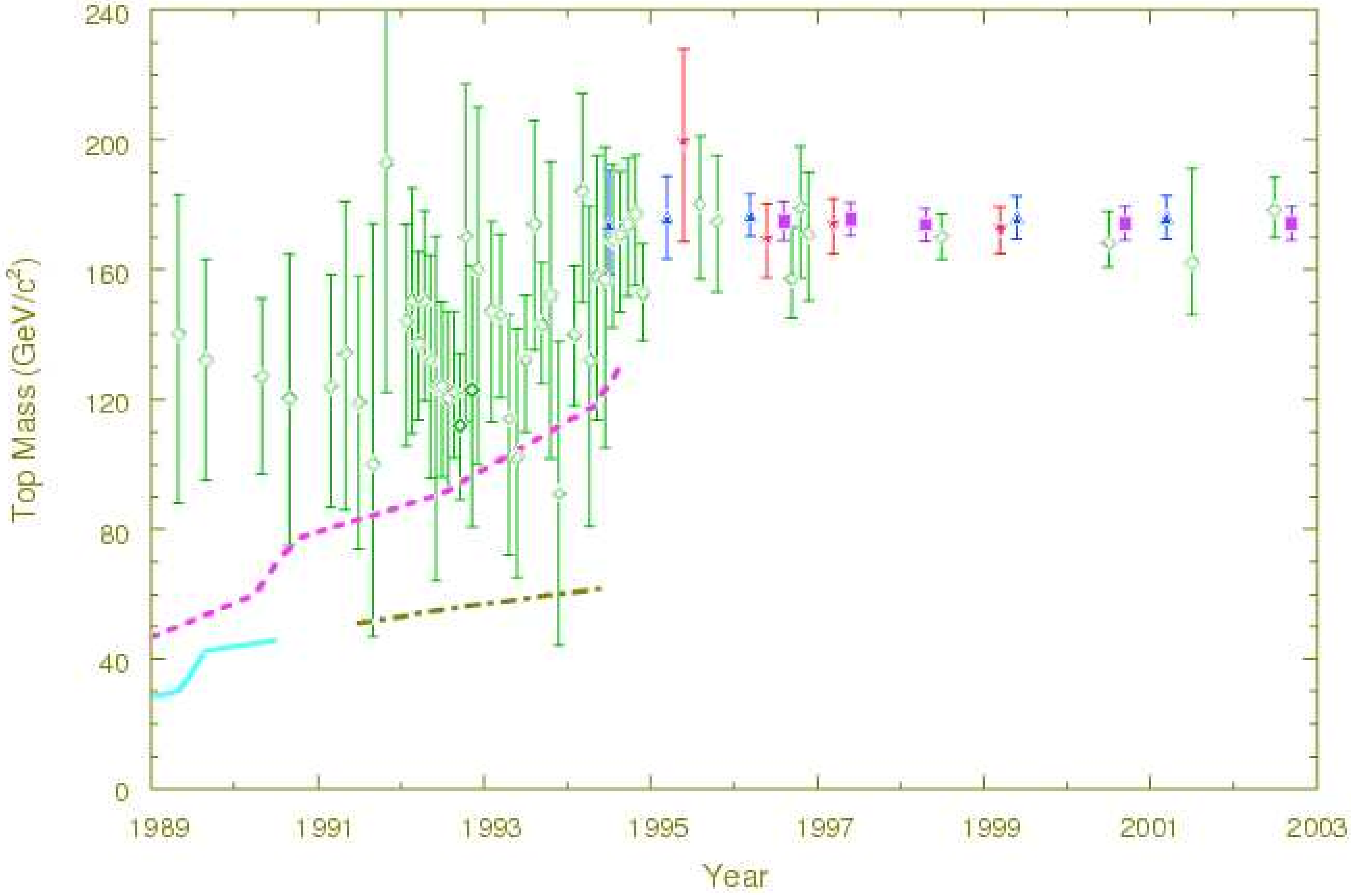
Phys. Lett. B 182, 388 (1986)



ABSTRACT

Within the standard model with three generations, the experimental data on the rate of W versus Z events in $p\bar{p}$ collisions favour $m_t < m_W$. The bound is sharpened for $N_\nu > 3$. We discuss the virtues as well as the shortcomings in the procedure to determine the t-quark mass from such data. Neutrino experiments sensitive to u(x)/d(x) structure function ratios can help.

Was the top-quark mass predicted?



Quigg

Look at the predictions in Sept. 1992. . .

The real evidence. . . (1995)

THE NEW YORK TIMES **NATIONAL** FRIDAY, MARCH 3, 1995

Elusive Particle Found By Scientists in Illinois

Continued From Page A1

The Top Quark: What Is It?

Hydrogen Atom

The 6 Quarks

UP

CHARM

TOP

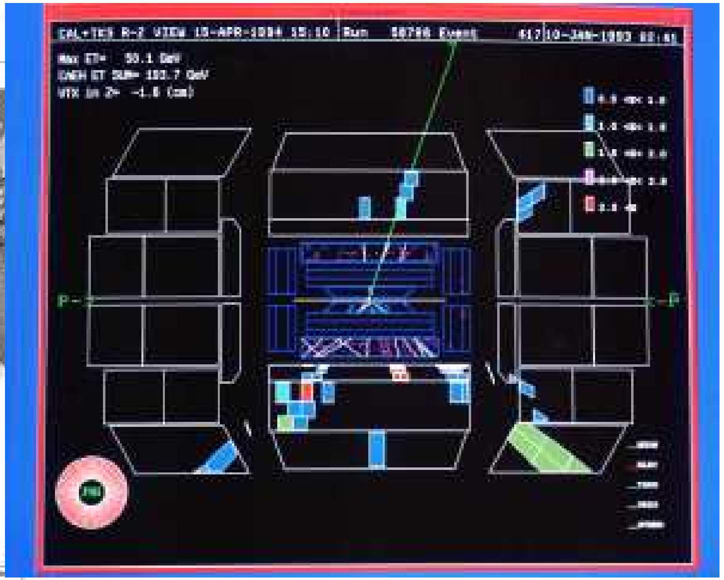
DOWN

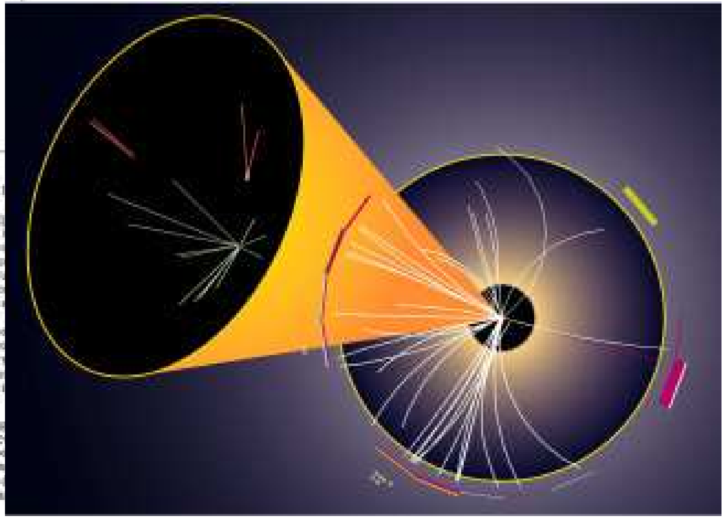
STRANGE

BOTTOM

Trying to understand the fundamentals of the universe.

Forging a link between the physical and the metaphysical.





What is a heavy-quark?

A heavy quark is a quark with $m_q \gg \Lambda_{\mathrm{QCD}}$.

