Lecture on Deep Inelastic Scattering

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- Part I:
 - Introduction to DIS formalism
 - Physics Results from DIS experiments
- Part II:
 - Relevance of DIS measurements for LHC physics
 - Example of QCD fits and impact of DIS data

Outline

- Yesterday have presented the basis of DIS formalism:
 - Kinematic variables to describe the process
 - Differential Cross Section in terms of Structure Functions for different processes
 - Relation of Structure Functions to PDFs (factorisation theorem)
- Some Milestones of Experimental Results:
 - Discovery of gluon
 - Electroweak Unification

• Today:

- Will continue with more experimental results
- Applicability of DIS measurements: determination of PDFs
 - \diamond importance of precision measurements and what does it involves
- From Low x to High x
- Relation between DIS to LHC
 - ♦ Most recent data sensitive to PDFs
- Outlook

Kinematics of DIS

• Proton can be probed via elementary particles as:

- o neutrinos (fixed target experiments) interact only weakly
- o electrons (fixed target and collider experiments) interact electroweakly
- Deep Inelastic Scattering (DIS) is the cleanest probe to study the substructure of nucleon
 - o scattering of a lepton off the quarks within the proton resulting into a hadronic shower and a lepton
- Kinematic Lorenz Invariant Variables:

$$\begin{split} Q^2 &= -q^2 = -(k-k')^2 \\ \text{Virtuality of the exchanged boson} \\ x &= \frac{Q^2}{2p \cdot q} \\ \text{Bjorken scaling parameter} \\ y &= \frac{p \cdot q}{p \cdot k} \\ \text{Inelasticity parameter} \\ s &= (k+p)^2 = \frac{Q^2}{xy} \\ \text{Invariant c.o.m.} \end{split}$$



e,ν,μ

x, Q²

For ep colider $s = 4E_e E_p$

For fixed target
$$s = 2M_p E_l$$

DIS Cross Section at HERA

General Form for the Differential cross section: $xF_3 \sim \sum (xq_i - x\overline{q}_i)$ $\frac{d^2\sigma}{dxdQ^2} = A^i \left\{ (1 - y - \frac{x^2y^2M^2}{Q^2})F_2^i + y^2xF_1^i \mp (y - \frac{y^2}{2})xF_3^i \right\} \qquad F_L \sim \alpha_S g \\ F_2 \sim \sum e_i^2(xq_i + x\overline{q}_i)$ $CC: e p \rightarrow v_a X$ **NC**: $e p \rightarrow e' X$ Aⁱ: process dependent NC: <u>p</u> _______ >γ/Z $\frac{d^2\sigma_{NC}^{\pm}}{dxdQ^2} = \frac{2\pi\alpha^2}{x} \left[\frac{1}{Q^2}\right]^2 \phi_{NC}^{\pm}(x,Q^2)$ =}**x** $\phi_{NC} = Y_{\pm} \tilde{F_2^{\pm}}(x, Q^2) - y^2 \tilde{F_L^{\pm}}(x, Q^2) \mp Y_{-} x \tilde{F_3^{\pm}}(x, Q^2),$ $\tilde{F_2^{\pm}} = F_2 - (v_e \pm P_e a_e) \kappa_Z F_2^{\gamma Z} + (v_e^2 + a_e^2 \pm 2P_e v_e a_e) \kappa_Z^2 F_2^Z, \quad \kappa_Z(Q^2) = \frac{1}{4sin^2(\theta_W)cos^2(\theta_W)} \frac{Q^2}{Q^2 + M_Z^2}$ $xF_3^{\pm} = -(a_e \pm P_e v_e)\kappa_Z xF_3^{\gamma Z} + (2v_e a_e \pm P_e (v_e^2 + a_e^2))\kappa_Z^2 xF_3^Z,$ CC: At LO $\frac{d^2 \sigma_{CC}^{\pm}}{dx dQ^2} = (1 \pm P_e) \frac{G_F^2}{2\pi x} \left[\frac{M_W^2}{Q^2 + M_W^2} \right]^2 \phi_{CC}^{\pm}(x, Q^2). \quad \begin{array}{l} e^+ : \quad \phi_{CC}^+ = x[(\bar{u}(x) + \bar{c}(x)) + (1 - y)^2(d(x) + s(x))]_{\bar{c}} \\ e^- : \quad \phi_{CC}^- = x[(u(x) + c(x)) + (1 - y)^2(\bar{d}(x) + \bar{s}(x))]_{\bar{c}} \\ \end{array}$

DIS Kinematic plane of HERA

• A vast extension of the kinematic range: both to high Q2 and to low-x.



