

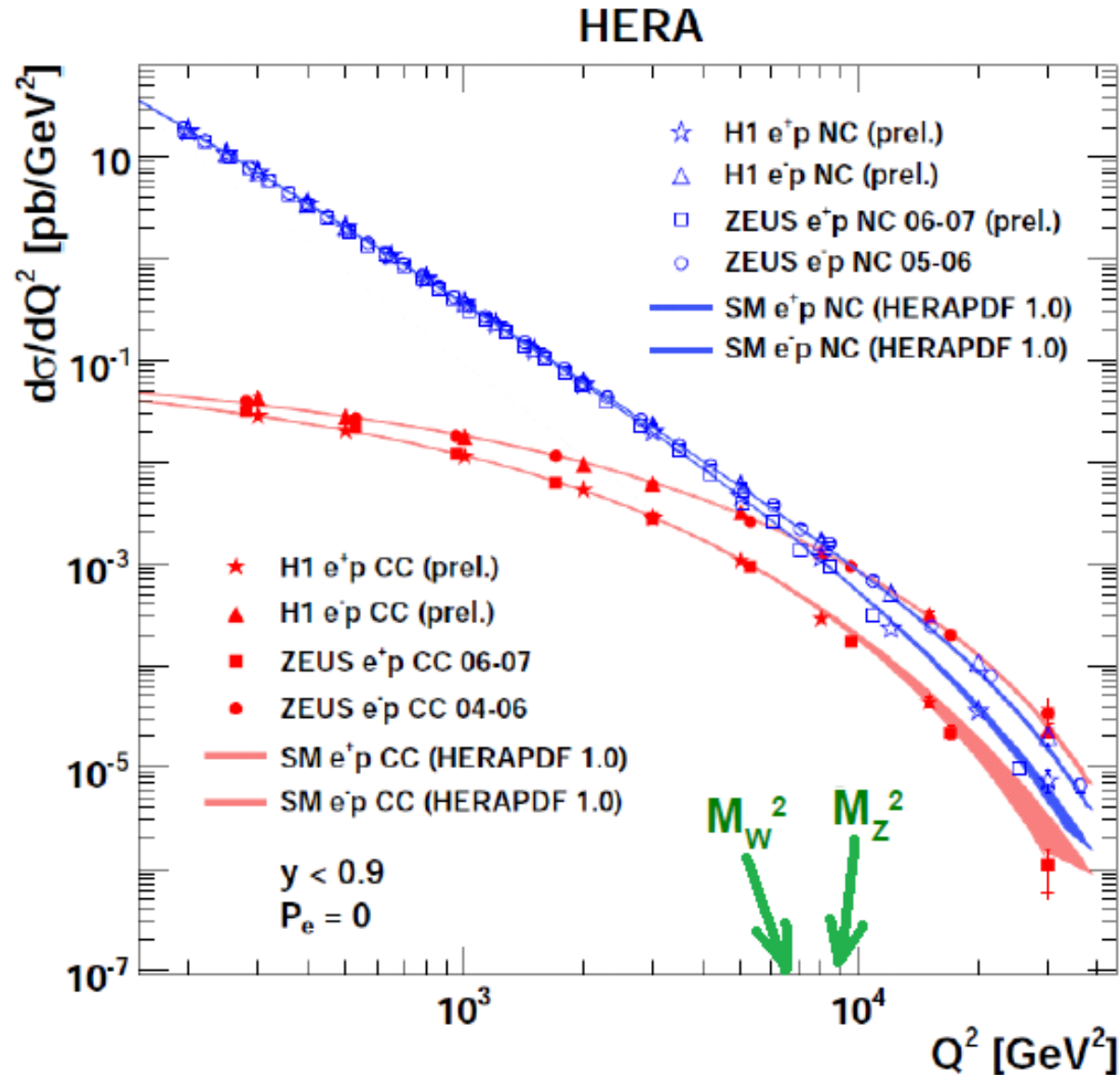
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"ONE SHOULD NEVER UNDERSTIMATE
THE PLEASURE WE FEEL FROM
HEARING SOMETHING WE ALREADY KNOW"

ENRICO FERMI

“UNIFICATION” OF E.M AND WEAK FORCES

PHOTON
+ HEAVY W

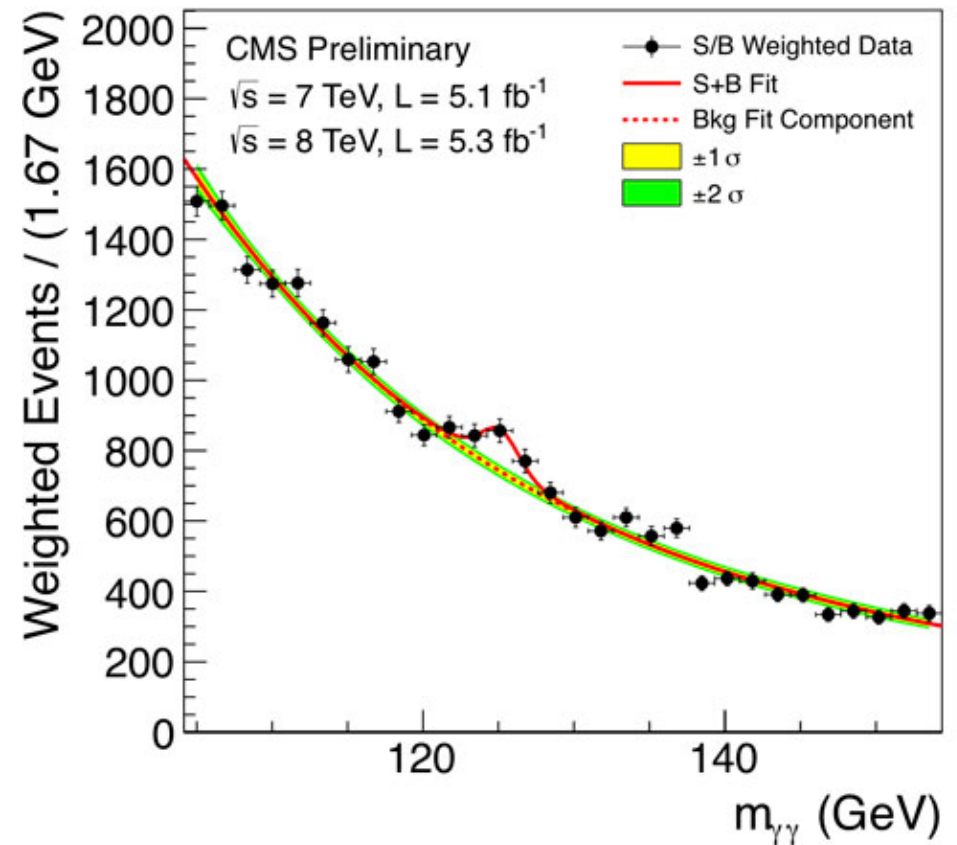
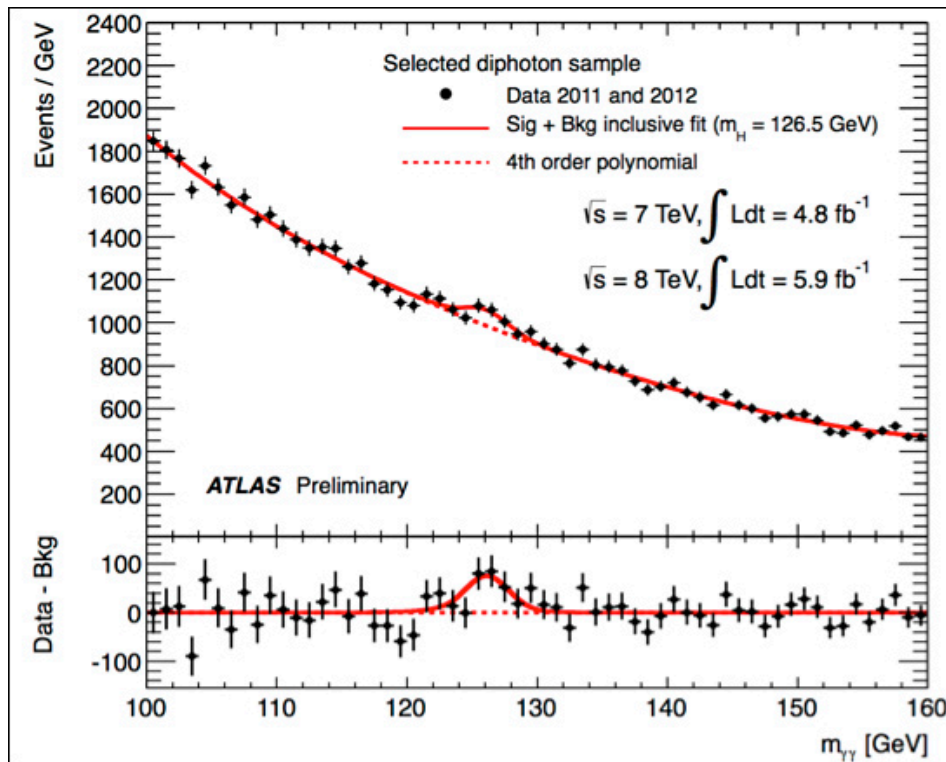


