2

PHENOMENOLOGY

(TO PROVOQUE)

- i) FORWARD - BACKWARD ASYMMETRY

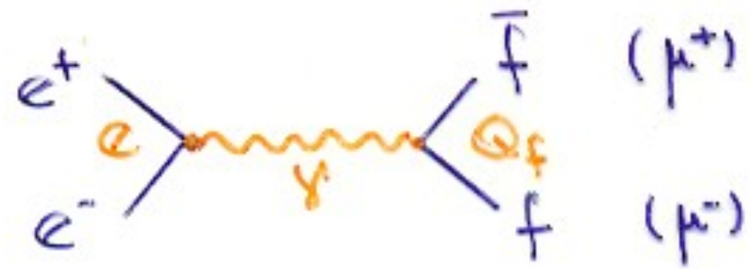
  $(e^+e^- \to f\bar{f})$

- ii) $R = \dfrac{(e^+e^- \to \text{HADRONS})}{(e^+e^- \to \mu^+\mu^-)}$

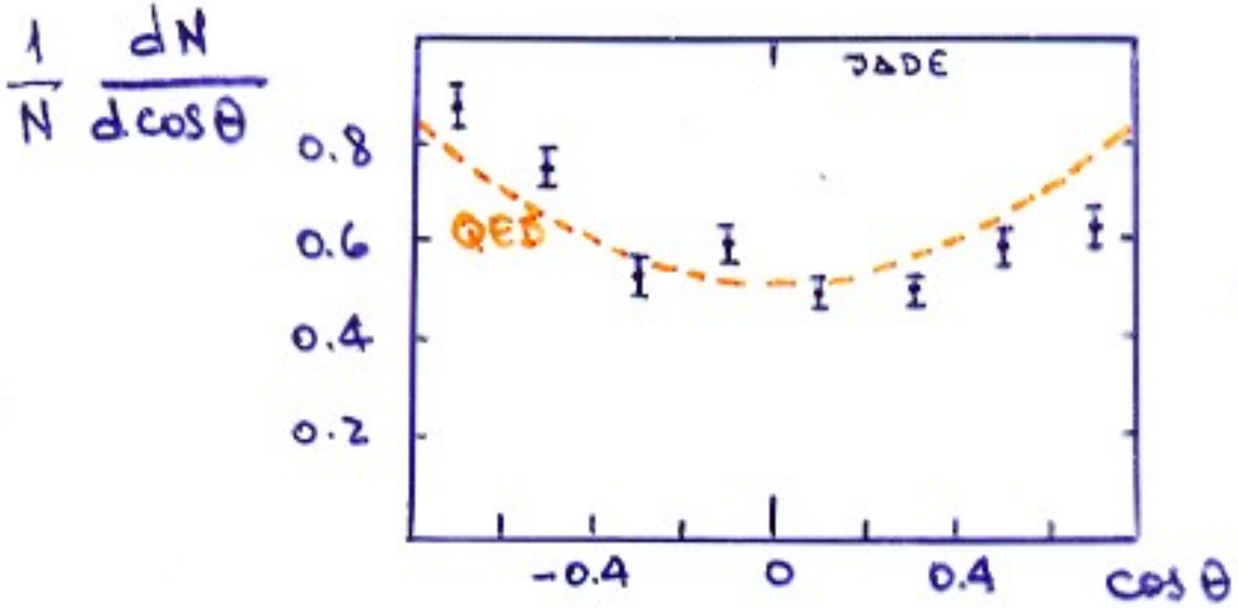
- iii) JETS

- iv) M_W; M_{Z^0}

i)

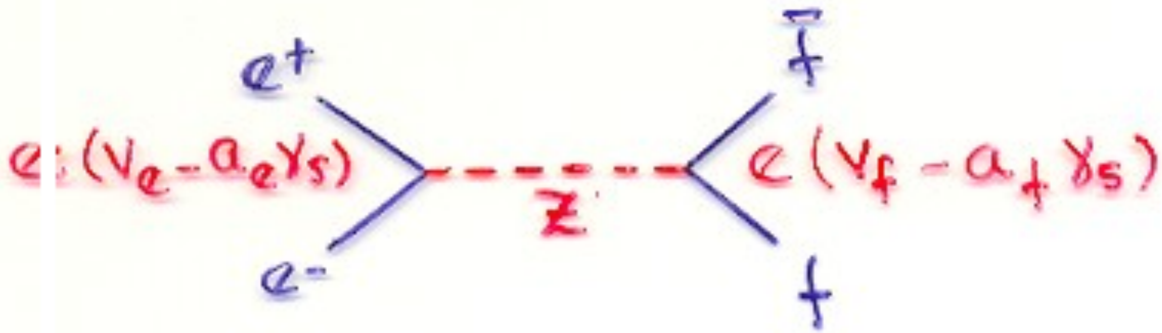


$$\frac{d\sigma}{d\Omega} = \frac{e^4}{64\pi^2 s}\left(1+\cos^2\theta\right)$$

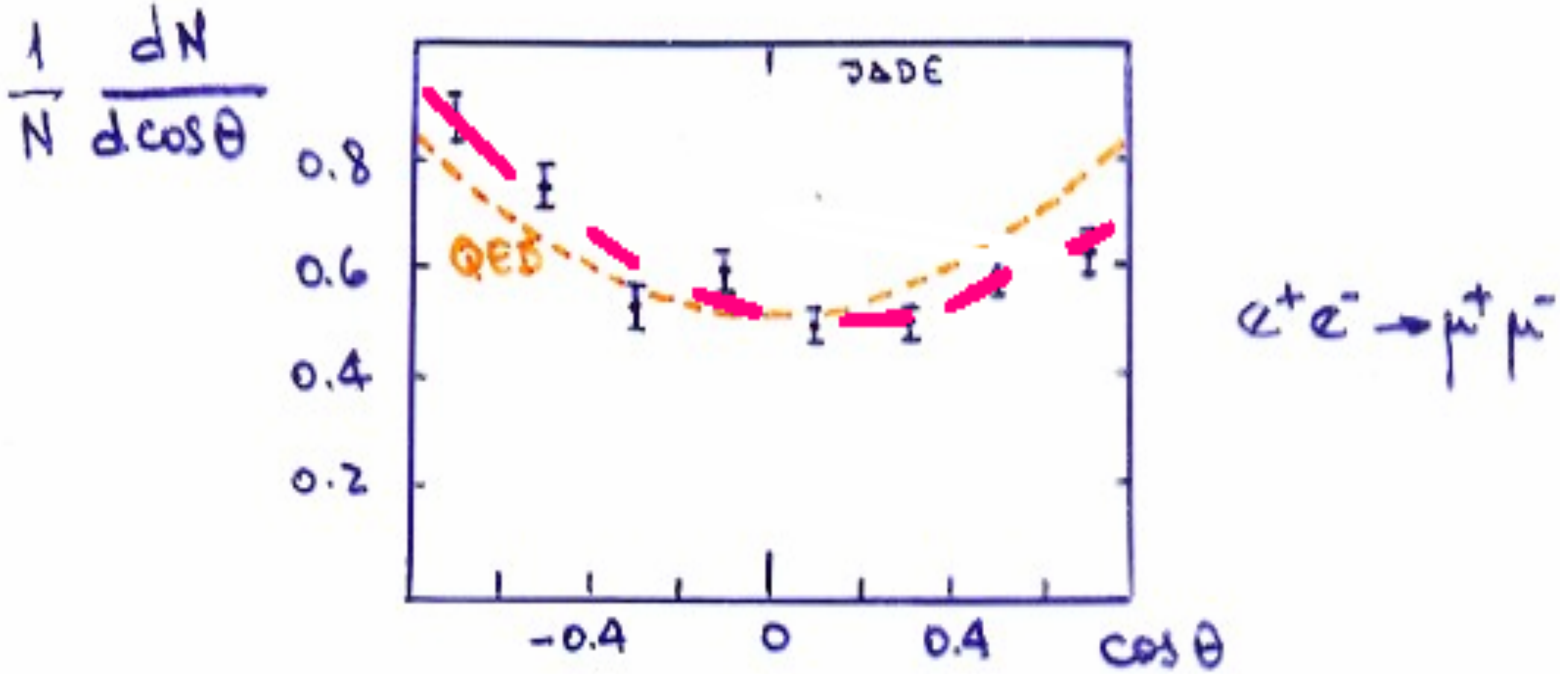


$e^+e^- \to \mu^+\mu^-$

INTERFERENCE γ-Z

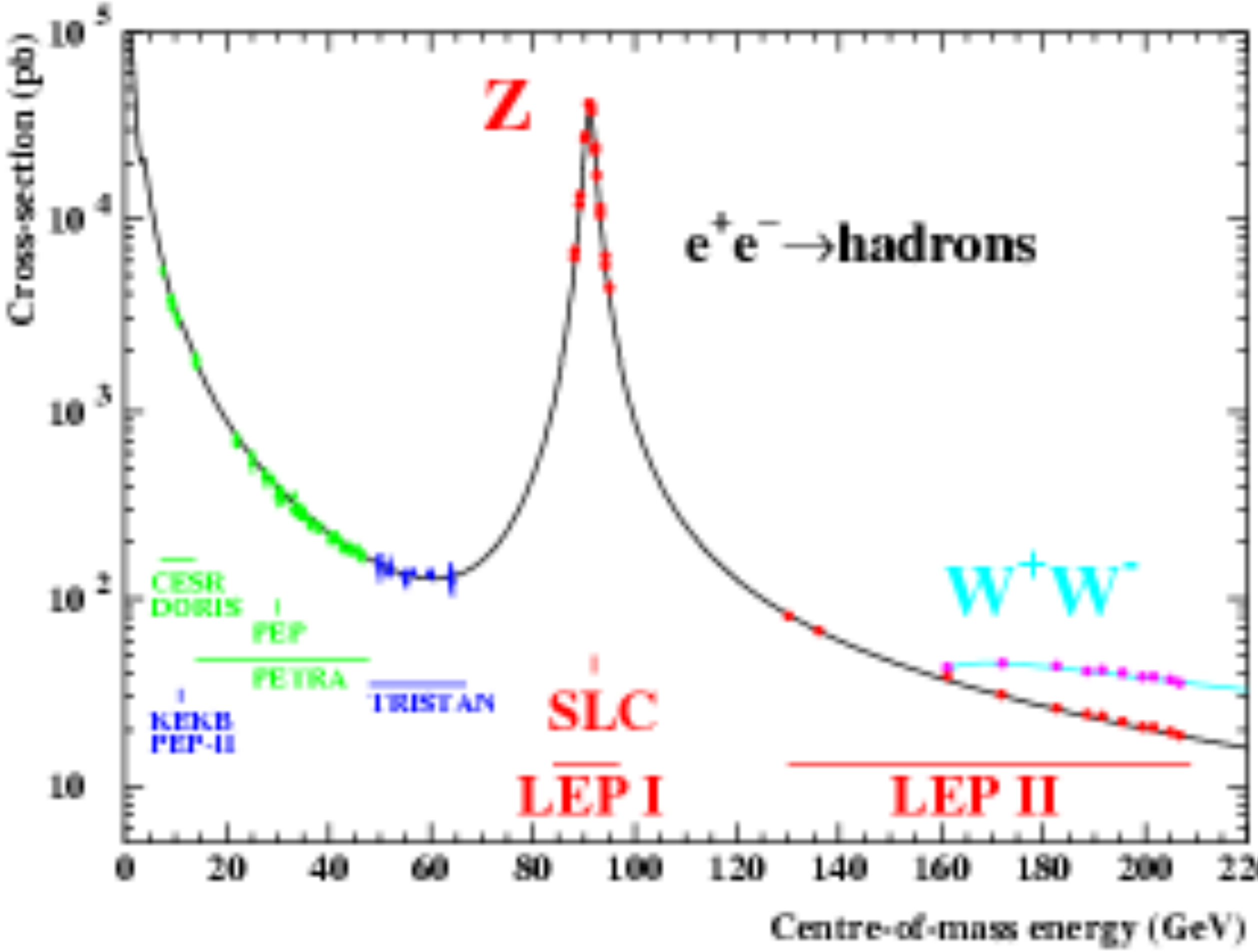


$$\frac{d\sigma}{d\Omega} = f(\cos^2\theta, \underline{\cos\theta}) \Rightarrow \underline{\text{ASYMMETRY}}$$