Inclusive Cross Sections



Mechanism of extracting PDFs



- Parametrise PDFs at starting scale
 - ♦ Observables are sensitive to different linear combinations of PDFs
 - ♦ For the inclusive $ep \rightarrow eX$ scattering, the dominant contribution is given by the structure function F2
- Evolve to the scale corresponding to data point
- Calculate the cross section
- Compare with data
- Minimize with respect to PDF parameters
- a full coverage of kinematic plane in x is needed
- Fast calculations are needed (employ grid techniques)
- → www.herafitter.org

PDF determination at HERA

- HERA PDFs are determined from QCD Fits to solely HERA data
 - NLO (and NNLO) DGLAP evolution equations, RT-VFNS (as for MSTW08) for dealing with heavy quarks

 \diamond Other schemes were investigated as well: RT (optimal), ACOT (full and χ), FFNS

The QCD settings are optimised for HERA measurements of proton structure functions

$$F_2(x,Q^2) = rac{4}{9}(xU+x\overline{U}) + rac{1}{9}(xD+x\overline{D})$$

- ▶ PDF parametrised at the starting scale Q_0^2 : $xg, xu_{val}, xd_{val}, x\bar{U} = x\bar{u}(+x\bar{c}), x\bar{D} = x\bar{d} + x\bar{s}(+x\bar{b})$
 - Simple Functional form: $xq_i(x) = A_i x^{B_i} (1-x)^{C_i} P_i(x)$
- A normalisation
- B low x behaviour
- C high x behaviour
- D,E medium x tuning
- Where Pi(x) are polynomials in powers of x and only terms that bring significant improvement to the fit quality are retained
- QCD sum rules:

$$\int_0^1 dx \cdot (xu_v + xd_v + x\bar{U} + x\bar{D} + xg) = 1$$

$$dx\cdot 2u_v=2$$
 $\int_0^1 dx\cdot d_v=1$

Additional Constraints:

$$x\overline{s} = f_s x\overline{D}$$
 strange sea is a fixed fraction f_s of \overline{D} at Q_0^2
 $B_{Ubar} = B_{Dbar}$
sea = 2 x (Ubar +Dbar)
Ubar = Dbar at x=0

Impact of data on PDFs – simple exercise

 A visualisation of the impact of CC ep data is shown below (using HERA I data): (only size of uncertainties are relevant in this exercise)



 Errors assigned to the data points translate into errors assigned to the fit parameters and thus to errors on the parton distribution functions (PDFs)

Chi square definition

- Typical measurements sensitive to PDFs are precise, with statistical uncertainties < 10%, so it follows normal distribution which allows use of chi square minimization for determining optimal PDF parameters.
- The simplest situation is when the errors are Gaussian and there are no correlations among different data points:

$$\chi^2(p_k) = \sum_i \left(\frac{m_i(p_k) - \mu_i}{\Delta_i}\right)^2$$

 p_k are parameters describing PDFs $m_i(p_k)$ are predictions

- The measurements are, however, correlated with each other:
 - ♦ Bin-to-bin correlations due to systematic uncertainties are larger than statistical correlations
 - > A convenient way to represent systematic correlations is by using nuisance parameters:
 - Influence of correlated uncertainty sources j on data points i can be described by a matrix rij such that if a source moves up by 1o all data points move by rij

$$\chi^2_{exp}(\mathbf{m}, \mathbf{b}) = \sum_i \frac{\left(m_i - \mu_i - \sum_j \Gamma^j_i b_j\right)^2}{\Delta_i^2} + \sum_j b_j^2.$$

 b_j are nuisance parameters corresponding to correlated systematic $\chi^2_{min}/N_{D,F} \approx 1$ if model is consistent with the data.

Data Combination

[JHEP01 (2010) 109]

- Typically several experiments provide their data in a similar kinematic phase space
 - One could combine them to gain ultimate precision
- The combination procedure is performed using χ² minimisation, in the same fashion as done when comparing data to theoretical models:
 Constant term → tests consistency of combination

$$\chi^2_{exp,1}(\mathbf{m}, \mathbf{b}) + \chi^2_{exp,2}(\mathbf{m}, \mathbf{b}) \equiv \chi^2_0 + \chi^2_{ave}(\mathbf{m}, \mathbf{b}')$$

 Combination of measurements provides consistency check of the measurements in a model independent way.