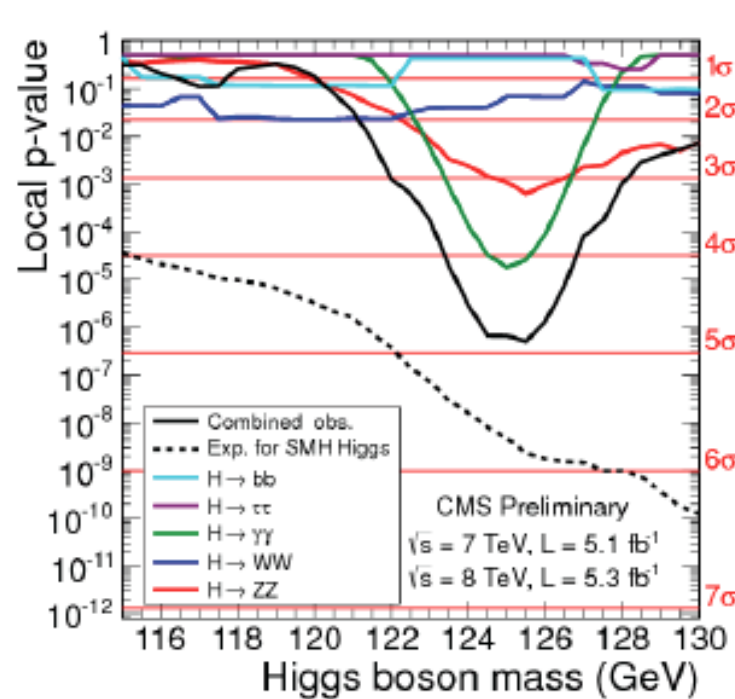
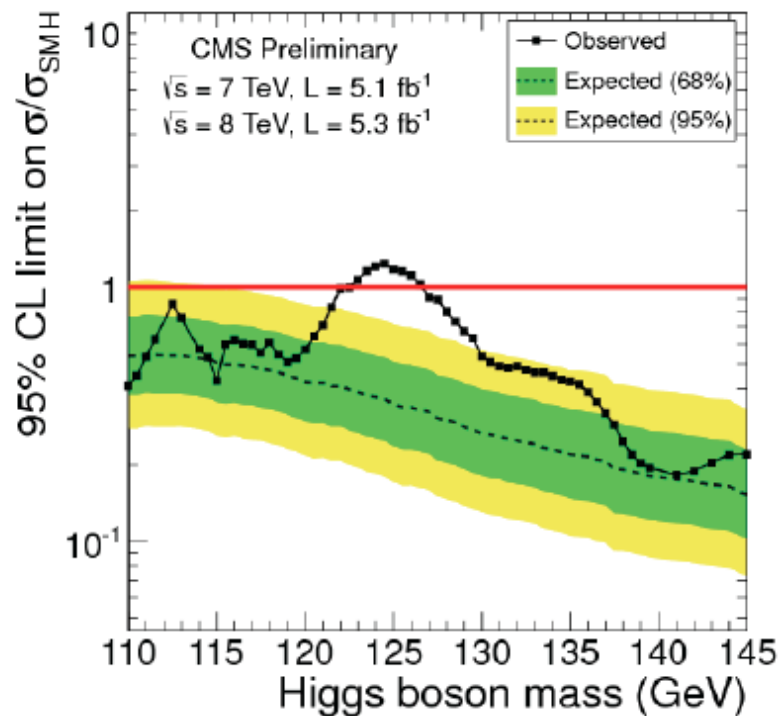
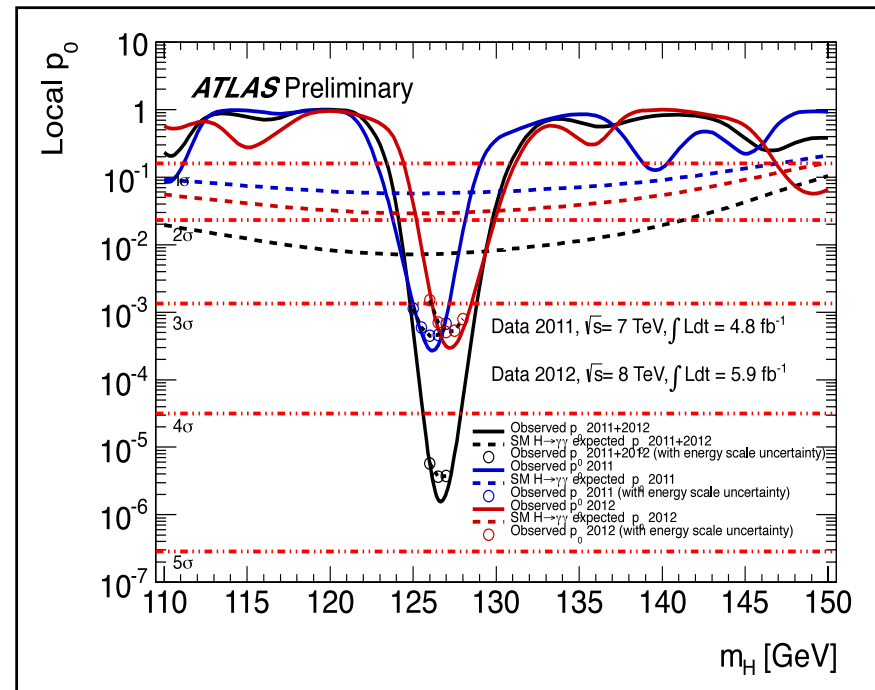
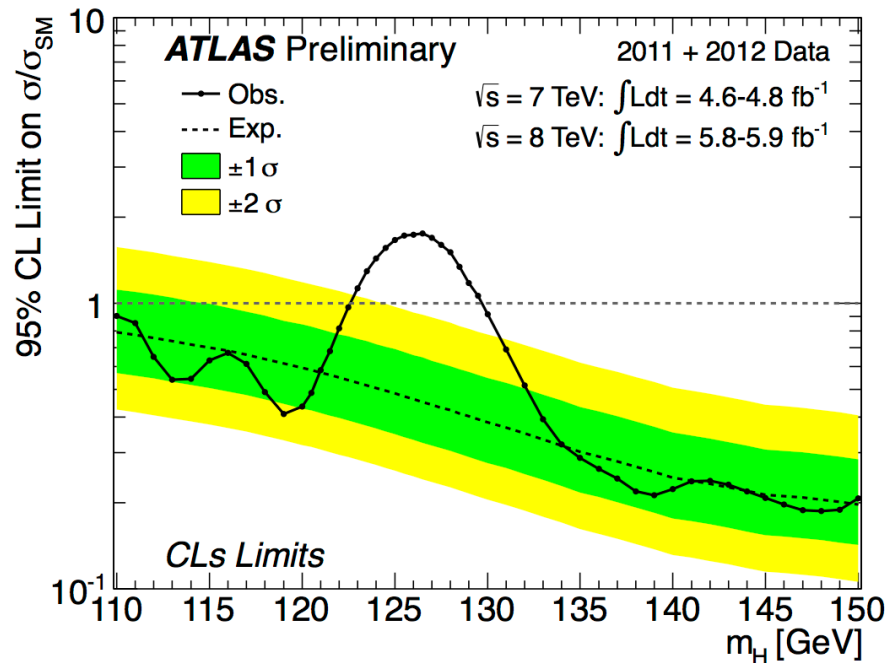
Unpolarized neutral and charged currents single differential cross section for combined HERA-I & HERA-II, e-p and e+p data measured by H1 and ZEUS

HIGGS ARRIVING...