$\frac{1}{N} \frac{dN}{d\cos\theta}$
JADE
0.8
0.6
0.4
0.2
QED
-0.4
0
0.4
$\cos\theta$
$e^+e^- \to \mu^+\mu^-$

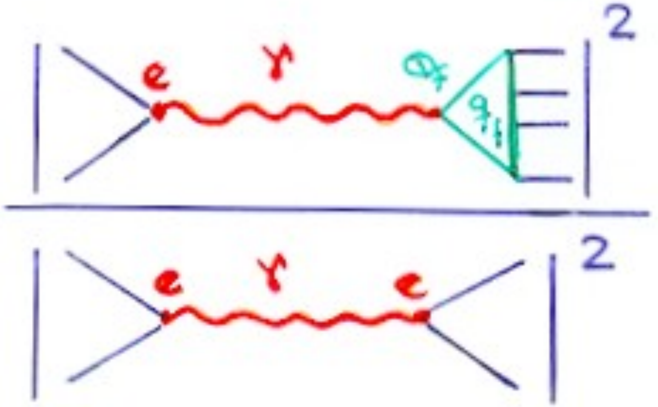

e+
e-
γ
f̄
f
e+
e-
Z
f̄
f


Cross-section (pb)
Z
e+e-→hadrons
CESR
DORIS
PEP
PETRA
TRISTAN
KEKB
PEP-II
SLC
LEP I
LEP II
W+W-
Centre-of-mass energy (GeV)

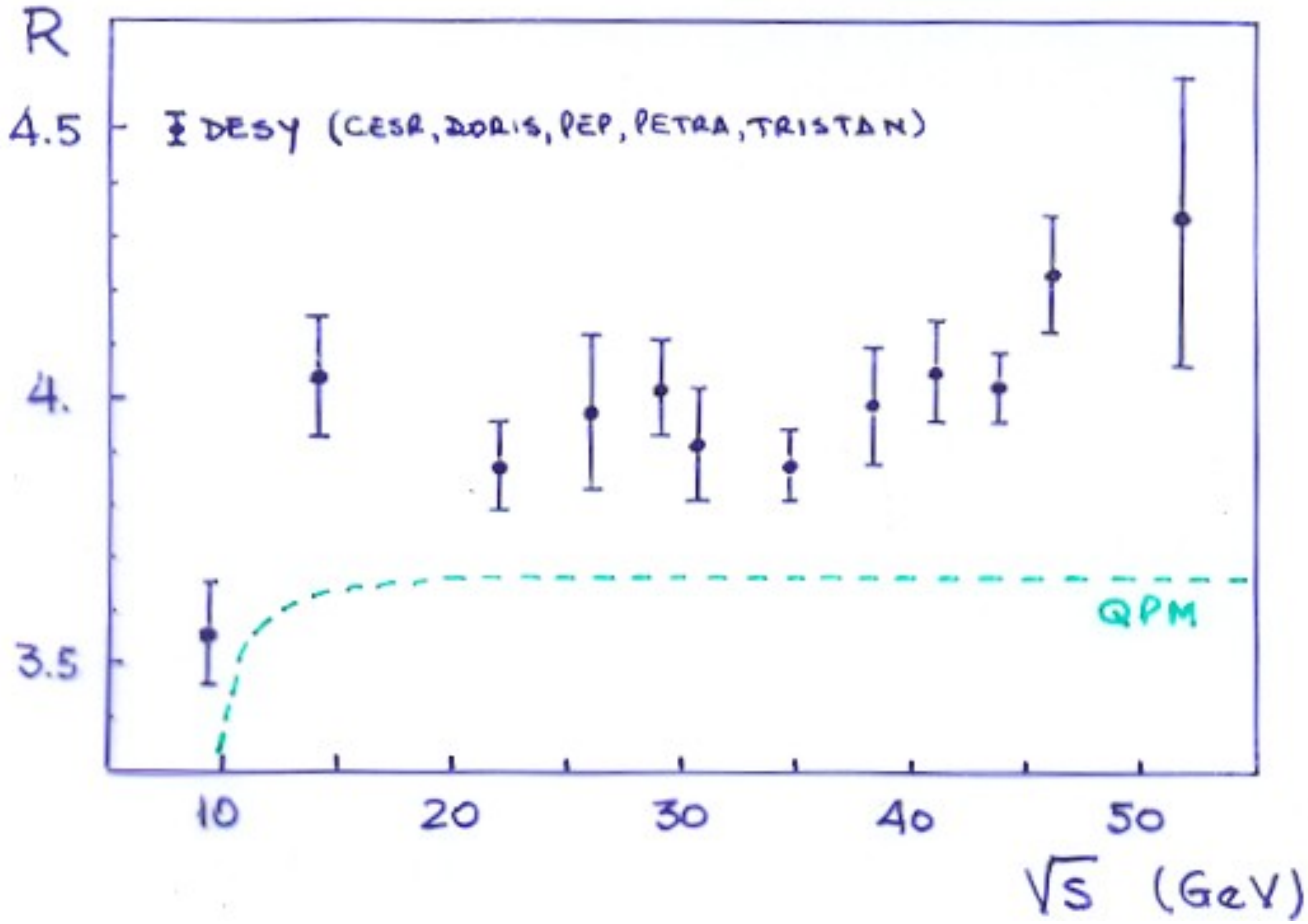
ii)

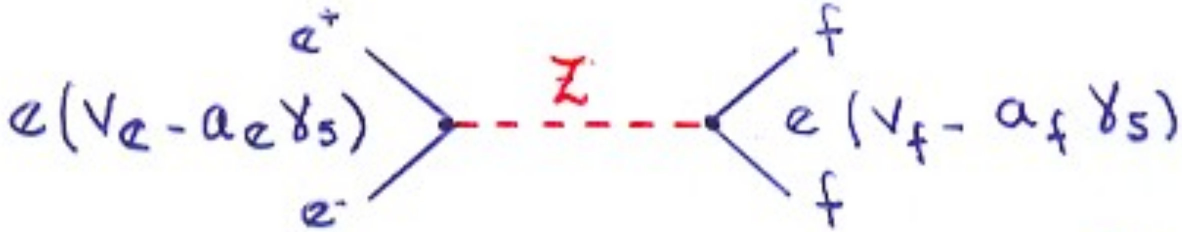
$$R = \frac{\sigma(e^+ + e^- \to \text{HADRONES})}{\sigma(e^+ + e^- \to \mu^+ + \mu^-)}$$

$R_{QPM} =$



$$R_{QPM} = 3 \sum_f Q_f^2$$

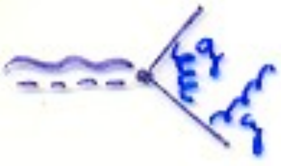

R
4.5
4.
3.5
DESY (CESR, DORIS, PEP, PETRA, TRISTAN)
QPM
10
20
30
40
50
√s (GeV)



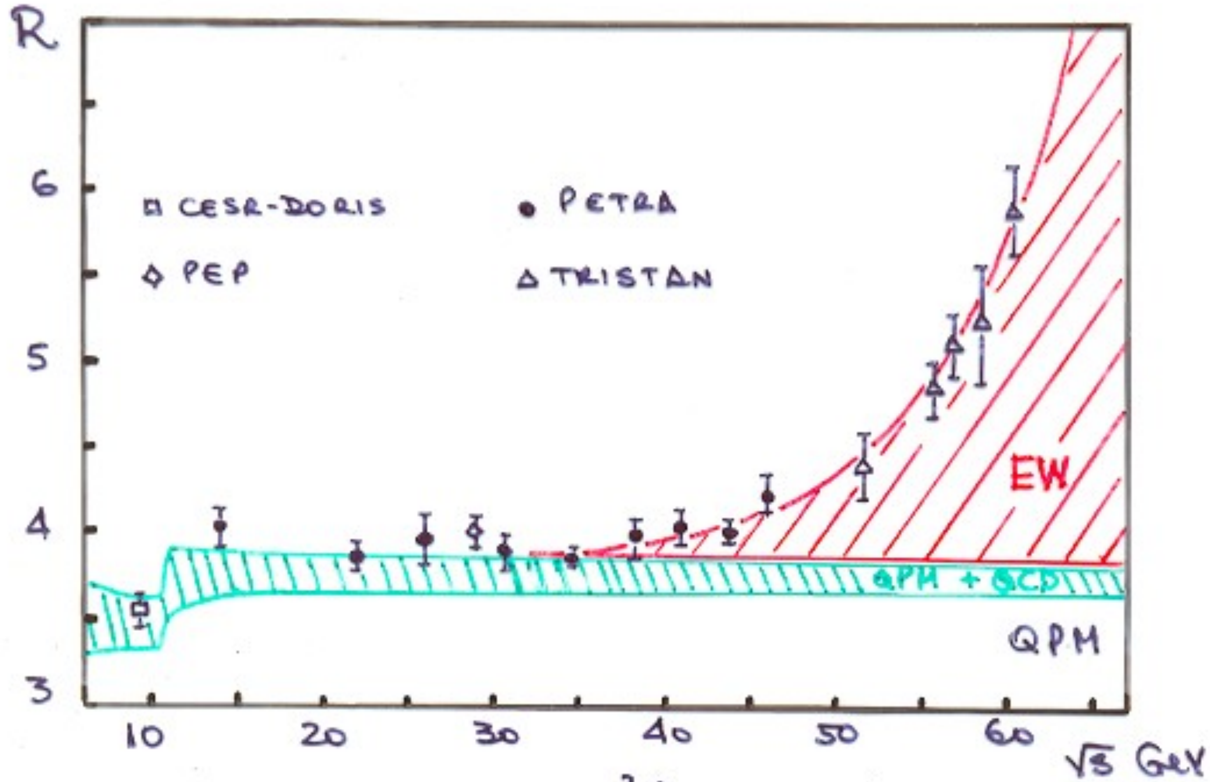
$$R = 3\sum_f Q_f^2 \left\{ 1 + C_1^V\left(\frac{\alpha_s}{\pi} + C_2^V\left(\frac{\alpha_s}{\pi}\right)^2\right) C_{VV} + [V \to A] \cdots \right\}$$

$$\begin{cases} C_{VV} = Q_f^2 - 2Q_f v_e v_f \,\mathrm{Re}(\chi) + (v_e^2 + a_e^2) v_f^2 |\chi|^2 \\ C_{AA} = (v_e^2 + a_e^2)\, a_f^2\, |\chi|^2 \\ \chi(s) = \dfrac{G_F}{8\sqrt{2}\pi\alpha} \dfrac{s\, m_Z^2}{s - m_Z^2 + i m_Z \Gamma_Z} \end{cases}$$

