Neutrino oscillation physics

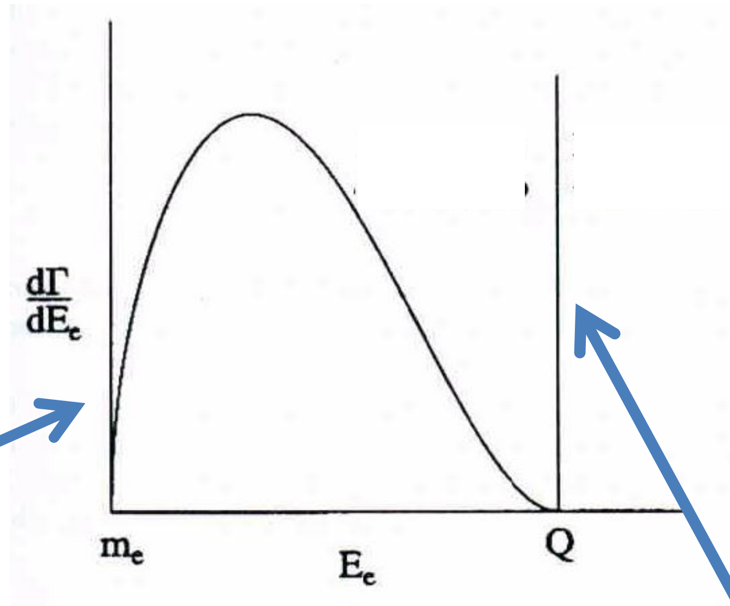
Alberto Gago
PUCP
CTEQ-FERMILAB School 2012
Lima, Perú - PUCP

Outline

- Introduction
- Neutrino oscillation in vacuum
- Neutrino oscillation in matter
- Review of neutrino oscillation data
- Conclusions

Historical introduction ν

(1930) W. Pauli propose a new particle for saving the incompatibility between the observed electron energy spectrum and the expected.



$$(A,Z) \rightarrow (A,Z+1) + e^- + X$$

Observed Spectrum – Continuos three body decay

$$E_{e^-} = Q \cong M(A,Z) - M(A,Z+1) - E_X$$

$$(A,Z) \rightarrow (A,Z+1) + e^-$$

Expected Spectrum-Monoenergetic two body decay

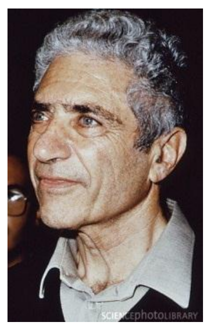
$$E_{e^-} = Q \cong M(A,Z) - M(A,Z+1)$$

Historical introduction ν

(1956)- F. Reines y C.Cowan detected for the first time a neutrino through the reaction $\bar{\nu}_e + p^+ \longrightarrow n + e^+$

This search was called as poltergeist project.

First reactor neutrino experiment



First neutrino experiment that used

$$\pi^- \rightarrow \mu^- \bar{\nu}_\mu$$

(1962)-Lederman-Schwartz-Steinberger, discovered in Brookhaven the muon antineutrino through the reaction:

$$\bar{\nu}_\mu + p \rightarrow \mu^+ + n$$

they did not observe: $\bar{\nu}_\mu + p \rightarrow e^+ + n$

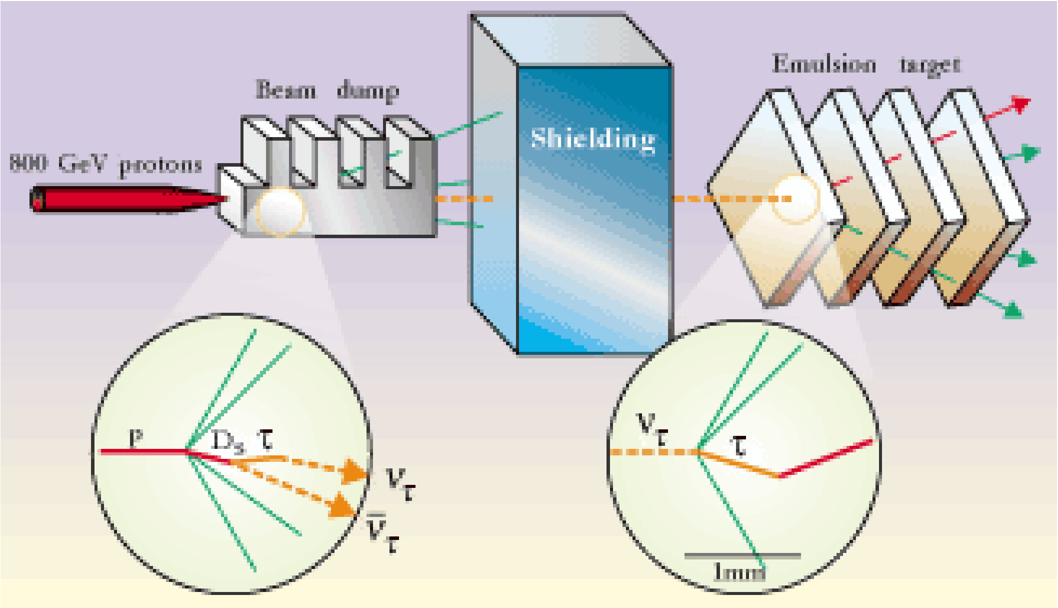
Confirming that

$$\nu_\mu \neq \nu_e$$

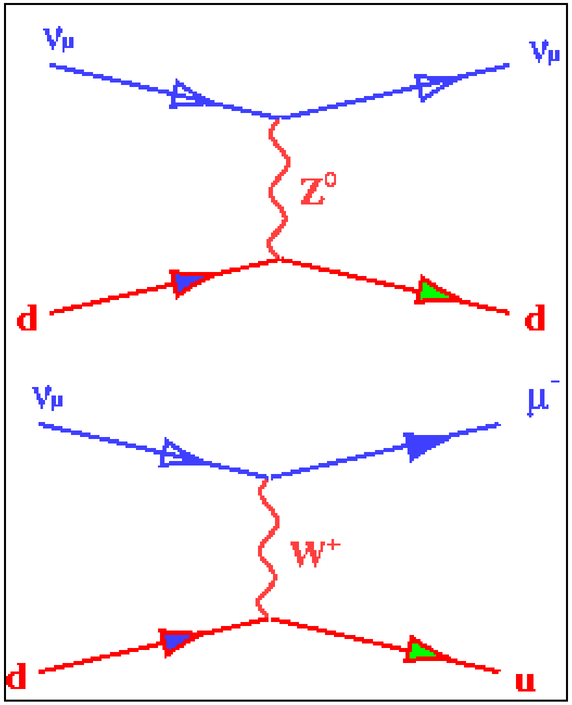
Historical introduction ν

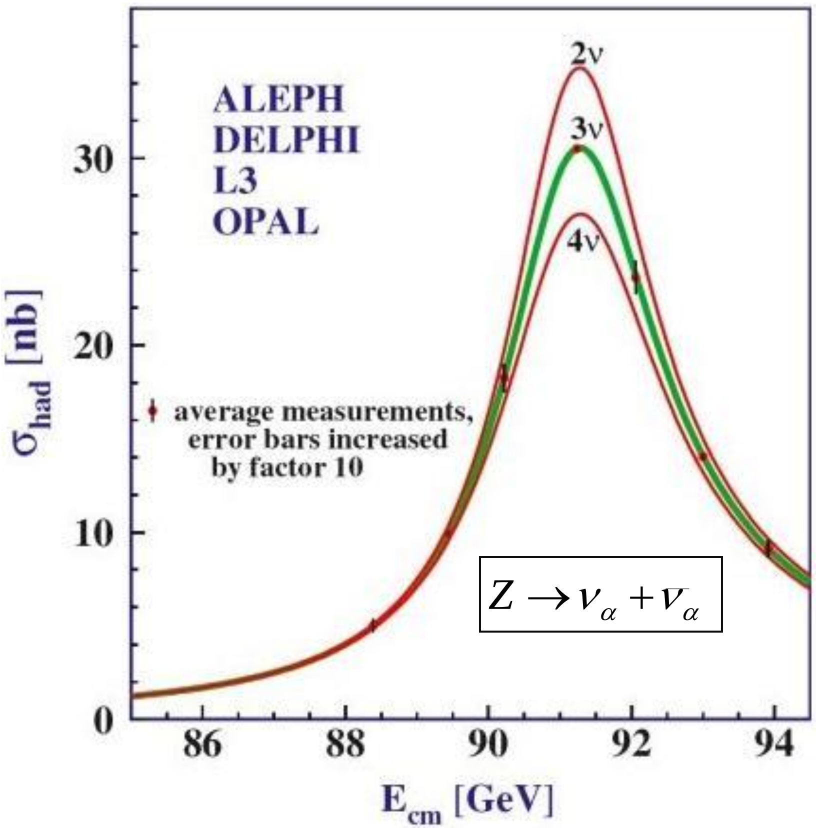
(2000) In Fermilab the DONUT collaboration observed for the first time events of ν_τ. They detected four events.