	Pole mass M	$\overline{\mathrm{MS}}$ mass $\overline{m}(\overline{m})$
Charm	~ 1.3–1.7 GeV	$1.27^{+0.07}_{-0.11}$ GeV
Bottom	~ 4.5–5 GeV	$4.20^{+0.17}_{-0.07}$ GeV
Top	$173.1 \pm 0.6 \pm 1.1$ GeV (?)	~ 163 GeV

PDG; TevEWWG

Pole Mass: $\sim \frac{1}{\not{p} - M}$

$\overline{\mathrm{MS}}$ Mass: Related to pole mass by

$$\frac{M}{\overline{m}(\overline{m})} = 1 + \frac{4}{3}\left(\frac{\alpha_s}{\pi}\right) + \left(\frac{\alpha_s}{\pi}\right)^2 (-1.0414 \ln(M^2/\overline{m}^2) + 13.4434) + \ldots$$

It seems kind of funny to list 2 different masses. . .

What is the top-quark mass?

Answer 1: A parameter of the Lagrangian $L \sim m_t \bar{t} t$

Answer 2: An effective coupling between t–t–h
$m_t = Y_t/(2\sqrt{2}G_F)^{1/2} \approx 1$ in the SM

Answer 3: The kinematic mass seen by the experiments

Right after the discovery of the top quark, Martin Smith and Scott Willenbrock asked this question about the "pole mass" of the top quark. They showed that a renormalon (the closest pole of the Borrel transform) induced an ambiguity of $\mathcal{O}(\Lambda_{QCD})$ in the definition of the pole mass.

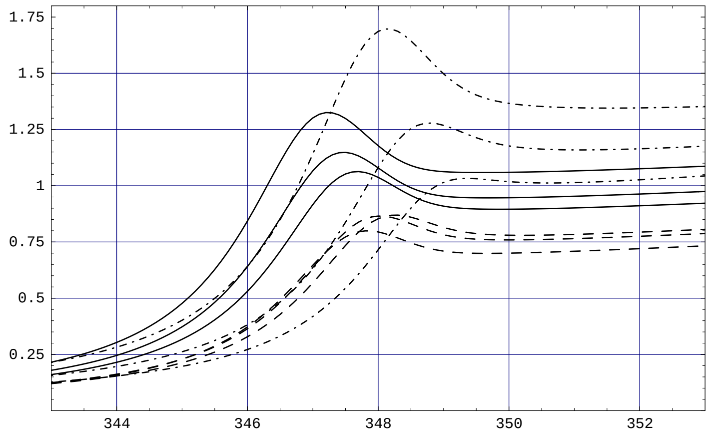
This led to the recommendation to use the $\overline{\mathrm{MS}}$ mass for top quarks as a standard.

We theorists are good at setting standards that make our life easier . . .
most perturbative calculations use the $\overline{\mathrm{MS}}$ mass for simplicity.

Of course mass is NOT measured directly. Instead, it affects the distribution of events that are measured, and that distribution is used to INFER the mass.

At the ILC, we hope to measure m_t to about 100 MeV by scanning over the $t\bar{t}$ threshold.

$t\bar{t}$ threshold at a linear collider (LC)



Yakovlev,Groote PRD63, 074012(01)

There is a subtle question when you try to make a precision measurement of QCD: What mass do you use?

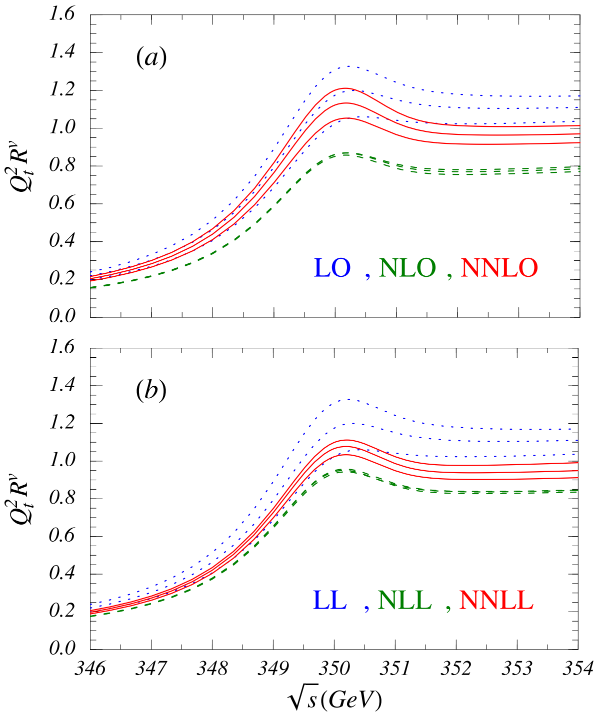
The pole mass is not defined beyond Λ_{QCD}.

In fact it is not well-defined at all, since there are no free quarks.

Solution: Use the 1S mass (pseudo bound state)

There are large non-relativisitic corrections

$$\sigma_{t\bar t} \;\propto\; v\sum\left(\frac{\alpha_s}{v}\right)\times\left\{\begin{matrix}1\\ \sum(\alpha_s\ln v)\end{matrix}\right\} \times\left\{\begin{matrix}\mathrm{LO}(1)+\mathrm{NLO}(\alpha_s,v)+\mathrm{NNLO}(\alpha_s^2,\alpha_s v,v^2)\\ \mathrm{LL}+\mathrm{NLL}+\mathrm{NNLL}\end{matrix}\right\}$$



Hoang, Manohar, Stewart, Teubner

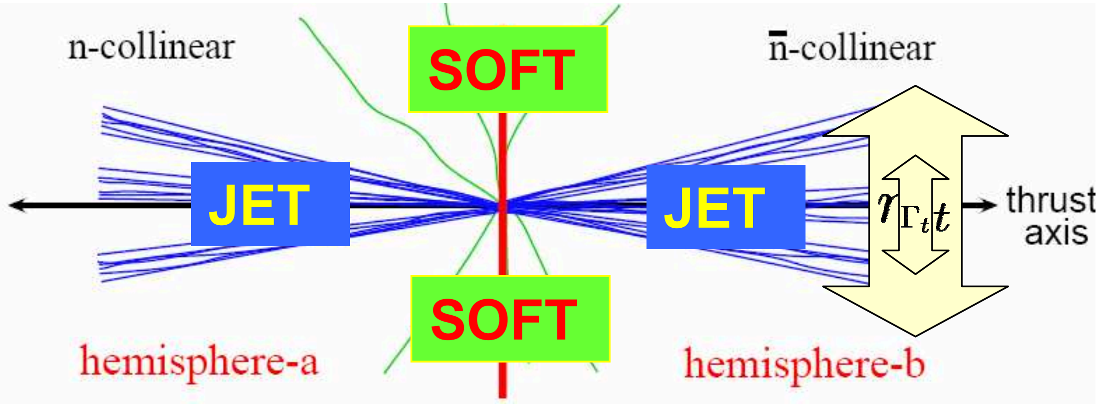
Normalization changes, but peak stable.

$\delta\sigma_{t\bar t}$ is $\pm 6\%$ before ISR/beamstrahlung

$\delta m_t \sim 100$ MeV is attainable

$t\bar{t}$ continuum mass

A recent idea based on Soft Collinear Effective field Theory (SCET) recommends just summing all of the QCD radiation into a top-quark jet mass.



$$\boxed{Q \gg m_t \gg \Gamma_t > \Lambda_{\mathrm{QCD}}}$$

Fleming, Hoang, Mantry, Stewart, PRD 77, 114003 (08)

Factorization of the effective field theories into hard, jet, and ultra-soft pieces was shown.

$$\frac{d^2\sigma}{dM_t^2\, dM_{\bar{t}}^2} = \sigma_0\, H_Q(Q,\mu) \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, J_n(s_t - Q\ell^+,\mu)\, J_{\bar{n}}(s_{\bar{t}} - Q\ell^-,\mu)\, S_{\mathrm{hemi}}(\ell^+,\ell^-,\mu)$$

If correct you could have another stable and accurate mass definition.