4th JULY 2012







STANDARD MODEL

"STANDARD MODEL" : THEORETICAL FRAMEWORK

- STARTS FROM OBSERVATION
- PREDICTS AND CORRELATES NEW DATA

EXCELLENT "APPROXIMATION" AT A GIVEN SCALE

→ TO GO BEYOND!

"STANDARD MODEL"

- EVOLUTION -

- MENDELEEV'S PERIODIC TABLE CHEMISTRY
- QUANTUM MECHANICS ATOMS
- 3! PARTICLES (FIELDS)
($\sim 10^{-18}$ m)

CONTAINS (IN AN EXTREMELY ECONOMICAL WAY)
THE INGREDIENTS TO DESCRIBE (ALMOST) EVERYTHING

BASED UPON

GAUGE SYMMETRY

- * ENGINE OF THE PRESENT PHYSICS DEVELOPMENT
- UNIFIED TREATMENT OF THE FUNDAMENTAL FORCES

GAUGE THEORY

● WHAT IS A GAUGE THEORY?

A SYNTHESIS OF QUANTUM MECHANICS

⊕ SYMMETRY

(GAUGE SYMMETRY)

- IDEA OF GAUGE SYMMETRY: HERMANN WEYL
(1919)

(ONLY ELECTRON AND PROTON WERE KNOWN!)

WIGNER DIXIT (ABOUT EINSTEIN'S RELATIVITY)

"EINSTEIN'S WORK SETS THE INVERSION OF A TENDENCY: BEFORE IT, INVARIANCE PRINCIPLES CAME FROM THE DYNAMICAL LAWS. NOW, IS NATURAL FOR US, TO OBTAIN THE LAWS OF NATURE FROM INVARIANCE PRINCIPLES"

PEOPLE LOVES SYMMETRY

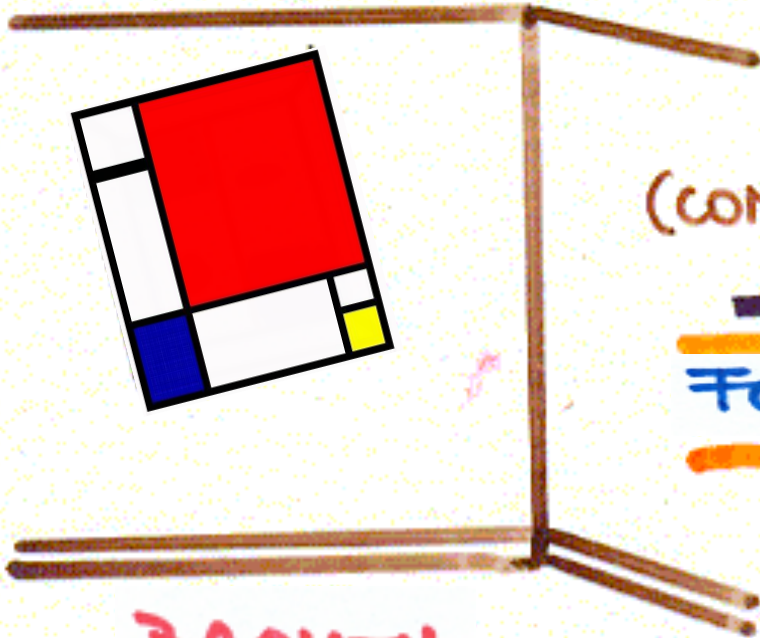
NATURE LOVES SYMMETRY

PEOPLE LOVES SYMMETRY

OUR MODEL OF

NATURE LOVES SYMMETRY

CROOKED



BROKEN

FUNDAMENTAL

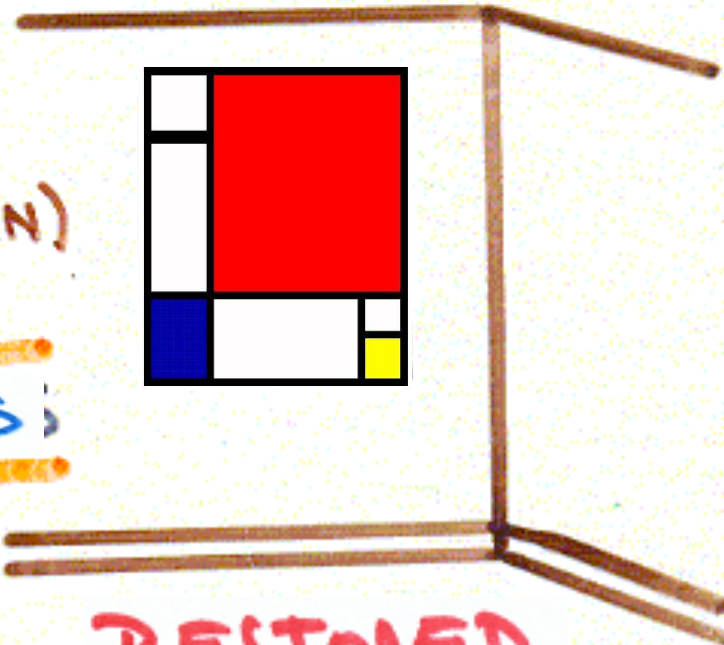
FORCES

(COMPULSION)



FORCES

UPRIGHT



RESTORED

FUNDAMENTAL

SYMMETRY



Gauge Symmetry

EVERYDAY LIFE: SYMMETRY \equiv HARMONY

(RIGHT - LEFT)

SYMMETRY IN PHYSICS

GEOMETRICAL

(ON THE SPACE-TIME COORDINATES)

OBSERVATIONAL SITUATIONS

INTERNAL

(ON THE DYNAMICAL VARIABLES AND OPERATORS)

QUALITY OF OBSERVABLES

GLOBAL

LOCAL

(PARAMETERS
CONNECTED WITH
SPACE-TIME)

GAUGE

ASYMMETRIES

TO GUARANTEE
GAUGE SYMMETRY



TO INTRODUCE
EXTRA FIELDS



INTERACTIONS

THE FRAMEWORK FOR FORMALIZING THE
INTERACTIONS IN NATURE

3!



$$3! =$$

3

⊗

2

⊗

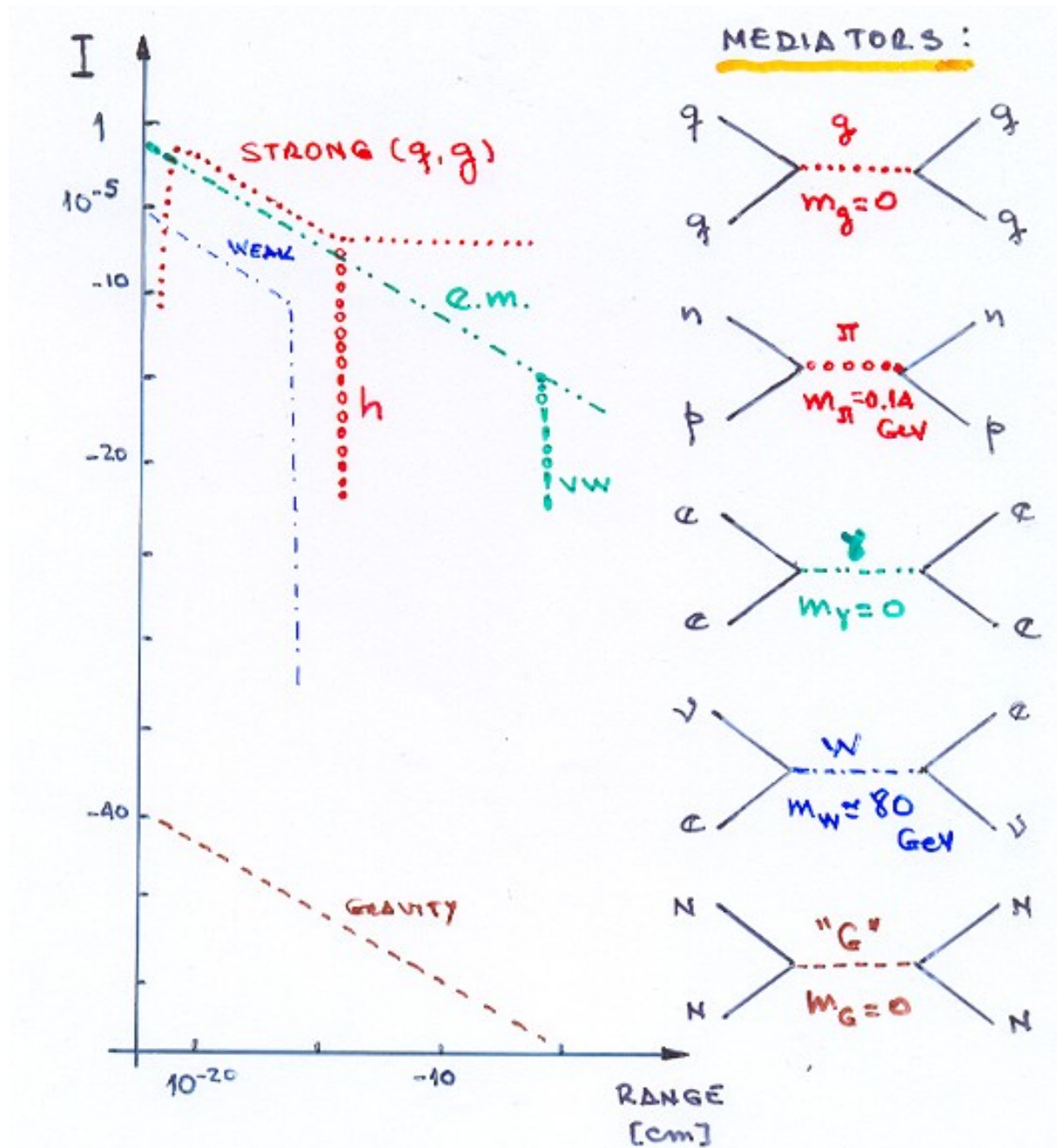
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$$3! = SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$$