$$D_S \rightarrow \tau + \overline{\nu}_\tau$$
$$\tau \rightarrow \nu_\tau + X$$



Neutrinos





$$Z \rightarrow \nu_\alpha + \nu_\alpha$$

$$\frac{\sigma(e^- e^-)}{\sigma(\nu_e e^-)} \cong \frac{10^{-33}\text{cm}^2}{10^{-41}\text{cm}^2} = 10^8$$

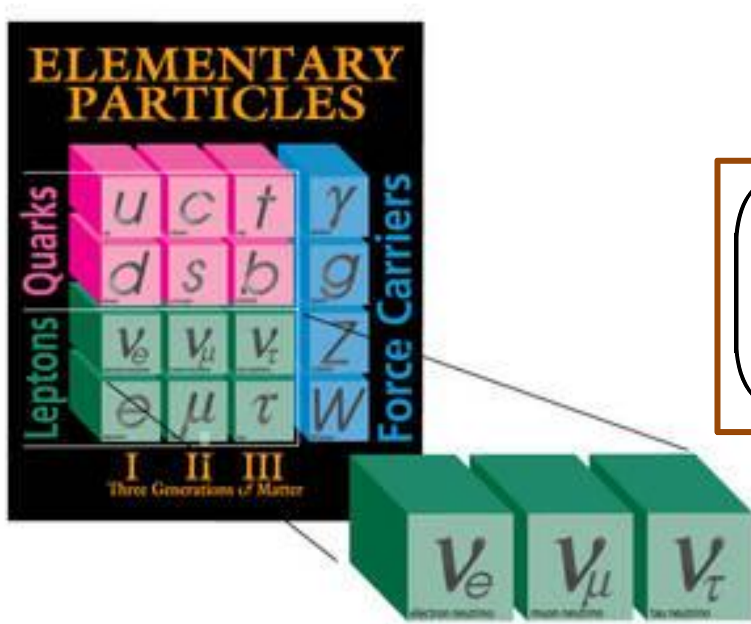
$$N_\nu = 2.9840 \pm 0.0082$$

Active neutrinos

$$N_\nu = \frac{\Gamma_{inv}}{\Gamma_l}\left(\frac{\Gamma_l}{\Gamma_\nu}\right)_{SM}$$

Neutrinos -SM

Therefore in the Standard Model (SM) we have:



Left-handed doublet

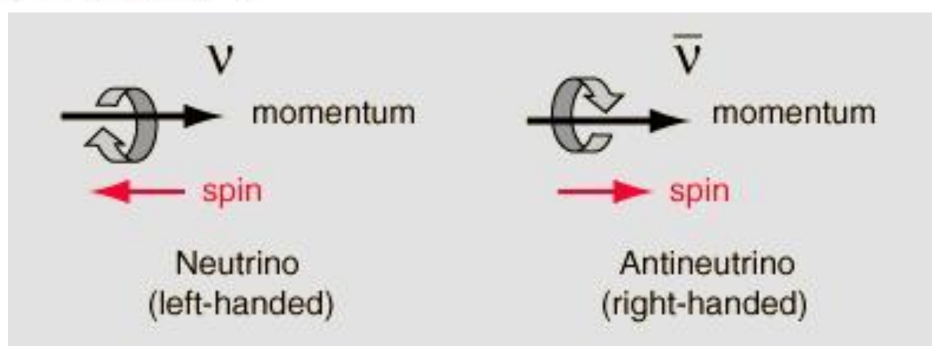
$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$

Neutrinos are fermions and neutrals

Only neutrinos of two kinds have been seen in nature:

left-handed –neutrino

right-handed- antineutrinos



Neutrino –SM

In the SM the neutrinos are consider massless (ad-hoc assumption)

….BUT we know that they have non-zero masses because they can change flavour or oscillate …

Neutrino oscillations -history

(1957) Bruno Pontecorvo suggested for the first time the idea of $\nu \rightarrow \bar{\nu}\ (\nu \leftarrow \bar{\nu})$ in analogy to $K_0 \rightarrow \bar{K}_0\ (K_0 \leftarrow \bar{K}_0)$.

Progress of Theoretical Physics, Vol. 28, No. 5, November 1962

Remarks on the Unified Model of Elementary Particles

Ziro MAKI, Masami NAKAGAWA and Shoichi SAKATA

Institute for Theoretical Physics
Nagoya University, Nagoya

(Received June 25, 1962)

a) The weak neutrinos must be re-defined by a relation

$$\left.\begin{aligned} \nu_e &= \nu_1 \cos\delta - \nu_2 \sin\delta, \\ \nu_\mu &= \nu_1 \sin\delta + \nu_2 \cos\delta. \end{aligned}\right\} \quad (2\cdot18)$$

The leptonic weak current (2·9) turns out to be of the same form with (2·1). In the present case, however, weak neutrinos are *not stable* due to the occurrence of a virtual transmutation $\nu_e \rightleftarrows \nu_\mu$ induced by the interaction (2·10). If the mass difference between ν_2 and ν_1, i.e. $|m_{\nu_2} - m_{\nu_1}| = m_{\nu_2}$ *) is assumed to be a few Mev, the transmutation time $T(\nu_e \rightleftarrows \nu_\mu)$ becomes $\sim 10^{-18}$ sec for fast neutrinos with a momentum of ~Bev/c. Therefore, a chain of reactions such as[10])

$$\pi^+ \rightarrow \mu^+ + \nu_\mu, \quad (2\cdot19a)$$

$$\nu_\mu + Z(\text{nucleus}) \rightarrow Z' + (\mu^- \text{ and/or } e^-) \quad (2\cdot19b)$$

is useful to check the two-neutrino hypothesis only when $|m_{\nu_2} - m_{\nu_1}| \lesssim 10^{-6}$ Mev

(1962) Maki, Nakagawa and Sakata proposed the mixing

Neutrino oscillation in vacuum

Flavour conversion ⟺

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ . \\ . \end{pmatrix} \neq \begin{pmatrix} \nu_1 \\ \nu_2 \\ . \\ . \end{pmatrix}$$

Non-diagonal

$$\left|\nu_\alpha\right\rangle = \sum_k U^*_{\alpha k}\left|\nu_k\right\rangle$$

$$\left\langle\nu_\alpha\middle|\nu_\beta\right\rangle = \delta_{\alpha\beta}$$

$$\left\langle\nu_i\middle|\nu_k\right\rangle = \delta_{ik}$$

flavour eigenstates $\alpha = e, \mu, \tau$

mass eigenstates $k = 1, 2, 3$

The flavour (weak) eigenstates are coherent superposition of the mass eigenstates

$$\left|\nu_\alpha\right\rangle = U_{\alpha 1}\left|\nu_1\right\rangle + U_{\alpha 2}\left|\nu_2\right\rangle + U_{\alpha 3}\left|\nu_3\right\rangle$$