Personal Opinion: Even if this case does not work, these EFT techniques will be central to theoretical physics in the future.

What mass do we measure?

The statement has been made that you measure a 1S mass at threshold, and a top-jet mass in the continuum (using the new calculations).

Other masses have been mentioned: $\overline{MS}$ mass, pole mass, (could have mentioned peak mass, Breit-Wigner mass, . . .)

Which mass do we measure? None of them.

We measure line-shapes or particle flow or invariant masses with cuts and ISR/FSR effects.

To the extent experimentalists use LO Monte Carlo programs, the mass is dominated by kinematics, and hence is close to the pole mass.

What we really care about is a mass we can use in many calculations.
The most convenient mass is the $\overline{\rm MS}$ mass.
— So whatever you extract, translate to that!

The challenge going forward will be to ensure that, whatever you use, the experimental and theoretical definitions agree.

Is this focus on masses just better bookkeeping?

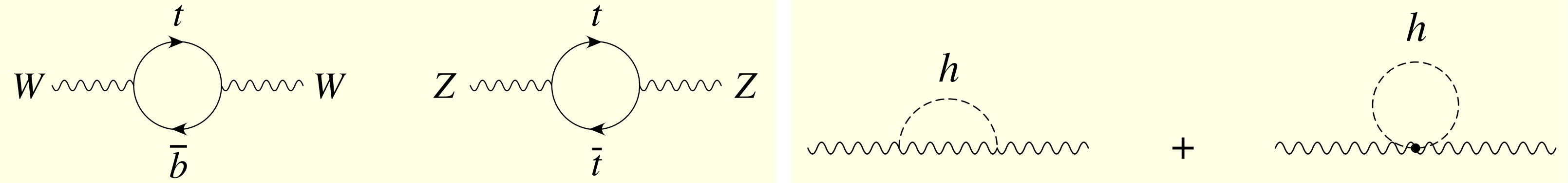
The top-quark mass offers us more

Why study the top-quark mass?

Answer: Electroweak (EW) precision physics

EW radiative corrections depend on the top-quark mass (m_t).
Using the value measured at the Fermilab Tevatron, EW precision fits constrain the Higgs boson mass M_H.

Both the top quark and Higgs contribute at 1-loop to the W/Z propagtors.



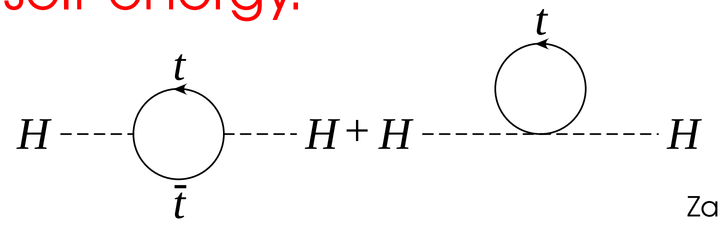
Assuming α, G_F, and M_Z as inputs, M_W^2 at 1-loop is:

$$M_W^2 = \frac{\pi\alpha}{\sqrt{2}G_F \sin^2\theta_W} \frac{1}{1 - \Delta r(m_t, m_H)}$$

where $\Delta r(m_t, m_H) \approx c_t m_t^2 = c_H \ln(M_H^2/M_Z^2) + \cdots$

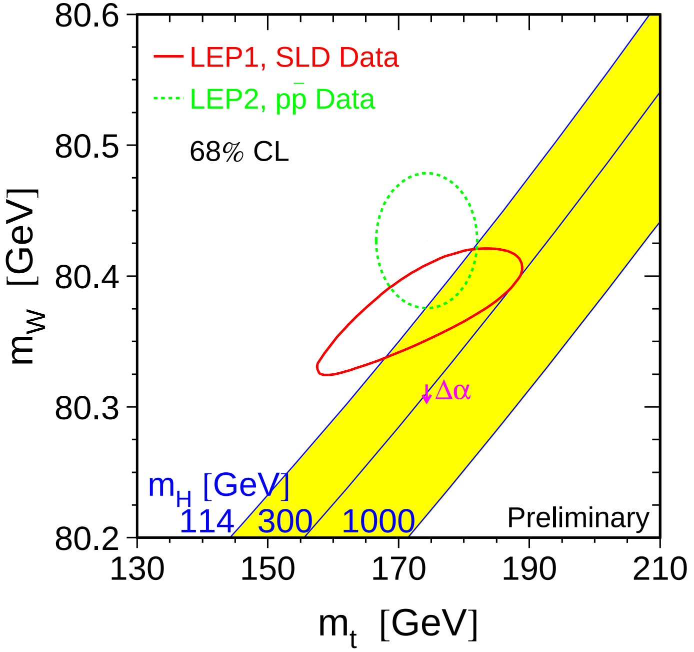
Inverting the formula provides a logarithmic contraint on M_H.

Higgs searchers put it differently: the top quark provides a large correction to the Higgs self-energy.



Constraints on Higgs mass from W and t

M_H is logarithmically sensitive to variations of M_W and m_t.



End of Run I

$m_t = 174.3 \pm 5.1$ GeV (3%)

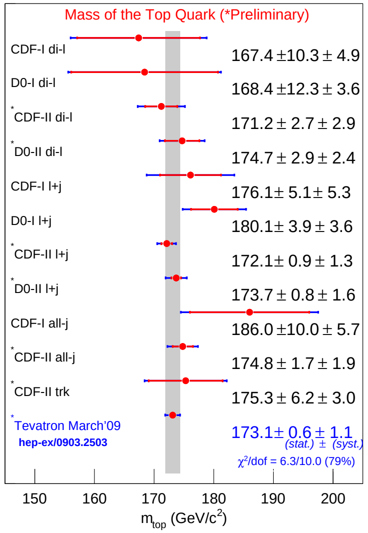
(Better than EW precision)

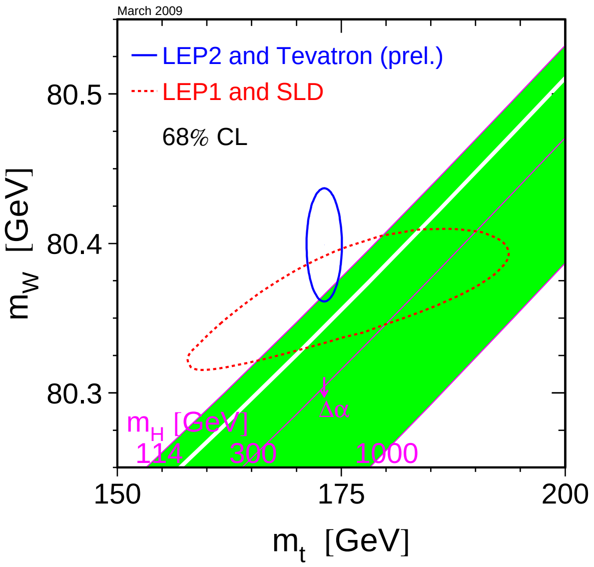
Early Summer 2005

$m_t = 178.0 \pm 4.3$ GeV (fishy)