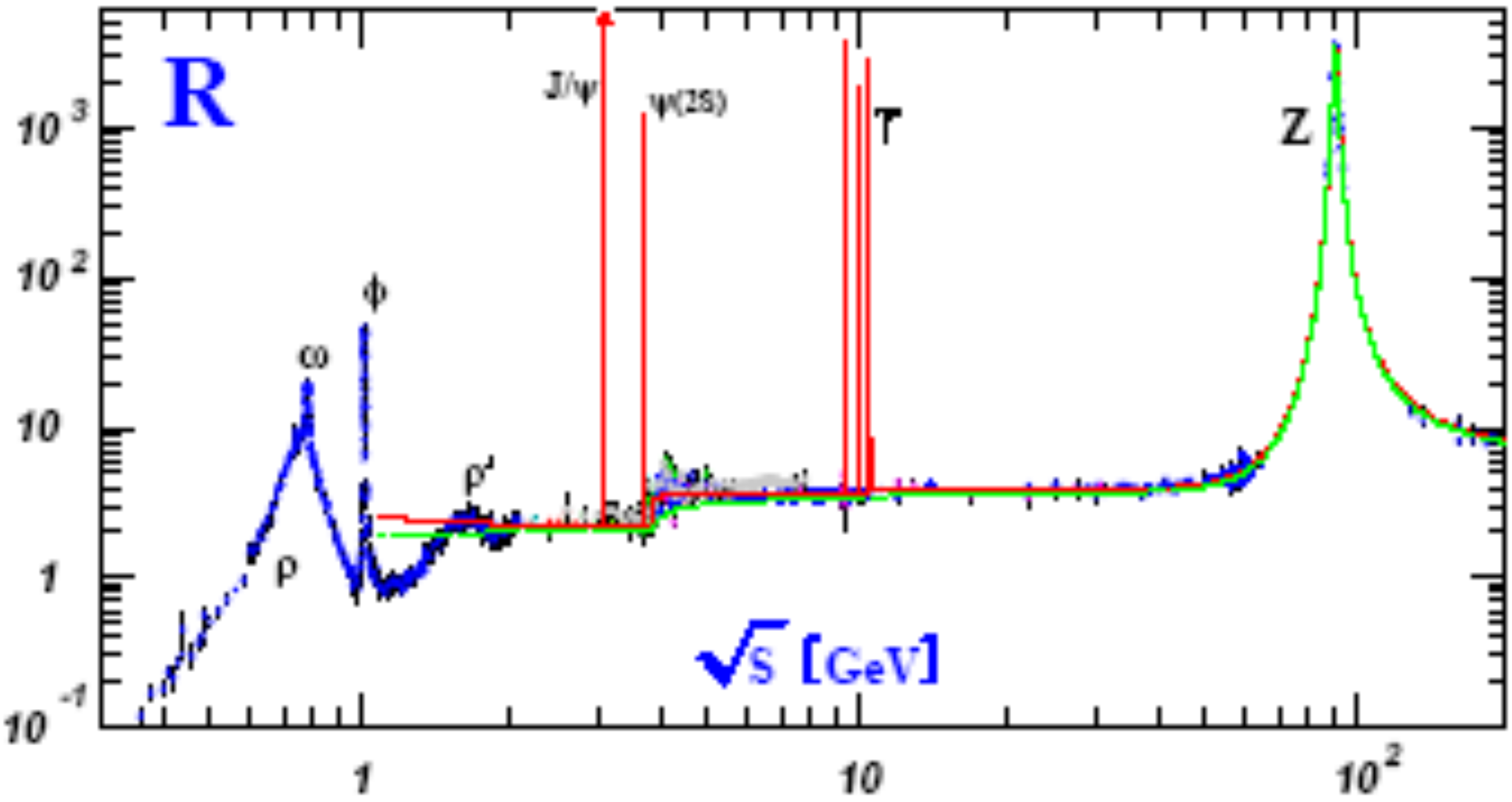
$$R_{QCD} = R_{QPM} \left\{ 1 + \underbrace{\frac{\alpha_s}{\pi}}_{\sim 5\%} + C_2 \underbrace{\left(\frac{\alpha_s}{\pi}\right)^2}_{\sim 0.4\%} + \cdots \right\}$$



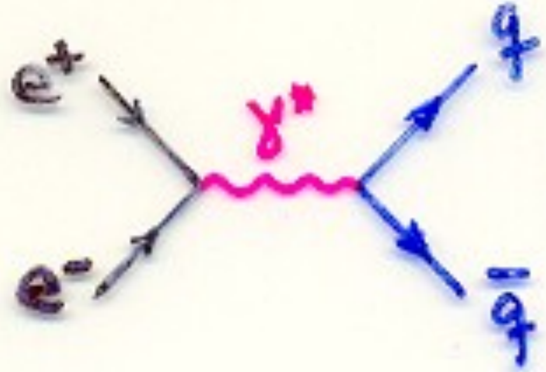
$$C_2(\overline{MS}) = 1.986 - 0.115\, N_f$$



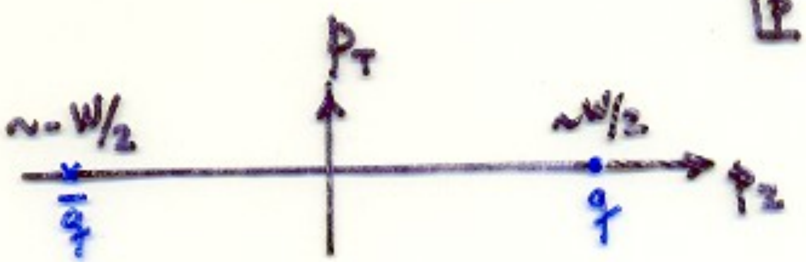
$\alpha_s = 0.142 \pm 0.018$ $\sin^2\theta_W = 0.220 \pm 0.025$