$$3! \equiv \underbrace{\text{STRONG}}_{SU(3)_c} \oplus \underbrace{\text{WEAK} \oplus \text{E.M.}}_{SU(2)_L \otimes U(1)_Y}$$

FUNDAMENTAL INTERACTIONS

$$m_M \propto \frac{1}{\text{RANGE}}$$



FERMIONS

3 GENERATIONS

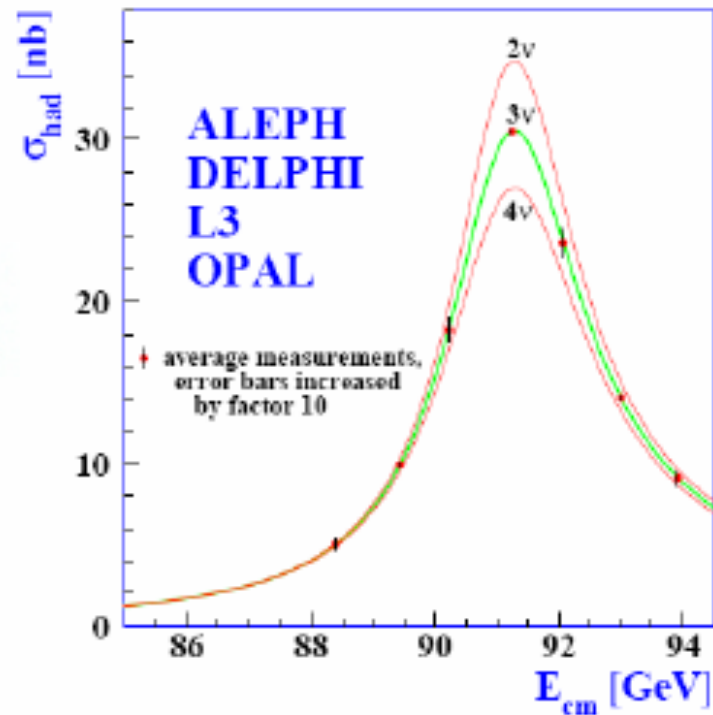
- 1st GENERATION.

$$\begin{array}{c} \leftarrow \text{SU}(3) \rightarrow \\ \text{SU}(2) \updownarrow \left(\begin{array}{ccc} u_L & u_L & u_L \\ d_L & d_L & d_L \end{array} \right) \end{array}$$

$$\begin{array}{ccc} (u_R & u_R & u_R) \\ (d_R & d_R & d_R) \end{array}$$

$$\text{SU}(2) \updownarrow \left(\begin{array}{c} \nu_L \\ e_L \end{array} \right)$$

$$e_R \quad \nu_R$$



* TYPICAL LAGRANGIAN (GAUGE)

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \bar{\Psi}_i i \gamma^\mu D_\mu \Psi_i$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c$$

$$D_\mu = \partial_\mu - ig T^a A_\mu^a$$

• g : GAUGE COUPLING $\alpha = \frac{g^2}{4\pi}$

• $a = 1, \dots, 8$ (SU(3)); $a = 1, 2, 3$ (SU(2)); $a = 1$ (U(1))

• i : ALL NECESSARY QUANTUM NUMBERS

• T^a : GROUP GENERATORS : $[T^a, T^b] = if^{abc} T^c$

• f^{abc} : STRUCTURE CONSTANTS OF THE GROUP

• $m_\gamma = m_g = 0$
(GAUGE)

• $m_w \neq 0 ; m_z \neq 0$
(\Rightarrow HIGGS)

SYMMETRIES IN FIELD THEORY

NATURAL LANGUAGE FOR PARTICLE PHYSICS:

QUANTUM FIELD THEORY

- FUNDAMENTAL EXCITATION \rightarrow QUANTUM FIELD

SYMMETRIES : INCORPORATED IN THE LAGRANGIAN

$$\mathcal{L} = \mathcal{L}(\phi, \partial_\mu \phi)$$

* CONTINUOUS SYMMETRIES

(TRANSFORMATIONS OF FIELDS UNDER A GROUP G)

$\psi_\alpha(x)$: QUANTUM FIELD

α : INTERNAL INDEX (COMPONENTS)

MEMBER OF AN IRREDUCIBLE MULTIPLIET

GROUP TRANSFORMATION ($a \in G$) \rightarrow

- $\psi_\alpha(x) \rightarrow \psi'_\alpha(x) = R_{\alpha\beta}(a) \psi_\beta(x)$

$R_{\alpha\beta}(a)$: MATRIX REPRESENTATION OF G

$$\text{If } \psi_\alpha(x) \xrightarrow{a} \psi'_\alpha(x) \xrightarrow{a'} \psi''_\alpha(x) \sim \psi_\alpha(x) \xrightarrow{a''} \psi''_\alpha(x)$$

EQUIVALENT

$$\rightarrow R_{\alpha\beta}(a') R_{\beta\gamma}(a) = R_{\alpha\gamma}(a'')$$

IN HILBERT SPACE:

* TRANSFORMATION \rightarrow UNITARY OPERATOR $U(a)$

$$\bullet U^{-1}(a) \psi_\alpha(x) U(a) = \psi'_\alpha(x) = R_{\alpha\beta}(a) \psi_\beta(x)$$

$$U(a) U(a') = U(a'')$$

CONTINUOUS SYMMETRIES

→ TO STUDY INFINITE SMALL



GENERATORS

G_i

$$* U(\delta a) = 1 + i \delta a_i G_i$$

$\delta a_i : \text{REAL}$

G_i : HERMITIAN

COMPOSITION OF $U(a) \rightarrow$

$$\underline{[G_i, G_j] = i C_{ijk} G_k}$$

LIE ALGEBRA

- C_{ijk} : STRUCTURE CONSTANTS OF G

↓

- $R_{\alpha p}(\delta a) = \delta_{\alpha p} + i \delta a_i (g_i)_{\alpha p}$

— g_i : REPRESENTATIONS OF GENERATORS. OBEY THE ALGEBRA

$$\text{using } (U^{-1} \chi U = R \chi)$$

$$* [G_i, \chi_\alpha(x)] = - (g_i)_{\alpha\beta} \chi_\beta(x)$$

(HOW THE FIELDS TRANSFORM UNDER G)

INVARIANCE OF A THEORY UNDER $G \rightarrow$

QUANTUM ACTION INVARIANT \rightarrow

THE CURRENTS $* J_i^\mu(x) = \frac{\partial \mathcal{L}}{\partial \partial_\mu \psi(x)} \cdot i (g_i)_{\alpha\beta} \psi_\beta(x)$

