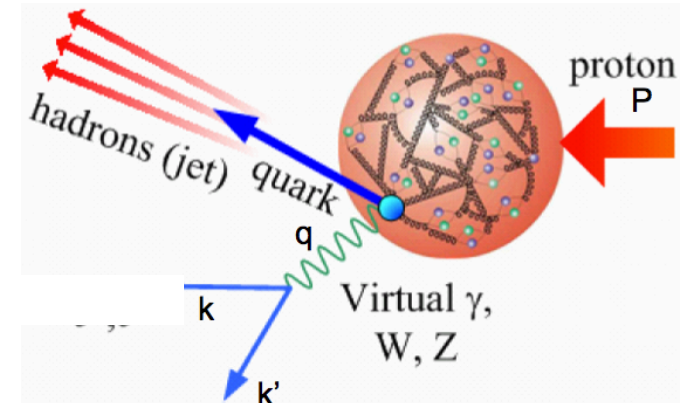


Lectures on Deep Inelastic Scattering

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(DESY)



- **Part I:**
 - Introduction to DIS formalism
 - Physics Results from DIS experiments
- **Part II:**
 - impact of DIS measurements
 - Relevance of DIS to LHC physics
 - Outlook

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PhD from Pitt in 2006

Today's Lecture

Lectures will present state of the art in the field blended in with new experimental results*

- ◆ Motivation
- ◆ A leap into history
- ◆ Quark Parton Model
- ◆ Parton Distribution Functions (PDFs)
- ◆ QCD add on features
- ◆ Selected Experimental measurements

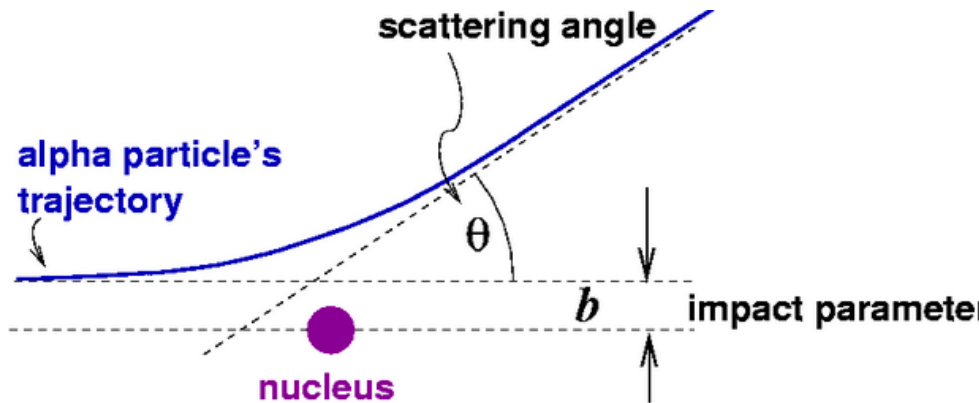
* Disclaimer: more coverage of H1, NuTeV and ATLAS is given due to my biases...

Motivation

- ◆ Deep inelastic scattering is the ideal process for the determination of the quark and gluon distributions in the proton.
 - ▶ Studies of the proton substructure of the nucleon are of great interest for the development of strong interaction theory
- ◆ With high energy and luminosity, the LHC search range will be extended to high masses, up to 5 TeV in pair production. At correspondingly large momentum the constituents of proton are unknown to a considerable extent.
 - ▶ Accurate knowledge of constituents of protons also a necessary input for new physics searches and studies at the Large Hadron Collider

Introduction to Deep Inelastic Scattering (DIS)

- ◆ Rutherford's gold foil experiment 1909 (performed by Geiger and Marsden)



Geiger and Rutherford

Rutherford's gold foil experiment set the scene for a century of ever-deeper and more precise resolution of the constituents of the atom, the nucleus and the nucleon.

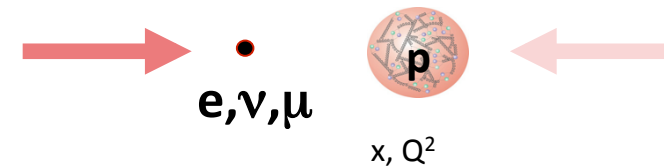
→ **Ideas for detecting quarks were formulated:**

To probe the interiors of target, pointlike and easily produced particle needed to be used.

Probing the Proton Structure

- Proton can be probed via elementary particles as:

- neutrinos (fixed target experiments) - interact only weakly
- electrons (fixed target and collider experiments) - interact electroweakly



- Deep Inelastic Scattering (DIS) is the cleanest probe to study the substructure of nucleon**

- scattering of a lepton off the nucleon involving a large momentum transfer and resulting into a hadronic shower and a lepton

- Kinematic Lorentz Invariant Variables:**

- virtuality of exchanged boson

$$Q^2 = -q^2 = -(k - k')^2$$

- proton momentum fraction of the scattered quark (Bjorken scaling variable)

$$x = \frac{Q^2}{2p \cdot q}$$

- inelasticity parameter:

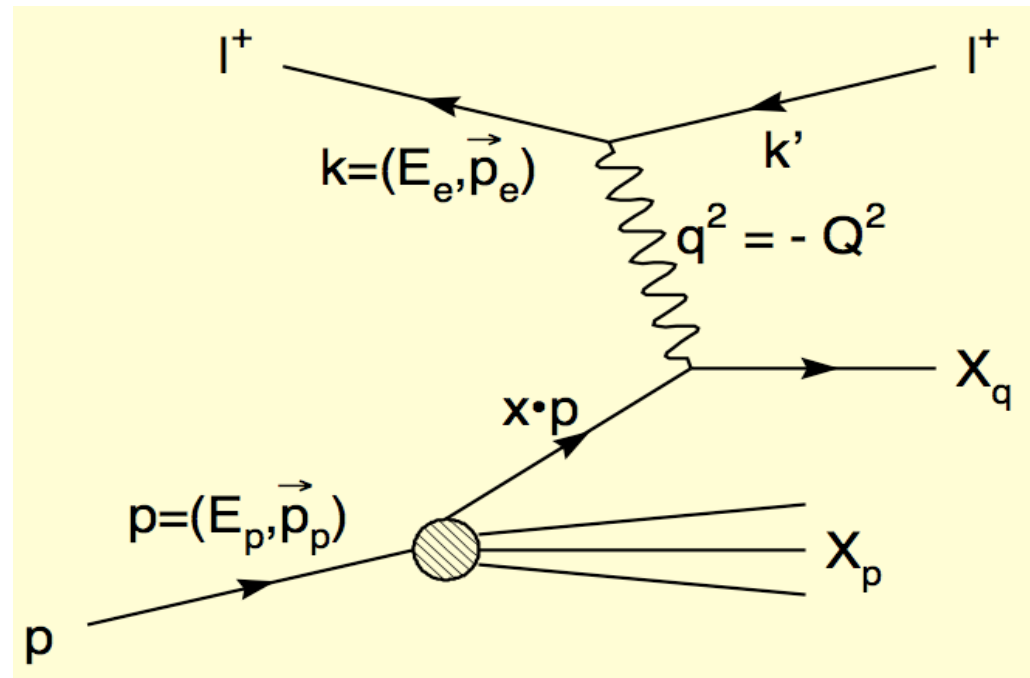
$$y = \frac{p \cdot q}{p \cdot k}$$

- invariant centre of mass energy:

$$s = (k + p)^2 = \frac{Q^2}{xy}$$

- Invariant centre of mass energy of the virtual boson-proton system)

$$W^2 = (P + q)^2 = m_p^2 - Q^2 + 2P \cdot q = ys - Q^2 + m_p^2(1 - y).$$



DIS Cross Sections

- Factorisable nature of interaction: Inclusive scattering cross section is a product of leptonic and hadronic tensors times propagator characteristic of the exchanged particle:

$$\frac{d^2\sigma}{dx dQ^2} = \frac{2\pi\alpha^2}{Q^4 x} \sum_j \eta_j L_j^{\mu\nu} W_j^{\mu\nu}$$

$$\eta_\gamma = 1; \quad \eta_{\gamma Z} = \left(\frac{G_F M_Z^2}{2\sqrt{2}\pi\alpha} \right) \left(\frac{Q^2}{Q^2 + M_Z^2} \right); \quad \eta_Z = \eta_{\gamma Z}^2;$$

$$\eta_W = \frac{1}{2} \left(\frac{G_F M_W^2}{4\pi\alpha} \frac{Q^2}{Q^2 + M_W^2} \right)^2,$$

For NC: j=γ, Z, γZ
For CC: j=W+, W-

Leptonic tensor: related to the coupling of the lepton with the exchanged boson

- contains the electromagnetic or the weak couplings
- can be calculated exactly in the standard electroweak U(1) × SU(2) theory.

Hadronic tensor: related to the interaction of the exchanged boson with proton

- can't be calculated, but only be reduced to a sum of structure functions:

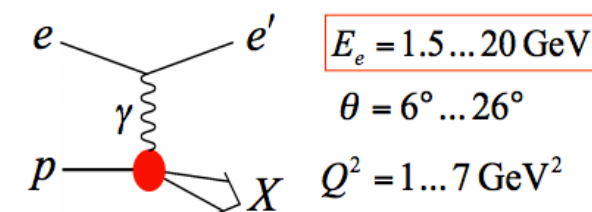
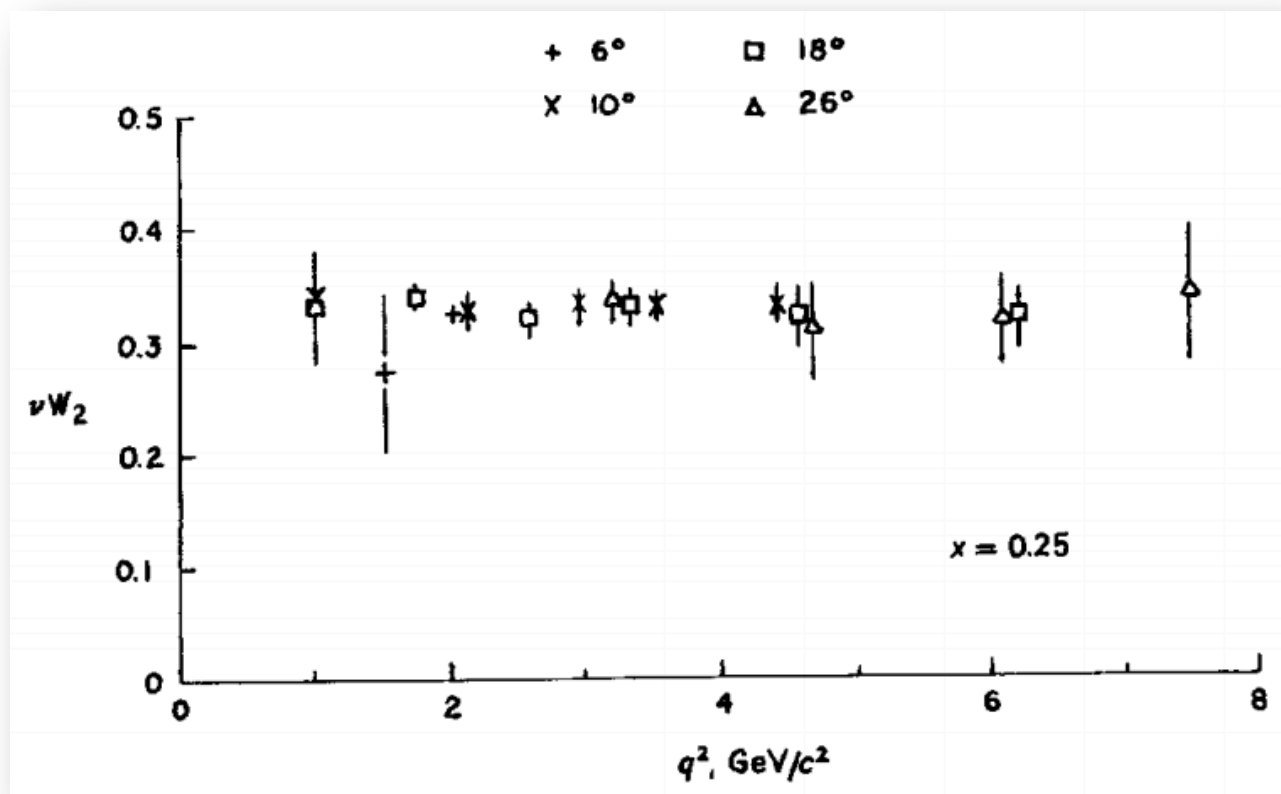
$$W^{\alpha\beta} = -g^{\alpha\beta} W_1 + \frac{p^\alpha p^\beta}{M^2} W_2 - \frac{i\epsilon^{\alpha\beta\gamma\delta} p_\gamma q_\delta}{2M^2} W_3 + \frac{q^\alpha q^\beta}{M^2} W_4 + \frac{p^\alpha q^\beta + p^\beta q^\alpha}{M^2} W_5 + \frac{i(p^\alpha q^\beta - p^\beta q^\alpha)}{2M^2} W_6 \quad \sim m_{\text{lepton}}$$

$$\frac{d^2\sigma}{dx dQ^2} = A^i \left\{ \left(1 - y - \frac{x^2 y^2 M^2}{Q^2}\right) F_2^i + y^2 x F_1^i \mp \left(y - \frac{y^2}{2}\right) x F_3^i \right\} \quad A^i: \text{process dependent}$$

Scaling of the structure functions

Structure functions can be extracted experimentally by looking at x, y, Q^2 dependence of the cross-section

- ◆ Experimental observation of scaling behaviour of F_2 is first evidence for a partonic sub-structure in the nucleon:



[MIT-SLAC Collab. 1970]

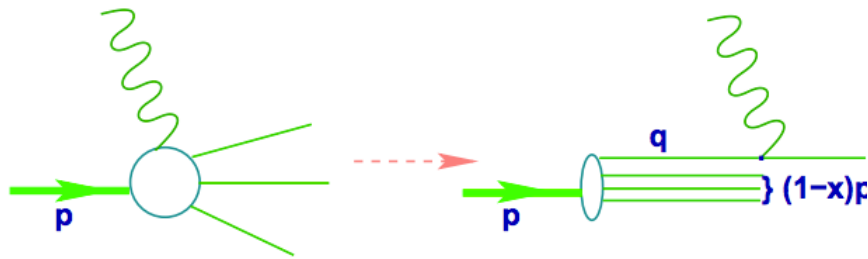
Scaling refers to the dependence of the structure functions on a single dimensionless variable x : Bjorken scaling

Once able to look into nucleon, can look into the properties of those partons...