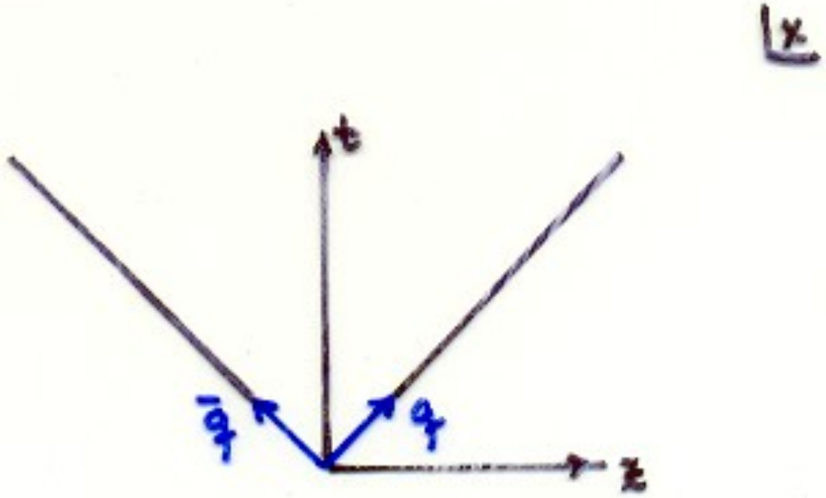

R
J/ψ
ψ(2S)
ϒ
Z
ϕ
ω
ρ'
ρ
√s [GeV]
10³
10²
10
1
10⁻¹
10⁻¹
1
10
10²

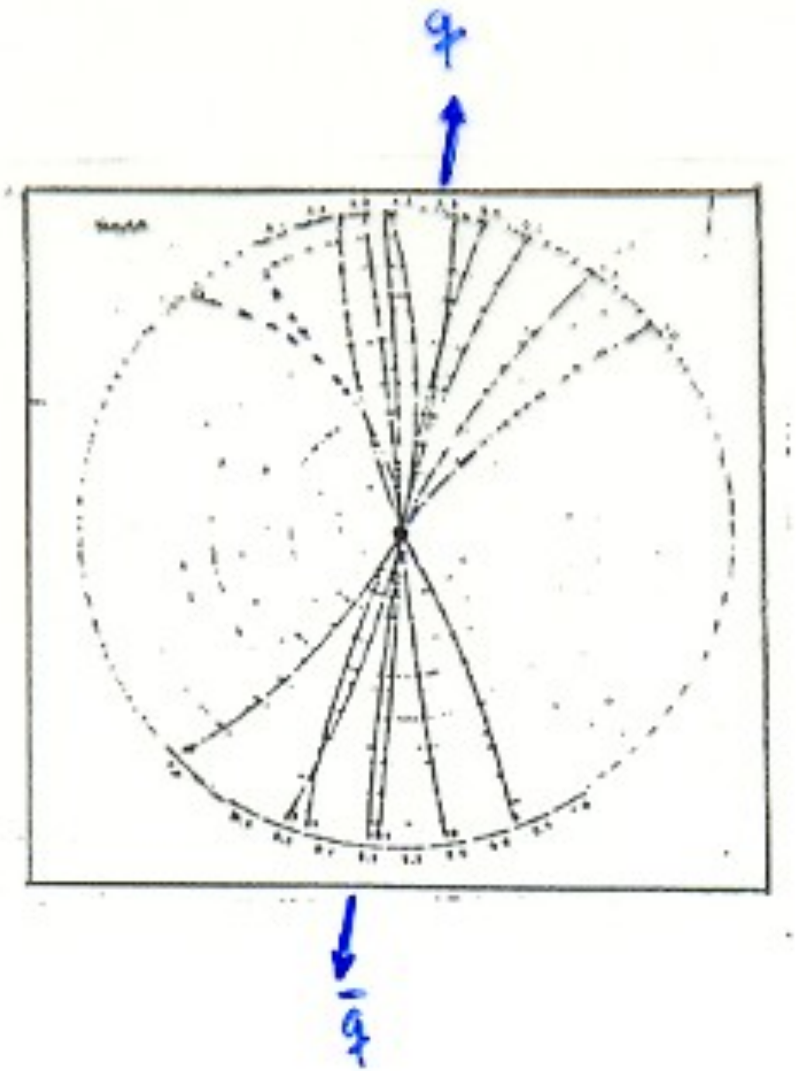
iii) # JETS

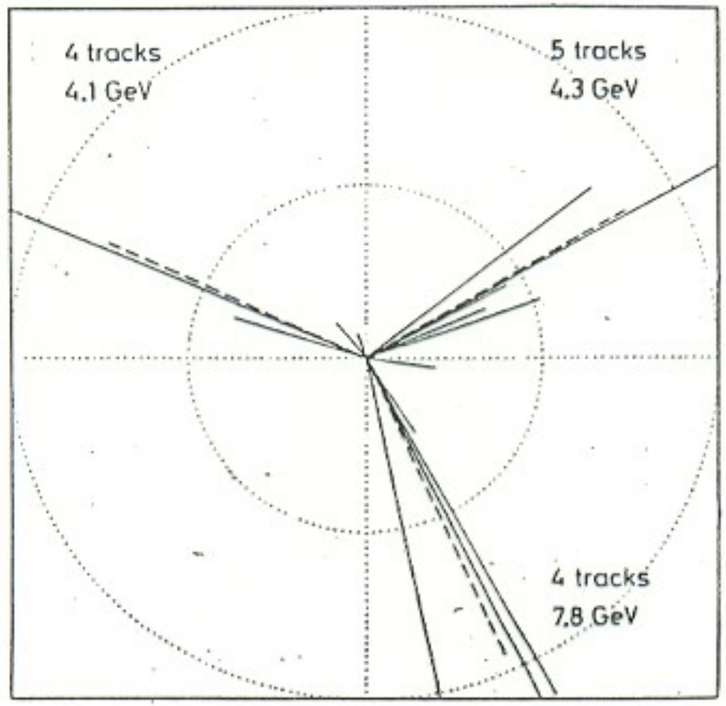


W = TOTAL ENERGY





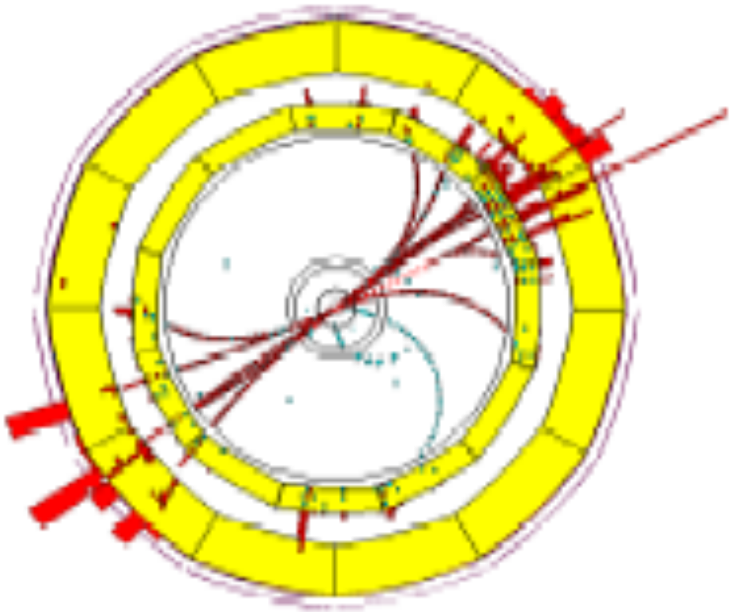




(TASSO)

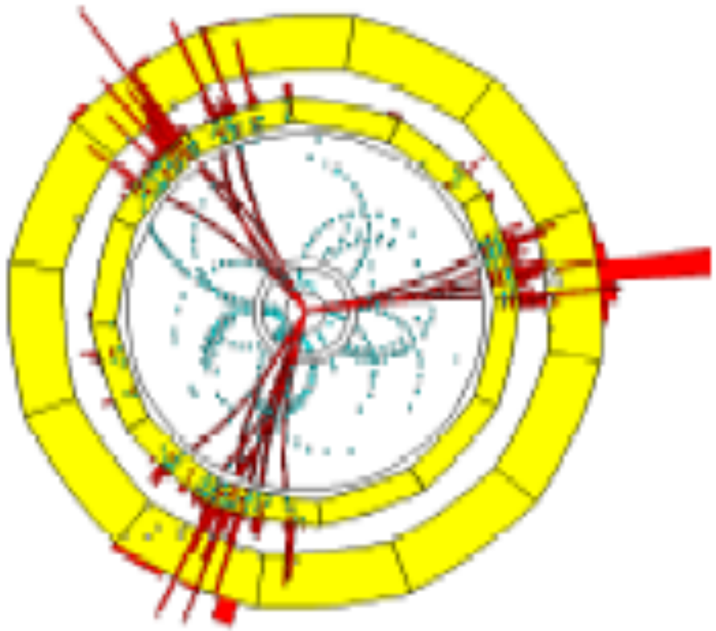
THIS THIRD JET IS PRESENT IN 10% OF THE FINAL STATES

ALEPH



$Z \longrightarrow q\,\bar{q}$

$Z \longrightarrow q\,\bar{q}\,G$



THE THIRD JET

PRESENT IN 10% OF THE FINAL STATES

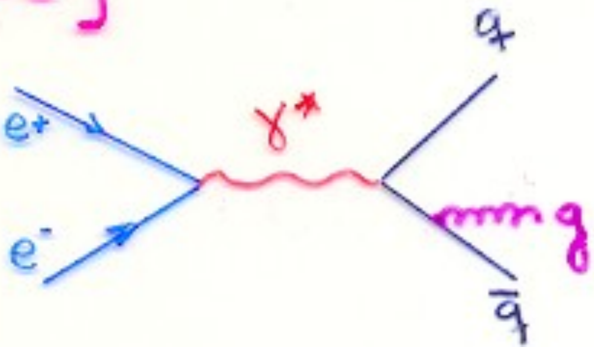
QCD:

- $$\frac{d^2\sigma}{dx_1\,dx_2}(3\,\text{jets}) = \frac{2\alpha_s}{3\pi}\,\sigma(2\,\text{jets})\,\frac{x_1^2+x_2^2}{(1-x_1)(1-x_2)}$$

$$\left[x_i = 2E_i/E \quad ; \quad x_1+x_2+x_3=2\right]$$

$\alpha_s(Q^2 \sim 25\,\text{GeV}^2) \simeq 0.1 - 0.2$

$\Downarrow$



GLUON SPIN

- VECTOR GLUON:

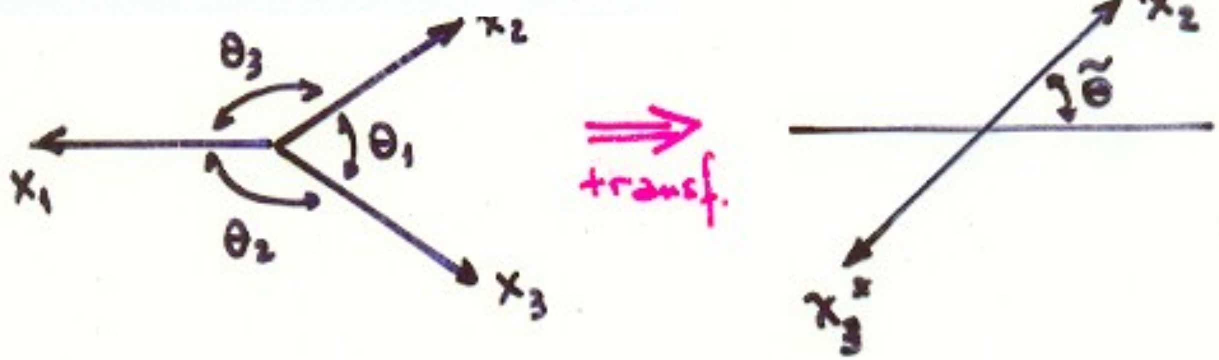
$$\frac{d^2\sigma}{dx_1\,dx_2} = \frac{2\alpha_s}{3\pi}\,\sigma_0 \left\{ \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)} + \text{p.c.} \right\}$$

- SCALAR GLUON:

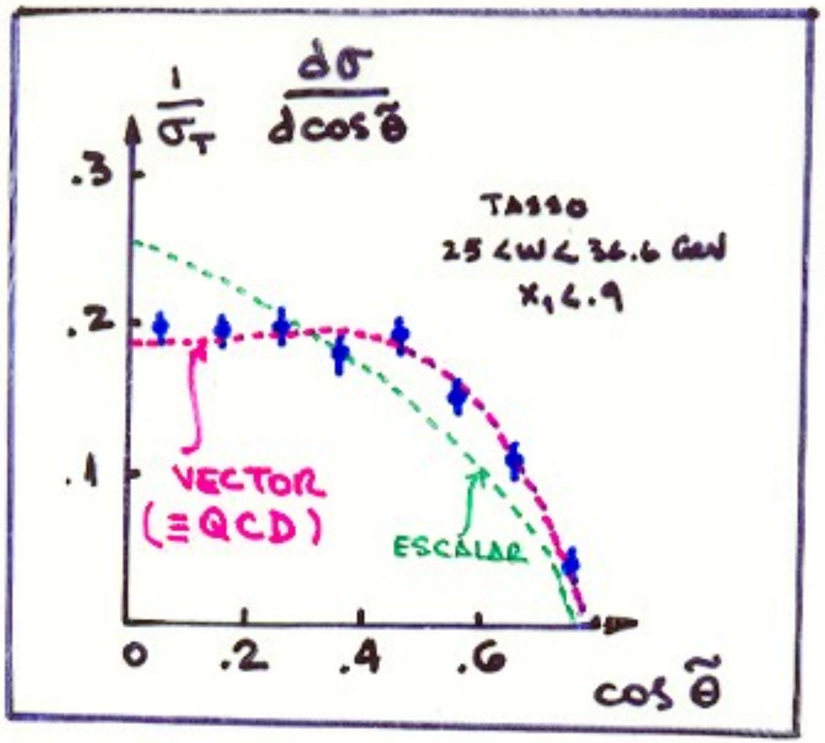
$$\frac{d^2\sigma}{dx_1\,dx_2} = \frac{\tilde{\alpha}_s}{3\pi}\,\sigma_0 \left\{ \frac{x_3^2}{(1-x_1)(1-x_2)} + \text{p.c.} \right\}$$

$$x_i = \frac{2\sin\theta_i}{\sin\theta_1 + \sin\theta_2 + \sin\theta_3}$$

X_1 REFERENCE FRAME



$$\cos\tilde{\theta} = \frac{x_2 - x_3}{x_1} = \frac{\sin\theta_2 - \sin\theta_3}{\sin\theta_1}$$



$s_g = 1$

iv)

$$M_W^{exp} = \begin{cases} 82.15 \pm 1.1 \pm 2.3 \text{ GeV} & \text{UA1/UA2} \\ 80.22 \pm 0.26 \text{ GeV} & \text{CDF} \end{cases}$$

$$M_Z^{exp} = \begin{cases} 92.75 \pm 1.4 \pm 2.3 \text{ GeV} & \text{UA1/UA2} \\ 91.173 \pm 0.020 \text{ GeV} & \text{LEP} \end{cases}$$