ARE CONSERVED :

$$\partial_\mu J_i^\mu = 0$$

IDENTIFICATION:

$$G_i = \int d^3x J_i^0(x)$$



CONSERVATION

$$[H, G_i] = 0$$

HAMILTONIAN

STATES

$|p; \alpha\rangle$: ONE PARTICLE STATE WITH $p^2 = -m^2$

PARTICLE STATES OF $\chi_\alpha(x)$ TRANSFORM AS $\chi_\alpha(x)$

— IF AND ONLY IF THE VACUUM IS G INVARIANT —

- $U(a) |0\rangle = |0\rangle$



- $G_j |0\rangle = 0$

LSZ:

$$|p; \alpha\rangle = \lim_{x^0 \rightarrow \pm\infty} \int d^3x e^{i p x} \frac{1}{i} \overleftrightarrow{\partial}_0 \psi_\alpha(x) |0\rangle$$



$$U^{-1}(a) |p; \alpha\rangle = \lim_{x^0 \rightarrow \pm\infty} \int d^3x e^{i p x} \frac{1}{i} \overleftrightarrow{\partial}_0 U^{-1}(a) \psi_\alpha(x) |0\rangle$$

$$U^{-1}(a) \psi_\alpha(x) \underbrace{U(a) |0\rangle}_{|0\rangle}$$

$$U \psi U^{-1} = R \psi$$

$$R_{\alpha\beta}(a) \psi_\beta(x) |0\rangle$$

$$* U^{-1}(a) |p; \alpha\rangle \stackrel{\downarrow}{=} R_{\alpha\beta}(a) |p; \beta\rangle *$$

WIGNER-WEYL REALIZATION OF SYMMETRY

THE VACUUM IS G INVARIANT

$$* U^{-1}(a) |p; \alpha\rangle = R_{\alpha\beta}(a) |p; \beta\rangle *$$



ALL STATES OF THE MULTIPLYTE $|p; \alpha\rangle$

* * HAVE THE SAME MASS *

$G_j |0\rangle = 0 \Rightarrow$ PARTICLES BELONGING TO A MULTIPLYTE
IN AN IRREDUCIBLE REPRESENTATION
HAVE THE SAME MASS

If $\phi^A \equiv (m_\alpha, \vec{0}) \rightarrow$ THE REST STATE

$$H |\phi; \alpha\rangle_{\text{REST}} = m_\alpha |\phi; \alpha\rangle_{\text{REST}}$$

$$[H, G_i] = 0 \rightarrow [H, U^{-1}(a)] = 0 \rightarrow$$

$$0 = [H, U^{-1}(a)] |\phi; \alpha\rangle_{\text{REST}}$$

$$0 = R_{\alpha\beta}(a) \{m_\beta - m_\alpha\} |\phi; \alpha\rangle_{\text{REST}}$$



$$\underline{m_\beta = m_\alpha}$$

$$[G_j, \chi_\alpha(x)] = - (g_j)_{\alpha\beta} \chi_\beta(x) \rightarrow$$

$$\langle 0 | [G_j, \chi_\alpha(x)] | 0 \rangle = - (g_j)_{\alpha\beta} \langle 0 | \chi_\beta(x) | 0 \rangle$$



W - W



$$\underline{\langle 0 | \chi_\beta(x) | 0 \rangle = 0}$$

* Nambu-Goldstone realization of symmetry *

* vacuum is NOT G invariant

$$* U(a) |0\rangle \neq |0\rangle$$



$$U^{-1}(a) |p; \alpha\rangle \neq R_{\alpha\beta}(a) |p; \beta\rangle$$

N-G



$$\underline{\langle 0 | \chi_{\beta}(x) | 0 \rangle \neq 0}$$



GOLDSTONE THEOREM:

FOR EACH GENERATOR G_j THAT DOES NOT
ANNIHILATE VACUUM \rightarrow ONE BOSON MASSLESS
"GOLDSTONE BOSON"

$$\left\{ G_j^{NA} |0\rangle = |\phi; j\rangle_G \quad (m^2=0) \right\}$$

Gauge Theory

- HERMANN WEYL : IDEA OF GAUGE INVARIANCE
(1919)

(ONLY ELECTRON AND PROTON)

PHYSICAL INTERPRETATION WRONG!



GAUGE INVARIANCE SURVIVED AS
MAXWELL EQUATIONS SYMMETRY

~ 1950 : YANG AND MILLS : EXTENSION OF
GAUGE SYMMETRY
(BEYOND Q.M.)

* RELATIVITY: \rightarrow NO ABSOLUTE FRAME OF REFERENCE
(S. & G.) IN THE UNIVERSE

• S: LORENTZ SYMMETRY GROUP OF SPECIAL RELATIVITY
IS A GLOBAL SYMMETRY

• G: A REFERENCE FRAME CAN ONLY BE DEFINED
LOCALLY (AT A SINGLE POINT)



IN G. TO RELATE FRAMES ONE NEEDS
A CONNECTION

① GRAVITATIONAL FIELD \rightarrow CONNECTION BETWEEN LOCAL FRAMES IN SPACE-TIME

• WEYL: ELECTROMAGNETISM CAN BE ASSOCIATED WITH A CONNECTION?

TO RELATE THE LENGTHS OF VECTORS AT DIFFERENT POINTS



SCALE (OR GAUGE) INVARIANCE

* LOCAL *

- WEYL: GAUGE CONNECTION \equiv ELECTROMAGNETIC

A_μ

- $\partial_\mu S \rightarrow \partial_\mu S + \partial_\mu \Lambda$

- $A_\mu \rightarrow A_\mu + \partial_\mu \Lambda$



MAGNITUDE OF A VECTOR: CHANGES ALONG A
TIME-LIKE WORLD-LINE



NO WAY OF DEFINING A STANDARD CLOCK
(RATE OF CLOCK DEPENDS ON THE HISTORY OF CLOCK)



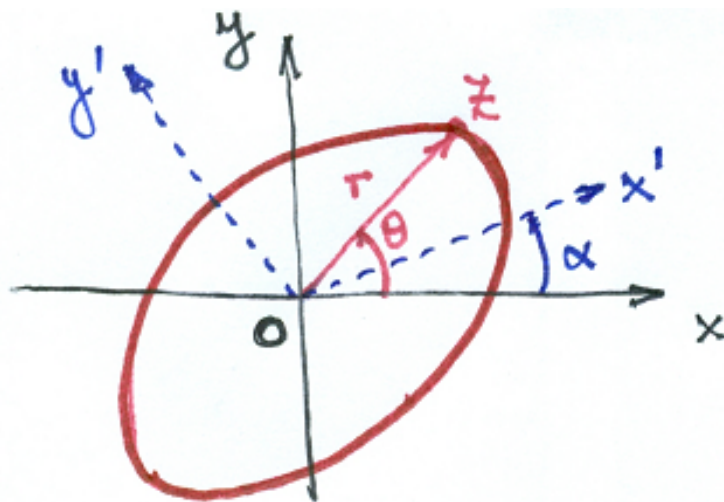
ATOMS AT DIFFERENT POINTS \Rightarrow DIFFERENT
SPECTRA !

NOT OBSERVED

BASICS IDEAS

(CONCATENATE...)

- HARMONIC OSCILLATOR (IN THE PLANE)



$$\begin{cases} y = A \sin \omega t \\ x = B \cos \omega t \end{cases}$$



— COMPLEX VARIABLE

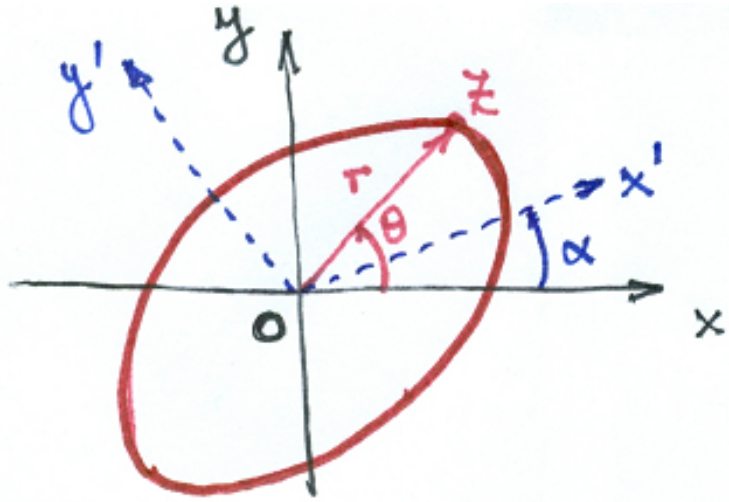
- $Z = x + iy$

- $(Z = r e^{i\theta}) \cdot r = \sqrt{x^2 + y^2}; \theta = \arctan \frac{y}{x}$

* OSCILLATOR EQUATION

- $\frac{d^2 Z}{dt^2} + \omega^2 Z = 0$

$$(T = \frac{2\pi}{\omega})$$



BUT: TO MEASURE θ FROM X-AXIS
ARBITRARY!