For example, at HERA:

Improvement of Statistical precision:

- H1 and ZEUS collected similar amounts of physics data.
- Improvement of Systematic precision:
 - H1 and ZEUS are different detectors and use different analysis techniques;
 - The H1 and ZEUS cross sections have different sensitivities to similar sources of correlated systematic uncertainty: can take best features from eachother

https://wiki-zeuthen.desy.de/HERAverager

Combined HERA-I Data



Combination of data is now actively used at LHC for ex W, Z for muon and electron channels Voica Radescu | DESY 🙀 | CTEQ 2013 DIS

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Combined HERA-I Data



Combination of data is now actively used at LHC for ex W, Z for muon and electron channels

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Modern Understanding of PDF

- At HERA PDF can be determined solely from HERA data:
 - use consistent data set: H1+ZEUS
 - proper treatment of error correlations



Gluons and sea quarks: dominant partons at low x

A summary of PDF sets

Table shows a summary of the current status of available PDFs (not included the high x PDFs)

	MSTW08	CTEQ6.6/CT10	NNPDF2.0/2.1	HERAPDF1.0/1.5	ABKM09/ABM11	GJR08/JR09
PDF order	LO, NLO, NNLO	LO, NLO, NNLO	LO, NLO, NNLO	NLO, NNLO	NLO, NNLO	NLO, NNLO
HERA DIS	🖌 (old)	✔ (old/new)	🖌 (new)	✔ (new/newest)	✔ (new)	🖌 (new)
Fixed target DIS	~	v	v	-	v	~
Fixed target DY	~	v	v	-	v	v
Tevatron W, Z	~	v	v	-	-	-
Tevatron jets	~	v	v	-	v	~
HF Scheme	RTGMVF	SACOT GMVFN	FONLL GMVFN	RT GMVFN	BMSN FFNS/FFNS	FFNS
Alphas (NLO)	0.120	0.118(f)	0.119	0.1176(f)	0.1179	0.1145
Alphas (NNLO)	0.1171	0.118(f)	0.1174	0.1176(f)	0.1147	0.1124

The analyses differ in many areas:

- different treatment of heavy quarks
- inclusion of various data sets and account for possible tensions
- different alphas assumption

http://mstwpdf.hepforge.org/pdf4lhc/

Some commonly used plots from G. Watt

Additional Constraints to PDFs: on gluon

- Sensitivity to gluon PDF arise from the coupled singlet-gluon QCD evolution
 - scaling violation:

$\frac{\partial F_2}{\partial \log Q^2} \sim \alpha_S x g(x). \quad F_2 \sim (\sigma_T + \sigma_L).$

- And it can be cross checked with other measurements:
 - ▶ From the inclusive structure function *FL*: pure QCD effect

$$F_{L} = \frac{\alpha_{s}}{4\pi} x^{2} \int_{x}^{1} \frac{dz}{z^{3}} \left[\frac{16}{3} F_{2} + 8 \sum_{q} e_{q}^{2} (1 - \frac{x}{z}) zg(z) \right]$$

quarks gluons
radiating a gluon splitting into quarks

$$F_L \sim \sigma_L$$

- ightarrow Interesting case also for study physics at low Q2
 - ♦ heavy flavour schemes, evolutions, higher twists
- From the measurement of semi-inclusive structure function *Fc*
- From the inclusive jet cross section (for ep and pp collisions) :
 - \diamond Able to decorrelate gluon and strong coupling

Measurement of Longitudinal Structure Function (1)

Direct measurement of F_{L} at HERA requires differential cross sections at same **x** and **Q**² but different **y**:

- larger difference in y, better sensitivity to FL
- Q²=xys → different y means different s(CME) → different beam energies Ep = 460, 575, 920 GeV

$$\sigma_{NC}(x,Q^2,y) \propto F_2(x,Q^2) - \frac{y^2}{1+(1-y)^2}F_L(x,Q^2)$$

• Reduced proton beam energy runs at the end of HERA operation were dedicated to measure F_{L} .

At given x and Q^2 : $\rightarrow F_2$ is the intercept at y-axis $\rightarrow F_L$ is the negative slope



Measurement of Longitudinal Structure Function (2)

Direct measurement of F_{L} at HERA requires differential cross sections at same **x** and **Q**² but different **y**:

- ▶ larger difference in y, better sensitivity to FL
- ▶ $Q^2 = xys \rightarrow different y$ means different s(CME) $\rightarrow different beam energies Ep = 460, 575, 920 GeV$

$$\sigma_{NC}(x,Q^2,y) \propto F_2(x,Q^2) - \frac{y^2}{1+(1-y)^2}F_L(x,Q^2)$$

• Reduced proton beam energy runs at the end of HERA operation were dedicated to measure F_{L} .