Quark Parton Model (QPM)

◆ In Quark Parton Model:

- ▶ inelastic scattering with nucleon is viewed as elastic scattering between lepton and a pointlike constituent of the target – **partons** (non-interacting) – explicitly assumed to be spin-1/2 particles



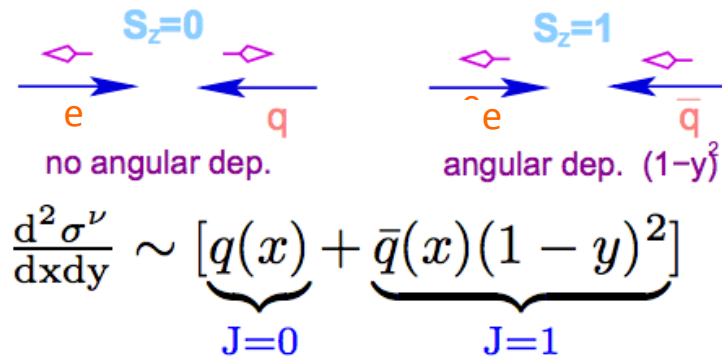
Each parton carries the fraction x with a probability $q(x)$

$$\left(\frac{d\sigma}{dx dQ^2}\right)_{ep \rightarrow eX} = \sum_i \int dx e_i^2 q_i(x) \left(\frac{d\sigma}{dx dQ^2}\right)_{eq_i \rightarrow eq_i}$$

- ▶ Bjorken- x has a meaning of momentum fraction carried by the struck quark:

The elastic scattering cross section for spin 1/2:

$$\begin{aligned} (\epsilon p + q)^2 &= m_{lepton} \approx 0 \\ q^2 + 2\epsilon p q^2 &\approx 0 \\ \epsilon &= \frac{Q^2}{2pq} = x \end{aligned}$$



- ▶ Considering probability distribution for the quark to have momentum fraction x , $xq(x)$,

Callan-Gross relation

$$F_2(x) = 2xF_1(x)$$

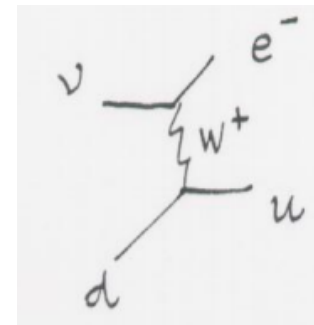
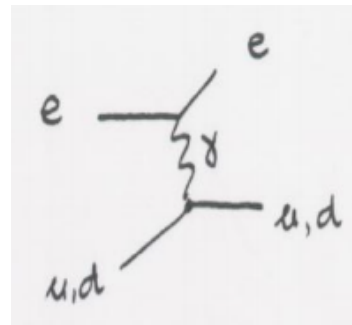
$$F_2(x) = \sum_q e_q^2 x q(x), \quad F_L(x) = 0.$$

Verification of QPM: fractional electric charge

- Using different probes (e, nu) in DIS processes: can probe electric charge of the partons

proton: uud
neutron: ddu

$$F_2(x) = \sum_i e_i^2 x [q_i(x) + \bar{q}_i(x)]$$



Neutrinos:

- interact only weakly
- left handed particles

$$\begin{aligned} F_2^{ep}(x) &= x[e_u^2(u + \bar{u}) + e_d^2(d + \bar{d})] \\ F_2^{en}(x) &= x[e_u^2(d + \bar{d}) + e_d^2(u + \bar{u})] \\ F_2^{eN}(x) &= \frac{1}{2}(F_2^{ep} + F_2^{en}) \\ &= x \frac{e_u^2 + e_d^2}{2} [u + \bar{u} + d + \bar{d}] \end{aligned}$$

$$\begin{aligned} F_2^{\nu p}(x) &= 2x[d + \bar{u}] \\ F_2^{\nu n}(x) &= 2x[u + \bar{d}] \\ F_2^{\nu N}(x) &= \frac{1}{2}(F_2^{\nu p} + F_2^{\nu n}) \\ &= x[u + \bar{u} + d + \bar{d}] \end{aligned}$$

Confirmed by experimental measurements

$$\frac{F_2^{eN}}{F_2^{\nu N}} = \frac{1}{2}(e_u^2 + e_d^2) = \frac{5}{18} = 0.28 \quad \longleftrightarrow \quad \frac{\text{SLACeN}}{\text{GGM}\nu N} = 0.29 \pm 0.05$$

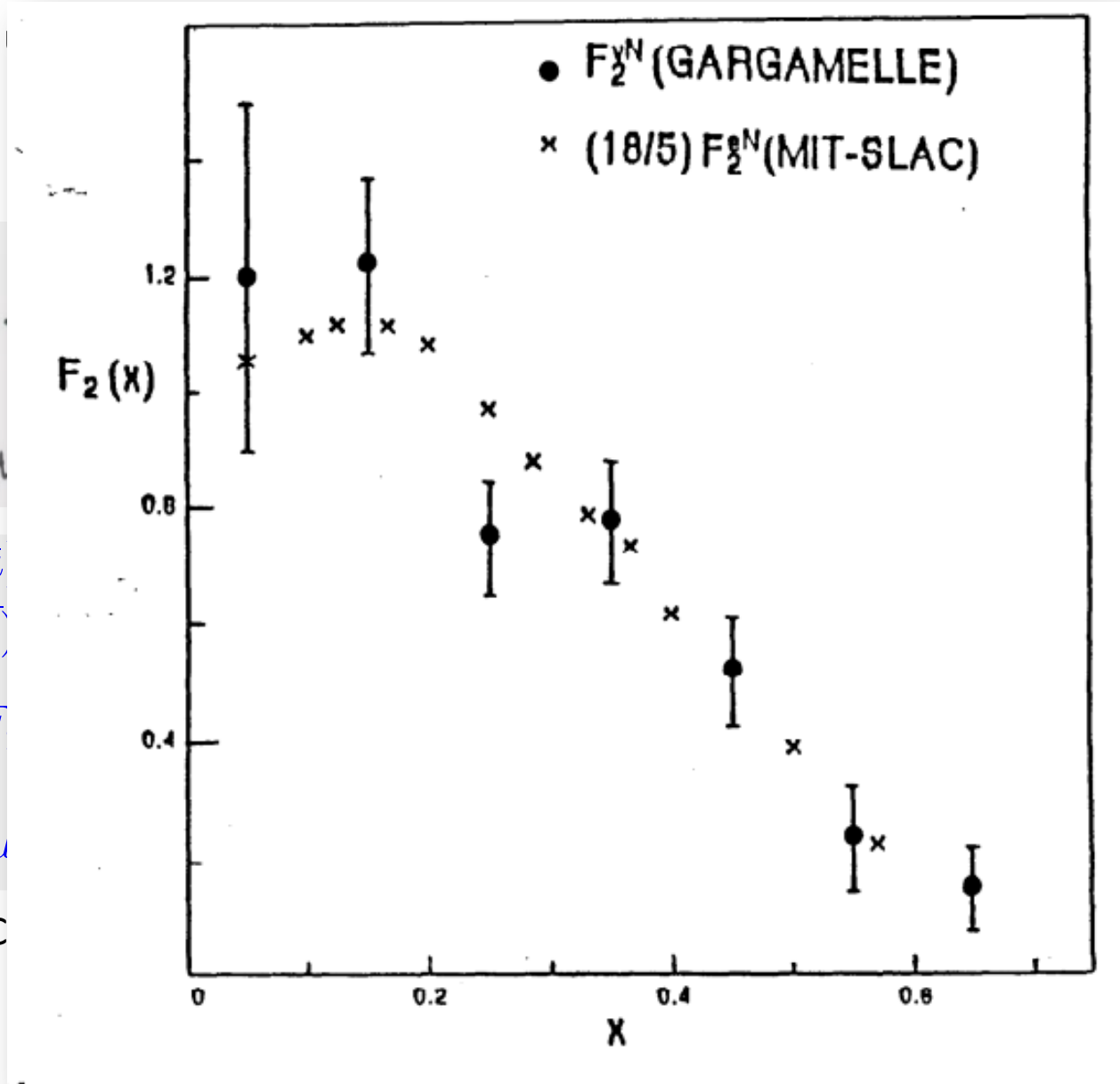
Verification of QPM: fractional electric charge

- Using different probes (e, n)
 - proton: uud
 - neutron: ddu

e
u,d

$$\begin{aligned}
 F_2^{ep}(x) &= x[e_u^2(u + \bar{u})] \\
 F_2^{en}(x) &= x[e_d^2(d + \bar{d})] \\
 F_2^{eN}(x) &= \frac{1}{2}(F_2^{ep} + F_2^{en}) \\
 &= x \frac{e_u^2 + e_d^2}{2} [u + \bar{u} + d + \bar{d}]
 \end{aligned}$$

$$\frac{F_2^{eN}}{F_2^{\nu N}} = \frac{1}{2}(e_u^2 + e_d^2) = \frac{5}{18}$$



Verification of QPM: valence, sea quarks

◆ Partons: valence and sea $u = u_{val} + u_{sea}; \quad u_{sea} = \bar{u}$

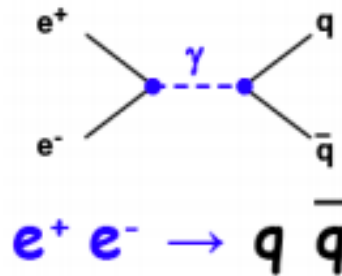
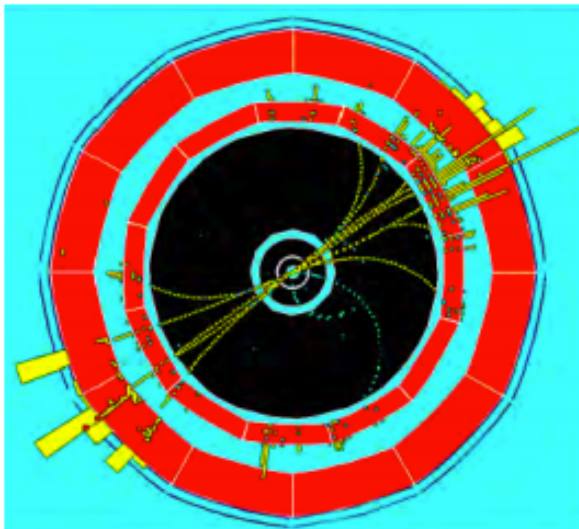
$$d = d_{val} + d_{sea}; \quad d_{sea} = \bar{d} = \bar{u}$$

▶ Gross-Llewellyn-Smith sum rule: counting the net number of quarks in the nucleons

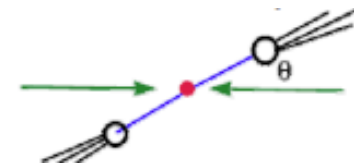
$$\begin{aligned} xF_3^{\nu p} &= 2x(d - \bar{d}) = 2xd_v \\ xF_3^{\nu n} &= 2x(u - \bar{u}) = 2xu_v \end{aligned} \quad \int_0^1 xF_3^{\nu N} \frac{dx}{x} = \int_0^1 (u_v + d_v) dx$$

QPM predicts that GLS=3; experimental findings agree within errors (Gargamelle).

▶ The observation of jet production was a major success of the Quark Parton Model approach:



The lowest order reaction leads to two jets of particles which are back-to-back in azimuth as predicted for spin- $\frac{1}{2}$ quarks



Some of the puzzles of the QPM:

- ◆ If the proton would be solely constituted of charged quarks, it was expected that

$$\int_0^1 dx x \sum_i q_i(x) = 1$$

- ▶ Experimentally was found that half of momentum of proton is NOT carried by quarks

✧ Gargamelle: 0.49 ± 0.07

- ◆ Initial phase of multi-hadron production is similar to muon pair production through e^+e^- annihilation:

- ▶ Measures directly the sum of the squares of the quarks charges (number of quark flavours)

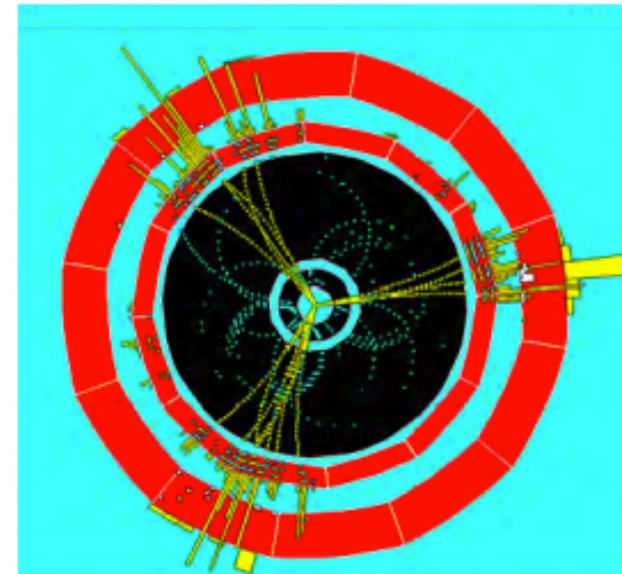
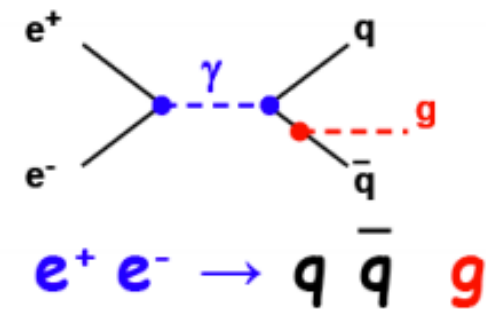
$$R_\gamma = \frac{\sigma(e^+e^- \rightarrow q\bar{q})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = \sum_q e_q^2 = \frac{11}{9}$$

✧ But actual experimental result is $\sim 11/3$

➔ **Indication that colour is more than just a quantum number:**

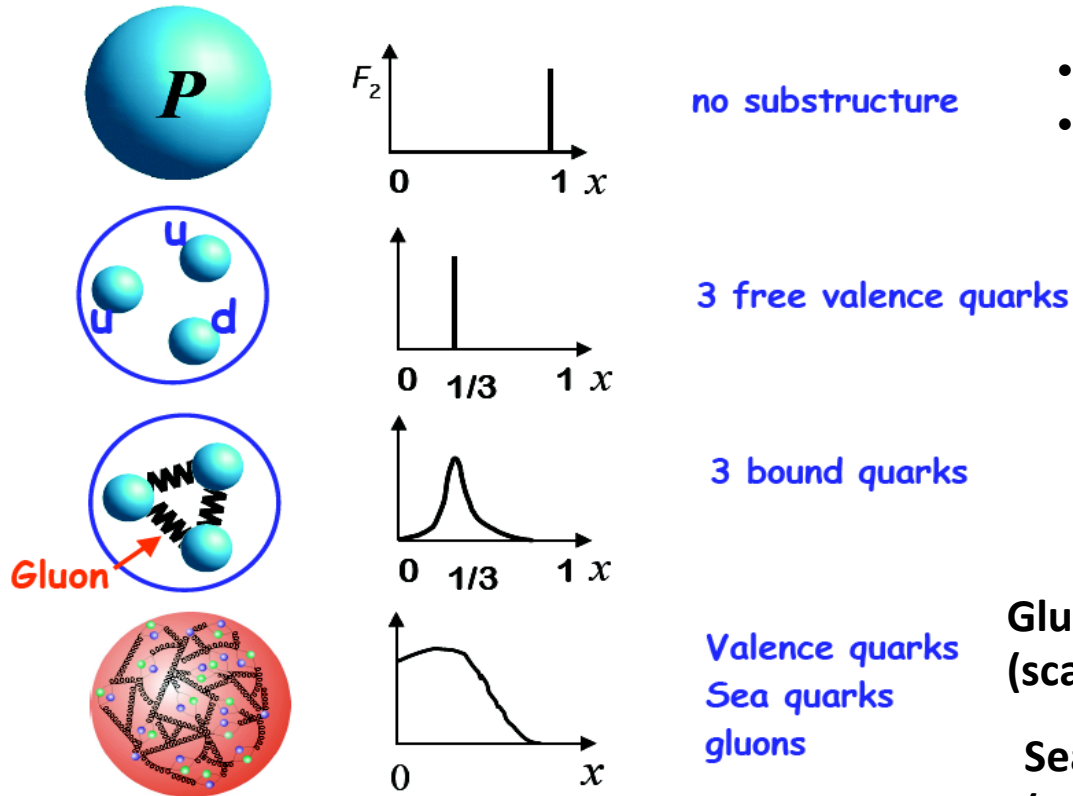
✧ discovery of the gluons at PETRA: 3 jet events