Late Summer 2005

$m_t = 172.7 \pm 2.9$ GeV





Winter 2009

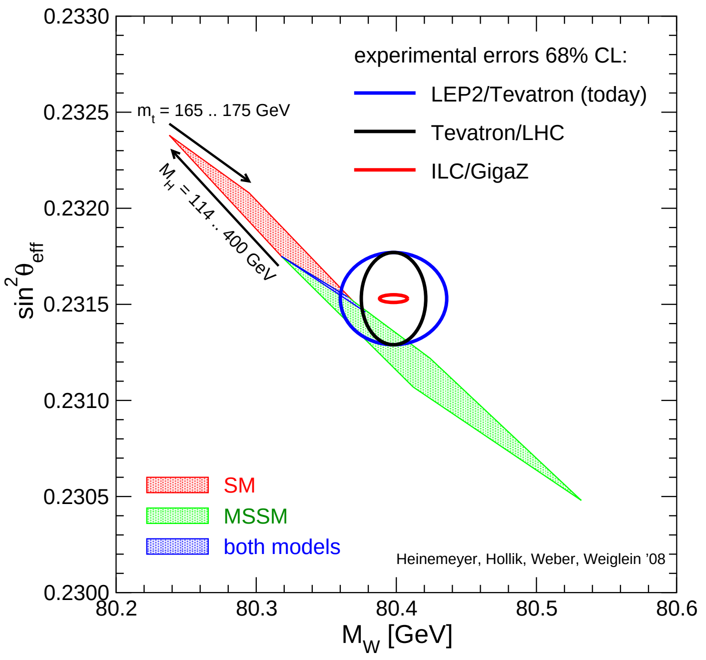
$m_t = 173.1 \pm 0.6 \pm 1.1$ GeV

(Close to the first $t\bar{t}$ event. . .)

Tevatron EWWG

How well do we need to know m_t?

There is a better way than “blue band plots” to look at this in the SM.



Heinemeyer *et al.*

- Assume M_H is known.
- M_W will be measured to ~ 20 MeV

  $\Rightarrow$ Need m_t to ~ 3 GeV at LHC.

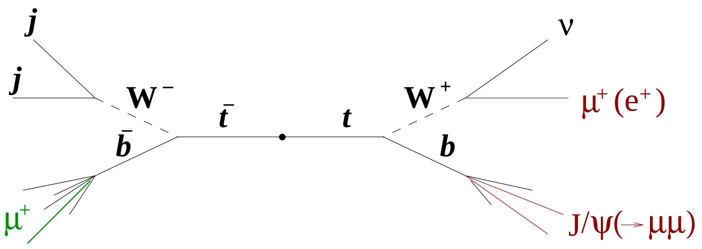
  (We already know it to 1.3 GeV.)
- A linear collider can measure M_W to ~ 6 MeV.

  Giga-Z can measure $\sin^2\theta_W \sim 10^{-5}$

  $\Rightarrow$ Need m_t to ~ 1 GeV.

At the LHC:

- Several channels can reach < 1 GeV (stat.)
- To reach systematics < 1 GeV use:
  $M_{J/\Psi\ell\nu}$ w/ template for m_t. (~ 300 fb^{-1})



The bottom line: We have already saturated the information we can extract about a SM Higgs from top-quark measurements given any near-term collider (i.e., LHC).

How well do we want to know m_t?

Most excitement about Higgs production has nothing to do with the SM.

Models of new physics predict different sensitivity to the top-quark mass.

SUSY Higgs masses are VERY sensitive to the top-quark mass

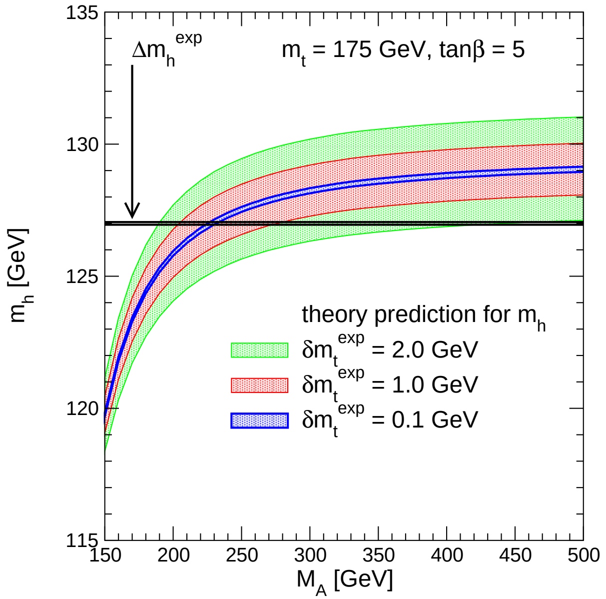
$$\Delta M_H^2 \approx \frac{3G_F m_t^4}{\sqrt{2}\pi^2 \sin^2\beta} \ln\left(\frac{\overline{m}_{\tilde{t}}^2}{m_t^2}\right)$$

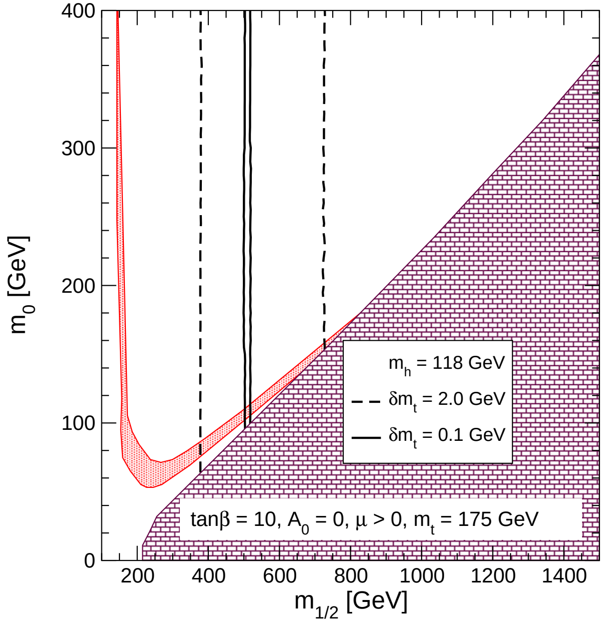
- Experimental error from LHC *may* reach ~ 200 MeV (using rare decays)
- $\delta M_H \sim \delta m_t$, so we will want $\delta m_t \sim 100$ MeV.

  Warning: 4-loop corrections are comparable in size.

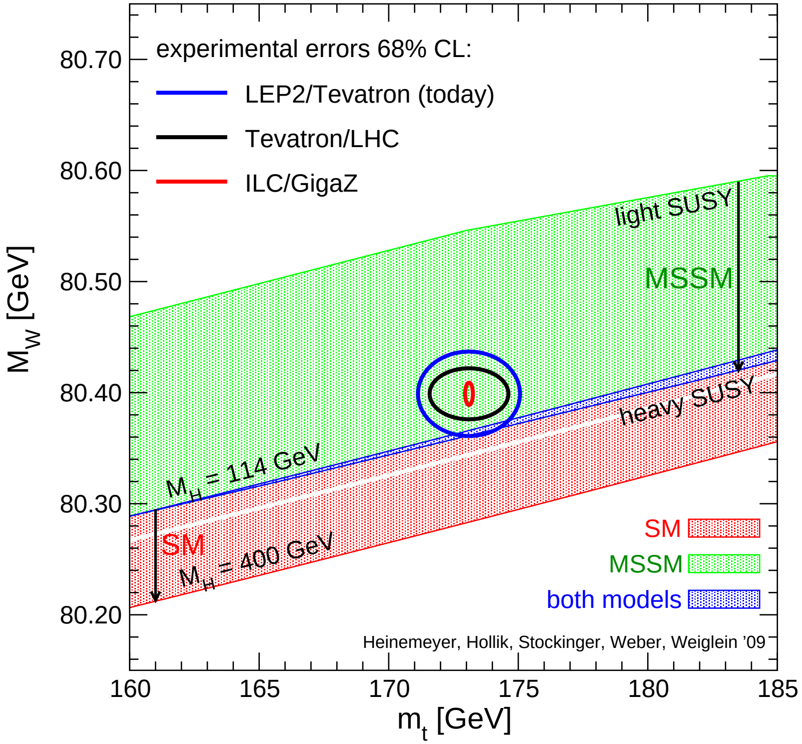
  This needs major effort

If a smaller error in m_t is achieved, we gain indirect access to M_A, A_t, $m_{1/2}$, etc.