Neutrino mixing

- The mixing matrix is appearing in the *charged current* interaction of the SM :

analog to the quark mixing case

$$L_{SM}^{CC} = -\frac{g}{\sqrt{2}} \sum_{\alpha=e,\mu,\tau} \sum_{k} \left(\overline{\nu}_{\alpha L} \gamma^{\mu} l_{\alpha L} W_{\mu} + \overline{l}_{\alpha L} \gamma^{\mu} \nu_{\alpha L} W_{\mu}^{+} \right)$$

$$L_{SM}^{CC} = -\frac{g}{\sqrt{2}} \sum_{\alpha=e,\mu,\tau} \sum_{k} \left(U_{\alpha k}^{*} \overline{\nu}_{kL} \gamma^{\mu} l_{\alpha L} W_{\mu} + U_{\alpha k} \overline{l}_{\alpha L} \gamma^{\mu} \nu_{kL} W_{\mu}^{+} \right)$$

$$U_{PMNS} = V_L^{l+} V_L^{\nu} \equiv V_L^{D+} V_L^{U} = U_{CKM}$$

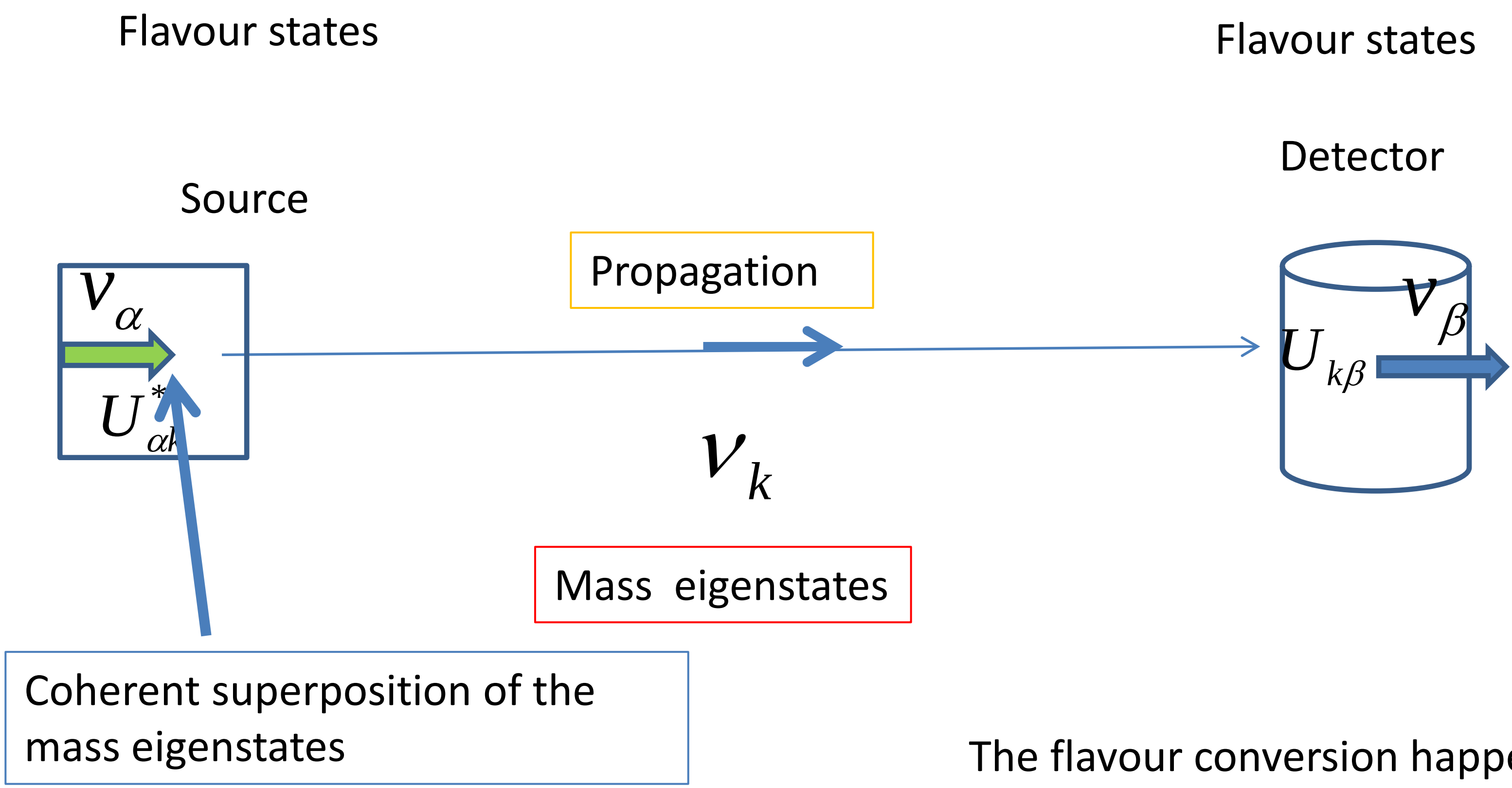
$$l_L^- \to \nu_{kL}$$
$$\to \nu_{kL} l_L^+$$

$$l_L^+ \to \overline{\nu}_{kL}$$
$$\to \overline{\nu}_{kL} l_L^-$$

PMNS=Pontecorvo-Maki-Nakagawa-Sakata

D= down quarks
U= up quarks

Neutrino oscillation in vacuum the scheme



The flavour conversion happens at long distances!

Neutrino oscillation in vacuum

- Starting point：

$$\left|\nu_\alpha\right\rangle = \sum_i U^*_{\alpha i}\left|\nu_i\right\rangle \quad \Leftrightarrow \quad \left|\nu_k\right\rangle = \sum_{\beta=e,\mu,\tau} U_{\beta k}\left|\nu_\beta\right\rangle \qquad (t=0,\ x=0)$$

$$i,k = \text{mass eigenstates} \qquad \alpha,\beta = \text{flavour eigenstates}$$

- Evolving the mass eigenstates in time and position (Plane wave approximation):

$$\left|\nu_\alpha(t,x)\right\rangle = \sum_i U^*_{\alpha i}\exp\left(-i E_i t + i p_i x\right)\left|\nu_i\right\rangle$$

$$= \sum_{\beta=e,\mu,\tau} \underbrace{\sum_i U^*_{\alpha i}\exp\left(-i E_i t + i p_i x\right) U_{\beta i}}_{A_{\alpha\to\beta}(t,x) = probability\ \ amplitude}\left|\nu_\beta\right\rangle \neq \left|\nu_\alpha\right\rangle$$

$$P_{\nu_\alpha\to\nu_\beta} = \left|A_{\nu_\alpha\to\nu_\beta}(t,x)\right|^2 = \left|\left\langle\nu_\beta\middle|\nu_\alpha(t,x)\right\rangle\right|^2 = \left|\sum_i U^*_{\alpha i}\exp\left(-i E_i t + i p_i x\right) U_{\beta i}\right|^2$$

Neutrino oscillation in vacuum

We are using *L* instead of x

Analyzing:

$$-E_i t + p_i L = \underbrace{-(E_i - p_i)L}_{\text{assumption } t \cong L} = -\frac{\left(E_i^2 - p_i^2\right)L}{\left(E_i + p_i\right)} \cong -\frac{m_i^2}{2E}L$$

E is the neutrino energy neglecting mass contributions

$$P_{\nu_\alpha \to \nu_\beta} = \sum_{i,j} U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* \exp\left(-i\frac{\Delta m_{ij}^2}{2E}L\right)$$

$$\Delta m_{ij}^2 = m_i^2 - m_j^2$$

Neutrino oscillation in vacuum

$$P_{\nu_\alpha \to \nu_\beta}(L,E) = \underbrace{\sum_i |U_{\alpha i}|^2 |U_{\beta i}|^2}_{} + \underbrace{2\,\mathrm{Re}\sum_{i>j} U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j} \exp\left(-i\frac{\Delta m^2_{ij}}{2E}L\right)}_{}$$

constant term

oscillating term

L : source-detector distance
E : neutrino energy

$$\frac{\Delta m^2_{ij}}{2E} L_{osc} = 2\pi \Rightarrow L_{osc} = \frac{4\pi E}{\Delta m^2_{ij}}$$

Oscillation wavelength

$$\sum_{\alpha=e,\mu,\tau} P_{\nu_\alpha \to \nu_\beta} = \sum_{\beta=e,\mu,\tau} P_{\nu_\alpha \to \nu_\beta} = 1$$

Valid if there is no sterile neutrinos

Neutrino oscillation in vacuum

$$P_{\nu_\alpha \to \nu_\beta}(L,E) = \sum_i |U_{\alpha i}|^2 |U_{\beta i}|^2 + 2\,\mathrm{Re} \sum_{i>j} U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j} \exp\left(-i \frac{\Delta m^2_{ij}}{2E} L \right)$$

using

$$\sum_i |U_{\alpha i}|^2 |U_{\beta i}|^2 = \delta_{\alpha\beta} - \sum_{i>j} 2\,\mathrm{Re}\left[U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j} \right]$$

$$P_{\nu_\alpha \to \nu_\beta}(L,E) = \delta_{\alpha\beta} - \sum_{i>j} 2\,\mathrm{Re}\left[U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j} \right] + 2\,\mathrm{Re}\left(\sum_{i>j} U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j} \left(\cos\left(\frac{\Delta m^2_{ij}}{2E} L \right) - i \sin\left(\frac{\Delta m^2_{ij}}{2E} L \right) \right) \right)$$

$$P_{\nu_\alpha \to \nu_\beta}(L,E) = \delta_{\alpha\beta} - 4 \times \sum_{i>j} \mathrm{Re}\left[U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j} \right] \sin^2\left(\frac{\Delta m^2_{ij}}{4E} L \right) + 2 \times \sum_{i>j} \mathrm{Im}\left[U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j} \right] \sin\left(\frac{\Delta m^2_{ij}}{2E} L \right)$$