THEORY

$$M_Z^2 = \frac{M_W^2}{\cos^2\theta_W} \; ; \quad M_W^2 = \frac{\pi\alpha}{\sqrt{2}\, G_F \sin^2\theta_W}$$

$$\Downarrow$$

3σ

$$M_Z \simeq 88.5 \text{ GeV} \qquad M_W \simeq 77.5 \text{ GeV}$$

e.m. RADIATIVE CORRECTIONS

(QUANTUM)

$$M_W^2 = \frac{\pi \alpha}{\sqrt{2}\, G_F \sin^2\theta_W} \left[\frac{1}{1-\Delta r} \right] \qquad (M_Z^2 = \ldots)$$

$$\underline{\Delta r \simeq 0.07}$$

$$\Downarrow$$

$$\underline{M_W \quad (M_Z)} \qquad \checkmark \text{ o.k.}$$

WEAK INTERACTIONS

- RESPONSIBLE OF:
  - NUCLEAR β DECAY
  - HADRON DESINTEGRATIONS
  - EVERYTHING INDUCED BY NEUTRINOS

- CHARACTERIZED BY:
  - $G_F \simeq \dfrac{10^{-5}}{m_p} \simeq 1.4 \times 10^{-49}\ \mathrm{erg\ cm^3}$
  - TO VIOLATE $\mathbb{P}$

- SUGGESTED: (TO PAULI)

  THE EXISTENCE OF NEUTRINO

NEUTRINOS

— "I HAVE DONE A TERRIBLE THING.
I HAVE POSTULATED A PARTICLE THAT CANNOT
BE DETECTED" —

- W. PAULI

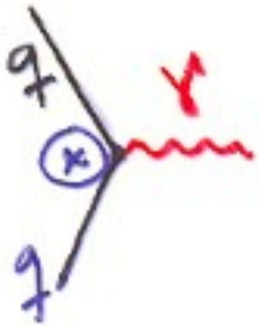
— "NEUTRINO IS THE MOST TINY QUANTITY OF
REALITY EVER IMAGINED BY A HUMAN BEING" —

- FREDERICK REINES
(NOBEL PRIZE 1995 : DETECCION $\bar{\nu}_e$)

PHENOMENOLOGICAL ANALYSIS

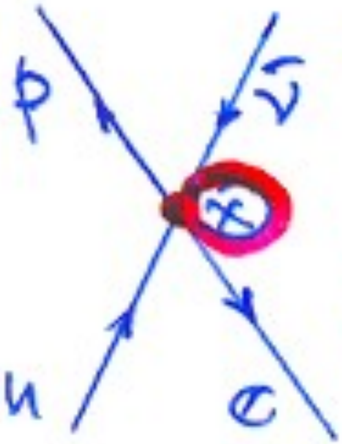
$$n \longrightarrow p + e + \bar{\nu}_e \qquad \bullet\; e \longrightarrow e + \gamma$$

FERMI



$$\bullet\; \mathcal{L}_{em} = e\, j^{\mu}(x)\, A_{\mu}(x) = e\, \bar{\psi}_q(x)\gamma^{\mu}\psi_q(x) A_{\mu}(x)$$

$\downarrow$



$$\bullet\; \mathcal{L}_F = G_F\, J^{\mu}_{np}(x) \cdot J_{\mu\, e\bar{\nu}}(x)$$

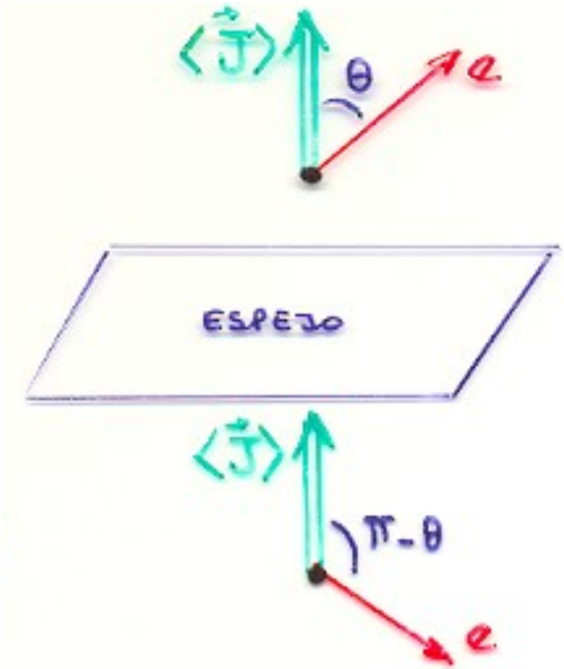
$$\bullet\; \mathcal{L}_F = G_F \left[\bar{\psi}_p(x)\gamma^{\mu}\psi_n(x)\right]\left[\bar{\psi}_e(x)\gamma^{\mu}\psi_{\nu_e}(x)\right]$$

— POINT INTERACTION —

$\cancel{P}$

ANGULAR DISTRIBUTION OF e (β) WITH RESPECT TO NUCLEUS SPIN

$$\bullet\ Co^{60} \longrightarrow Ni^{*60} + e^{-} + \bar{\nu}_e$$



$$\cos\theta = \frac{\vec{p}_e \cdot \vec{J}}{|\vec{p}_e|\, J}$$

$$\vec{J} \xrightarrow{P} \vec{J}$$

$$\vec{p}_e \xrightarrow{P} -\vec{p}_e$$

$$\Downarrow$$

$$\cos\theta \xrightarrow{P} -\cos\theta$$

$$W_e(\theta) = 1 + \alpha \cos\theta$$

EXPERIMENT: $\alpha = -0.4 \rightarrow \cancel{P}$

$\Downarrow$

- $\mathcal{L}_F[\gamma^\mu] \rightarrow$ • $\mathcal{L}_F[\gamma^\mu\gamma^5]$

* V-A INTERACTION

$$J^\mu_{(x)} = \bar{\psi}_f(x)\,\gamma^\mu(1-\gamma^5)\,\psi_{f'}(x)$$

(e) $\qquad$ (ν_e)

$$\underbrace{\underbrace{\underbrace{\gamma^\mu}_{V}\,(1-\gamma^5)}_{A}}_{V-A}$$

$\Downarrow$

NEUTRINOS ARE LEFT-HANDED !

~~FERMI~~

$\mathcal{L}_F$: PHENOMENOLOGICAL EFFECTIVE LAGRANGIAN •

- FIRST ORDER O.K.
- HIGHER ORDERS ~~O.K.~~

* NON-RENORMALIZABLE "THEORY"

* WHERE DIVERGENCES ARE HARMFUL ?

Ej: $\bar{\nu}_\mu + \mu^- \to \bar{\nu}_e + e^-$

- FIRST ORDER
  - $\sigma \sim 4\pi G_F^2 E^2$
- OPTICAL THEOREM (UNITARITY)
  - $\sigma_T \simeq \frac{1}{E^2} \operatorname{Im} A(\epsilon^2, 0)$
- NO SPIN
  - $A(\epsilon^2, 0) = 16\pi \sum_{l=0}^{\infty} (2l+1) P_l(\cos\theta)\, e^{i\delta_l(\epsilon^2)} \sin\delta_l(\epsilon^2)$
- ONLY S-WAVE
  - $\operatorname{Im} A(\epsilon^2, 0) \simeq 16\pi \sin^2\delta_0(\epsilon^2) \leq \underline{16\pi}$

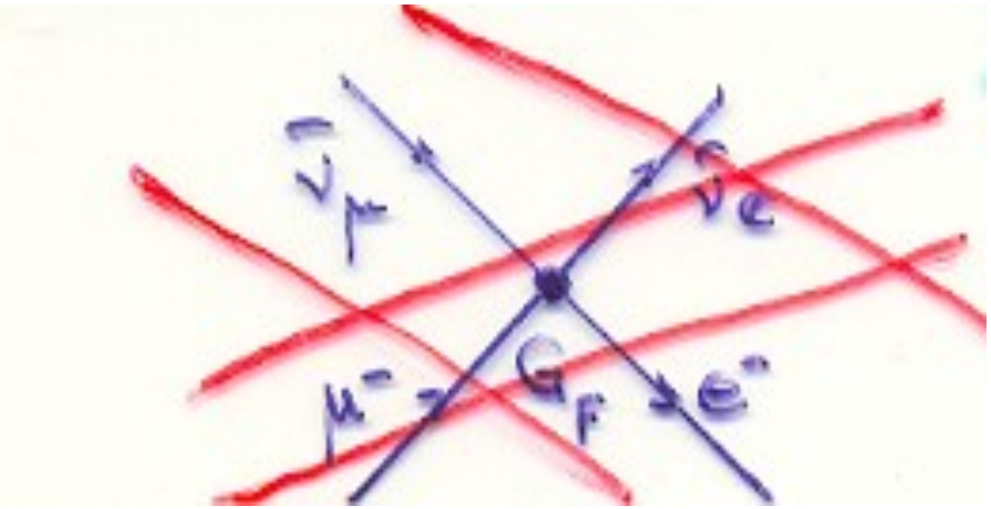
$\Downarrow$

$$\sigma_T \leq \frac{16\pi}{E^2}$$

$$4\pi G_F^2 E^2 \leq \sigma_T \leq \frac{16\pi}{E^2}$$

* FIRST ORDER VALID IF $E \leq 300$ GeV

UNITARITY LIMIT



~~4 FERMIONS MODEL~~

INSPIRATION:

QED

Ej: $e^+e^- \to \mu^+\mu^-$

- FIRST ORDER

$$\sigma = \frac{4\pi}{3}\,\frac{\alpha^2}{\underline{E^2}}$$

{ NO CONFLICT WITH UNITARITY ! }

- DIMENSIONAL ANALYSIS

$\hbar = c = 1 \quad \to \quad [M] = [L]^{-1}$

$[\mathcal{L}] = 4$

$[\psi_f] = 3/2$

$$\left\{ \begin{array}{l} [e] = 0 \\ [G_F] = -2 \end{array} \right\}$$

TO LOOK FOR A W.I. THEORY WITH

- ADIMENSIONAL COUPLING CONSTANT
- CARRIER OF THE INTERACTION

- $\mathcal{L}_F \longrightarrow$ • $\mathcal{L}_W = g\, J^{\lambda}(x)\, W_{\lambda}(x) + h.c.$