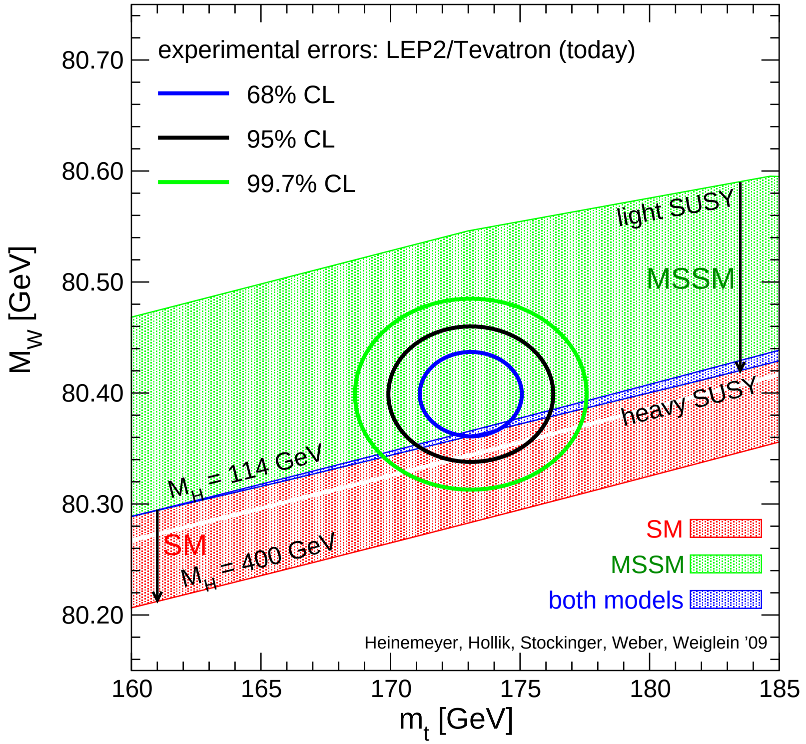




M_W vs. m_t for MSSM Higgs



“SUSY Higgs is favored”



No one tends to show this plot.

It is clear that whatever physics explains electroweak symmetry breaking, there is at least an effective interaction whose mass scale is low.

To extract masses we depend on well-defined predictions of obervables

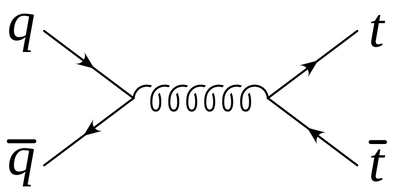
Let's look at the total top-quark cross section

Top-quark pair ($t\bar{t}$) production

$q\bar{q} \to t\bar{t}$

Leading contribution at Tevatron

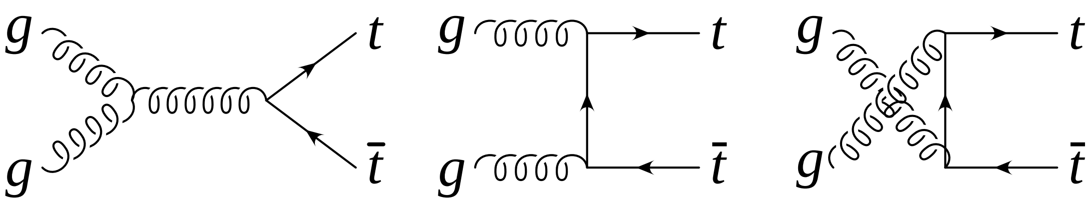
Tev (RunII)	85%
LHC	10%



$g\bar{g} \to t\bar{t}$

Leading contribution at LHC

Tev (RunII)	15%
LHC	90%



At the Tevatron, $t\bar{t}$ is produced close the the kinematic threshold $\hat{s} \approx 4m_t^2$, so $x \sim 0.2$. At LHC $x \sim 0.02$.

A few dozen reconstructed $t\bar{t}$ pairs in Run I of the Tevatron was enough for discovery.

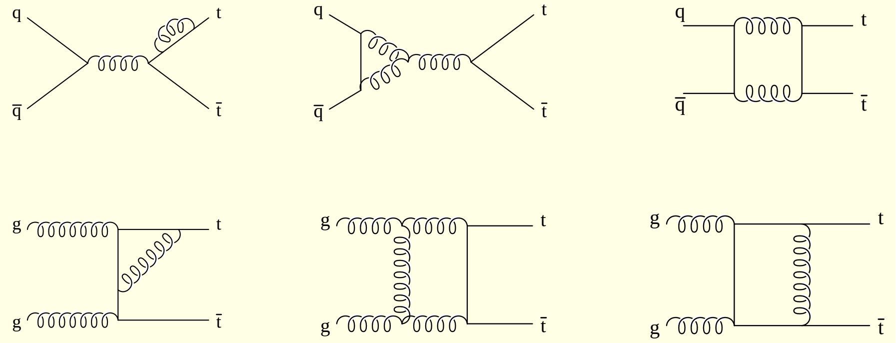
At Run II there are already hundreds.

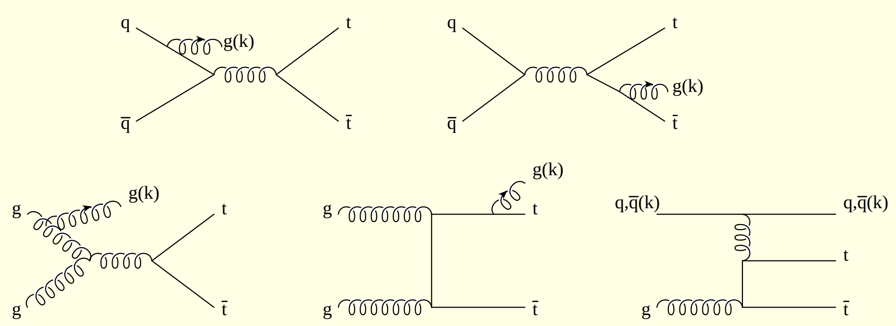
At LHC there will be about 1 pair/second produced!

NLO calculations

- The production rate of $t\bar{t}$ is a sensitive probe of strong interactions.
- $t\bar{t}$ production is already becoming a precision measurement.

  $\Rightarrow$ Very precise theory is required to understand the dynamics and match the experimental precision that will be available.





Complete NLO calculations exist for total and differential cross sections.

Nason, Dawson, Ellis, NPB 303, 607 (88), NPB 327, 49 (89);
Beenakker, Kuijf, van Neerven, Smith, PRD 40, 54 (89);
plus Meng, Schuler, NPB 351, 507 (91)

But this is not enough at the Tevatron. . .

Large threshold corrections in $t\bar{t}$

The top-quark decays before the bound state forms. However, pseudo-bound states of $t\bar{t}$ near threshold ($\hat{s} = 4m_t^2$) cause large logarithmic enhancements to the cross section.