Neutrino oscillation in vacuum

- For the antineutrino case:

$$\left|\nu_\alpha\right\rangle = \sum_i U_{\alpha i}\left|\nu_i\right\rangle \Rightarrow U \rightarrow U^*$$

$$P_{\nu_\alpha\rightarrow\nu_\beta}(L,E) = \delta_{\alpha\beta} - 4\times\sum_{i>j}\mathrm{Re}\left[U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}\right]\sin^2\left(\frac{\Delta m^2_{ij}}{4E}L\right)$$

$$- 2\times\sum_{i>j}\mathrm{Im}\left[U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}\right]\sin\left(\frac{\Delta m^2_{ij}}{2E}L\right)$$

This is the only difference…we will return to this later

Two neutrino oscillation

$$U_{(\theta)} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$|\nu_e\rangle = \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle$$

$$|\nu_\mu\rangle = -\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle$$

$$P_{\nu_e \to \nu_\mu} = \left| \sum_{\beta=e,\mu} \sum_i^2 U_{\alpha i}^* \exp\left(-i\frac{m_i^2}{2E}L\right) U_{\beta i} \right|^2$$

$$= \left| U_{e1}^* U_{\mu 1} + U_{e2}^* U_{\mu 2} \exp\left(-i\frac{\Delta m_{21}^2}{2E}L\right) \right|^2$$

$$= \left| \cos\theta\sin\theta - \sin\theta\cos\theta\exp\left(-i\frac{\Delta m_{21}^2}{2E}L\right) \right|^2$$

$$= \cos^2\theta\sin^2\theta \left| 1 - \cos\left(\frac{\Delta m_{21}^2}{2E}L\right) + i\sin\left(\frac{\Delta m_{21}^2}{2E}L\right) \right|^2$$

$$= \frac{1}{2}\sin^2 2\theta \left(1 - \cos\left(\frac{\Delta m_{21}^2}{2E}L\right)\right)$$

$$= \sin^2 2\theta \sin^2\left(\frac{\Delta m_{21}^2}{4E}L\right)$$

Two neutrino oscillation

$$P_{\nu_e \to \nu_e}(L,E) = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2_{21}}{4E} L\right) \quad \text{survival probability}$$

$$P_{\nu_e \to \nu_\mu}(L,E) = \underbrace{\sin^2 2\theta}_{\text{oscillation amplitude}} \sin^2 \underbrace{\left(\frac{\Delta m^2_{21}}{4E} L\right)}_{\text{oscillation phase}} \quad \text{transition probability}$$

In the two neutrino framework we have:

$$P_{\nu_\alpha \to \nu_\beta}(L,E) = P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta}(L,E)$$

$$P_{\nu_\alpha \to \nu_\beta}(L,E) = P_{\nu_\beta \to \nu_\alpha}(L,E)$$

Two neutrino oscillation

- Introducing units to *L* and *E*

$$\frac{\Delta m_{21}^2}{4E} L \rightarrow 1.27 \frac{\Delta m_{21}^2(\mathrm{eV}^2)}{E(\mathrm{MeV})} L(\mathrm{m}) \quad \text{or} \quad 1.27 \frac{\Delta m_{21}^2(\mathrm{eV}^2)}{E(\mathrm{GeV})} L(\mathrm{km})$$

$$L_{osc} = \frac{4\pi E}{\Delta m_{21}^2} = 2.47 \frac{E(\mathrm{MeV})}{\Delta m_{21}^2(\mathrm{eV}^2)} \mathrm{m} = 2.47 \frac{E(\mathrm{GeV})}{\Delta m_{21}^2(\mathrm{eV}^2)} \mathrm{km}$$

Oscillation regimes

$$\frac{\Delta m_{21}^2}{4E} L = \frac{\Delta m_{21}^2}{4\pi E} \pi L = \pi \frac{L}{L_{osc}}$$

- Oscillation starting $$\frac{\Delta m_{21}^2}{4E} L << 1 \equiv \frac{L}{L_{osc}} << 1$$
- Ideal case $$\frac{\Delta m_{21}^2}{4E} L \approx \vartheta(1) \equiv \frac{L}{L_{osc}} \sim \frac{1}{2}$$
- Fast oscillations $$\frac{\Delta m_{21}^2}{4E} L >> 1 \equiv \frac{L}{L_{osc}} >> 1$$

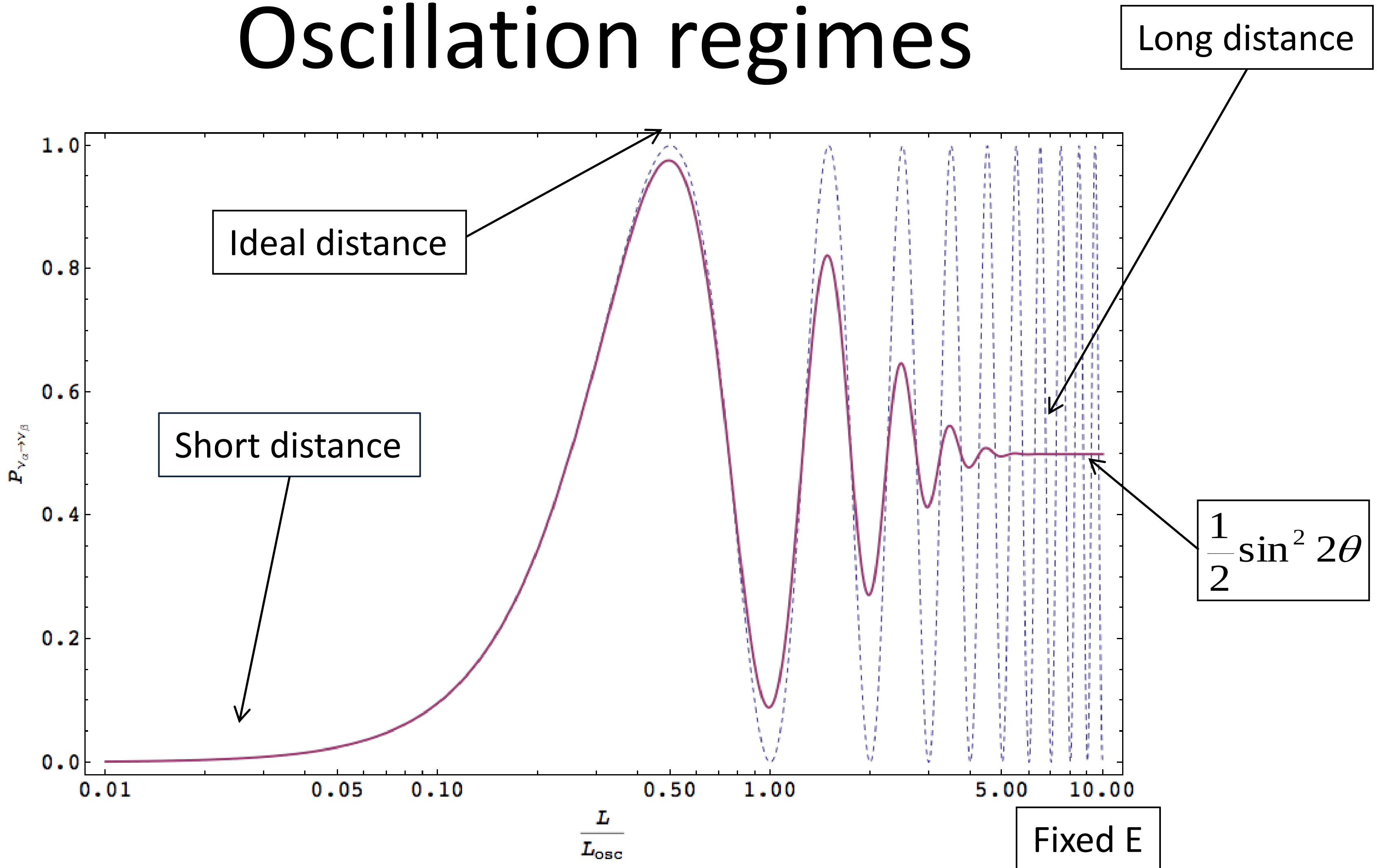
No oscillation

$$\text{sen}^2 2\theta \to 0$$

$$\Delta m_{21}^2 = 0 \Rightarrow m_2 = m_1 \text{ or } m_2 = m_1 = 0$$

$$\frac{\Delta m_{21}^2 L}{4E} \to 0$$

Oscillation regimes



$$\langle P_{\nu_e \to \nu_\mu} \rangle = \sin^2 2\theta \frac{\int \sin^2\left(1.27 \frac{\Delta m_{21}^2 L}{E}\right) e^{-\frac{(E-E')^2}{2\sigma_E^2}} dE'}{\int e^{-\frac{(E-E')^2}{2\sigma_E^2}} dE'}$$