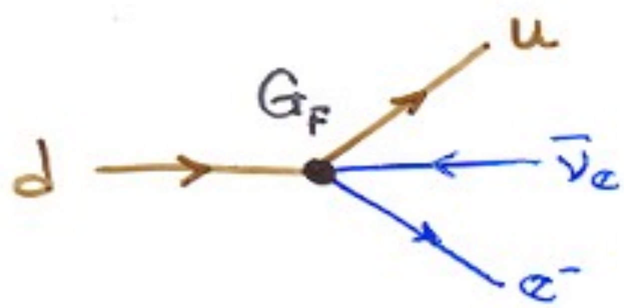
INTERMEDIATE BOSON $W^{\pm}$

INTERACTION CARRIER : CHARGED BOSON

FERMI THEORY : EFFECTIVE AT LOW ENERGY

$$\left(q^2 \ll M_W^2 \right)$$

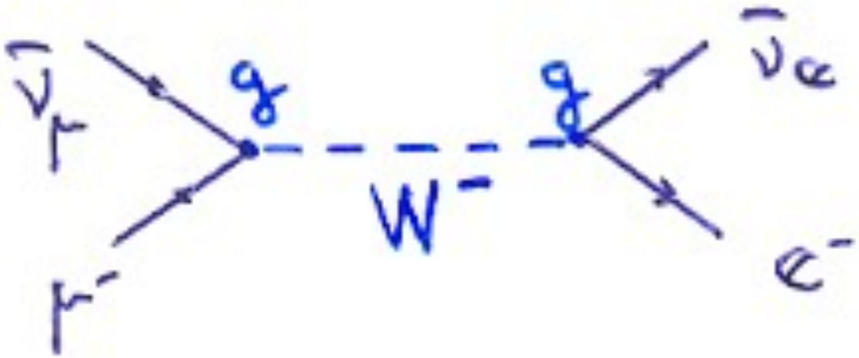
β-DECAY



$n \to p + e^- + \bar{\nu}_e$

~~W~~

Ej: $\bar{\nu}_\mu + \mu^- \rightarrow \bar{\nu}_e + e^-$



$$g\,\bar{\psi}_e \gamma_\lambda (1-\gamma_5)\,\psi_{\nu_e} W^\lambda \cdot g\,\bar{\psi}_{\nu_\mu} \gamma_\lambda (1-\gamma_5)\,\psi_\mu W^\lambda$$

$[\psi_f] = \frac{3}{2}$; $[W] = 1 \Rightarrow \underline{[g] = 0}$

SHORT RANGE OF W.I $\rightarrow$

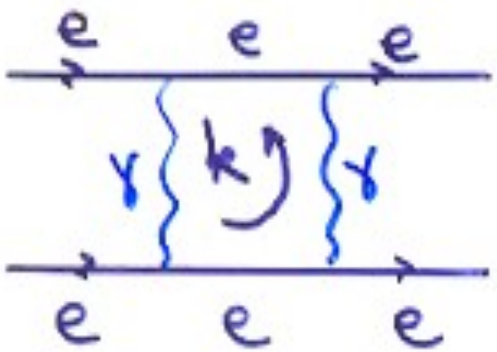
$$\underline{m_W \neq 0}$$

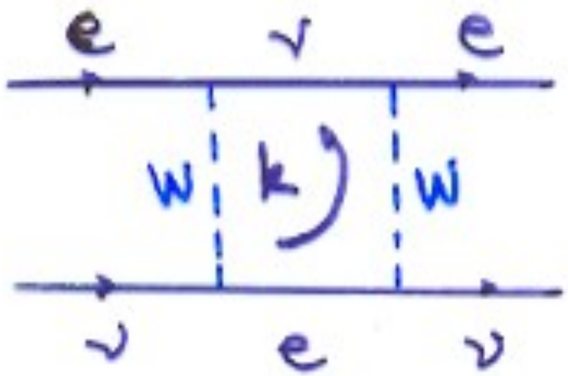
$\downarrow$

W-PROPAGATOR $\left\{ \quad \dfrac{-g_{\mu\nu} + \dfrac{p_\mu p_\nu}{m_W^2}}{p^2 - m_W^2} \right.$

LONGITUDINAL POLARIZATION

$$\xrightarrow[p \to \infty]{} \text{CONSTANT}$$





$k \to \infty$

$\int \frac{d^4k}{k^6} \to$ FINITE

QED

$\int \frac{d^4k}{k^2} \to$ DIVERGES

$\cancel{W}$

- SHORT RANGE → $M_W = 80.4 \pm 0.1$ GeV (exp.)

  ⁂ RANGE $\sim \frac{1}{M_W} \sim 0.002$ fm ⁂

- ELECTRIC CHARGE OF $W^{\pm}$ →

  "CHARGED CURRENT"

- CHANGES FLAVOUR

  (ONLY CHARGED CURRENTS !)

- VIOLATES P

QUARK-FLAVOR MIXING

$n \to p + e + \bar{\nu}_e \Rightarrow$

CHARGED (V-A) CURRENTS CONSTRUCTED FROM $\begin{pmatrix} u \\ d \end{pmatrix}_L$

LEPTONS $\begin{matrix} (\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu) \\ (\mu^- \to e^- + \bar{\nu}_e + \nu_\mu) \end{matrix} \Rightarrow \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L ; \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L$

ALL COUPLE WITH $G_F \equiv$ UNIVERSALITY

$\Downarrow$

* NATURAL TO TRY TO EXTEND TO $\begin{pmatrix} c \\ s \end{pmatrix}_L$

BUT : NOT CORRECT !

EX: $K^+ \to \mu^+ \nu_\mu$: OCCURS

$(u\bar{s})$

$\Rightarrow$ MUST BE A WEAK CURRENT COUPLING $\underline{u}$ TO $\underline{\bar{s}}$

BUT ABOVE SCHEME : ONLY $u \leftrightarrow d$

$c \leftrightarrow s$

$\Rightarrow$ TO INTRODUCE NEW COUPLINGS

OR

MODIFY QUARK DOUBLETS !

→ "ROTATED" QUARK STATES :

$$\begin{pmatrix} u \\ d' \end{pmatrix} ; \begin{pmatrix} c \\ s' \end{pmatrix} ; \dots \quad \begin{cases} d' = d\cos\theta_c + s\sin\theta_c \\ s' = -d\sin\theta_c + s\cos\theta_c \end{cases}$$

θ_c : QUARK MIXING ANGLE ≡ CABIBBO ANGLE

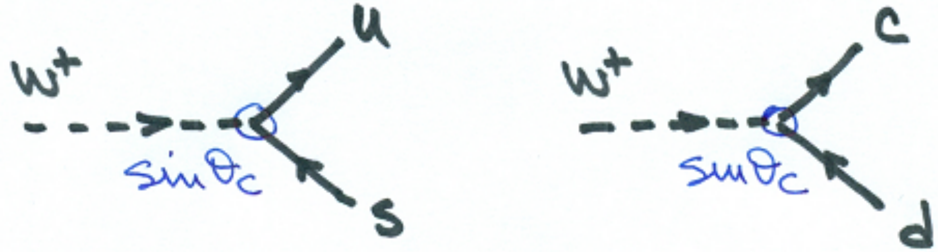
EX :

$$\begin{array}{l} \Delta S = 1 : \\ \Delta S = 0 : \end{array} \quad \frac{\Gamma(K^+ \to \mu^+ \nu_\mu)}{\Gamma(\pi^+ \to \mu^+ \nu_\mu)} \sim \sin^2\theta_c$$

$$\frac{\Delta S = 1}{\Delta S = 0} \sim \frac{1}{20} \Rightarrow \sin\theta_c = 0.2255$$

CABIBBO FAVOURED:

CABIBBO SUPPRESSED:



CHARGED CURRENT: $$J^\mu = (\bar{u}\ \bar{c})\,\frac{\gamma^\mu(1-\gamma^5)}{2}\,U\begin{pmatrix} d \\ s \end{pmatrix}$$

- $$U = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix}$$

3 DOUBLETS :

$$u \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$U = \begin{pmatrix} U_{ud} & U_{us} & U_{ub} \\ U_{cd} & U_{cs} & U_{cb} \\ U_{td} & U_{ts} & U_{tb} \end{pmatrix} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

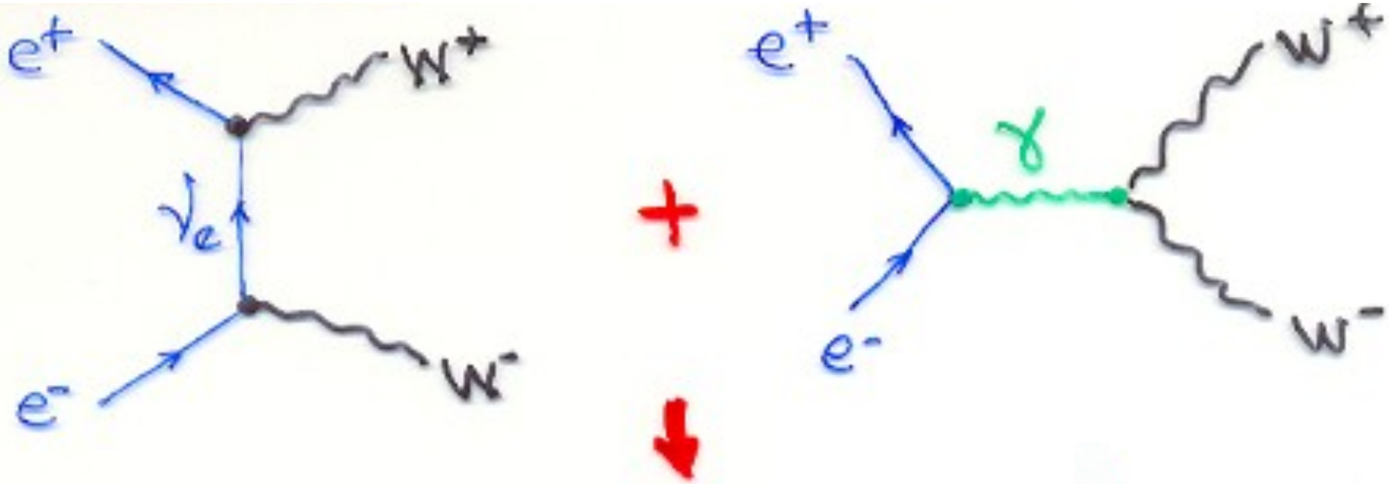
$$\lambda = \sin\theta_c$$

ELECTROMAGNETIC - WEAK "UNIFICATION"