Schematically, the $t\bar{t}$ NLO cross section is

$$\sigma_{ij}^{\mathrm{NLO}}(m_t^2,\mu) = \frac{\alpha_s^2(\mu)}{m_t^2}\left\{c_{ij}^0 + 4\pi\alpha_s(\mu)\left[c_{ij}^1(\rho) + \bar{c}_{ij}^1(\rho)\ln\left(\frac{\mu^2}{m_t^2}\right)\right]\right\};\ \rho = \frac{4m_t^2}{\hat{s}}$$

Near threshold, the LO cross section vanishes:

$$c_{q\bar{q}}^0(\rho) \approx \frac{T_R C_F}{2N_c}\pi\beta \xrightarrow{\beta\to 0} 0;\quad c_{gg}^0(\rho) \approx \frac{T_R}{N_c^2-1}(C_F - C_A/2)\pi\beta \xrightarrow{\beta\to 0} 0$$

At NLO there are soft and collinear singularities:

$$c_{q\bar{q}}^1(\rho) \xrightarrow{\beta\to 0} \frac{1}{4\pi^2}c_{q\bar{q}}^0(\rho)\left[(C_F - C_A/2)\frac{\pi^2}{2\beta} + 2C_F\ln^2(8\beta^2) - (8C_F + C_A)\ln(8\beta^2)\right]$$

$$c_{gg}^1(\rho) \xrightarrow{\beta\to 0} \frac{1}{4\pi^2}c_{gg}^0(\rho)\left[\frac{N_c^2+2}{N_c(N_c^2-2)}\frac{\pi^2}{4\beta} + 2C_A\ln^2(8\beta^2) - \frac{(9N_c^2-20)C_A}{N_c^2-2}\ln(8\beta^2)\right]$$

$$\bar{c}_{q\bar{q}}^1(\rho) \xrightarrow{\beta\to 0} \frac{1}{4\pi^2}c_{q\bar{q}}^0(\rho)\left[-2C_F\ln(4\beta^2) + \overline{C}_2(\mu^2/m_t^2)\right]$$

$$\bar{c}_{gg}^1(\rho) \xrightarrow{\beta\to 0} \frac{1}{4\pi^2}c_{gg}^0(\rho)\left[-2C_A\ln(4\beta^2) + \overline{C}_3(\mu^2/m_t^2)\right]$$

Threshold resummation

Threshold logarithms can be resummed via exponentiation, similar to the case of Drell-Yan (DY) or e^+e^- →jets.
Challenges are IS/FS interference, scale difference between m_t and v_t.

Historically, logs are resummed in moment space (Mellin-transform space)
The cross section for the N-th moment under a Mellin-transform is:

$$\sigma_N(m_t^2) = \int_0^1 d\rho\, \rho^{N-1} \sigma(\rho, m_t^2)$$

The threshold region corresponds to the $\lim N \to \infty$, which leads to threshold corrections of the form:

$$\sigma_N^{\mathrm{LO}} \left[1 + \sum_{n=1}^{\infty} \alpha_s^n \sum_{m=1}^{2n} c_{n,m} \ln^m N \right]$$

In Drell-Yan, this structure exponentiates to a radiative form factor $\Delta_{DY,N}$:

$$\begin{aligned} \Delta_{DY,N}(\alpha_s) &= \exp\left[\sum_{n=1}^{\infty} \alpha_s^n \sum_{m=1}^{n+1} G_{n,m} \ln^m N \right] \\ &= \exp\Big[\underbrace{g_{DY}^{(1)} \alpha_s \ln^2 N}_{\mathrm{LL}} + \underbrace{g_{DY}^{(2)} \alpha_s \ln N}_{\mathrm{NLL}} + \underbrace{g_{DY}^{(3)} \alpha_s^2 \ln N}_{\mathrm{NNLL}} + \cdots \Big] \end{aligned}$$

Realization of threshold resummation in $t\bar{t}$

Generalizing Drell-Yan-like resummation to $t\bar{t}$ requires:

— Dealing with soft-gluons from IS, FS, and IS/FS interference.
— Dealing with gg color octet states.

The solution is to recast the cross section for moment N in the form:

$$\sigma_{ij} = \sum_{I,J} M^{\dagger}_{ij,I,N} [\Delta_{ij,N}]_{I,J} M_{ij,J,N}$$

where the sum on I, J is over all color states, $[\Delta_{ij,N}]_{I,J}$ is the radiation form factor, and M are matrices in color space.

The advantage is that it describes a formal expansion of the logarithms that can be improved to NNLL, NNNLL, NNNNLL, (and then you collapse)

Formalism: Kidonakis, Sterman, PLB 387, 867 (96)

Implementation:
Bonciani, Catani, Mangano, Nason, NPB 529, 424 (98)
Kidonakis, Vogt, PRD 68, 114014 (03)
Cacciari, Frixione, Mangano, Nason, JHEP 04, 68 (04)

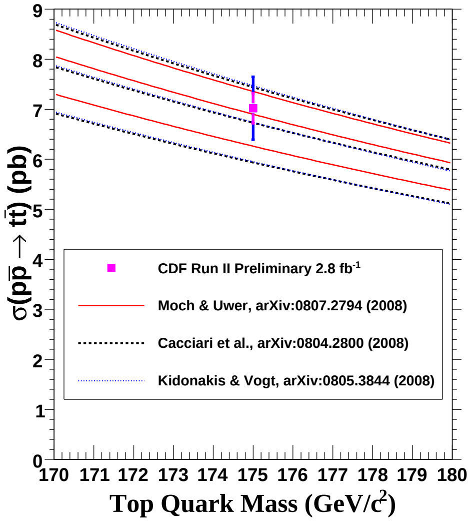
Prior to this formalism there were 2 competing calculations that performed the integrations by truncating the moments. This was mathematically inconsistent, but gave reasonable numerical results. May we never go back. . .

Berger, Contapaganos, PRD 54, 2085 (96)
Catani, Mangano, Nason, Trentadue, NPB 478, 273 (96)

Nomenclature and uncertainties

Bad nomenclature

"NNLO-NNNLL"



This is really NLO+the Sudakov-like resummation we saw above, where the exponent is re-expanded to the 3rd LL.

There is nothing NNLO about it.

Moch and Uwer do "NNLO$_{\text{approx}}$"
This is NLO+real NNLL resummed.

Unusual uncertainties

NLO scale uncertainty of $\pm 10\% \longrightarrow \pm 5\%$ w/ NLL correction
Including PDF uncertainty, $\longrightarrow \pm 15\%$ at Tevatron

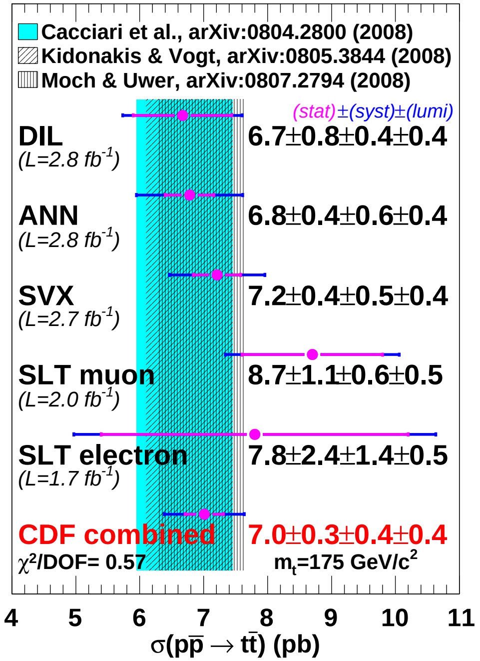
There is an additional uncertainty due to expansion kinematics:

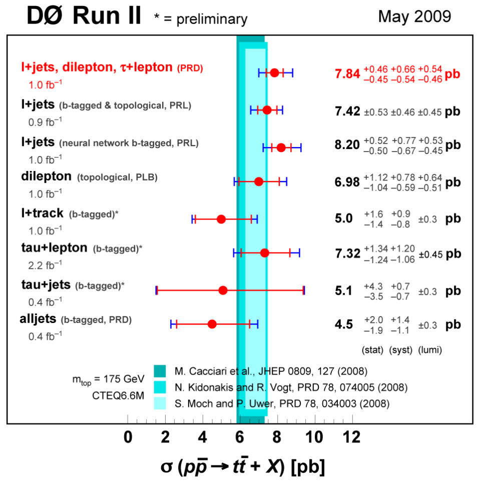
- 1 particle inclusive (1PI): $s = (p_q + p_{\bar{q}})^2$
- Pair invariant mass (PIM): $s = M_{t\bar{t}}^2 = (p_t + p_{\bar{t}})^2$

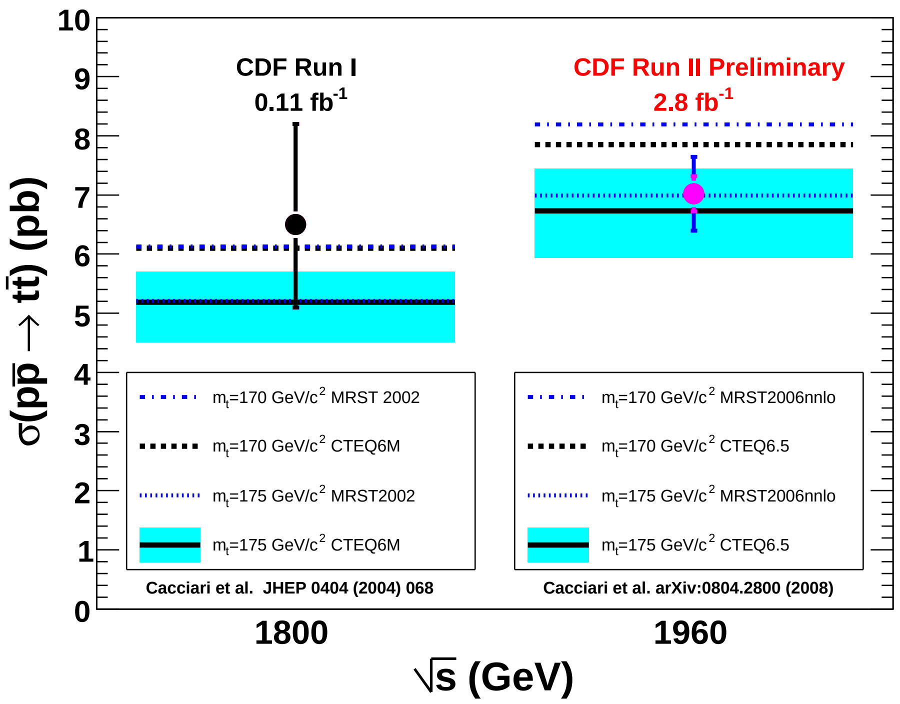
	σ	±1PI/PIM	±scale	± PDF
Run I	5.24±	0.31	± 0.2	±0.6 pb
Run II	6.77±	0.42	± 0.1	±0.7 pb

LHC is not dominated by threshold kinematics:
$\sigma = 825 \pm 50 \pm 100 \pm 90$ pb.
Full NNLO is needed!

Tevatron data







Great agreement so far!

Lighter top-quark mass preferred.

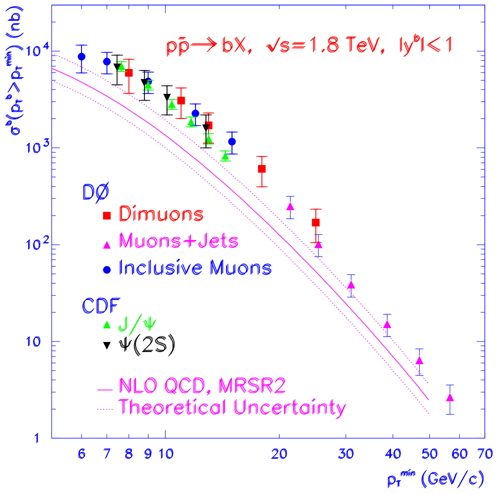
Experiment will be better than theory soon.

QCD is flavor blind

Let's apply the same calculations to the bottom-quark cross section

How about the b-quark cross section?

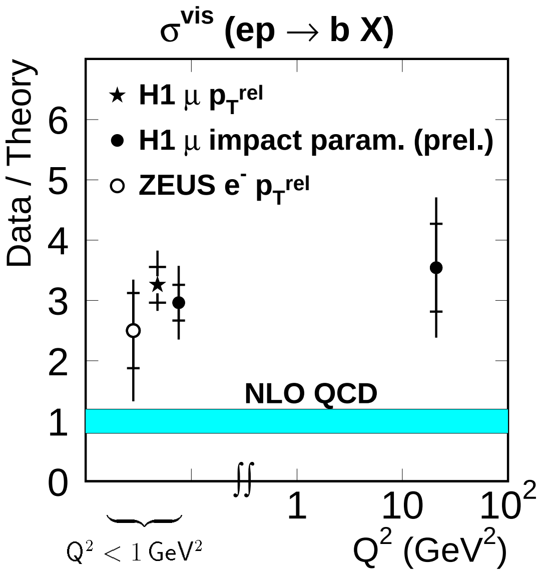
There was a long-standing (20 year) problem that the b-quark cross section was consistently understimated by factors of 2–3.



Tevatron

Several modes;
$b \to J/\psi$ most precise

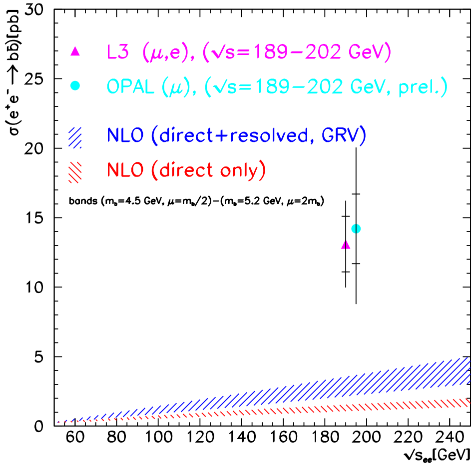
CDF and DØ



HERA

Excess in both DIS & photoproduction

Sefkow, hep-ex/0109038



LEP

$$\sigma(e^+e^- \to e^+e^- b\bar{b})$$
$$\approx \gamma\gamma \to b\bar{b}$$

We do not see b-quarks

Unlike t quarks, b quarks hadronize to long-lived B hadrons

What we actually have is:

$$\frac{d\,\sigma(b \to B \to J/\psi)}{d\,p_T} = \int \frac{dz}{z}\frac{d\,\sigma(b)}{d\,\hat{p}_T} \otimes D(b \to B; z) \otimes D(B \to J/\psi)$$

Fragmentation functions D are non-perturbative!

Nevertheless, a modified perturbative approximation called “Peterson fragmentation” was historically used.