Sensitivity plot

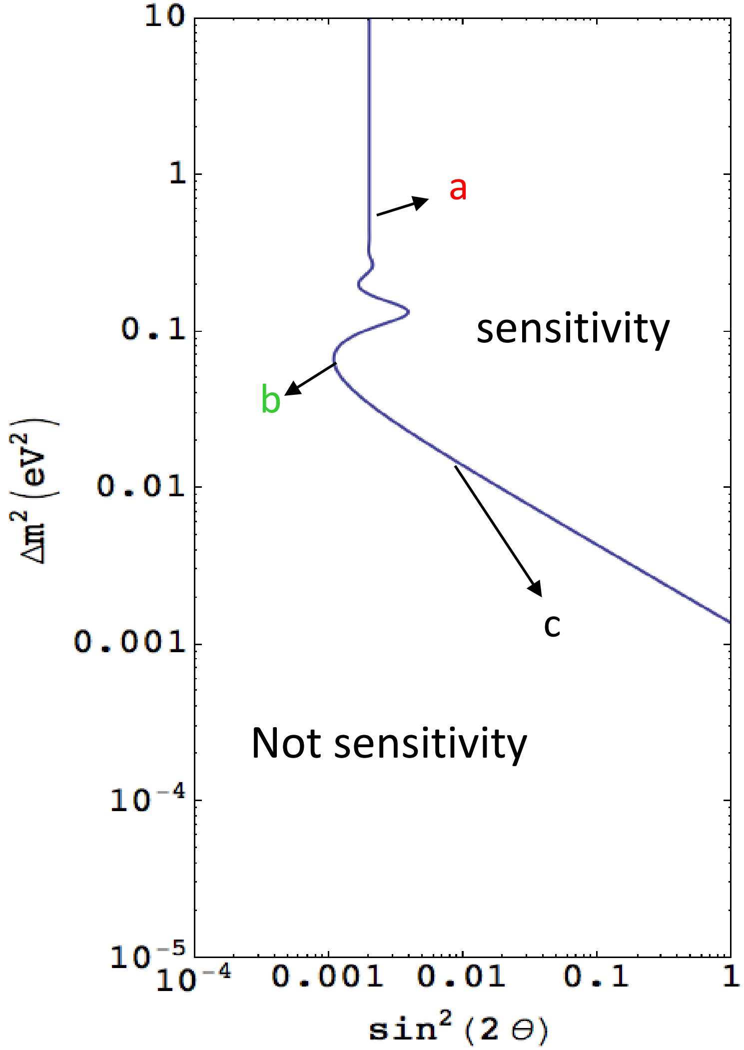
a) $P_L = \frac{1}{2}\sin^2 2\theta$

b) $P_L = \sin^2 2\theta \left\langle \sin^2\left(1.27\frac{\Delta m_{21}^2 L}{E}\right)\right\rangle$

c) $P_L = \sin^2 2\theta \left(1.27\Delta m_{21}^2 \left\langle \frac{L}{E}\right\rangle\right)^2 \Rightarrow \Delta m_{21}^2 = \frac{1}{\sin 2\theta} 1.27 \left\langle \frac{E}{L}\right\rangle \sqrt{P_L}$

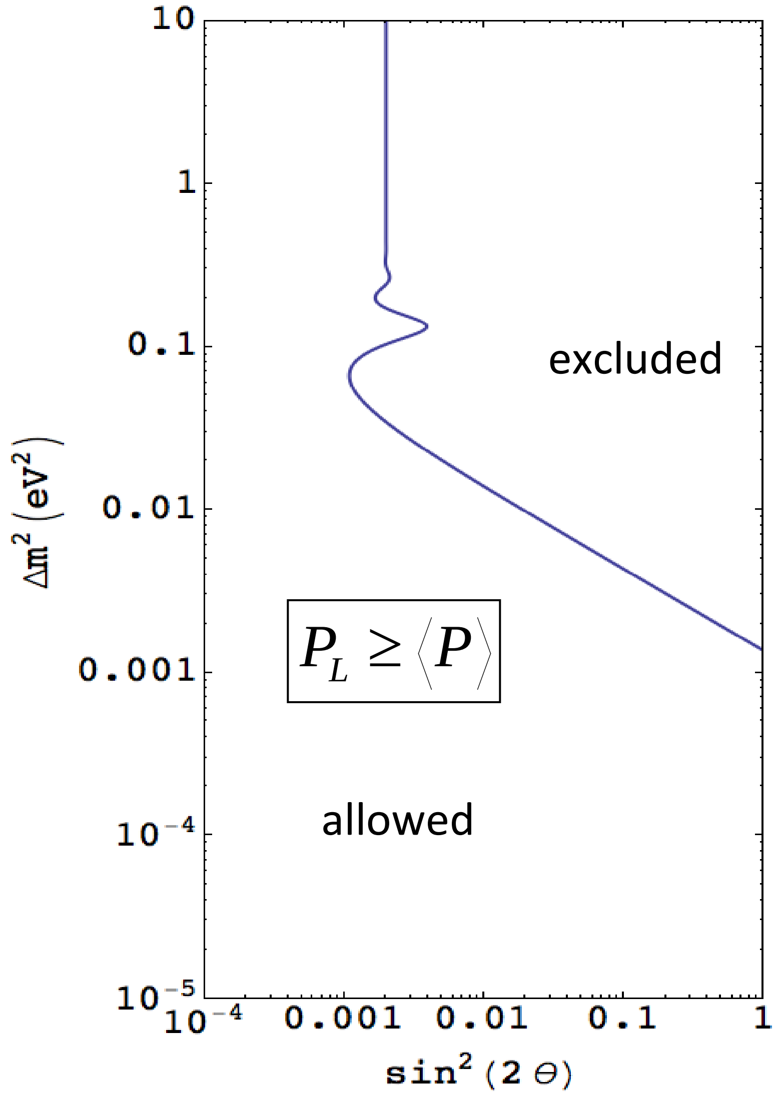
$\Rightarrow \log(\Delta m_{21}^2) = -\frac{1}{2}\log(\sin^2 2\theta) + \log\left(1.27\left\langle \frac{E}{L}\right\rangle \sqrt{P_L}\right)$

@b maximum sensitivity $\left(1.27\Delta m^2 \left\langle \frac{L}{E}\right\rangle \sim \frac{\pi}{2}\right)$



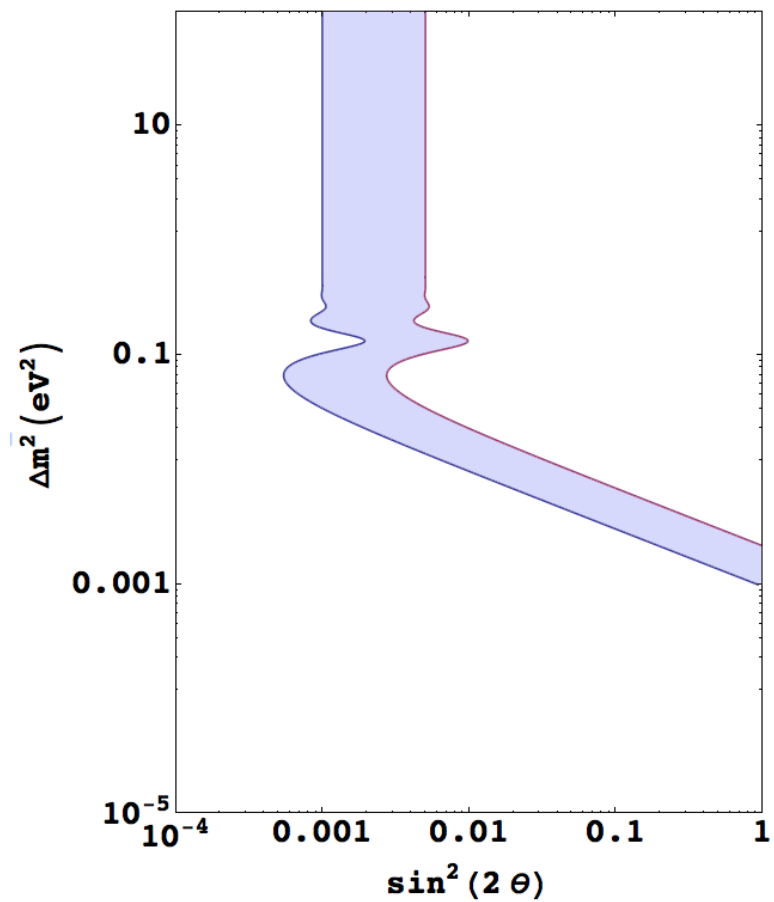
$P_L \equiv$ lower probability limit for having a positive signal in a detector