3 jets discovered at DESY in 1979

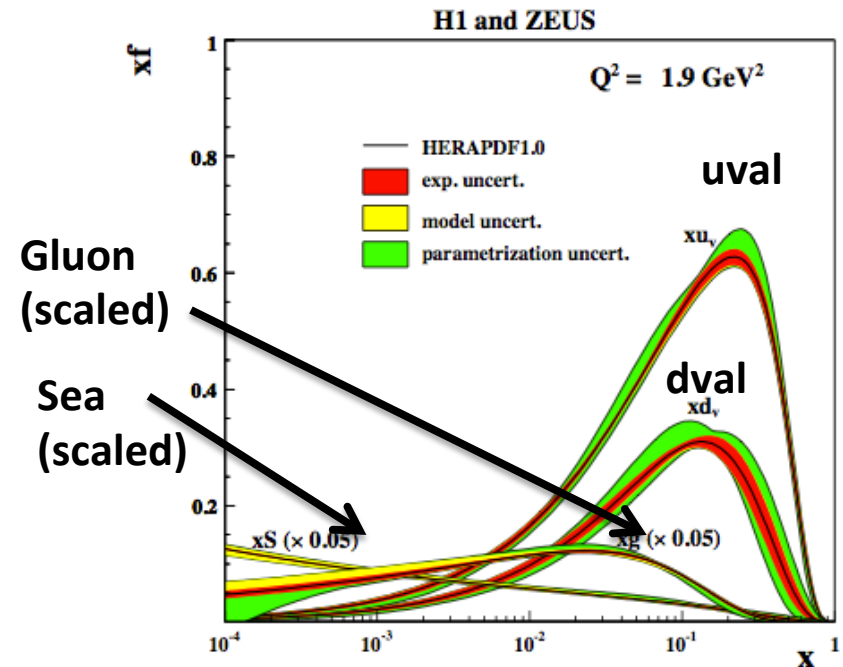


Parton Distribution Functions (PDFs)

The proton has a dynamic structure determined by the resolving power of the process



- In QPM: there is no Q^2 dependence
- In QCD: the Q^2 dependence is determined by the QCD evolution.



- $2xF_1^{\nu(\bar{\nu})} = 2 \left[xq^{\nu(\bar{\nu})} + \bar{x}q^{\nu(\bar{\nu})} \right] \propto \sigma_T$
- $F_2^{\nu(\bar{\nu})} = 2 \left[xq^{\nu(\bar{\nu})} + x\bar{q}^{\nu(\bar{\nu})} + 2xk^{\nu(\bar{\nu})} \right] \propto \sigma_T + \sigma_L$
- $xF_3^{\nu(\bar{\nu})} = 2 \left[xq^{\nu(\bar{\nu})} - x\bar{q}^{\nu(\bar{\nu})} \right]$

QCD features

- ◆ Quantum Chromo Dynamics is theory of strong interactions among quarks and gluons
 - ▶ The charge of the strong interaction is a new quantum number called colour with 3 d.o.f (RGB)
 - ▶ The gauge bosons of the strong interactions are 8 massless gluons with no electric nor weak charge, gluons carry colour charges and are therefore able to self-interact
 - ▶ **The strong interaction is characterised by a strong coupling parameter:**

Characteristics:

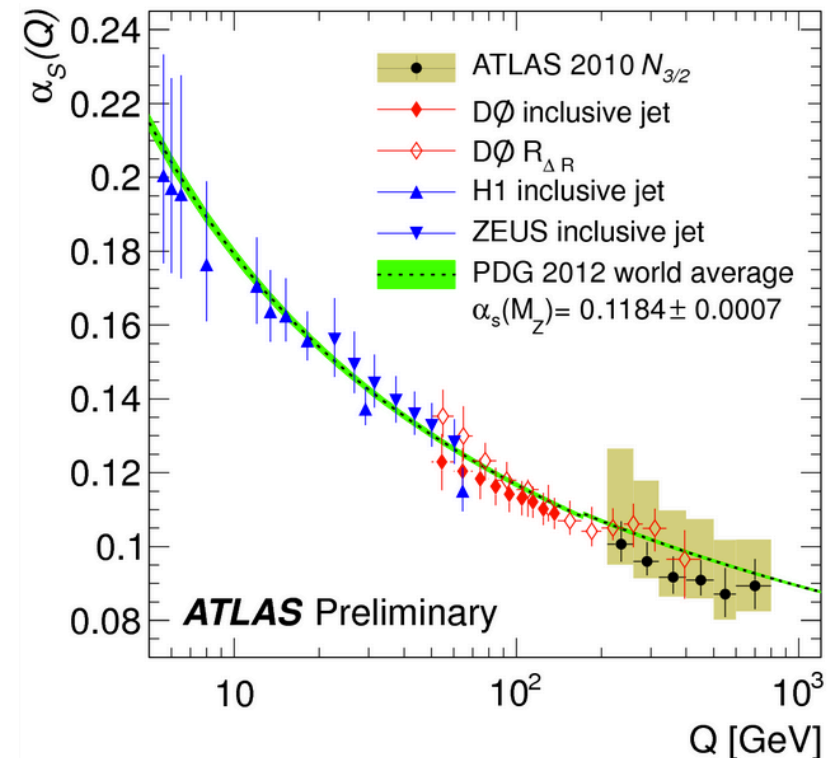
■ Quarks are bound inside protons, strongly coupled, cannot measure directly their distributions: **confinement** (strength at large distance → at low Q)

■ At large scattering scales the coupling of strong force decreases and quarks become quasi-free partons:

asymptotic freedom

(weakness at short distance → at large Q)

- interactions of quarks and gluons at large scales can be calculated perturbatively in running strong coupling.



Renormalisation and running coupling

- ◆ Calculation of a scattering cross section in pQCD reduces to summing over the amplitudes of all possible intermediate states:
 - ▶ 4-momentum conserved at each vertex, however inclusion of loop diagram leads to divergences
 - **Renormalisation method: introducing a scale for which UV divergence is removed**

However any observable (R) should be free of such scale:

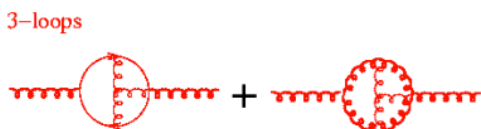
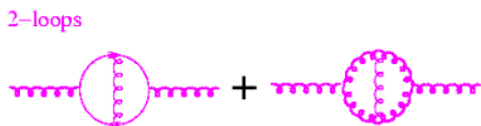
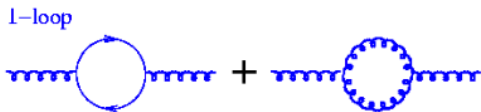
$$\mu \frac{d}{d\mu^2} R\left(\frac{Q^2}{\mu^2}, \alpha\right) = \left(\mu^2 \frac{\partial}{\partial \mu^2} + \mu^2 \frac{\partial \alpha}{\partial \mu^2} \frac{\partial}{\partial \alpha} \right) R = 0$$

This way we obtain the equation for running alpha:

$$t = \log\left(\frac{Q^2}{\mu^2}\right) \text{ and } \beta(\alpha) = \mu^2 \frac{\partial \alpha}{\partial \mu^2}$$

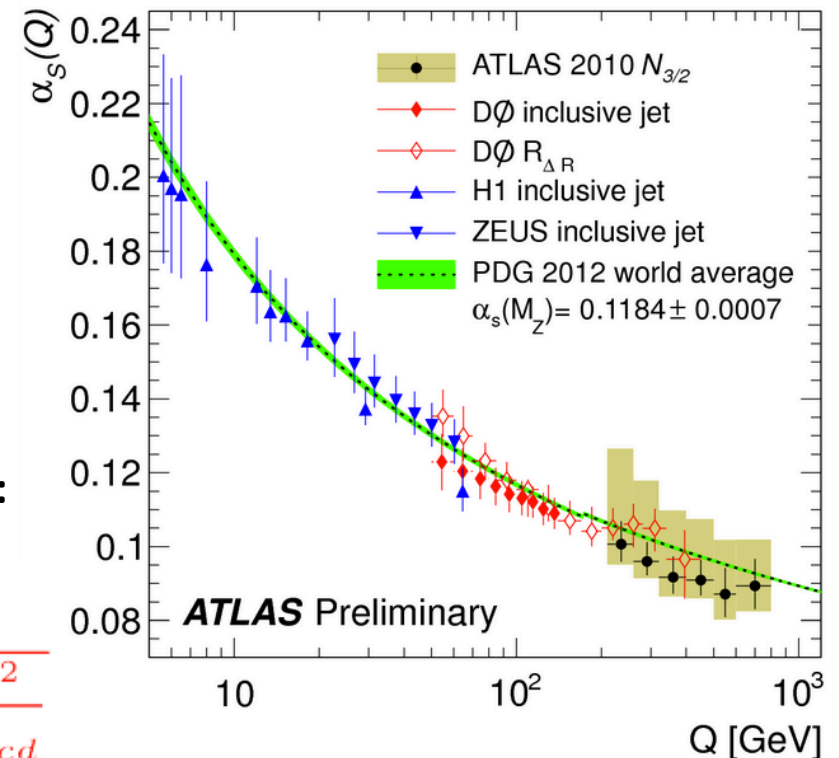
- ▶ Perturbation expansion of beta function:

$$\beta(\alpha_s) = -b\alpha_s^2 (1 + b'\alpha_s + b''\alpha_s^2 + \dots)$$



Running coupling in one loop:

$$\alpha_s = \frac{12\pi}{(33 - 2n_f) \log \frac{Q^2}{\Lambda_{qcd}^2}}$$



Factorisation theorem

Perturbative calculations are performed in context of the factorisation theorem:

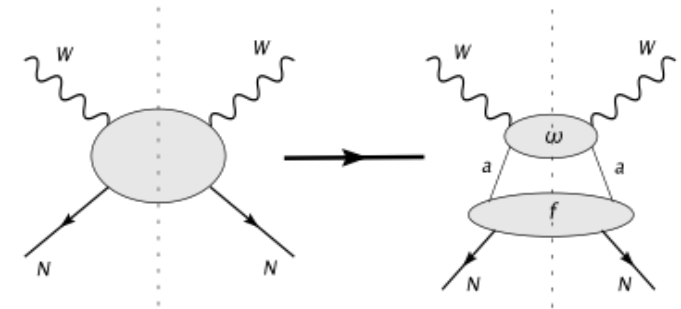
- o extended to the case of heavy quarks [Collins 1998]

Factorisation Theorem: short and long distances processes are separable → introduce μ_f

- ▶ soft part: PDFs – parametrised and determined from data
- ▶ hard part: process dependent - calculable

$$F_2(x, Q^2) \sim \sum_a f_a(x, \mu_f) \otimes \hat{F}_2^a(x, \frac{Q}{\mu_f})$$

physical
PDF
partonic



⇒ Structure Functions (F_i) are a convolution of PDFs (f_a) with hard scattering coefficient function

- ▶ Physical Structure Function is INDEPENDENT of choice of the scale:
 - ✧ both, pdf's and the short-dist. coefficient depend on μ_f (long distance physics)
 - ✧ There is also short distance physics: we can insert perturbative corrections to loops μ_r

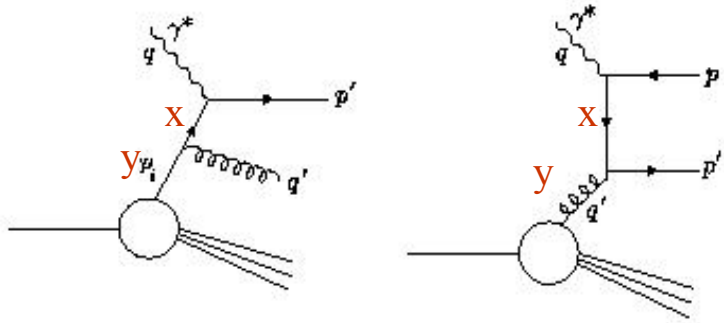
a measurable cross section $d\sigma$ has to be independent of μ_r and μ_f

$$\mu_{r,f} \frac{d\sigma}{d\mu_{r,f}} = \frac{d\sigma}{d \ln \mu_{r,f}} = 0 \quad \rightarrow \quad \text{renormalization group equations}$$

Determination of QCD Evolution equations

- ◆ Illustration of what could happen before the quark is struck

We already stated that physical quantity should be independent of choice of the factorisation scale:



$$y > x, z = x/y$$

now we can compute $\frac{dF_2(x, Q^2)}{d \ln \mu_f} = 0$:

$$\frac{dq(n, \mu_f)}{d \ln \mu_f} \hat{F}_2(n, \frac{\mu_f}{Q}) + q(n, \mu_f) \frac{d\hat{F}_2(n, \frac{\mu_f}{Q})}{d \ln \mu_f} = 0$$



- ◆ Theory can predict the rate at which the parton distributions (both quarks and gluons) evolve with Q^2 -BUT it does not predict their shape at Q^2_0

calculable in pQCD

$$\frac{d}{d \ln \mu} \begin{pmatrix} q(x, \mu) \\ g(x, \mu) \end{pmatrix} = \int_x^1 \frac{dz}{z} \begin{pmatrix} \mathcal{P}_{qq} & \mathcal{P}_{qg} \\ \mathcal{P}_{gq} & \mathcal{P}_{gg} \end{pmatrix} (z, \alpha_s) \cdot \begin{pmatrix} q(x/z, \mu) \\ g(x/z, \mu) \end{pmatrix}$$

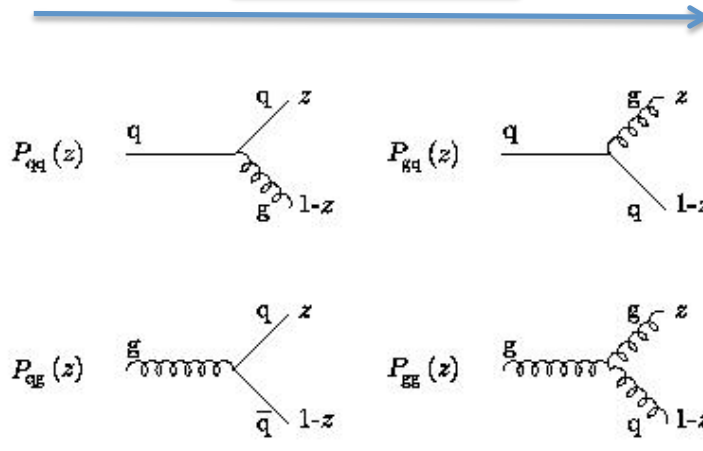
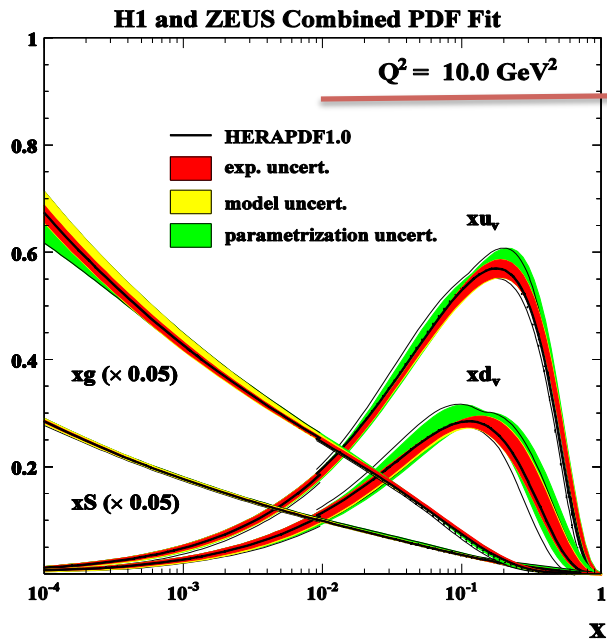
Dokshitzer Gribov Lipatov Altarelli evolution eq.