Peterson, Schlatter, Schmitt, Zerwas, PRD 27, 105 (83)

$$D(b \to B; z) \sim \frac{1}{z[1 - 1/z - \epsilon_Q/(1-x)]^2} \text{ with } \epsilon_Q \sim \frac{\Lambda^2}{m_Q^2} \text{and} < 1 - z > \sim \sqrt{\epsilon_Q}$$

NOTE: $\epsilon_b = \frac{m_b^2}{m_B^2}$, however it is floated in practice to approximate unknown non-perterturbative physics. . .

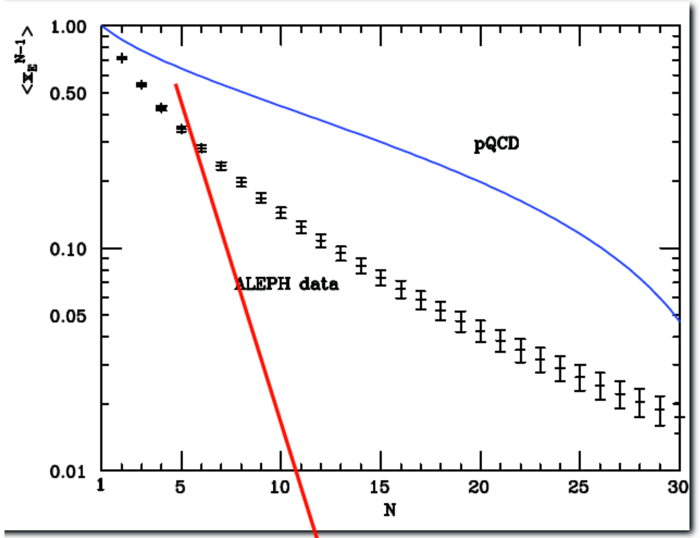
Perhaps it is not surprising this did not work too well

Better idea: Extract non-perturbative fragmentation from data

Cacciari and Nason fit LEP $Z \to b\bar{b}$ data to directly extract $D_{\rm np}$.

Cacciari, Nason, PRL 89, 122003 (02)

Translating the LEP data to Mellin space:



This gap:
non-perturbative QCD

$$D_N \equiv \int_0^1 x^{N-1} D(x)\, dx = < x^{N-1} >$$

In this space:

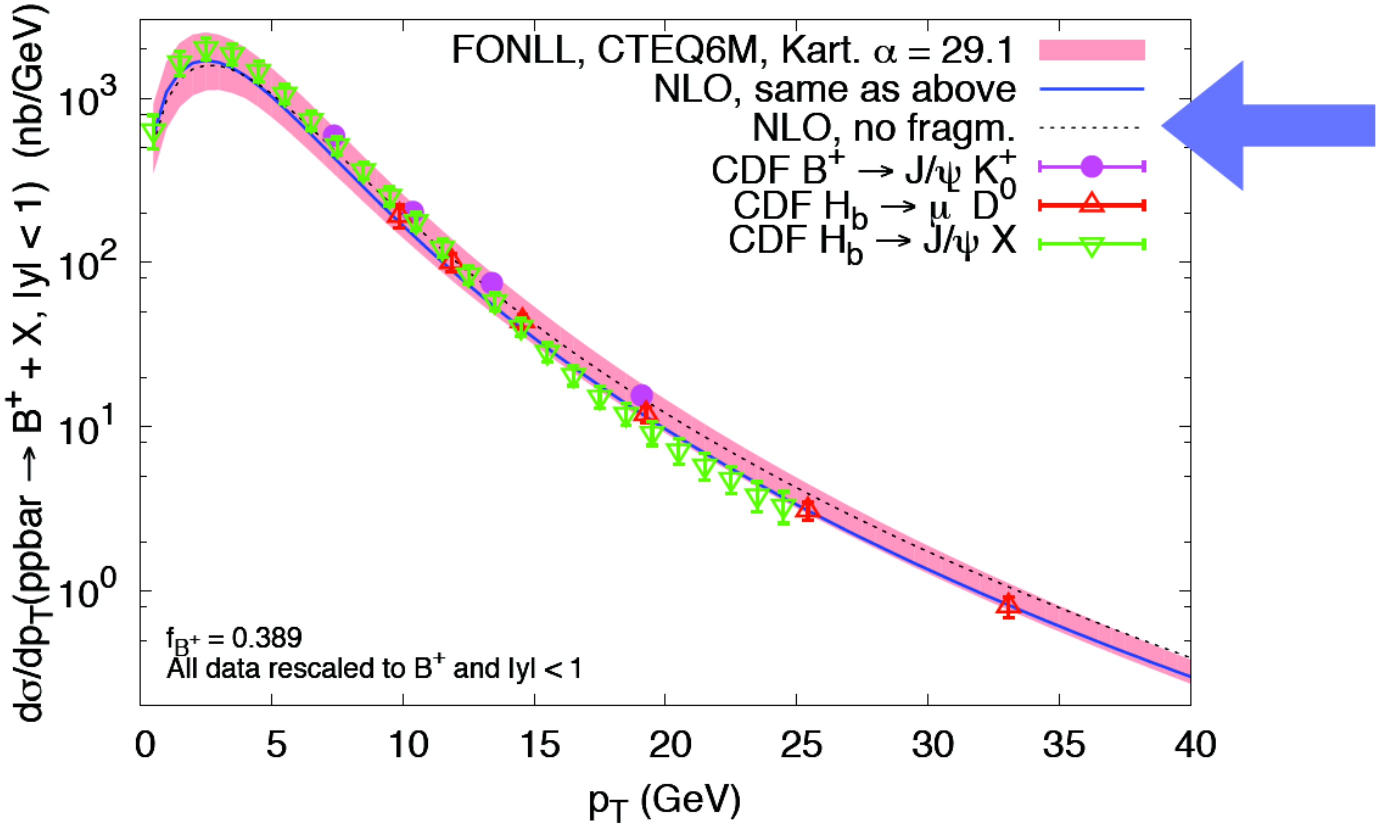
$$< x >_{\rm expt} = < x >_{\rm pQCD} < x >_{\rm np}$$

LEP is sensitive to $N = 2$.

Tevatron is sensitive to $N \sim 4$.

This is a HUGE extrapolation

B-meson cross sections at Tevatron



Conclusion: NLO with corrected fragmentation has "excellent agreement with the data"

Do you agree? — Look at $B \to j/\psi$. Data and theory are indpendently correlated at each bin.

Perhaps one of you will check this "solution"

Conclusions

1. The study of $t\bar{t}$ has become a game of precision measurements.
    - The top-quark now has the best measured mass (1%) of any quark.

      $$m_t = 173.1 \pm 1.3 \text{ GeV}$$

    - The measured top-quark cross section has uncertainties comparable in size to the theoretical calculations.

      $\sigma_{\text{exp}} = 7.3 \pm 0.8$ pb, $\sigma_{\text{th}} = 6.8 \pm 0.8$ pb at Run II (175 GeV)

We are theory and physics modeling constrained!

- We need a better handle on W+heavy-quark final states — dominates mass uncertainty.
- We need even higher order calculations valid near threshold — NNLO/NNNNLL
- To utilize this information we need higher-order (3-loop, soon 4-loop) calculations of EW processes.

Conclusions

2. The b-quark production saga has taught us we have to think carefully about the final state as well as the matrix element.

   We broke the cross section up:

$$\frac{d\sigma(B)}{dp_T} \Rightarrow \frac{d\sigma(b)}{dp_T} \otimes D(b \to B)$$

   Heavy quarks forced us to learn more about fragmentation, D.

3. We are in an age of <u>precision QCD</u>!

Whether we are looking at masses, or cross sections, the big lesson is we need to be certain theorists and experimentalists are discussing the same physics!

Your help will be needed in maximizing our understanding of the fantastic data we now have from the Tevatron and will have from LHC.