Exclusion plot



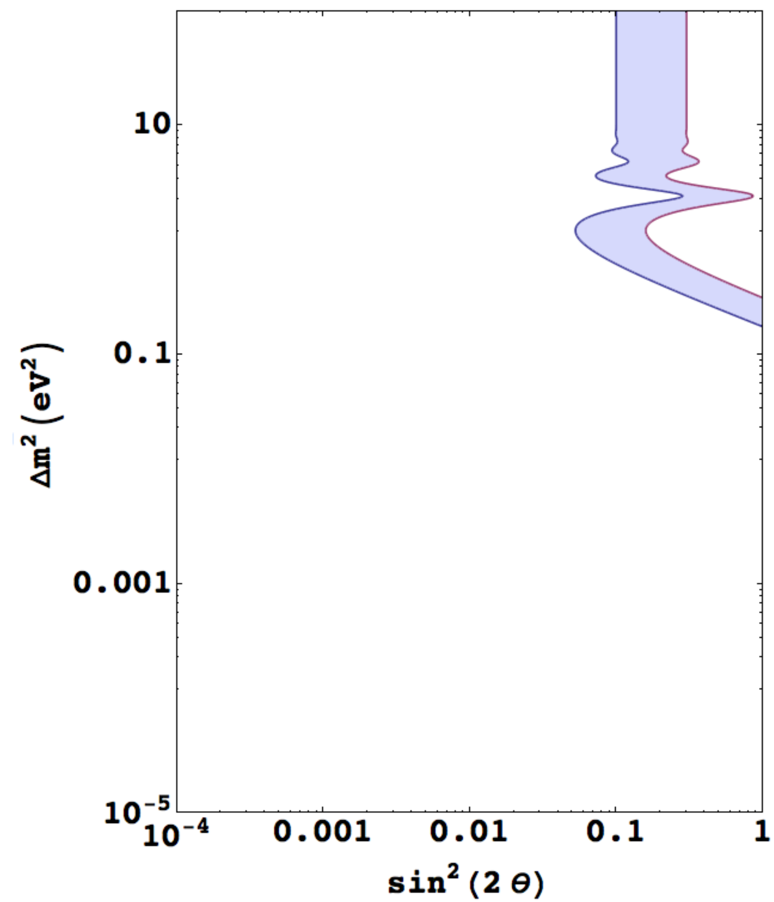
$P_L \equiv$ upper limit of the oscillation probability

Positive signal



$0.001 < P < 0.005$

$\langle L/E \rangle$ = 18km/GeV



$0.05 < P < 0.15$

$\langle L/E \rangle$ = 1km/GeV

The mixing matrix

A general complex $U_{N\times N}$ matrix has $2N^2$ parameters

$$\Rightarrow 2N^2 - \underbrace{\left(\underbrace{N}_{\text{unitarity length of each row}} + \underbrace{N(N-1)}_{\text{orthogonality between different rows}} \right)}_{\text{from } \sum_i U^*_{\alpha i} U_{\beta i} = \delta_{\alpha\beta}} = N^2$$

the mixing matrix is unitary and satisfies the condition $U^+U = UU^+ = 1$

$$N^2 - \underbrace{\frac{N(N-1)}{2}}_{\text{rotation angles}} = \underbrace{\frac{N(N+1)}{2}}_{\text{complex phases}}$$

$$\Rightarrow \text{unitary mixing matrix has} \begin{cases} \frac{N(N-1)}{2} & \text{rotation angles} \\ \frac{N(N+1)}{2} & \text{complex phases} \end{cases}$$

In 3ν scheme:

$$\frac{3\times(3-1)}{2} = 3 \text{ rotation angles}$$

$$\frac{3\times(3+1)}{2} = 6 \text{ complex phases}$$

The mixing matrix

written a unitary matrix 3×3 representation within $U^*_{\alpha k}\overline{\nu}_{kL}\gamma^\mu l_{\alpha L}$:

$$\begin{pmatrix}\overline{\nu}_{1L} & \overline{\nu}_{2L} & \overline{\nu}_{3L}\end{pmatrix}\times\begin{pmatrix}e^{i\phi_1} & 0 & 0\\ 0 & e^{i\phi_2} & 0\\ 0 & 0 & 1\end{pmatrix}\times U_{CKM-TYPE}\times e^{i\sigma}\times\begin{pmatrix}e^{i\alpha_e} & 0 & 0\\ 0 & e^{i\alpha_\mu} & 0\\ 0 & 0 & 1\end{pmatrix}\times\begin{pmatrix}e_L\\ \mu_L\\ \tau_L\end{pmatrix}$$

$U \equiv U_{CKM}$ (3 rotation angles & 1 complex phase)

* The phases α_e, α_μ and σ can be absorbed by rephasing

$$l_{L(l=e,\mu)} \rightarrow e^{-i(\alpha_l+\sigma)} l_L \text{ and } \tau_L \rightarrow e^{-i\sigma}\tau_L$$

Other terms of the lagrangian are invariant

*IF ν's are Dirac neutrinos $\equiv$ neutrinos $\neq$ antineutrinos

$\Rightarrow \phi_1$ and ϕ_2 can be absorbed by rephasing

The mixing matrix

- If neutrinos are equal than antineutrinos

Charge conjugation operator

$$\Rightarrow \underbrace{\nu}_{\text{neutrino}} = \underbrace{\nu^C}_{\text{antineutrino}} = C\,\bar{\nu}^T \quad \text{Majorana condition}$$

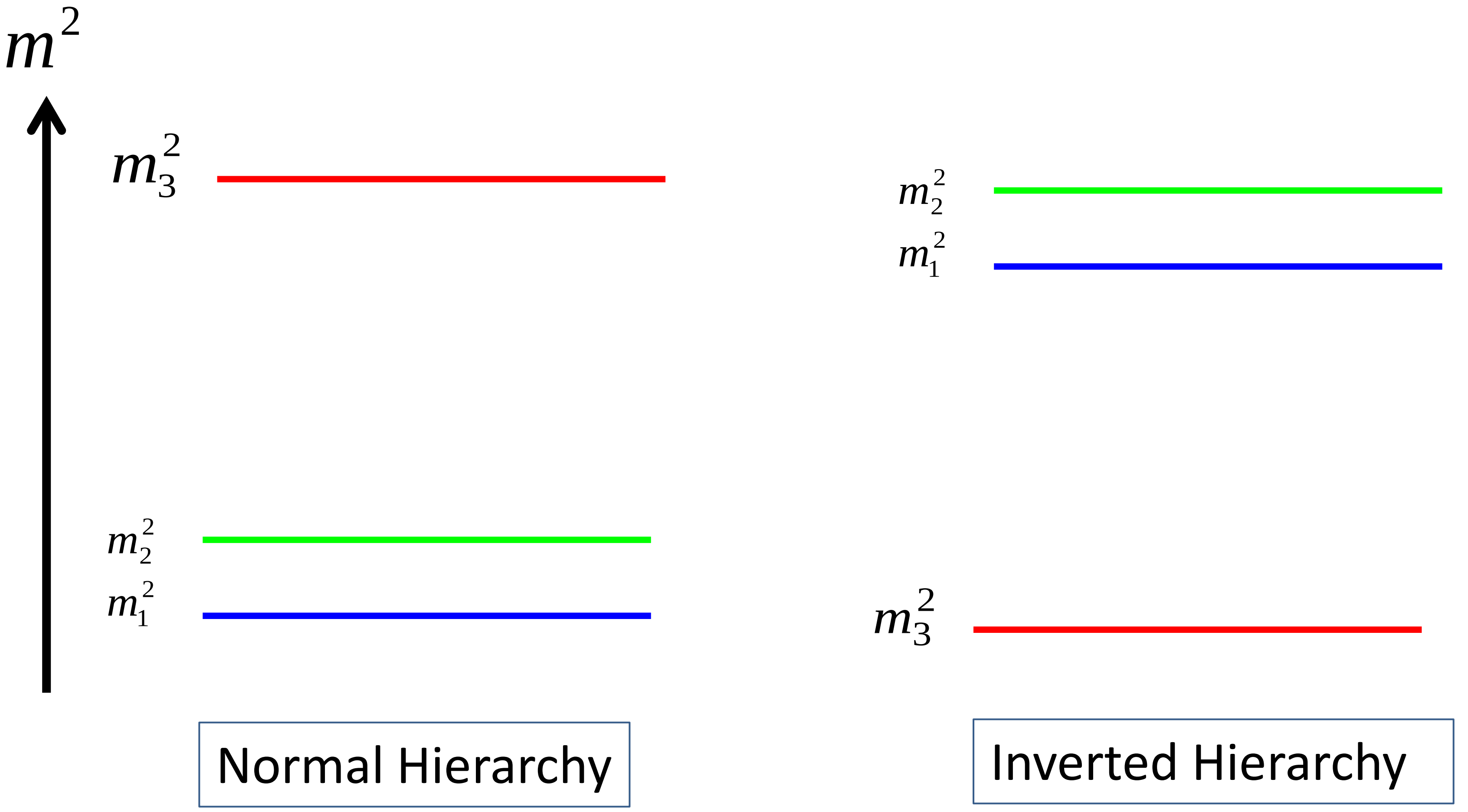
ϕ_1 and ϕ_2 can not be absorbed since the neutrino phases are fixed