think of evolution as the effect of increasing the resolution scale

QCD Evolution equations

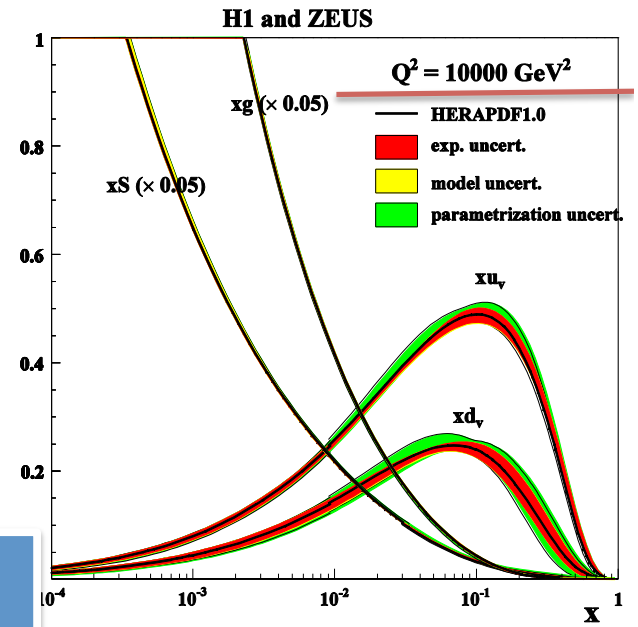
- ◆ Parton momentum distributions change with the scale of the probe:
 - ▶ $Q^2=p^2-E^2 \sim 10 \text{ GeV}^2$ is typical scale for low energy experiments
 - ▶ $Q^2=p^2-E^2 \sim 10000 \text{ GeV}^2$ is the scale that we are now starting to probe at the LHC

Q2 evolution



$y > x, z = x/y$

- Valence quarks are radiating gluons
- Gluons are splitting into quark antiquark pairs



Total momentum carried by the valence quarks is $\sim 0.5 \Rightarrow$ the rest is the gluon and sea quarks.

PDF parametrisation

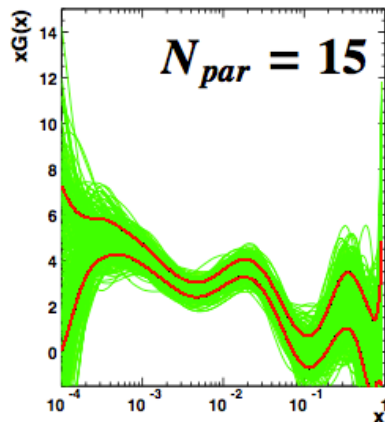
PDFs are parametrised at a starting scale and QCD evolution evolve them to any scale!

$$\begin{aligned}
 xg(x) &= A_g x^{B_g} (1-x)^{C_g} (1 + D_g x + E_g x^2 + F_g \sqrt{x} + \dots) \\
 xu_v(x) &= A_{u_v} x^{B_{u_v}} (1-x)^{C_{u_v}} (1 + D_{u_v} x + E_{u_v} x^2 + F_{u_v} \sqrt{x} + \dots) \\
 xd_v(x) &= A_{d_v} x^{B_{d_v}} (1-x)^{C_{d_v}} (1 + D_{d_v} x + E_{d_v} x^2 + F_{d_v} \sqrt{x} + \dots) \\
 x\bar{u}(x) &= A_{\bar{u}} x^{B_{\bar{u}}} (1-x)^{C_{\bar{u}}} (1 + D_{\bar{u}} x + E_{\bar{u}} x^2 + F_{\bar{u}} \sqrt{x} + \dots) \\
 x\bar{d}(x) &= A_{\bar{d}} x^{B_{\bar{d}}} (1-x)^{C_{\bar{d}}} (1 + D_{\bar{d}} x + E_{\bar{d}} x^2 + F_{\bar{d}} \sqrt{x} + \dots)
 \end{aligned}$$

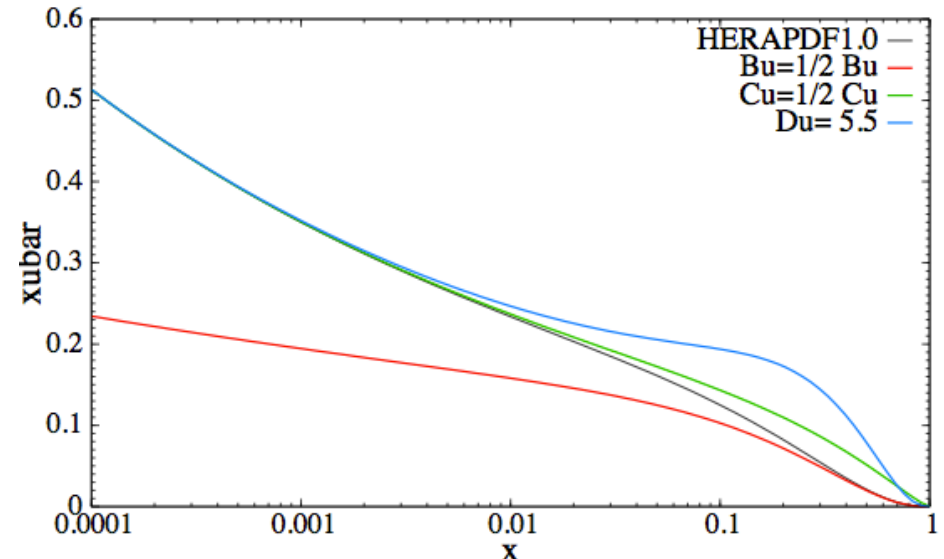
- B → low x behaviour
 - B>0 for valence like shape
 - B<0 for sea
- C → high x behaviour
- D, E, F → interpolate between low and high x

There are many studies done to assess biases due to parametrisation ansatz:

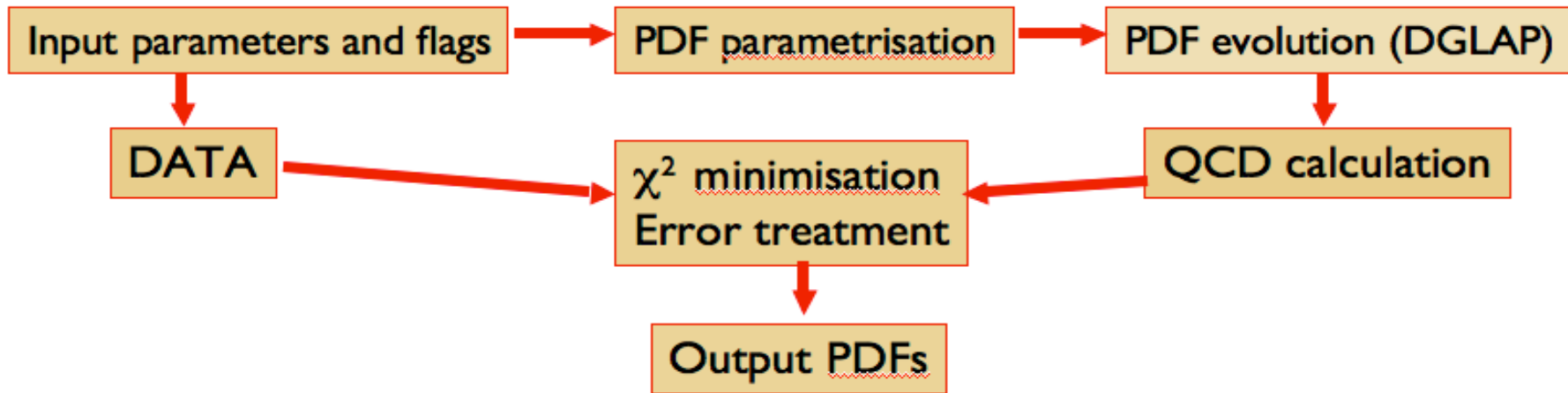
- Neural network PDF: very flexible parametrisation
- Use of Chebyshev Polynomial
 - These flexible parametrisation require though Regularisation Methods to smooth the PDFs



xubar from HERAPDF at starting scale:



Schematics of PDF extraction



PDFs are extracted from QCD fits to double differential cross section data:

- Parametrise PDFs at a starting scale by smooth functions with sufficient parameters;
- Evolve PDFs to other scales by the evolution equations (DGLAP)
- Compute cross sections for DIS (or other processes) at NLO (NNLO)
- Calculate χ^2 measure of agreement between data and theory model
- Obtain the best estimate of the PDFs by varying the free parameters to minimize χ^2

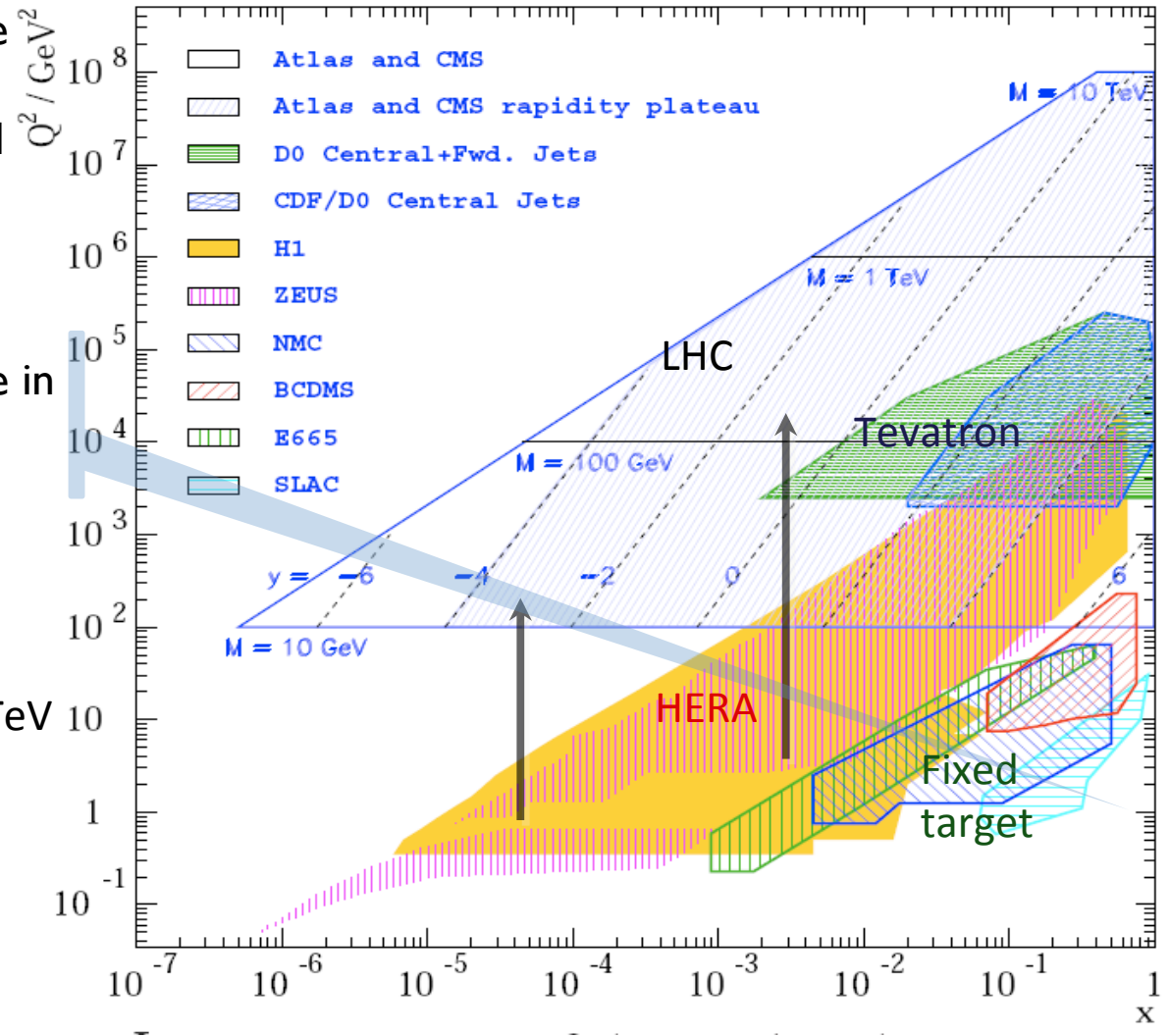
- For tomorrow..

HERAFitter Framework provides means to the experimentalist to assess the impact of measurements

www.herafitter.org

Experimental Data on the Proton Structure

- Persistent experimental effort over the last 40 years both by fixed-target and collider experiments around the world supported by the theoretical developments
 - Large extension in kinematic space in x and Q^2 from the original SLAC measurements
- **DIS experiments may be classified as:**
 - low energy: SLAC, now JLAB
 - medium: BCDMS, NMC, CCFR/NuTeV
 - high energy: HERA