→ ROTATE α TO (x', y') → TO MEASURE FROM x'



* $\theta \rightarrow \theta - \alpha$

[θ WAS "REGAUGED"]



- MULTIPLY THE EQUATION BY $e^{-i\alpha}$ (PHASE)
- REDEFINE $z' = z e^{-i\alpha}$

- $\frac{d^2 z}{dt^2} + \omega^2 z = 0$



- $\frac{d^2 z'}{dt^2} + \omega^2 z' = 0$

NOTHING CHANGED!

ABSOLUTE VALUE OF θ
IRRELEVANT!



GLOBAL
GAUGE INVARIANCE
(θ INDEPENDENT OF t)

IF $\alpha = \alpha(t)$

LOCAL GAUGE TRANSFORMATION

$e^{-i\alpha(t)}$

CANNOT BE ABSORBED IN z'

- $\frac{d^2 z'}{dt^2} = \frac{d^2}{dt^2} [z(t) e^{-i\alpha(t)}]$

$\frac{d\alpha(t)}{dt} \neq 0$

HOW TO REGAIN INVARIANCE?

(TO COMPENSATE $\frac{d\alpha(t)}{dt}$)

- REPLACE : $\frac{d}{dt} \rightarrow \frac{d}{dt} - i A(t)$
- WITH THE PROPERTY : IF : $\theta \rightarrow \theta - \alpha(t)$
THEN : $A(t) \rightarrow A(t) - \frac{d\alpha(t)}{dt}$
- $A(t)$: COMPENSATOR \equiv GAUGE FIELD

* AND?

$\alpha = \alpha(t) \Rightarrow$ TO ROTATE THE REFERENCE FRAME
WITH $\omega = \frac{d\alpha(t)}{dt}$

ROTATING FRAME \Rightarrow "FICTITIOUS" FORCES
(CENTRIFUGE - CORIOLIS)

A(t) GENERATES THESE FORCES!

MAXWELL GAUGE INVARIANCE

$$\bullet \begin{cases} \vec{\nabla} \wedge \vec{A} = \vec{B} \\ \vec{\nabla} \cdot \vec{A} = ? \end{cases}$$

$$\bullet \vec{E} = -\vec{\nabla} \phi - \kappa \frac{\partial \vec{A}}{\partial t}$$

* PANOFSKY - PHILLIPS (p.2)

POEMA "VECTORIAL"

Esto el Papa exclamó al firmar la bula
con que furioso excomulgó a Lutero:
La **divergencia** de un **rotor** es nula
y el **rotor** de un **gradiente** es siempre cero.
El gran fraile alemán invocó a Dios
y exclamó con su habitual vehemencia:
El **rotor** de un **rotor** mas **nabla dos**
da el **gradiente** de toda **divergencia**.

Enrique Loedel Palumbo (1901-1962), ¿físico o poeta?



$$* \left\{ \begin{array}{l} \vec{A} \rightarrow \vec{A}' = \vec{A} + \vec{\nabla} \Lambda \\ \phi \rightarrow \phi' = \phi - \kappa'' \frac{\partial \Lambda}{\partial t} \end{array} \right.$$

Λ : SCALAR FUNCTION

→ IDENTICAL \vec{E} AND \vec{B}

GAUGE SYMMETRY OF E.M.

* ARBITRARINESS OF A_μ (IN CLASSICAL PHYSICS)

* ABRAHONOV - BOHM EFFECT

(PHASE SHIFT DUE TO A SOLENOID)



POTENTIAL \vec{A} IS A PHYSICAL FIELD
DIRECTLY OBSERVED

$$\frac{e}{\hbar}(S_1 - S_2) = \frac{e}{\hbar} \left[\int_1 \vec{A} \cdot d\vec{x} - \int_2 \vec{A} \cdot d\vec{x} \right]$$

NO PROBLEM WITH AMBIGUOUSNESS OF \vec{A}

[OBSERVABLE: RELATIVE CHANGE OF PHASE DUE TO \vec{A}]

ELECTROMAGNETIC INTERACTIONS

(MEDIATED BY PHOTONS)

FROM THE REQUIREMENT OF

LOCAL ABELIAN GAUGE INVARIANCE

⊕ YANG-MILLS (FOR NON ABELIAN)



FUNDAMENTAL INTERACTIONS DESCRIBED BY

LOCAL GAUGE THEORIES

MEDIATED BY GAUGE FIELDS

$$\underline{m}_{\text{GAUGE FIELD}} \equiv 0$$

$$\mathcal{L}_m = \frac{1}{2} m A_\mu A^\mu \text{ NOT ALLOWED!}$$

* "STANDARD" WAY OF CONSTRUCTING
A GAUGE THEORY
(FOR MATTER FIELDS)

- CHOOSE AN APPROPRIATE UNITARY LIE GROUP $G \{g\}$
- PROMOTE AN ACTION WITH GLOBAL INVARIANCE UNDER G AS THE MATTER SYMMETRY GROUP



ACTION INVARIANT UNDER CONSTANT PHASE TRANSFORMATION



CONSERVATION OF A NOETHER CHARGE

- TO PROMOTE GLOBAL INVARIANCE TO LOCAL ONE

$$g \rightarrow g(x) \in \bar{G}$$

(PHASES SPACE-TIME DEPENDENT)



TO INTRODUCE GAUGE FIELDS THAT
COMPENSATE (VIA GAUGE TRANSFORM)
CHANGES PRODUCED BY LOCAL PHASE CHANGES



KINETIC PART OF THE MATTER LAGRANGIAN
WRITTEN IN TERMS OF COVARIANT DERIVATIVE

- A KINETIC TERM FOR GAUGE FIELDS HAS TO BE INCLUDED

$q(x)$ WELL BEHAVED \Rightarrow CONSERVED NOETHER CHARGE
UNCHANGED

(FROM THE GLOBAL CASE)

* BUILDING A GAUGE THEORY *

"THE STANDARD WAY"

EX:

* ONE PARAMETER GROUP SYMMETRY

COMPLEX SCALAR FIELD:

$$\bullet \mathcal{L}(\phi, \partial_\mu \phi) = \partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi$$

INVARIANT UNDER PHASE (CONSTANT) \Rightarrow U(1)

$$\phi(x) \rightarrow \phi'(x) = e^{i\alpha} \phi(x)$$

* α CONSTANT *

$$\text{INFINITESIMAL} \Rightarrow \begin{cases} \delta\phi = i\alpha\phi \\ \delta\phi^* = -i\alpha\phi^* \end{cases}$$

$$\text{NOETHER} \rightarrow \mathcal{J}_\mu = i [\partial_\mu \phi^* \phi - \phi^* \partial_\mu \phi]$$

CONSERVED



$$Q = \int d^3x \mathcal{J}_0 = \text{CONSTANT} \rightarrow \underline{\dot{Q} = 0}$$

Q : "CHARGE" OF THE FIELD

INTRODUCING:
$$\begin{cases} \phi_1 = \frac{1}{\sqrt{2}} (\phi + \phi^*) \\ \phi_2 = \frac{1}{\sqrt{2}} (\phi - \phi^*) \end{cases}$$



• $\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2$

• $\mathcal{L}_k = \frac{1}{2} \partial_\mu \phi_k \partial^\mu \phi_k - m^2 \phi_k^2 ; (k=1,2)$



PHASE TRANSFORMATION →

$$\begin{cases} \phi_1 \rightarrow \phi'_1 = \cos \alpha \phi_1 - \sin \alpha \phi_2 \\ \phi_2 \rightarrow \phi'_2 = \cos \alpha \phi_2 + \sin \alpha \phi_1 \end{cases}$$

ROTATION IN (ϕ_1, ϕ_2) -PLANE

- LOCAL PHASE INVARIANCE

$$\alpha \rightarrow \alpha(\underline{x})$$

YANG AND MILLS MOTIVATION:

"THE CONCEPT OF FIELD AND THE CONCEPT OF LOCAL INTERACTIONS IMPLY A SPREADING OF INFORMATION TO NEIGHBOURING POINTS AND ELIMINATE THE ACTION AT A DISTANCE. THEN GLOBAL PHASE INVARIANCE SEEMS TO CONTRADICT THE GENERALIZED-IDEA OF LOCALITY"...

$$\ast \phi(x) \rightarrow \phi'(x) = e^{i\alpha(x)} \phi(x) \quad \text{IN}$$

$$\bullet \mathcal{L}(\phi, \partial_\mu \phi) = \partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi$$

SYMMETRY IS PRESERVED? NO

MASS TERM : O.K. ✓

KINETIC TERM :

$$\bullet \partial_\mu \phi(x) \rightarrow e^{i\alpha(x)} [\partial_\mu + i \partial_\mu \alpha(x)] \phi(x) \quad \text{NO}$$

KINETIC TERM: NO

- $\partial_\mu \phi \partial^\mu \phi^* \rightarrow [\partial_\mu - i \partial_\mu \alpha(x)] \phi^* [\partial^\mu + i \partial^\mu \alpha(x)] \phi$



\mathcal{L} IS NOT INVARIANT UNDER
LOCAL PHASE TRANSFORMATIONS

* HOW TO TURN \mathcal{L} INVARIANT?

• FREEDOM OF A_μ (e.m. FIELD) •

REPLACE

• $\partial_\mu \phi$

BY • $D_\mu \phi \equiv (\partial_\mu - iq A_\mu) \phi$

(GAUGE INVARIANT DERIVATIVE)

WHEN $\phi(x)$ CHANGES BY $\alpha(x) \rightarrow$

• $A_\mu(x) \rightarrow A'_\mu(x) = A_\mu(x) - \frac{1}{q} \partial_\mu \alpha(x)$

THEN: • $D_\mu \phi(x) \rightarrow D'_\mu \phi'(x) = e^{i\alpha(x)} D_\mu \phi(x)$ •

TRANSFORMS AS THE FIELD

$$\mathcal{L} = [\partial_\mu + iq A_\mu(x)] \phi^*(x) [\partial^\mu - iq A^\mu(x)] \phi(x) - m^2 \phi^*(x) \phi(x)$$

IS LOCAL PHASE TRANSFORMATION INVARIANT!

\mathcal{L} NOW CONTAINS INTERACTIONS
(THE MINIMAL INTERACTION)

• $A_\mu(x)$: GAUGE FIELD



- COMPENSATING FIELD
(RESTORE THE SYMMETRY)
- COMPENSATIVE FIELD
(DISTINGUISHES CHARGES)

FERMIONS

$$* \mathcal{L}_{f-A} = \underbrace{i \bar{\Psi} \gamma^{\mu} (\partial_{\mu} - i e A_{\mu}) \Psi}_{i \bar{\Psi} \gamma^{\mu} \partial_{\mu} \Psi + e \bar{\Psi} \gamma^{\mu} \Psi A_{\mu}} - m \bar{\Psi} \Psi$$

PROPOSAL

(POSSIBLE AND NATURAL)

* TO INVERT THE PROCESS *

TO AVOID THE UKASE THAT PROMOTES GLOBAL
TO LOCAL SYMMETRY OF THE MATTER L.

"UKASE" (YK23)

(HAVE THE POWER OF LAWS BUT MAY NOT ALTER
THE REGULATIONS OF EXISTING LAWS)

"UPSIDE DOWN"

Building Gauge Theories: The Natural Way
Fundamental Journal of Modern Physics 2 (2011) 15.
C. A. García Canal, F.A. Schaposnik

* SYMMETRIES: OF FIELDS

• IMPOSING SYMMETRIES TO FIELDS

{ WHY STARTING FROM PHASE INVARIANCE
OF MATTER FIELDS?

TO BUILD THE THEORY OF FUNDAMENTAL
INTERACTIONS STARTING FROM THE
INTERACTION MEDIATORS: THE GAUGE FIELDS

SOURCES ARE ADDED IMPOSING LOCAL GAUGE
INVARIANCE AND LORENTZ INVARIANCE
(OF MAXWELL EQUATION OR NON-ABELIAN ONES)

● STARTING POINT:

ELECTRODYNAMICS

* MAXWELL EQUATIONS

NO SOURCES
↓
(PHYSICAL PAIR)

● $\partial_\mu F^{\mu\nu} = 0$

(THE OTHER PAIR GIVEN BY BIANCHI IDENTITY)

(VIA U(1) GAUGE FIELD $A_\mu(x)$)

$$F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)$$

— GAUGE TRANSFORMATION:

- $A_\mu(x) \rightarrow \hat{A}_\mu(x) = A_\mu(x) + \partial_\mu \Lambda(x)$

$$F_{\mu\nu}(x) \quad \text{INVARIANT}$$

- $\partial_\mu F^{\mu\nu} = 0 \quad \text{(GAUGE INVARIANT)}$

- MAXWELL FROM THE LAGRANGIAN:

- $$L_M = -\frac{1}{4} F_{\mu\nu}(x) F^{\mu\nu}(x)$$

(LORENTZ AND GAUGE INVARIANT)

PRESENCE OF EXTERNAL SOURCE

j_{ext}^{μ}



MAXWELL →

$$\partial_{\mu} F^{\mu\nu} = e j_{\text{ext}}^{\nu}$$

$[j_{\text{ext}}^{\nu}(x) : \text{NON-DYNAMICAL}]$

$(e \in \mathbb{R} : \text{COUPLING OF SOURCE})$

NATURAL AND SIMPLEST LORENTZ INVARIANT
TERM IN THE LAGRANGIAN FORMULATION:

- $L_{\text{INT}} = e A_{\mu}(x) j_{\text{ext}}^{\mu}(x)$

IF

$$\underline{L = L_M + L_{INT}}$$

GAUGE INVARIANT



$$\int_{\text{ext}}^M(x) \xrightarrow{\Lambda} \underline{\int_{\text{ext}}^M \Lambda(x) = \int_{\text{ext}}^M(x)}$$

AND

• $\partial_\mu \int_{\text{ext}}^M(x) = 0$ (FORCED BY MAXWELL)

(L CHANGES AT MOST BY A TOTAL DERIVATIVE)
(UNDER GAUGE)

STRUCTURE OF MAXWELL PRESENTED BY
LORENTZ SYMMETRY



ONE CAN ANTICIPATE COUPLING OF MATTER
TO GAUGE FIELDS

* CONSIDER A DYNAMICAL DIRAC FERMION $\psi(x)$

AS THE ORIGIN OF THE CURRENT $j^\mu(x)$ ($\neq j_{\text{ext}}^\mu(x)$)



MOST ECONOMIC FORM: BILINEAR OF FERMIONS

CONSTRUCTION GUIDED BY LORENTZ AND GAUGE

i) LORENTZ:

$$\bullet \quad j^M(x) = \bar{\psi}(x) \gamma^M \psi(x)$$

ii) GAUGE:

(INVARIANCE OF $j^M(x)$)

$$\psi(x) \rightarrow \psi'(x) = e^{iq\Lambda(x)} \psi(x)$$

$(q \in \mathbb{R})$

FERMION DYNAMICAL \Rightarrow $L_D = i \bar{\psi}(x) \gamma^M \partial_\mu \psi(x)$



• $L = L_M + L_D + L_{INT}$: GAUGE INVARIANT

\mathbb{F} $q \equiv e$



⊕ COVARIANT DERIVATIVE : $D_\mu = \partial_\mu - eA_\mu(x)$



USUAL

$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\Psi} \gamma^\mu D_\mu \Psi$

(MINIMAL Q.M. COUPLING)

NOTICE:

1) IF $\int_{\mu}^{\gamma} = \bar{\Psi} \gamma_{\mu} \gamma^5 \Psi$ (INSTEAD OF $\int_{\mu} = \bar{\Psi} \gamma_{\mu} \Psi$)

PSEUDOVECTOR COUPLED TO $A^{\mu} \Rightarrow$ PSEUDOSCALAR

ONE NEEDS \perp SCALAR

2) NON MINIMAL COUPLING : CORRECTS MAXWELL AT CLASSICAL LEVEL!