For instance, if $\nu \rightarrow \nu\, e^{i\phi} \Rightarrow \nu\, e^{i\phi} = \left(\nu\, e^{i\phi}\right)^C \Rightarrow \nu = e^{-i2\varphi}\nu^C$ $\quad \varphi$ has to be zero

In the 3ν scheme:

	Dirac Neutrinos	Majorana Neutrinos
mixing angles	$\frac{N(N-1)}{2} = 3$	$\frac{N(N-1)}{2} = 3$
physical phases	$\frac{(N-1)(N-2)}{2} = 1$	$\frac{N(N-1)}{2} = 3$

3ν-mass scheme



$$\Delta m_{31}^2 \cong \vartheta\left(10^{-3}\,\text{eV}^2\right) \quad \Delta m_{21}^2 \cong \vartheta\left(10^{-5}\,\text{eV}^2\right)$$

The mixing matrix in 3ν scheme

$$U^M = \overbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \times \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{pmatrix} \times \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}^{U^D} \times \underbrace{\begin{pmatrix} e^{-i\phi_1} & 0 & 0 \\ 0 & e^{-i\phi_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{D^M}$$

Dirac CP phase

Majorana phases

$$U^M = U^D \times D^M \equiv U^D_{\alpha i} e^{-i\phi_i} \left(\text{with } \phi_3 = 0\right)$$

Notation: $c_{ij} = \cos\theta_{ij} \quad s_{ij} = \sin\theta_{ij}$

…Tomorrow we will see the current measurements of these angles done by the experiments

Majorana phases and neutrino oscillations

$$\left|A_{\nu_\alpha \to \nu_\beta}(t,x)\right|^2 = \left|\sum_i U^{*M}_{\alpha i}\, e^{(-i\,E_i t + i\,p_i x)}\, U^{M}_{\beta i}\right|^2 = \left|\sum_i U^{*D}_{\alpha i}\, e^{+i\phi_i}\, e^{(-i\,E_i t + i\,p_i x)}\, U^{D}_{\beta i}\, e^{-i\phi_i}\right|^2$$

$$= \left|\sum_i U^{*D}_{\alpha i}\, e^{(-i\,E_i t + i\,p_i x)}\, U^{D}_{\beta i}\right|^2$$

Only the dirac phase is observable in neutrino oscillation

CP violation

$$L_l^{CC} = -\frac{g}{\sqrt{2}} \sum_{\alpha=e,\mu,\tau} \sum_{k=1,2,3} \left(U_{\alpha k}^{*} \bar{\nu}_{kL} \gamma^{\mu} l_{\alpha L} W_{\mu} + U_{\alpha k} \bar{l}_{\alpha L} \gamma^{\mu} \nu_{kL} W_{\mu}^{+} \right)$$

$$U_{CP} L_l^{CC} U_{CP}^{-1} = -\frac{g}{\sqrt{2}} \sum_{\alpha=e,\mu,\tau} \sum_{k=1,2,3} \left(U_{\alpha k}^{*} \bar{l}_{\alpha L} \gamma^{\mu} \nu_{kL} W_{\mu}^{+} + U_{\alpha k} \bar{\nu}_{kL} \gamma^{\mu} l_{\alpha L} W_{\mu} \right)$$

~~CP~~ needs $U^{*} \neq U$

CP violation

- What does CP mean in neutrino oscillation ?:

$$\nu_{\alpha L} \to \nu_{\beta L} \overset{C}{\Rightarrow} \overline{\nu}_{\alpha L} \to \overline{\nu}_{\beta L} \overset{P}{\Rightarrow} \overline{\nu}_{\alpha R} \to \overline{\nu}_{\beta R}$$

$$P_{\nu_\alpha \to \nu_\beta} \overset{CP}{\Rightarrow} P_{\overline{\nu}_\alpha \to \overline{\nu}_\beta}$$

$$\text{if} \quad U_{\alpha i} \neq U^*_{\alpha i} \ \text{ and } \alpha \neq \beta$$

$$\Rightarrow P_{\nu_\alpha \to \nu_\beta} \neq P_{\overline{\nu}_\alpha \to \overline{\nu}_\beta} \ (CP \text{ is violated}$$

CP violation

$$\Delta P(\alpha,\beta) = P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta} = 4 \times \sum_{i>j} \mathrm{Im}\left[U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}\right] \sin\left(\frac{\Delta m^2_{ij}}{2E} L\right)$$

$$\mathrm{Im}\left[U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}\right] = (\pm) J_{CP}$$

Jarlskog invariant

$(+)$ cyclic permutations in (α,β) and (i,j)

$(-)$ anticyclic permutations in (α,β) and (i,j)

Independent of the mixing matrix parameterization =rephasing invariant

CP violation

$$\Delta P_{(\alpha,\beta)} = (\pm)4J_{CP}\left(\sin\left(\frac{\Delta m_{12}^2}{2E}L\right)+\sin\left(\frac{\Delta m_{23}^2}{2E}L\right)+\sin\left(\frac{\Delta m_{31}^2}{2E}L\right)\right)$$

$$= (\pm)16J_{CP}\underbrace{\left(\sin\left(\frac{\Delta m_{12}^2}{4E}L\right)\times\sin\left(\frac{\Delta m_{23}^2}{4E}L\right)\times\sin\left(\frac{\Delta m_{31}^2}{4E}L\right)\right)}_{S_{CP}}$$

$$J_{CP} = \cos^2\theta_{13}\,\mathrm{sen}\,\theta_{13}\cos\theta_{12}\,\mathrm{sen}\,\theta_{12}\cos\theta_{23}\,\mathrm{sen}\,\theta_{23}\,\mathrm{sen}\,\delta$$

CP violation phase

All the mixing angles have to be non-zero to have CP violated.

In particular we just found out that $\theta_{13} \neq 0$

CP violation

- It is interesting to note that $\Delta P_{CP} = \Delta P_{T-ODD}$:

$$\Delta P_{CP}(\alpha,\beta) = P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta} = P_{\nu_\alpha \to \nu_\beta} - P_{\nu_\beta \to \nu_\alpha} = \Delta P_{T-ODD}(\alpha,\beta)$$

- Checking for CPT

$$\bar{\nu}_{\alpha R} \to \bar{\nu}_{\beta R} \overset{C}{\Rightarrow} \nu_{\alpha R} \to \nu_{\beta R} \overset{P}{\Rightarrow} \nu_{\alpha L} \to \nu_{\beta L} \overset{T}{\Rightarrow} \nu_{\beta L} \to \nu_{\alpha L}$$

$$P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta} \overset{CPT}{=} P_{\nu_\beta \to \nu_\alpha} \quad \text{(CPT is conserved)}$$

Useful approximations

- In the three neutrino framework we have:

$$\Delta m^2_{32} \approx \Delta m^2_{31} >> \Delta m^2_{21} \text{ with } \frac{\Delta m^2_{21}}{\Delta m^2_{31}} = \varepsilon \sim \vartheta\left(10^{-2}\right)$$

$$\boxed{\Delta m^2_{32} = \Delta m^2_{31} - \Delta m^2_{21}}$$

- Case 1 : sensitive to the large scale :

$$\left(\frac{\Delta m^2_{32}}{4E} L\right) \sim \vartheta(1) \Rightarrow \left(\frac{\Delta m^2_{21}}{4E} L\right) \sim \varepsilon\, \vartheta(1) \rightarrow \vartheta\left(10^{-2}\right) \Rightarrow \text{we neglect } \Delta m^2_{21}$$