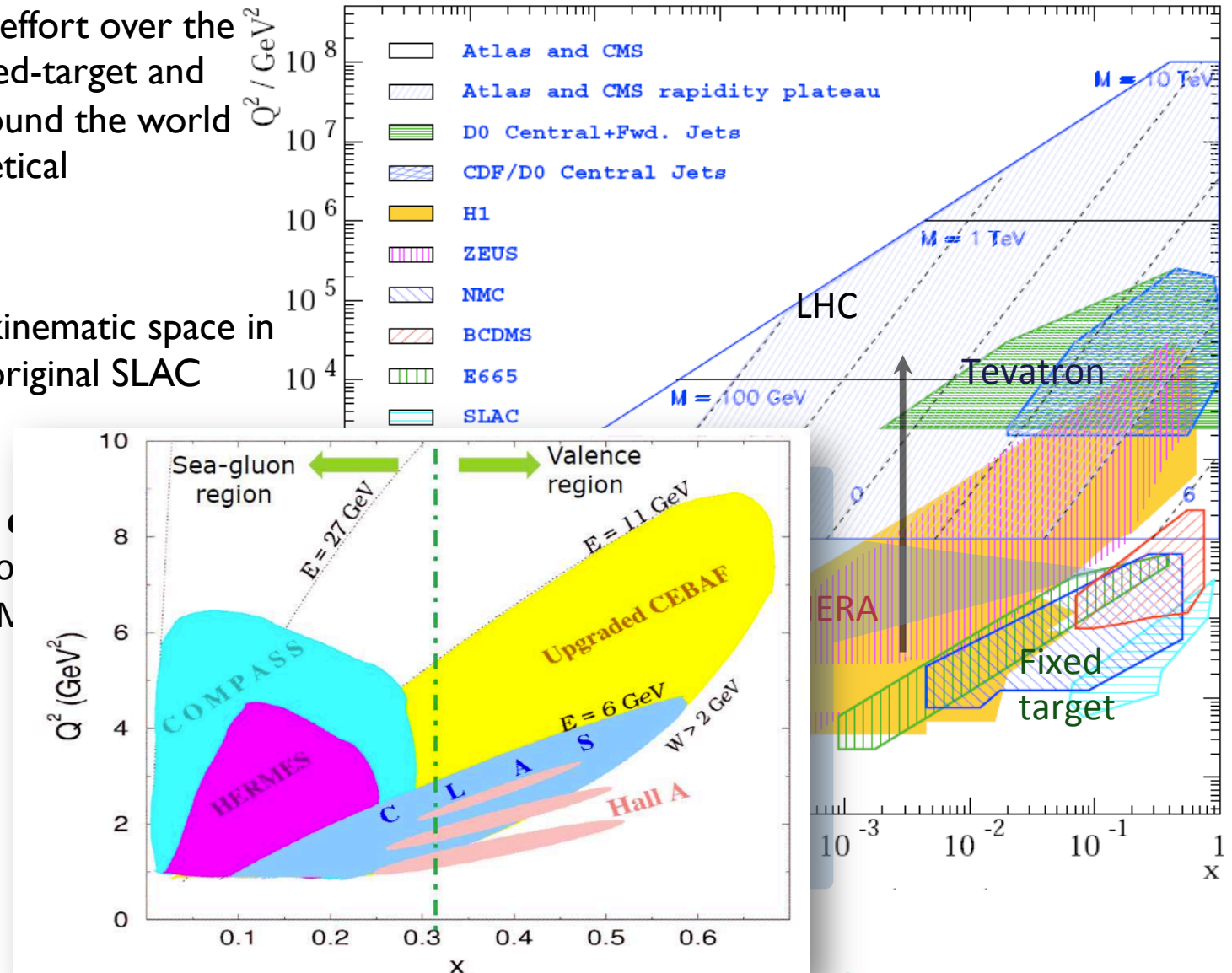
Experimental Data on the Proton Structure

- Persistent experimental effort over the last 40 years both by fixed-target and collider experiments around the world supported by the theoretical developments

- Large extension in kinematic space in x and Q^2 from the original SLAC measurements

- **DIS experiments may be**
 - low energy: SLAC, no
 - medium: BCDMS, NMC
 - high energy: HERA

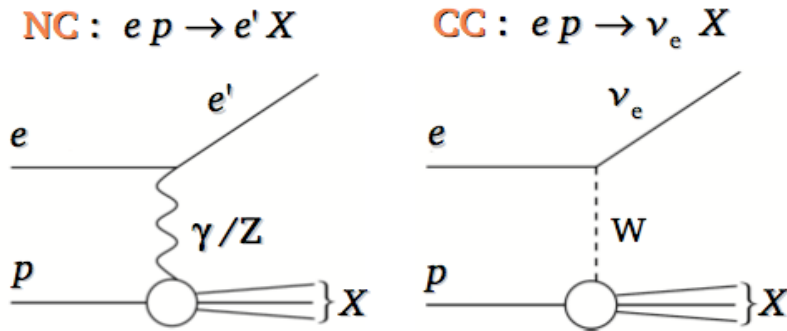
And High X kinematics:
(tomorrow)



HERA Kinematic plane

HERA provides a rich physics program to study DIS

HERA-I	1992-2000	$E_p=820,920$ GeV	$L\sim 110/\text{pb}$ per exp.
HERA-II	2003-2007	$E_p=920, 460,575$ GeV	$L\sim 500/\text{pb}$ per exp.



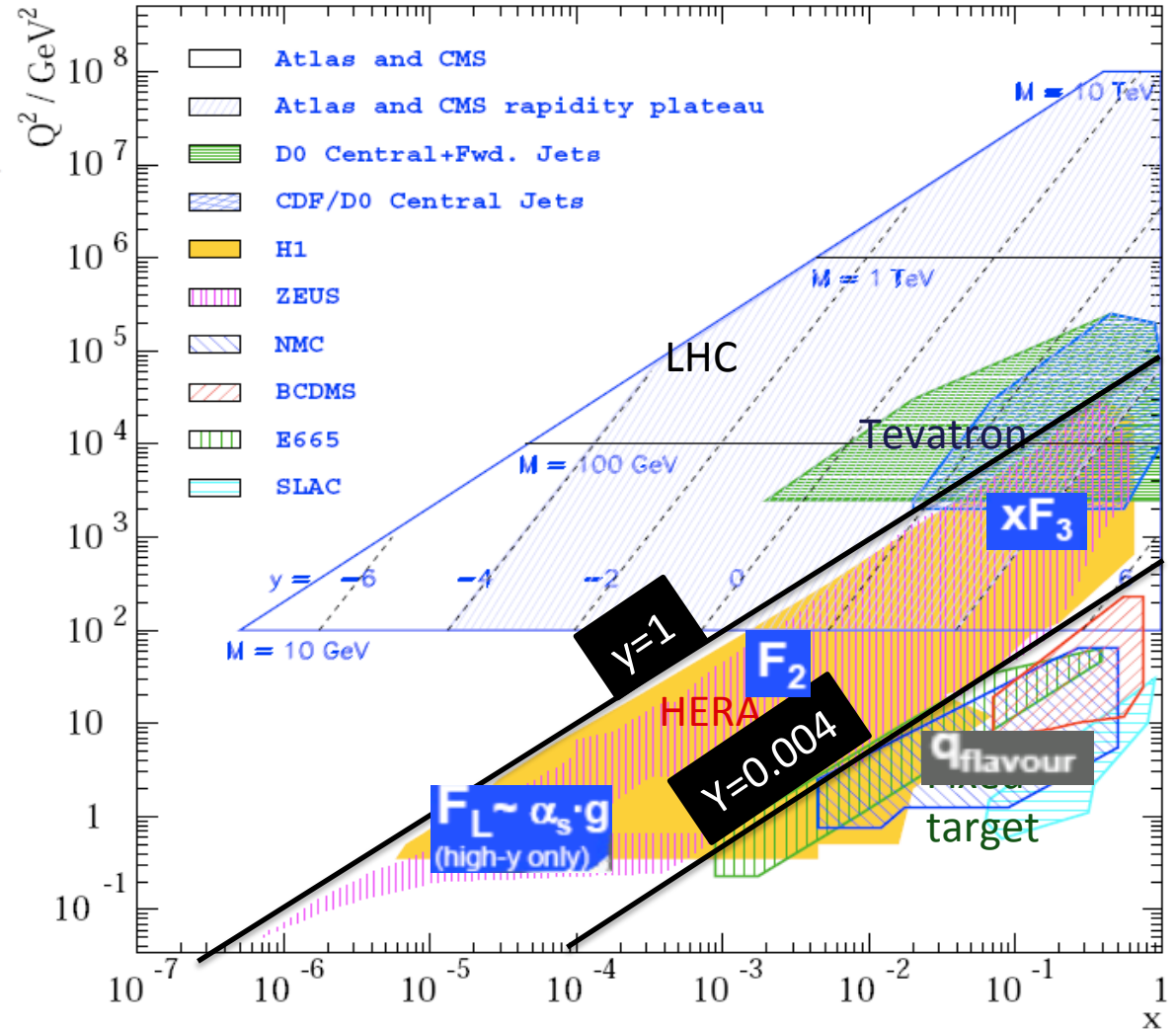
Kinematic limits for HERA

$$E_e = 27.6\text{GeV}, E_p = 920\text{GeV}$$

$$\sqrt{s} = 2\sqrt{E_e E_p} = 319\text{GeV}$$

$$Q^2 = sxy - \text{high}$$

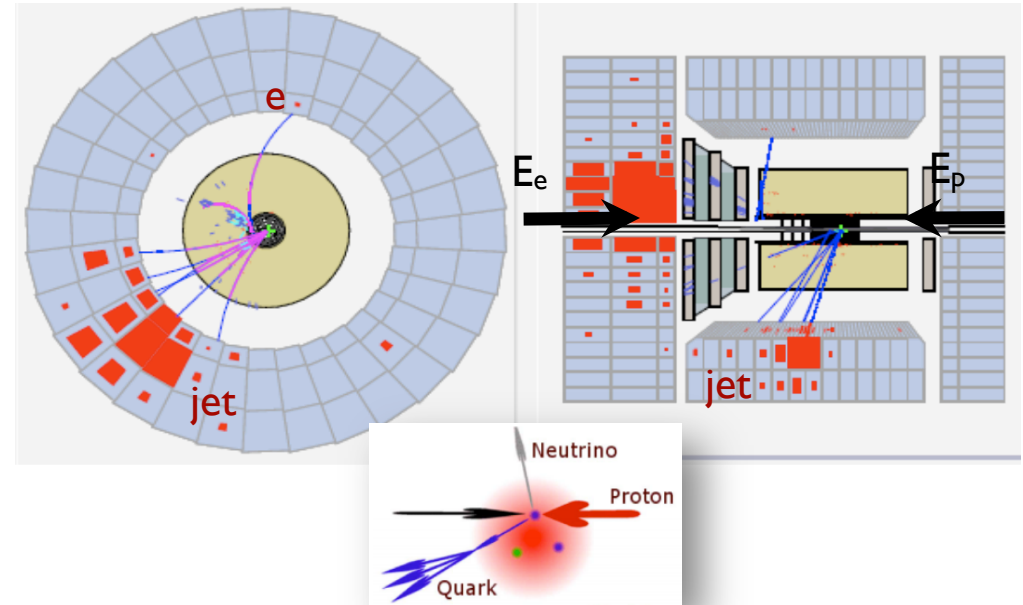
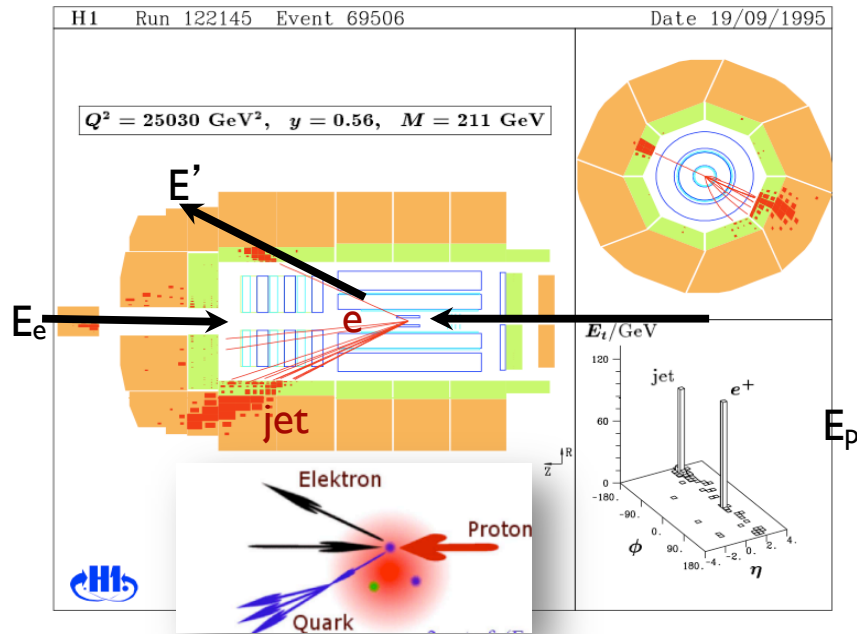
$$x = Q^2 / sy - \text{low}$$



Detector and Kinematics at HERA: NC and CC DIS

- Neutral Current event sample in H1 detector

- Charged Current event sample in ZEUS detector



- Determination of the Event Kinematics:
 - using lepton information (E_e', θ_e)
 - using hadronic final state particles**
 - using both lepton and hadronic final state variables

E_e', θ_e :
 E_h, γ_h :
 θ_e, γ_h :

$$s = 4E_e E_p$$

$$Q^2 = E_e E' (1 + \cos \theta_e)$$

$$y = 1 - \frac{E'}{E_e} \frac{1}{2} (1 - \cos \theta_e)$$

$$x = \frac{Q^2}{su}$$

$$\sum_h E_h - p_{z,h} + E_e' (1 - \cos \theta_e) \approx 2E_e$$

$$y_h = \frac{E_e' (1 - \cos \theta_e)}{2E_e}$$

Redundant reconstruction of the kinematics allows extension of kinematic coverage, extra checks of systematic uncertainties.

Inclusive differential cross sections at HERA

◆ **NC:**

$$\frac{d^2\sigma_{NC}^\pm}{dx dQ^2} \propto \left| \begin{array}{c} e \quad \sim e_e \quad e \\ \quad \quad \gamma \\ q \quad \sim e_q \quad q \end{array} + \begin{array}{c} e \quad \sim(v_e, a_e) \quad e \\ \quad \quad Z^0 \\ q \quad \sim(v_q, a_q) \quad q \end{array} \right|^2$$

$$\frac{d^2\sigma_{NC}^\pm}{dx dQ^2} = \frac{2\pi\alpha^2}{x} \left[\frac{1}{Q^2} \right]^2 \phi_{NC}^\pm(x, Q^2)$$

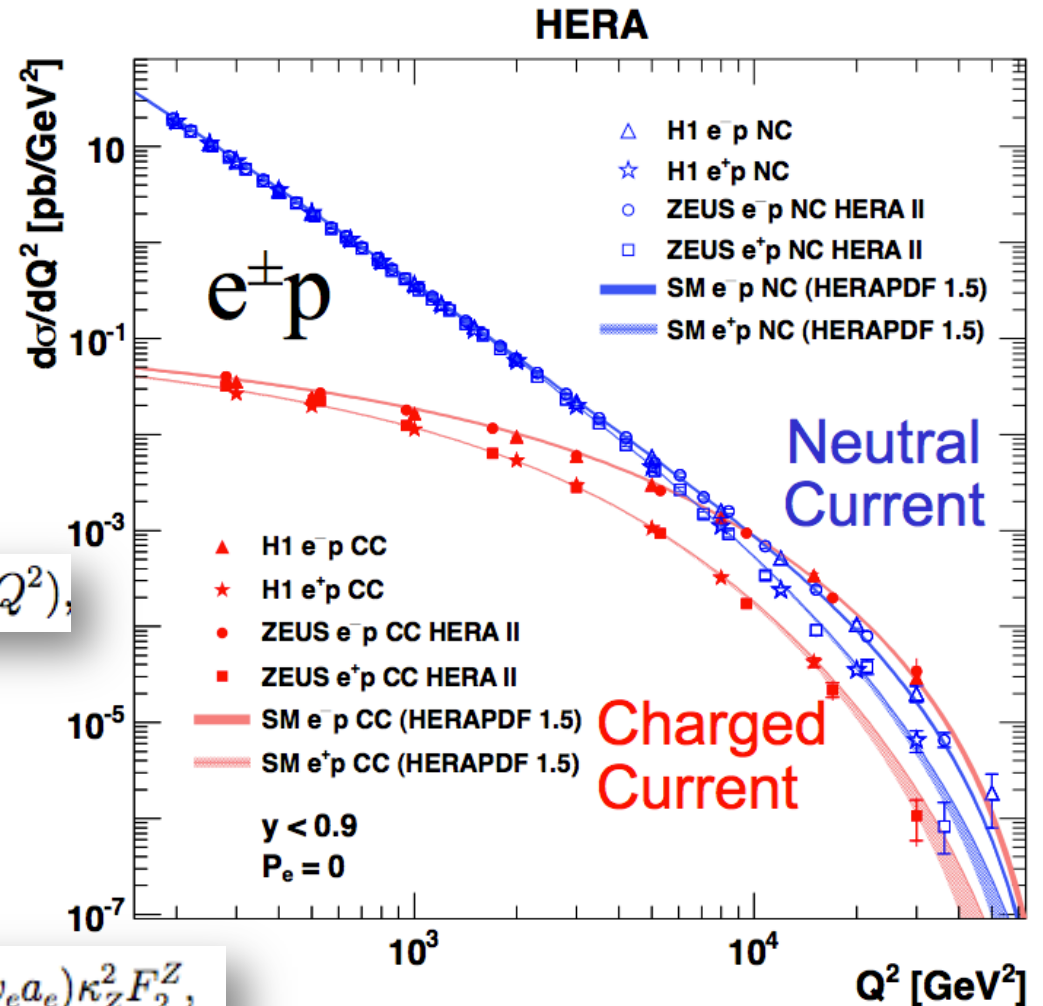
$$\phi_{NC} = Y_+ \tilde{F}_2^\pm(x, Q^2) - y^2 \tilde{F}_L^\pm(x, Q^2) \mp Y_- x \tilde{F}_3^\pm(x, Q^2),$$