* ELECTROWEAK *

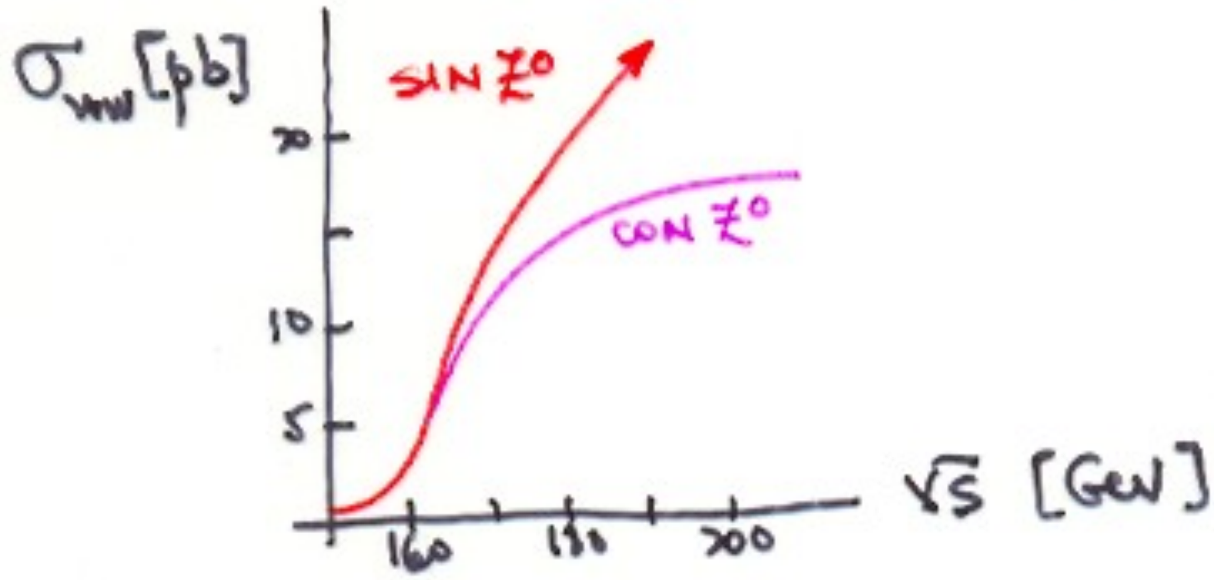
$W^{\pm}$ COUPLE TO PHOTON γ

$e^+ e^- \to W^+ W^-$

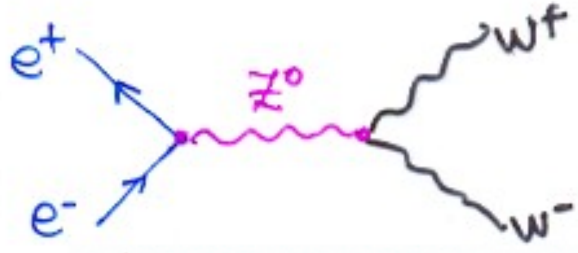


* CROSS-SECTION DIVERGES ! *

BUT

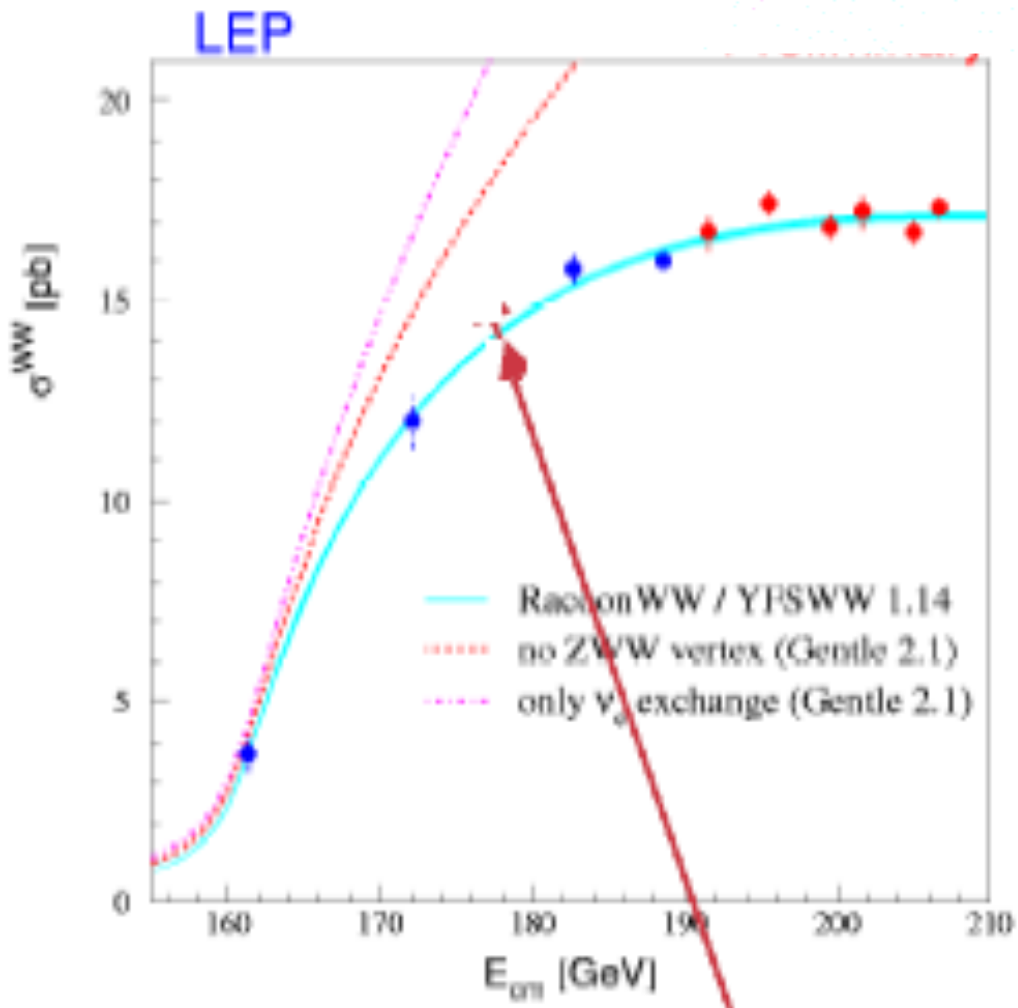


* ANOTHER CONTRIBUTION NEEDED



• **Z^0 CURED!**

INDISPENSABLE: γ, $W^\pm$, Z^0 ! → ELECTROWEAK



ZWW vertex exists!

• SHORT RANGE → $M_W = 80.4 \pm 0.1$ GeV (exp.)

※ RANGE $\sim \frac{1}{M_W} \sim 0.002$ fm ※

$$m_{\text{GAUGE FIELD}} \equiv 0$$

$\mathcal{L}_m = \frac{1}{2} m W^\mu W_\mu$ NOT ALLOWED!

?

LATENT SYMMETRY

SYMMETRY REALIZED À LA NAMBU-GOLDSTONE

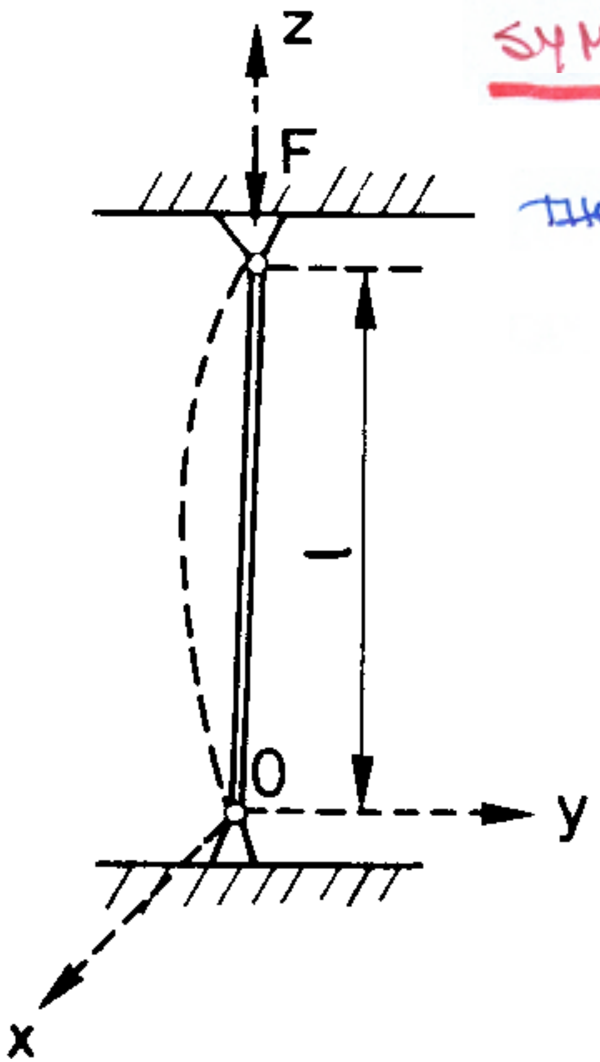
SYMMETRY PRESENTS IN THE LAGRANGIAN BUT NOT RESPECTED BY VACUUM EXPECTATION VALUES OF THE FIELDS

↓

NON TRIVIAL CONSEQUENCES IN SYSTEMS OF INFINITE EXTENSION

THE BENT ROD

CYLINDRICAL ROAD CHARGED



SYMMETRIC UNDER ROTATIONS AROUND Z-AXIS

THE SYSTEM HAS THE SYMMETRIC SOLUTION

$$X = Y = 0$$

BUT

IF $F > F_{CR} \left(= \frac{\pi^2 E I}{l^2} \right)$

→ **ASYMMETRIC SOLUTION**

(THE ROD IS BENT)

SYMMETRY OF EQUATIONS OF MOTION

BUT ASYMMETRIC SOLUTION

IN WHICH DIRECTION OF THE (X,Y) PLANE THE ROD IS GOING TO BEND?

IT CANNOT BE PREDICTED

A SYMMETRY TRANSFORMATION TO AN ASYMMETRIC SOLUTION $\Rightarrow$ ANOTHER ASYMMETRIC SOLUTION

* THE GROUND STATE (THE VACUUM) IS <u>DEGENERATE</u>

- THE SYMMETRY IS <u>LATENT</u> (<u>HIDDEN</u>)

<u>SPONTANEOUSLY BROKEN</u>

REALIZED À LA <u>NAMBU GOLDSTONE</u>

EX: HEISENBERG (EXCHANGE INTERACTION)

$$H = -K \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

- O(3) INVARIANT $(\vec{S}_i \to R\,\vec{S}_i)$
- GROUND STATE: $T < T_c$ (CRITICAL POINT)

NON SYMMETRIC

$\Downarrow$

SPINS ALIGNED

O(3) BROKEN !