- Case 2: sensitive to the small scale

$$\left(\frac{\Delta m^2_{21}}{4E} L\right) \sim \vartheta(1) \Rightarrow \left(\frac{\Delta m^2_{32}}{4E} L\right) \sim \frac{1}{\varepsilon} \vartheta(1) \rightarrow \vartheta\left(10^{2}\right) \Rightarrow \text{terms related are averaged out}$$

Useful approximations

- Case 1: sensitive to the large scale

$$\alpha \neq \beta$$

$$\Delta m_{31}^2 \approx \Delta m_{32}^2 \text{ and } \Delta m_{21}^2 \to 0 \; (S_{CP} \to 0)$$

$$\begin{aligned}
P_{\nu_\alpha \to \nu_\beta} &= -4 \times \left\{ \mathrm{Re}\left[U_{\alpha 3}^* U_{\beta 3} U_{\alpha 1} U_{\beta 1}^*\right] \sin^2\left(\frac{\Delta m_{31}^2}{4E} L\right) + \mathrm{Re}\left[U_{\alpha 3}^* U_{\beta 3} U_{\alpha 2} U_{\beta 2}^*\right] \sin^2\left(\frac{\Delta m_{31}^2}{4E} L\right) \right\} \\
&= -4 \times \left\{ \mathrm{Re}\left[U_{\alpha 3}^* U_{\beta 3} \left(U_{\alpha 1} U_{\beta 1}^* + U_{\alpha 2} U_{\beta 2}^*\right)\right] \sin^2\left(\frac{\Delta m_{31}^2}{4E} L\right) \right\} \\
&= \underbrace{4 \times |U_{\alpha 3}|^2 |U_{\beta 3}|^2}_{\sin^2 2\theta_{eff}} \sin^2\left(\frac{\Delta m_{31}^2}{4E} L\right)
\end{aligned}$$

Similar structure to the two generation formula

Useful approximations

- Case 1: sensitive to the large scale

Explicit formulas

$$P_{\nu_e\to\nu_e} = 1 - \text{sen}^2 2\theta_{13}\,\text{sen}^2\left(\frac{\Delta m^2_{31}}{4E}L\right)$$

$$P_{\nu_\mu\to\nu_\tau} = c^4_{13}\,\text{sen}^2 2\theta_{23}\,\text{sen}^2\left(\frac{\Delta m^2_{31}}{4E}L\right)$$

$$P_{\nu_\mu\to\nu_e} = s^2_{23}\,\text{sen}^2 2\theta_{13}\,\text{sen}^2\left(\frac{\Delta m^2_{31}}{4E}L\right)$$

$$P_{\nu_u\to\nu_\mu} = 1 - 4 s^2_{23}c^2_{13}\left(1 - s^2_{23}c^2_{13}\right)\text{sen}^2\left(\frac{\Delta m^2_{31}}{4E}L\right)$$

Useful approximations

- Case 2: sensitive to the small scale

averaged out

$$\left\langle \sin^2\left(\frac{\Delta m^2_{31(32)}}{4E}L\right)\right\rangle = \frac{1}{2}$$

$\alpha = \beta$

$$P_{\nu_\alpha \to \nu_\alpha} = 1 - 4\times|U_{\alpha 2}|^2|U_{\alpha 1}|^2\sin^2\left(\frac{\Delta m^2_{21}}{4E}L\right) - 4\times\left(|U_{\alpha 3}|^2|U_{\alpha 2}|^2\left(\frac{1}{2}\right) + |U_{\alpha 3}|^2|U_{\alpha 1}|^2\left(\frac{1}{2}\right)\right)$$

$$= 1 - 2\times|U_{\alpha 3}|^2\left(1-|U_{\alpha 3}|^2\right) - 4\times|U_{\alpha 2}|^2|U_{\alpha 1}|^2\sin^2\left(\frac{\Delta m^2_{21}}{4E}L\right)$$

$$= |U_{\alpha 3}|^4 + \left(1-|U_{\alpha 3}|^2\right)^2\left(1 - \underbrace{4\times\frac{|U_{\alpha 2}|^2|U_{\alpha 1}|^2}{\left(|U_{\alpha 2}|^2+|U_{\alpha 1}|^2\right)^2}}_{\sin^2 2\theta_{eff}}\sin^2\left(\frac{\Delta m^2_{21}}{4E}L\right)\right)$$

Similar structure for the two generation formula

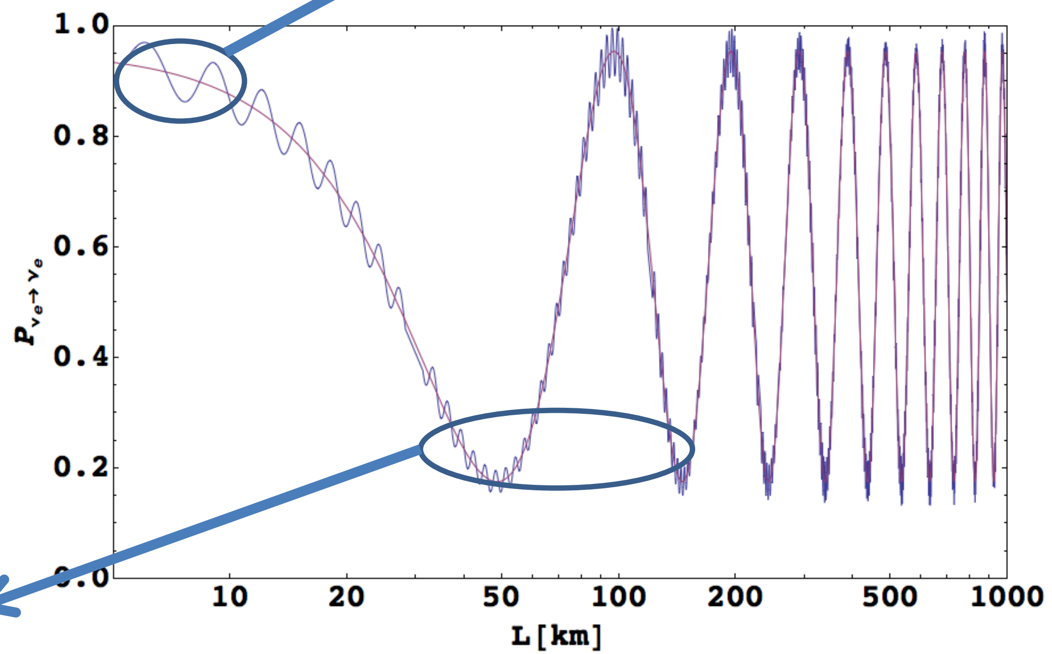
Useful approximations

- Case 2: sensitive to the small scale

$$P^{3\nu}_{\nu_e\to\nu_e} = s_{13}^4 + c_{13}^4\left(1 - \mathrm{sen}^2 2\theta_{12}\,\mathrm{sen}^2\left(\frac{\Delta m_{21}^2}{4E}L\right)\right)$$

$$P^{3\nu}_{\nu_e\to\nu_e} = s_{13}^4 + c_{13}^4\,P^{2\nu}_{\nu_e\to\nu_e}$$

Δm_{31}^2 scale



Approximation

Full Probability

Δm_{21}^2 scale

Useful approximations

- We can get a better approximation for the oscillation formula when we expand this in small parameters up to second order. We are in the case of sensitivity of the large scale.
- These small parameters are :

$$\theta_{13}, \frac{\Delta m_{21}^2}{\Delta m_{31}^2}, \frac{\Delta m_{21}^2 L}{2E_\nu}$$

leading term $-P_{2\nu}$ approximation

Now it is appearing δ and Δm_{12}^2

$$P_{\nu_e \to \nu_\mu}\ (\bar{\nu}_e \to \bar{\nu}_\mu) = s_{23}^2 \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m_{13}^2}{4E_\nu} L\right)$$

$$+ c_{23}^2 \sin^2 2\theta_{12} \sin^2\left(\frac{\Delta m_{12}^2}{4E_\nu} L\right)$$

$$+ \tilde{J} \cos\left(\pm\delta - \frac{\Delta m_{13}^2}{4E_\nu} L\right) \frac{\Delta m_{12}^2}{4E_\nu} \sin\left(\frac{\Delta m_{13}^2}{4E_\nu} L\right)$$

$$\tilde{J} = c_{13} \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23}$$