$x F_3$ only sensitive at large Q^2 ($\sim M_Z^2$)
 F_L only sensitive at low Q^2 and high y
 F_2 dominates

$$\tilde{F}_2^\pm = F_2 - (v_e \pm P_e a_e) \kappa_Z F_2^{\gamma Z} + (v_e^2 + a_e^2 \pm 2P_e v_e a_e) \kappa_Z^2 F_2^Z,$$

$$x \tilde{F}_3^\pm = -(a_e \pm P_e v_e) \kappa_Z x F_3^{\gamma Z} + (2v_e a_e \pm P_e (v_e^2 + a_e^2)) \kappa_Z^2 x F_3^Z,$$

$$\kappa_Z(Q^2) = \frac{1}{4\sin^2(\theta_W)\cos^2(\theta_W)} \frac{Q^2}{Q^2 + M_Z^2}$$



$$x F_3 \sim \sum (x q_i - x \bar{q}_i)$$

$$F_L \sim \alpha_S g$$

$$F_2 \sim \sum e_i^2 (x q_i + x \bar{q}_i)$$

Electro-Weak Unification

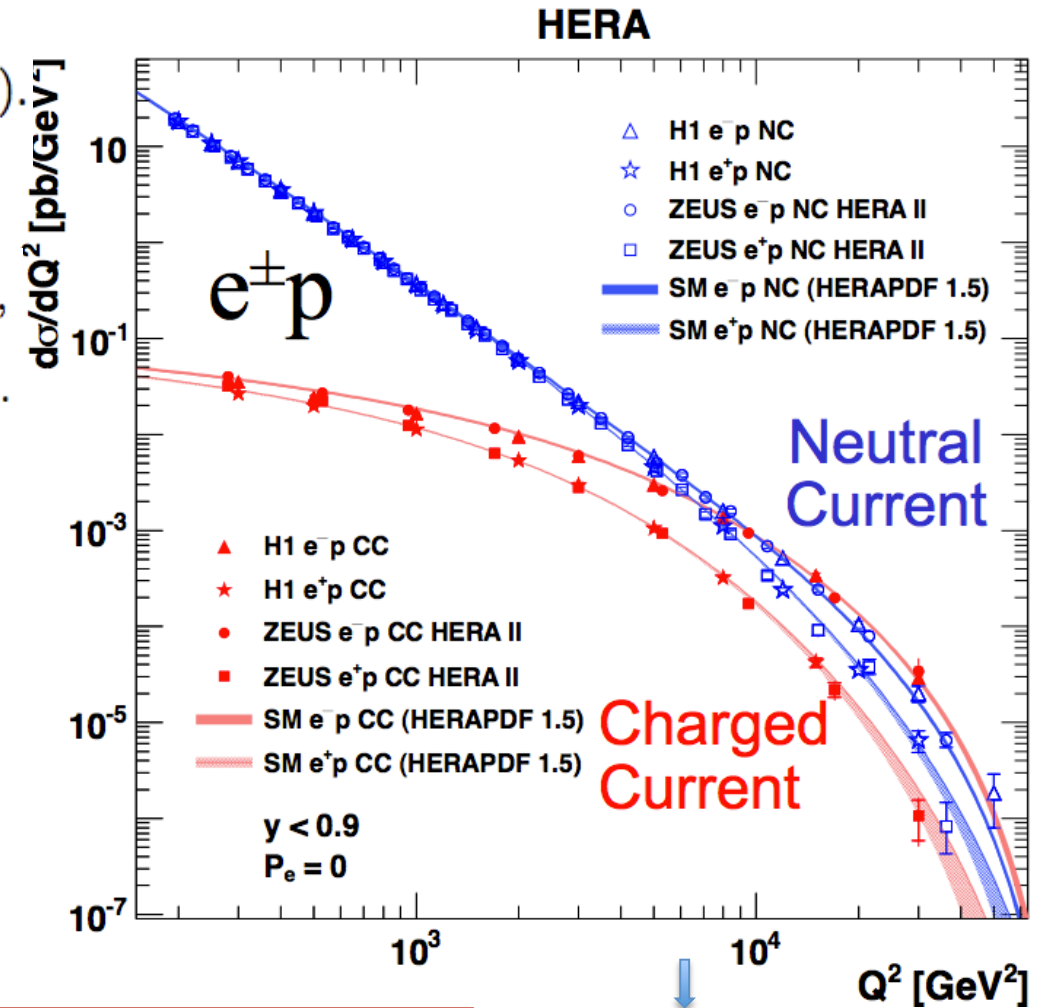
◆ CC:

$$\frac{d^2\sigma_{CC}^\pm}{dx dQ^2} = (1 \pm P_e) \frac{G_F^2}{2\pi x} \left[\frac{M_W^2}{Q^2 + M_W^2} \right]^2 \phi_{CC}^\pm(x, Q^2).$$

$$e^+ : \quad \phi_{CC}^+ = x[(\bar{u}(x) + \bar{c}(x)) + (1 - y)^2(\underline{d(x)} + s(x))],$$

$$e^- : \quad \phi_{CC}^- = x[(\underline{u(x)} + c(x)) + (1 - y)^2(\bar{d}(x) + \bar{s}(x))].$$

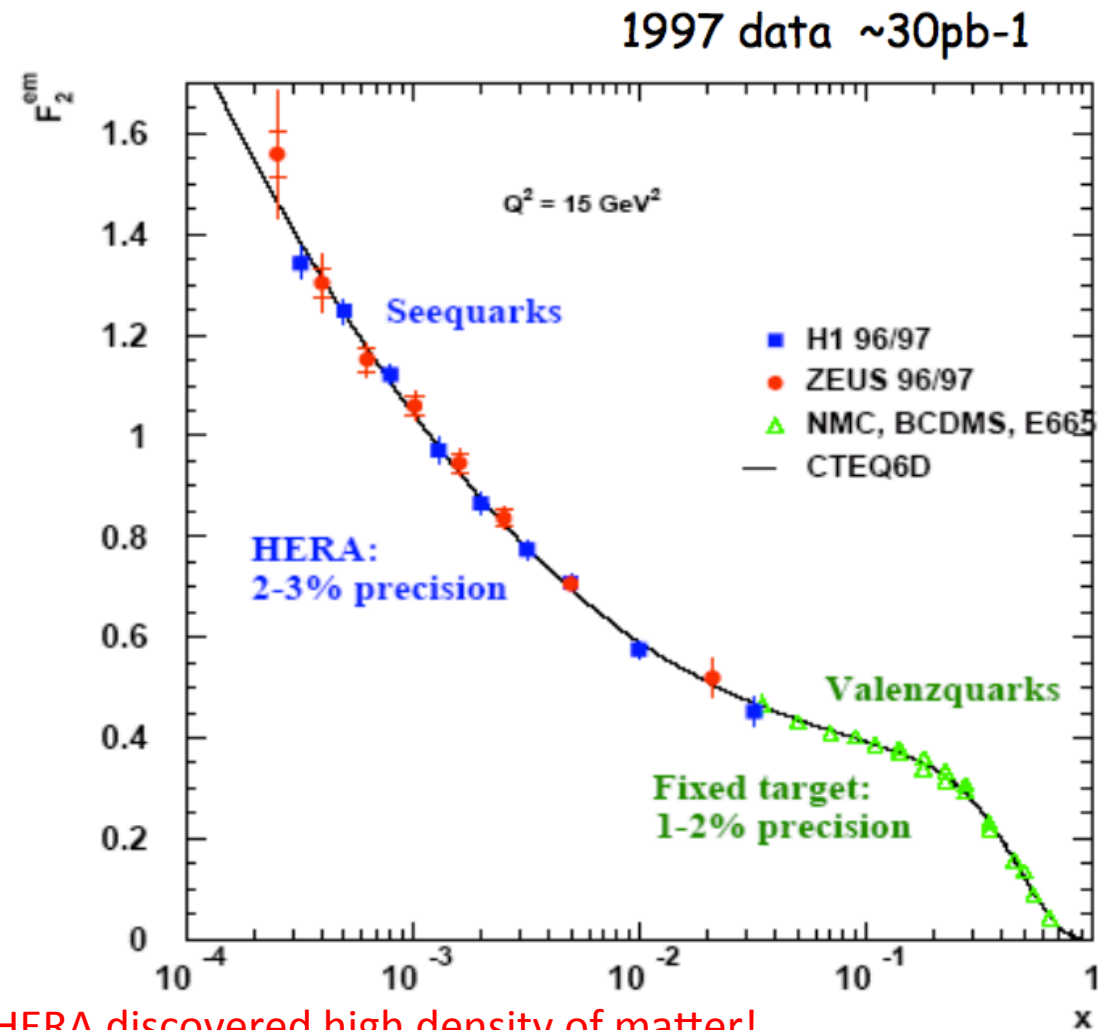
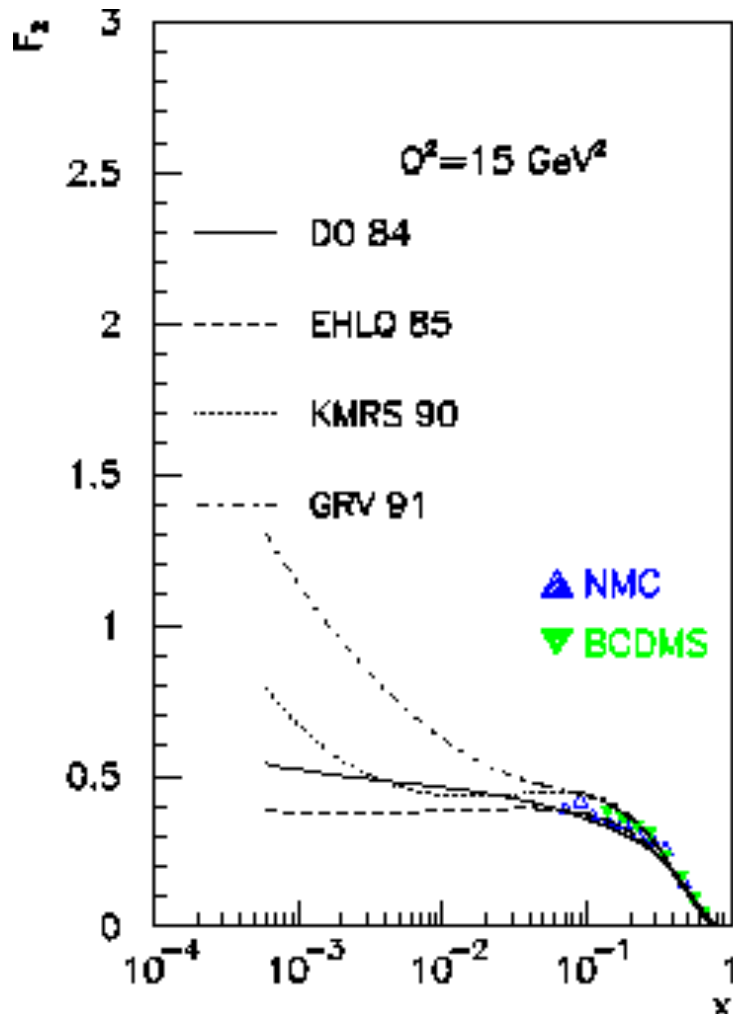
CC $e^-p > CC e^+p$ because W^- is exchanged for CCe^-p which couples to u and is more abundant in the proton



Electro-weak unification is clearly observed at $Q^2 \sim M_Z^2$ $\sigma_{NC} = \sigma_{CC}$

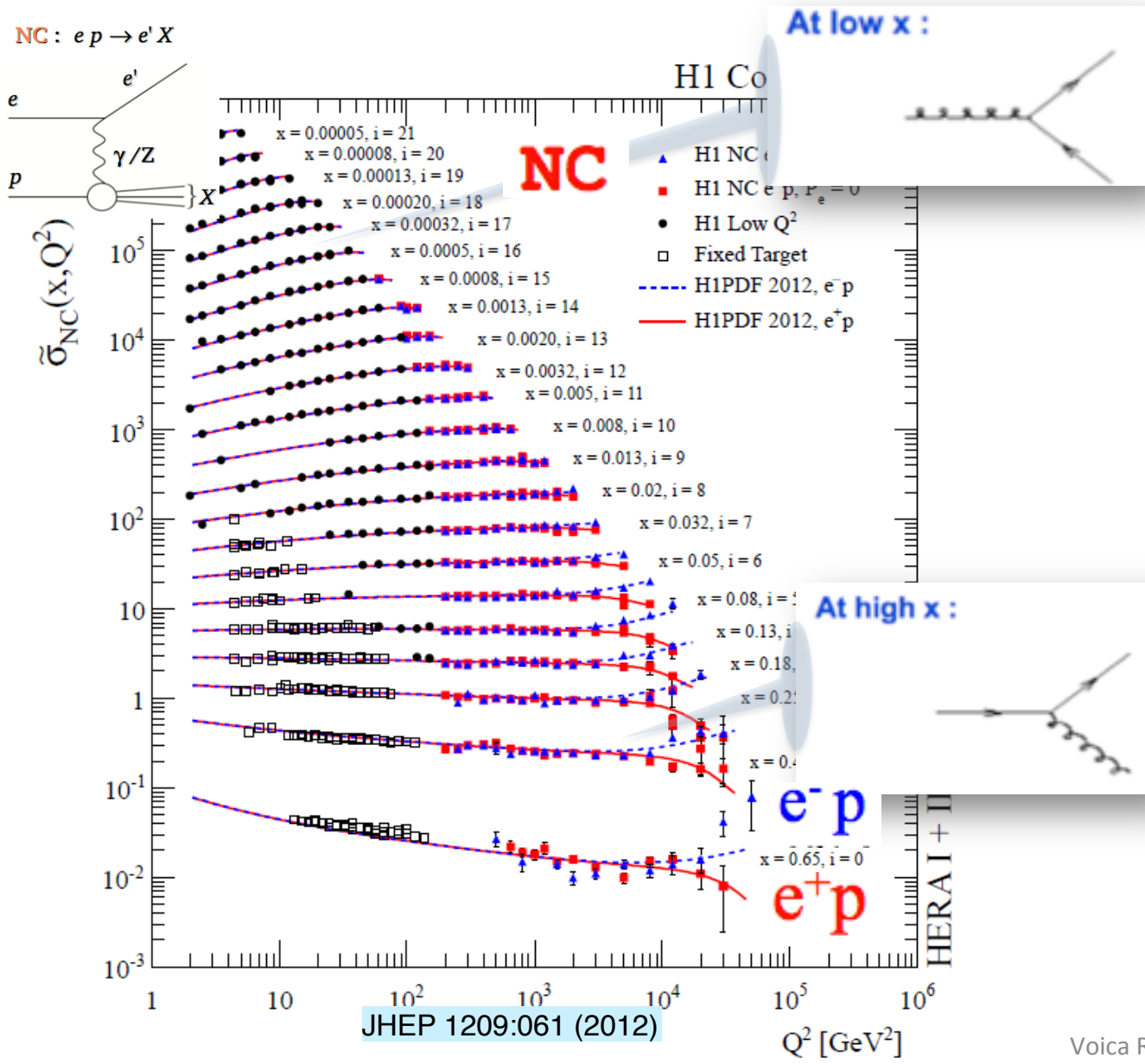
Rise of F_2 at low x seen at HERA

- ◆ Expectations on the density of partons before HERA.... And after HERA (high energy ep)
 - ▶ Before the HERA measurements most of the predictions for low- x were not rising!