$\Downarrow$

LATENT SYMMETRY: NO PREFERRED DIRECTION FOR ALIGNEMENT

EACH DIRECTION CONNECTED WITH ANY OTHER BY O(3) (DEGENERACY: ∞, J)

GOLDSTONE MODES: SPIN WAVES

CONTINUOUS GLOBAL LATENT SYMMETRY

- GOLDSTONE MODEL

$$\mathcal{L} = -\partial_\mu \phi^\dagger(x)\, \partial^\mu \phi(x) - \lambda \left[\phi^\dagger(x)\, \phi(x) - v \right]^2$$

- $\phi(x)$: COMPLEX SCALAR FIELD
- $\mathcal{L}$ INVARIANT $U(1)$ GLOBAL

$$\left\{ \phi(x) \rightarrow \phi'(x) = e^{i\alpha} \phi(x) \qquad \alpha = \text{CONSTANT} \right\}$$

- POTENTIAL :

$$V(\phi^\dagger \phi) = \lambda \left[\phi^\dagger \phi - v \right]^2$$

DEPENDS ON THE SIGN OF v

- CASE i) $v < 0$

– ONLY ONE MINIMUM AT $\phi = 0$ –

$\Downarrow$

WIGNER-WEYL REALIZATION OF $U(1)$

- CASE ii) $v > 0$

– INFINITE NUMBER OF MINIMA –

$\Downarrow$

NAMBU-GOLDSTONE REALIZATION OF $U(1)$

- CASE ii) $v > 0$

- MINIMA DEFINED BY $\phi^\dagger \phi = v$

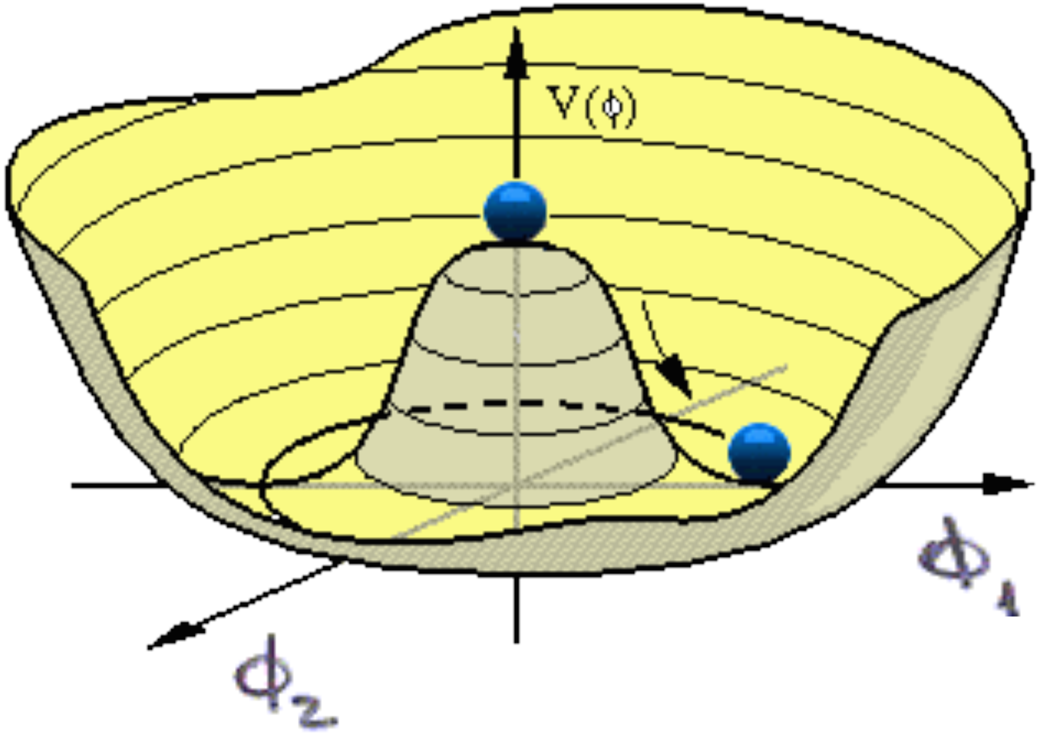
$$\phi_1^2 + \phi_2^2 = v$$

FAMOUS

"MEXICAN HAT"

OR

"CULO DE BOTELLA"



- LATENT SYMMETRY $\Rightarrow$ GOLDSTONE BOSON -

($m_G = 0$)

* NO SENSE TO DEVELOP AROUND $\phi = 0$

$\Downarrow$

PERTURBATION FROM

$$\underline{\phi = \sqrt{v}\, e^{i\theta}}$$

* θ: ARBITRARY (NON UNICITY OF VACUUM) *

($\theta = 0$)

→ DEFINE : $\underline{\phi(x) = \sqrt{v} + \chi(x)}$: $\underline{\langle 0 | \chi(x) | 0 \rangle = 0}$

$$V = \lambda v\,(\chi + \chi^{\dagger})^2 + 2\lambda\sqrt{v}\,\chi^{\dagger}\chi\,(\chi + \chi^{\dagger}) + \lambda(\chi\chi^{\dagger})^2$$

$$\chi_1 = \frac{1}{\sqrt{2}}(\chi + \chi^{\dagger}) \quad ; \quad \chi_2 = \frac{i}{\sqrt{2}}(\chi^{\dagger} - \chi)$$

$$\Downarrow$$

χ_1 MASSIVE $\longrightarrow$ $m_{\chi_1} = 4\lambda v$

(χ_2 MASSLESS GOLDSTONE BOSON) $\longrightarrow$ $m_{\chi_2} = 0$

$$\Downarrow$$

{ U(1) SYMMETRY IS NOT PRESENT IN THE SPECTRUM }

CHANGE: $\begin{cases} \chi_1 = \rho \cos\theta \\ \chi_2 = \rho \sin\theta \end{cases}$

$$U(1) \Rightarrow \quad \rho \to \rho \quad ; \quad \theta \to \theta + \alpha$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \rho)^2 + \frac{\rho^2}{2}(\partial_\mu \theta)^2 - V(\rho)$$

DISPLACE: $\rho' = \rho - \sqrt{v} \quad ; \quad \theta' = \theta$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \rho')^2 + \frac{1}{2}(\rho' + \sqrt{v})^2(\partial_\mu \theta)^2 - V(\rho' + \sqrt{v})$$

ρ: RADIAL EXCITATION

θ: MOVEMENT AROUND THE CIRCLE ($\omega = 0$) GOLDSTONE BOSON ($m=0$)

LOCAL LATENT SYMMETRY → HIGGS

LATENT GLOBAL SYMMETRY ⇒ LOCAL (GAUGE)

↓

$$\boxed{\begin{array}{l}\text{GOLDSTONE BOSONS DISAPPEAR}\\ \text{GAUGE FIELDS ACQUIRE MASS}\end{array}}$$

HIGGS MECHANISM

GOLDSTONE BOSONS DEGREES OF FREEDOM

TRANSFERRED TO

LONGITUDINAL POLARIZATION OF GAUGE FIELDS

$(m \neq 0)$

EX : U(1)

$$\mathcal{L} = -\partial_\mu \phi^\dagger \partial^\mu \phi - \lambda(\phi^\dagger\phi - v)^2$$

$v > 0 \Rightarrow$ LATENT SYMMETRY

NOW MAKE U(1) LOCAL $\Rightarrow$

$$\partial_\mu \to \partial_\mu - i g\, a_\mu(x) \equiv D_\mu$$

$a_\mu(x)$: MASSLESS GAUGE FIELD

$$\mathcal{L} = -(D_\mu \phi)^\dagger (D^\mu \phi) - \lambda(\phi^\dagger\phi - v)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

U(1) (LOCAL) →

- $\phi(x) \rightarrow e^{i\alpha(x)}\, \phi(x)$
- $a_\mu(x) \rightarrow a_\mu(x) + \frac{1}{g} \partial_\mu\, \alpha(x)$

⇓ INTERACTIONS :

- $\mathcal{L}_I^{(1)} = g\, a_\mu(x)\, J^\mu(x)$
- $\mathcal{L}_I^{(2)} = -g^2\, a_\mu(x)\, a^\mu(x)\, \phi^\dagger(x)\, \phi(x)$

LATENT SYMMETRY → $\langle 0|\phi|0\rangle = \sqrt{v} \neq 0$

$$\left[\phi(x) = \sqrt{v} + \chi(x)\right] \qquad \langle 0 | \chi(x) | 0 \rangle = 0$$

$\Downarrow$ (HIGGS MECHANISM)

$$\left\{ \begin{aligned} \mathcal{L}_M &= -g^2 v \, a^\mu a_\mu \\ &= -\frac{1}{2} m_a^2 \, a^\mu a_\mu \end{aligned} \right\}$$

GAUGE FIELD $a_\mu(x)$ ACQUIRE MASS : $m_a = g\sqrt{\frac{v}{2}}$

WITH EXPLICIT CONSERVATION OF DEGREES OF FREEDOM

CONSERVATION OF DEGREES OF FREEDOM

$$(2+2) \longrightarrow (3+1)$$

EXPONENTIAL PARAMETRIZATION OF $\phi(x)$:

$$\phi(x) = \left[v + \rho(x)\right] e^{i\sigma(x)/v}$$

$\left.\begin{array}{l}\rho(x)\\ \sigma(x)\end{array}\right\}$ REAL FIELDS

$\sigma(x)$: GOLDSTONE MODE

$$V(\phi^{+}\phi) \rightarrow V = \lambda\left(\rho^{2} + 2\rho\sqrt{v}\right)^{2}$$

$$\Downarrow$$

$$m_{\rho}^{2} = 8\lambda v$$

- AND $\sigma(x)$? :

- $$D_\mu \phi = e^{i\sigma/\sqrt{v}}\left[\partial_\mu \rho - i(\rho+\sqrt{v})\, g\left(a_\mu + \frac{1}{g\sqrt{v}}\partial_\mu \sigma\right)\right]$$

→
- $e^{i\sigma/\sqrt{v}}$ DISSAPEARS FROM $(D_\mu \phi)^\dagger D^\mu \phi$
- $a_\mu + \frac{1}{g\sqrt{v}}\partial_\mu \sigma$: GAUGE TRANSFORMED OF $a_\mu(x)$

  $(\alpha(x) = \sigma(x)/\sqrt{v})$

→ FIXING THE GAUGE

- $$A_\mu(x) = a_\mu(x) + \frac{1}{g\sqrt{v}}\partial_\mu \sigma(x)$$

$$\mathcal{L} = -\frac{1}{2}\partial_\mu \rho\, \partial^\mu \rho - \frac{1}{2} m_\rho^2 \rho^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$
$$- \frac{1}{2} m_a^2 A_\mu A^\mu - g^2 (\rho^2 + 2\sqrt{v}\,\rho) A_\mu A^\mu$$
$$- \lambda (\rho^4 + 2\sqrt{v}\,\rho^3)$$

- THE GOLDSTONE FIELD $\sigma(x)$ WAS GAUGED AWAY
- CLEARLY 4 DEGREES OF FREEDOM

[EXCHANGE OF DEGREES OF FREEDOM IS NOT A VIOLATION OF GOLDSTONE THEOREM : IT CANNOT BE APPLIED WHEN LONG RANGE FORCES ARE PRESENT (ONES DUE TO GAUGE FIELDS)]

PROPAGATOR OF A_μ : $D_{\mu\nu} = \left(g_{\mu\nu} + \frac{k_\mu k_\nu}{m_a^2} \right) \frac{1}{k^2 + m_a^2}$

$\Downarrow$

THEORY NON-RENORMALIZABLE ?

NO PROBLEM

$\mathcal{L}(\rho, A_\mu)$ GAUGE EQUIVALENT $\mathcal{L}(\phi^\dagger\phi, a_\mu)$ RENORMALIZABLE

$\downarrow$ PRACTICAL USE

$\downarrow$ FORMAL USE

YANG-MILLS THEORY WITH LATENT SYMMETRY : RENORMALIZABLE

SEE YOU AT 3