NON ABELIAN CASE

Gauge fields: $\underline{A_\mu(x) = A_\mu^a(x) t^a}$



• $L = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \bar{\Psi}_i (i \not{\partial} \delta_{ij} - e \not{A}^a T_{ij}^a) \Psi_j$

(GAUGE FIELD THEORY OF SU(N))

* SUMMARIZING

- USUAL WAY: TO PROMOTE THE GLOBAL UNITARY SYMMETRY OF THE MATTER LAGRANGIAN TO A LOCAL ONE \Rightarrow REQUIRING A GAUGE FIELD
- "NATURAL" WAY: START FROM THE PURE GAUGE FIELD AND IMPOSING LORENTZ AND GAUGE INVARIANCE WHEN COUPLING MATTER \Rightarrow SOME L AND CONSERVED J_μ

— PARALLEL TO THE GEOMETRIC APPROACH —

(STARTING POINT: DEFINE A CONNECTION IN A
PRINCIPAL FIBER BUNDLE AND
INTRODUCE MATTER FIELDS AS SECTIONS
IN THE ASSOCIATED VECTOR BUNDLE

UPSIDE DOWN



Upside Down Cake

You have some latitude with the fruits that you use. Just make sure that whatever you use covers the bottom in a substantial layer, around double-thickness, since the fruit will cook down while baking and settle nicely into place.

Berries and such as good nestled in the gaps between the slices of fruits.

For the fruit layer:

3 tablespoons butter (45g), salted or unsalted

3/4 cup packed (135g) light brown sugar

fruit: 8 quartered plums or apricots, 3-4 thickly-sliced pears or nectarines,
or 2 cups cranberries; add a handful of huckleberries, cherries, raspberries, or another bushberry

For the cake layer:

8 tablespoons (115g) unsalted butter

3/4 cup (150g) sugar

1 teaspoon vanilla extract

2 large eggs, at room temperature.

1 1/2 cups (210g) flour

1 1/2 teaspoon baking powder, preferably aluminum-free

1/4 teaspoon salt

1/2 cup (125ml) whole milk, at room temperature

1. Melt the 3 tablespoons (45g) of butter in a cast iron skillet, or cake pan.

Add the brown sugar and cook while stirring, until the sugar is melted and begins to bubble. Remove from heat and let cool.

2. Once cool, arrange the fruit in a pinwheel design, added berries if desired. Set aside.

3. To make the cake, preheat the oven to 350F. (190C)

4. Beat the 8 tablespoons (115g) of butter and sugar until fluffy. Add the vanilla, then the eggs, one at a time, until smooth.

5. Whisk or sift together the flour, baking powder, and salt.

6. Stir in half of the flour mixture, then the milk, then the remaining dry ingredients.

Do not overmix: stir just until the flour is barely incorporated into the batter.

7. Spread the batter over the fruit, then bake for 45 minutes to one hour (depending on the size of the pan, and the thickness of the batter). The cake is ready when it begins to pull away from the sides of the pan and the center feels just set.

8. Remove from oven, let cool about 20 minutes, then place a cake plate on top, and wearing oven mitts,

flip the cake out on to the plate.

Serving: Upside Down Cake is best served warm, perhaps with whipped cream or vanilla ice cream.

SEE YOU AT 2

CONNECTION BETWEEN
Q.M. FIELD STRENGTH
AND COVARIANT DERIVATIVE



GAUGE INVARIANCE
FUNDAMENTAL PROPERTY
OF ELECTRODYNAMICS

ALREADY IN NON-RELATIVISTIC QUANTUM DYNAMICS
(GAUGED RELATIVISTIC)



CHARGED PARTICLE IN Q.M. FIELD:

GAUGE INVARIANCE IMPOSES A PHASE IN THE
PARTICLE WAVE FUNCTION

PHASE FACTOR IS DIRECTLY ASSOCIATED TO THE
COVARIANT DERIVATIVE \rightarrow

~~STANDARD~~ GAUGE COVARIANCE IS TRANSMITTED FROM THE
FIELD STRENGTH TO THE WAVE FUNCTION

NON ABELIAN CASE

GAUGE FIELDS: $\underline{A_\mu(x) = A_\mu^a(x) t^a}$

VALUES IN THE LIE ALGEBRA OF SU(N) WITH GENERATORS $\underline{t^a}$

GAUGE TRANSFORMATION

- $A_\mu(x) \rightarrow \hat{A}_\mu(x) = g^{-1}(x) A_\mu(x) g(x) - \frac{i}{e} g^{-1}(x) \partial_\mu g(x)$

$$\left\{ \begin{array}{l} g(x) = e^{i\lambda(x)} \\ \lambda(x) = \lambda^a(x) t^a \end{array} \right.$$

INFINITESIMALLY:

$$A_\mu(x) \rightarrow \hat{A}_\mu(x) = A_\mu(x) + \frac{1}{e} D_\mu[A] \Lambda(x)$$

COVARIANT DERIVATIVE: $D_\mu[A] = \partial_\mu + ie [A_\mu(x),]$

TRANSFORMING AS:

- $D_\mu[A^\wedge] = g^{-1}(x) D_\mu[A] g(x)$

- $F_{\mu\nu} = F_{\mu\nu}^a t^a = \partial_\mu A_\nu - \partial_\nu A_\mu + e [A_\mu, A_\nu]$

MAXWELL

→
(NON- Λ)

YANG-MILLS

• $\mathcal{D}_\mu [A] F^{\mu\nu} = 0$

UNDER GAUGE $F_{\mu\nu}(x) \rightarrow F_{\mu\nu}^\wedge(x) = g^{-1}(x) F_{\mu\nu}(x) g(x)$
(COVARIANTLY)

* $\mathcal{D}_\mu [A] F^{\mu\nu} = 0$ ALSO

FROM • $\mathcal{L}_{YM} = -\frac{1}{2} \text{Tr} \left\{ F_{\mu\nu} F^{\mu\nu} \right\}$

INTRODUCE (AS BEFORE!) : $\int_{\text{ext}}^M \Rightarrow$

- $D_\mu F^{\mu\nu} = e \int_{\text{ext}}^\nu$

CONSISTENCY $\Rightarrow \int_{\text{ext}}^\nu$ TAKES VALUES IN THE LIE ALGEBRA OF $SU(N)$

- $\int_{\text{ext}}^\nu(x) = \int_{\text{ext}}^\nu(x) t^a$

- $L_{\text{INT}} = e \text{Tr} \left\{ A_\mu \int_{\text{ext}}^M \right\}$

$$L = L_{\text{YM}} + L_{\text{INT}} \quad \text{GAUGE INVARIANT IF}$$

$$\bullet \int_{\text{ext}}^M(x) \rightarrow \int_{\text{ext}}^{A \wedge}(x) = g^{-1}(x) \int_{\text{ext}}^M(x) g(x)$$

$$\text{AND SATISFIES: } \bullet \mathcal{D}_\mu[A] \int_{\text{ext}}^M(x) = 0$$

MAKING THE SOURCE DYNAMICAL: \Rightarrow DUAL FERMIONS!

MAKING THE SOURCE DYNAMICAL: \Rightarrow DUAL FERMIONS



DEFINE :

$$J_{\mu}^a(x) = \bar{\Psi}^i \gamma_{\mu} T_{ij}^a \Psi^j \quad i, j = 1, 2, \dots, N$$

(FERMIONS IN THE FUNDAMENTAL OF $SU(N)$ WITH GENERATORS T^a)

$$\Psi(x) \rightarrow \Psi^{\Lambda}(x) = e^{i q \Lambda(x)} \Psi(x)$$

$$q \equiv e$$

⊕ KINETIC TERM FOR FERMIONS

NOTICE: THE MATTER CURRENT IS COVARIANTLY CONSERVED \Rightarrow DOES NOT LEAD TO A CONSERVED CHARGE $[\partial_\mu[A]^\mu = 0]$

ONLY WHEN THE GAUGE NOETHER CURRENT

$$\left(j^{\text{GAUGE}} \right)_\mu^a = f^{abc} F_{\mu\nu}^b A^{c\nu}$$

(ASSOCIATED TO THE YANG-MILLS LAGRANGIAN)

IS INCLUDED

\rightarrow • $J_\mu = \left(j^{\text{GAUGE}} \right)_\mu + j_\mu$ IS CONSERVED

$$\underline{\partial^\mu J_\mu = 0}$$