HERA discovered high density of matter!

Scaling violations from F_2 at HERA (ep)



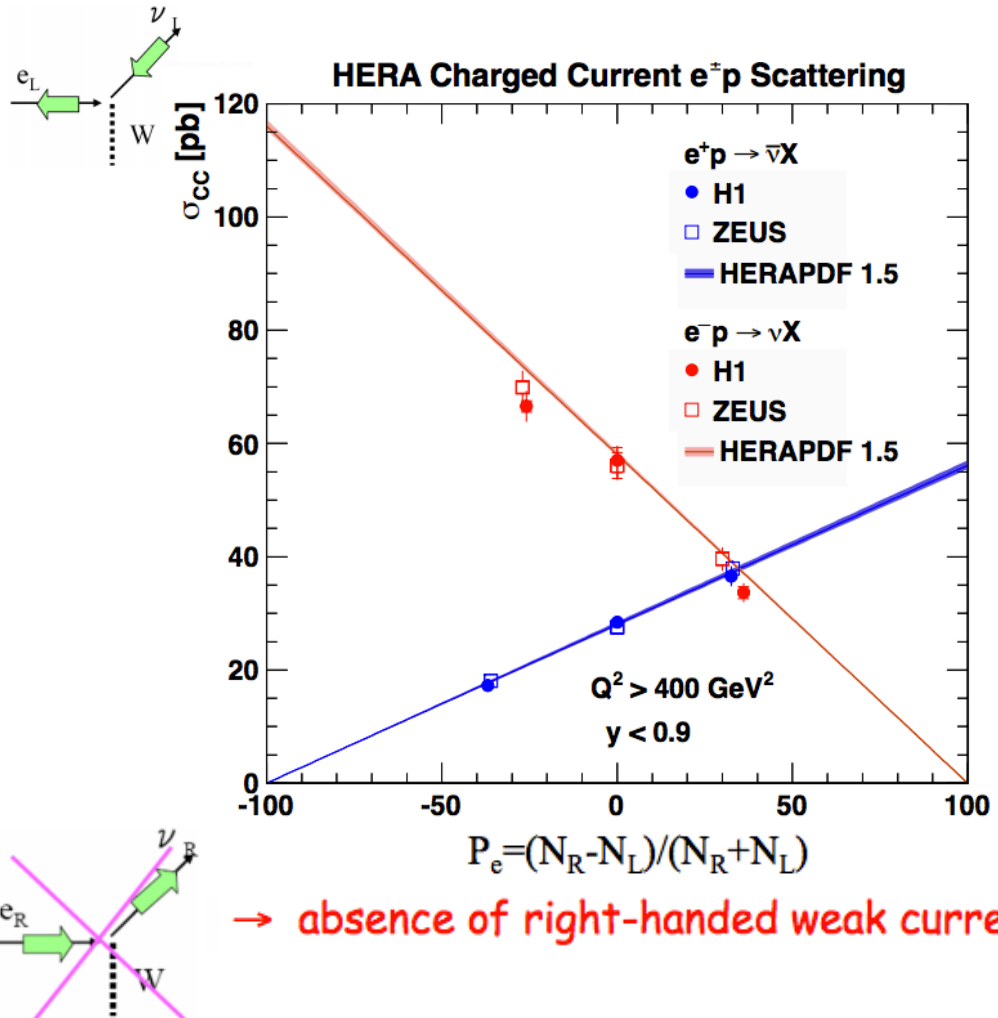
$$\frac{\partial F_2}{\partial \ln Q^2} \propto \alpha_s(Q^2) xg(x, Q^2)$$

$$\frac{\partial F_2}{\partial \ln Q^2} \propto \alpha_s(Q^2) q(x, Q^2)$$

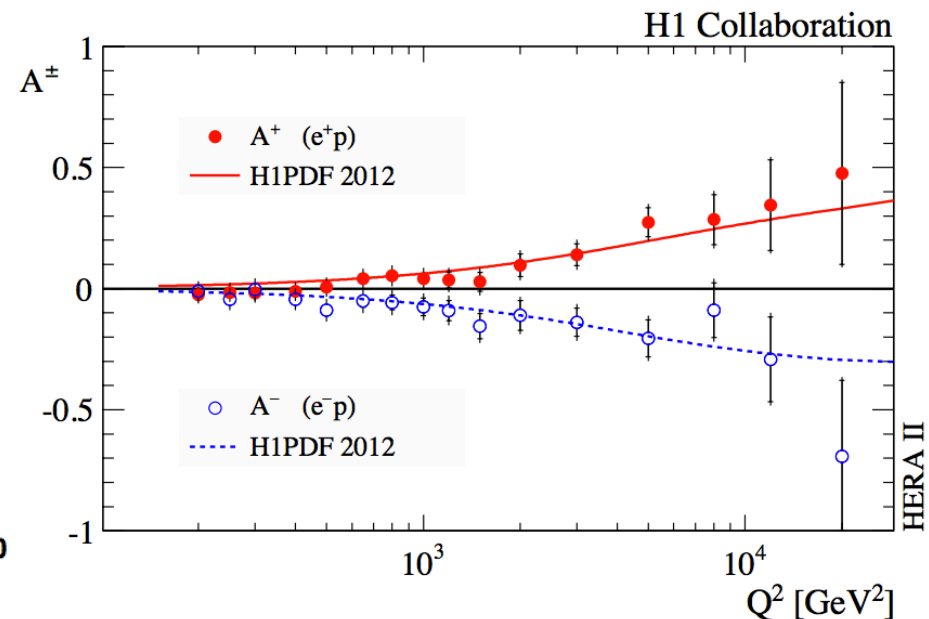
Polarisation effects in CC and NC

- SM predicts that CC cross section vanishes for right-handed electrons and left-handed positrons.

- SM predicts a difference in the NC cross section for leptons with different helicity states arising from the chiral structure of the neutral electroweak exchange



$$A^\pm = \frac{2}{P_L^\pm - P_R^\pm} \cdot \frac{\sigma^\pm(P_L^\pm) - \sigma^\pm(P_R^\pm)}{\sigma^\pm(P_L^\pm) + \sigma^\pm(P_R^\pm)}$$



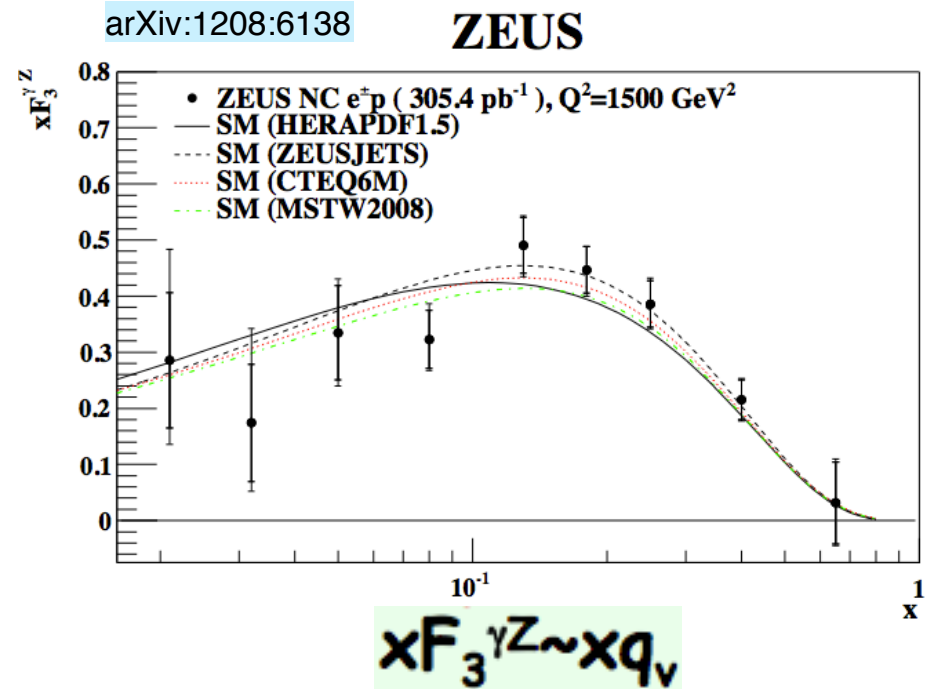
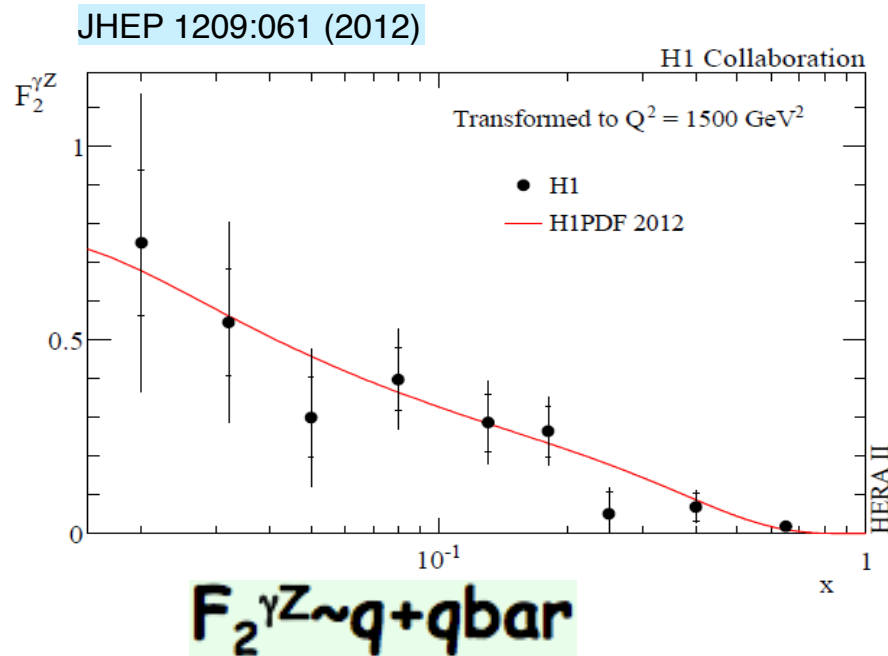
→ a direct measure of parity violation in NC

Measurements of Asymmetries from HERA

- ◆ Explore polarisation asymmetry to extract $F_2^{\gamma Z}$
- ◆ Explore charge asymmetry to extract $xF_3^{\gamma Z}$ (improved measurement from HERA I+II)

$$\tilde{F}_2^\pm \approx F_2 - (v_e \pm P_e a_e) \kappa \frac{Q^2}{Q^2 + M_Z^2} F_2^{\gamma Z}$$

$$\sigma_r^\pm = \tilde{F}_2^\pm \mp \frac{1 - (1 - y)^2}{1 + (1 + y)^2} x \tilde{F}_3 - \frac{y^2}{1 + (1 - y)^2} \tilde{F}_L$$



The shape of the distribution reflects their parton sensitivity

Summary Lecture I

- ◆ **Today have presented the basis of DIS formalism:**
 - ▶ Kinematic variables to describe the process
 - ▶ Differential Cross Section in terms of Structure Functions for different processes
 - ▶ Relation of Structure Functions to PDFs (factorisation theorem)
- ◆ **Some Milestones of Experimental Results:**
 - ▶ Discovery of gluon
 - ▶ Electroweak Unification

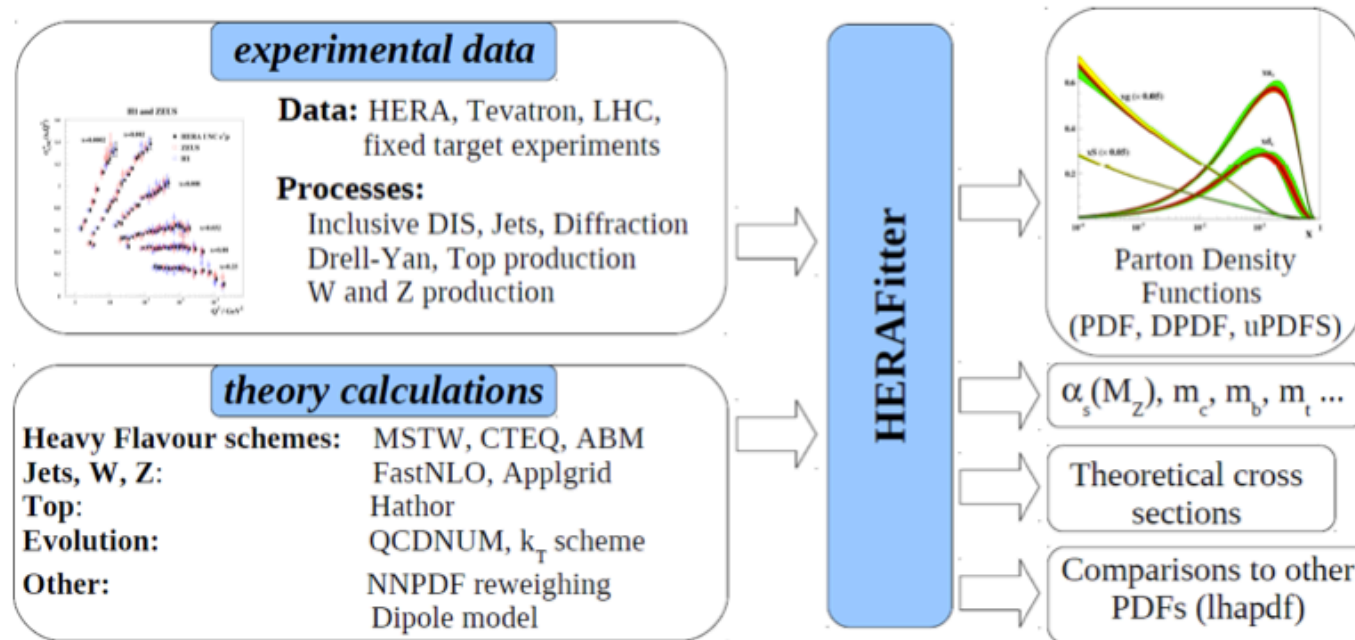
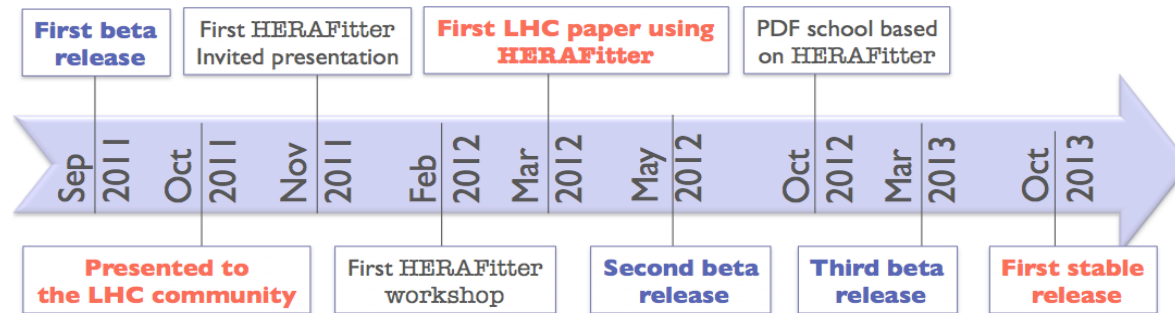
- ◆ **Tomorrow:**
 - ▶ Will continue with more Experimental results
 - ▶ Applicability of DIS measurements: determination of PDFs
 - ✧ importance of precision measurements and what does it involves
 - ▶ From Low x to High x
 - ▶ Relating DIS to LHC
 - ✧ Most recent data sensitive to PDFs
 - ▶ Outlook

HERAFitter QCD platform



Heritage of HERA transferred to LHC:

Open Source QCD Framework freely available at <https://www.herafitter.org>



DIS Cross Sections

- Factorisable nature of interaction: Inclusive scattering cross section is a product of leptonic and hadronic tensors times propagator characteristic of the exchanged particle:

$$\frac{d^2\sigma}{dx dQ^2} \sim \left| \begin{array}{c} \ell(k) \\ N(P) \end{array} \right. \begin{array}{c} \ell(k') \\ V^*(q) \\ X(P_X) \end{array} \left. \right|^2 L^{\mu\nu} W_{\mu\nu}$$

For NC: $V=\gamma, Z, \gamma Z$
For CC: $V=W^+, W^-$

Leptonic tensor: related to the coupling of the lepton with the exchanged boson

- contains the electromagnetic or the weak couplings
- can be calculated exactly in the standard electroweak $U(1) \times SU(2)$ theory.

Hadronic tensor: related to the interaction of the exchanged boson with proton

- can't be calculated, but only be reduced to a sum of structure functions:

$$W^{\alpha\beta} = -g^{\alpha\beta} W_1 + \frac{p^\alpha p^\beta}{M^2} W_2 - \frac{i\epsilon^{\alpha\beta\gamma\delta} p_\gamma q_\delta}{2M^2} W_3 + \frac{q^\alpha q^\beta}{M^2} W_4 + \frac{p^\alpha q^\beta + p^\beta q^\alpha}{M^2} W_5 + \frac{i(p^\alpha q^\beta - p^\beta q^\alpha)}{2M^2} W_6 \quad \sim m_{\text{lepton}}$$

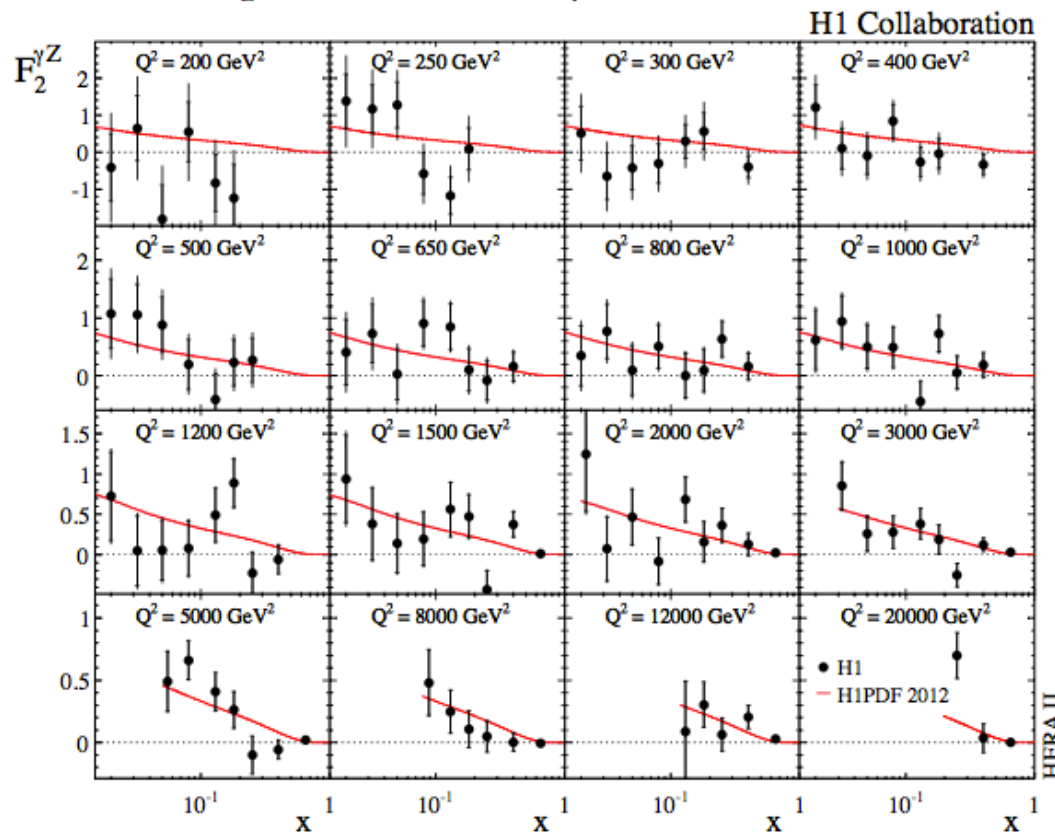
$$\frac{d^2\sigma}{dx dQ^2} = A^i \left\{ \left(1 - y - \frac{x^2 y^2 M^2}{Q^2}\right) F_2^i + y^2 x F_1^i \mp \left(y - \frac{y^2}{2}\right) x F_3^i \right\} \quad A^i: \text{process dependent}$$

The First Measurement of Parity Violating SF $F_2^{\gamma Z}(x, Q^2)$

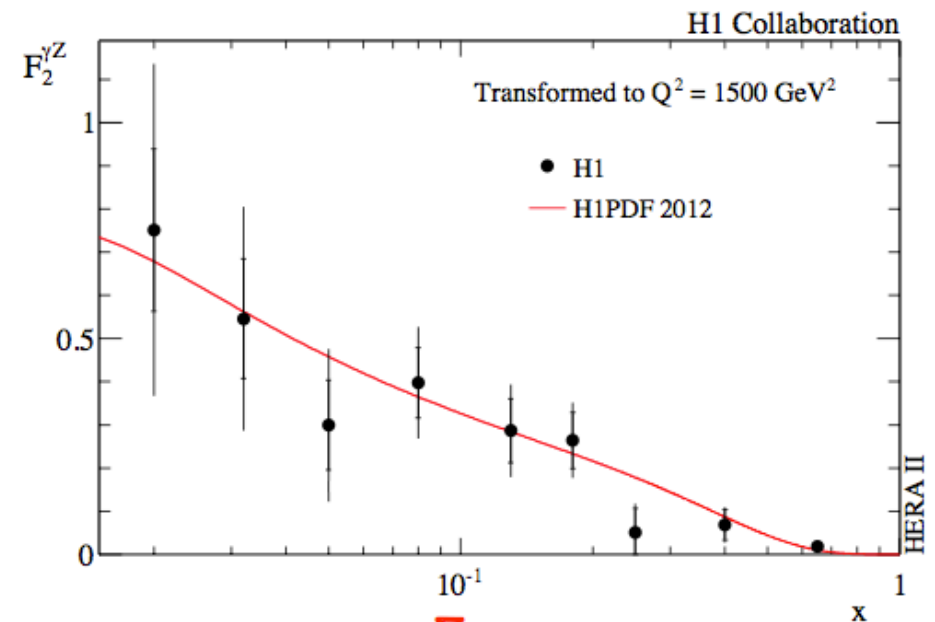
$$\frac{\sigma^\pm(P_L^\pm) - \sigma^\pm(P_R^\pm)}{P_L^\pm - P_R^\pm} = \frac{\kappa Q^2}{Q^2 + M_Z^2} \left[\mp a_e F_2^{\gamma Z} + \frac{Y_-}{Y_+} v_e x F_3^{\gamma Z} - \frac{Y_-}{Y_+} \frac{\kappa Q^2}{Q^2 + M_Z^2} (v_e^2 + a_e^2) x F_3^Z \right]$$

taking the difference for e^+p and e^-p , the terms with $x F_3^{\gamma Z}$ and $x F_3^Z$ cancel and $F_2^{\gamma Z}$ can be directly extracted from measured polarised cross sections

$$\kappa^{-1} = 4 \frac{M_W^2}{M_Z^2} \left(1 - \frac{M_W^2}{M_Z^2} \right)$$



transform the $F_2^{\gamma Z}(x, Q^2)$ measurements to $Q^2 = 1500 \text{ GeV}^2$ and average them to get $F_2^{\gamma Z}(x)$ at $Q^2 = 1500 \text{ GeV}^2$



$$\rightarrow F_2^{\gamma Z} = x \sum [2e_v v_v(q + q\bar{q})]$$

Structure Function $x\tilde{F}_3(x, Q^2)$

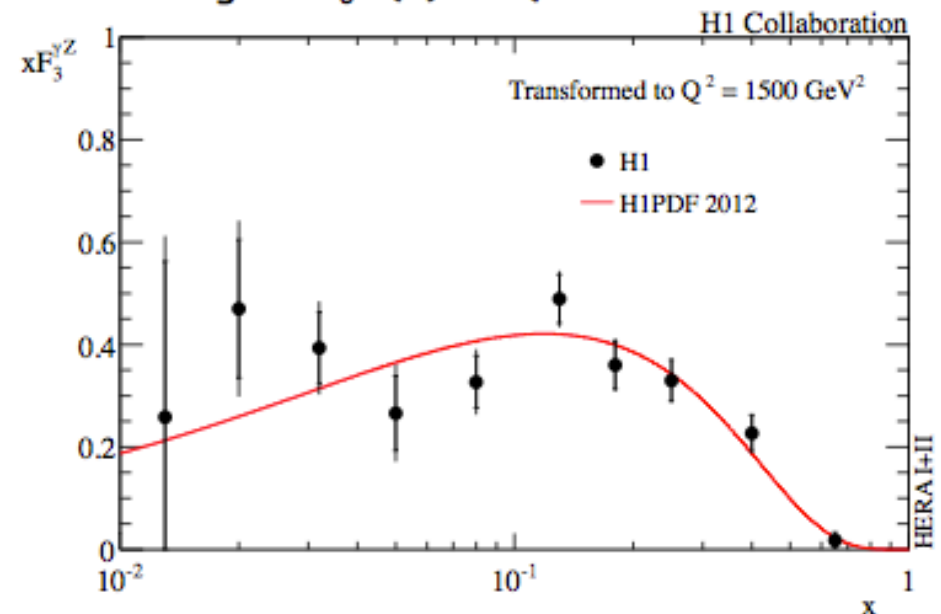
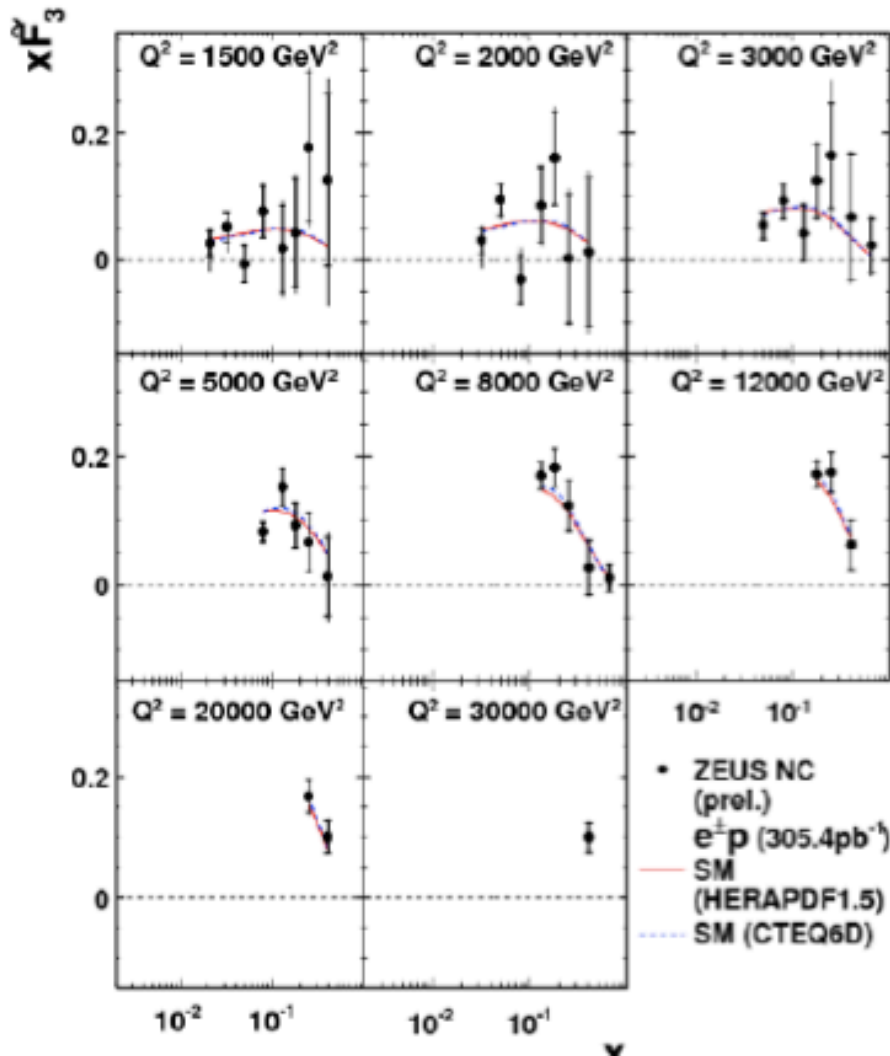
$$x\tilde{F}_3 = \frac{Y_+}{2Y_-} (\tilde{\sigma}_{NC}^- - \tilde{\sigma}_{NC}^+) \quad \text{- charge asymmetry of unpolarised } e^\pm p \text{ NC cross sections}$$

ZEUS

→ mostly due to γZ interference

$$xF_3^{\gamma Z} = -x\tilde{F}_3 \cdot (Q^2 + M_Z^2) / (a_e \kappa Q^2)$$

transform the $x\tilde{F}_3^{\gamma Z}(x, Q^2)$ measurements to $Q^2 = 1500 \text{ GeV}^2$ and average them to get $x\tilde{F}_3^{\gamma Z}(x)$ at $Q^2 = 1500 \text{ GeV}^2$



→ sensitive to valence quark: $F_3^{\gamma Z} \approx (2u_v + d_v)/3$

$$\int_{0.016}^{0.725} dx F_3^{\gamma Z}(x, Q^2 = 1500 \text{ GeV}^2) = 1.22 \pm 0.09(\text{stat}) \pm 0.07(\text{syst})$$

(H1PDF2012: 1.16+0.02-0.03)