Additional Constraints on PDFs: Charm at HERA

• Inclusive structure function F2 is sensitive to

$$F_2 \sim \frac{4}{9}(U + \bar{U}) + \frac{1}{9}(D + \bar{D}), U = u + c; D = d + s + b$$

- Up to 30% of the inclusive cross section is related to charm production (at low x)
- F₂ charm data provides a complementary way
 - to impact gluon *
 - to decompose u from c and study the theoretical matching schemes



At HERA, different measurements available using various ways to tag the charm:

- reconstruction of D* and D decays
- inclusive analysis of tracks lifetime information
- muons from charm semi-leptonic decays



Eur. Phys. J. C 73:2311 (2013), [arXiv:1211.1182]

Additional Constraints on PDFs: Charm at HERA

Combined HERA *Fcc* data reaches 5 – 10% precision per point, can be used to study different HF models:

All models use a parameter mc related to the charm pole mass \rightarrow can use it as a tuning parameter for different scheme



e p p p c, b c, b c, b c, b c, b

Inclusion of charm has impact on:

• gluon, charm and light sea:



From low x to high x

PDF flavour decomposition can be classified into regions:

Low x (x<0.01)</p>

- ▶ Gluon density is determined with high accuracy from the *F2* scaling violation using QCD evolution
 - ✤ Indirect determination cross checked using direct measurement of FL
 - ♦ Results are consistent for Q2>10, however still open questions left for low Q2 and low x region
- > Accurate measurements on F2 which probes linear combination of the quark distributions.
 - ♦ PDF decomposition is not so well constrained:

Little is known experimentally about u^{-}/d^{-} as well as s^{-}/d^{-} . (LHC can help further)

Heavy c and b are determined using semi-inclusive scattering process with tagged heavy quarks.

• High x (x>0.01)

- Gluon at high x is very sensitive to Higgs Production, hence improvement is crucial for precision measurements of properties of Higgs
 - ♦ scaling violation, pp jets
- Light quarks:
 - ♦ HERA's CC helps provide clean constraints on u and d at high x
 - \diamond Separation of *s* from neutrino scattering, however with large theoretical uncertainties.
 - \diamond Separation of u^{-} , d^{-} using p p, pd experiments.

Another handle on u/d: measurement of F2d.

Constraints from High x Measurements



JLAB has an intense program on polarised and unpolarised structure functions and more is to come with JLAB@12GeV

Halls A,C: dedicated small angle spectrometers. Hall B: large acceptance spectrometer CLAS.

Hall D: under construction for photon beam experiments.

- Structure functions sensitive:
 - ▶ to gluon density
 - to power corrections in Q2
 - to higher twist contributions
 - ▶ to high x pdfs

sensitive to d/u.

Exploit the difference between proton and deuteron structure functions:



Measurements at JLAB (hall B)



♦ Sensitive to nuclear medium effects (referred to as EMC)^{VcVoica Radescu| DESY} | CTEQ 2013 DIS
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Future Measurement at JLAB (hall A)

MARATHON will measure with *Ee* = 12 GeV F2n/F2p taking into account
 Fermi-motion and binding effects in deuterium
 Using of Super-Ratio method:

In terms of the proton and neutron momentum distributions:

F2n/2Fp from above does not depend on the size of the EMC effect in 3He or 3H, but rather on the ratio of the EMC effects in 3He and 3H.

From DIS to LHC



Pushing Energy Frontier

- LHC can provide with its multitude of new measurements
- PDF discrimination by confronting theory with data
- PDF improvement by using LHC data for more accurate



- 1. W and Z production
- 2. W+c production
- 3. Inclusive Jet and Di-Jet production
- 4. Drell-Yan: low and high invariant mass
- 5. Top, ttbar
- 6. Prompt Photon, + Jets
- 7. W,Z+b

Drell-Yan kinematics



Low and high x are linked together at the LHC

Probing the Proton Structure: Drell Yan



Flavour decomposition of W and Z and the LHC

 Additional constraints on PDFs come from DY and jet data at the LHC probe a bi-linear combination of quarks



 $\begin{array}{ll} Z & \sim 0.29(u\bar{u}+c\bar{c})+0.37(d\bar{d}+s\bar{s}+b\bar{b}) \\ \gamma^{*} & \sim 0.44(u\bar{u}+c\bar{c})+0.11(d\bar{d}+s\bar{s}+b\bar{b}) \end{array}$



Measurements of W, Z production differentially in y_z and η_{ℓ} provide information on light sea decomposition and can constrain better the strange distribution

Measurements at LHC sensitive to PDFs

- Standard Model LHC measurements can add PDF discrimination and PDF improvement:
 - W, Z inclusive:light quark sea is flavour symmetric:
 - ♦ Confirmed by preliminary W+c
 - \diamond High mass, low mass DY \rightarrow feedback on dbar-ubar
 - Exploiting different energy beams for inclusive jets brings forward sensitivity to the gluon PDFs.
 - \diamond Photon-jet measurements \rightarrow gluon PDF
 - \diamond First alphas from ATLAS from 3/2 jets 2010 \rightarrow consistent with world average
 - Top measurement is becoming a valuable player in the impact on PDFs (and alphas)
- More precision measurements from LHC are to come also with 2012 data:
 - **Top,** W,Z+ c,b, W,Z+jets, ...
- It is crucial to match data precision to available theoretical calculation:
 - require sometimes tedious work to interface/validate different packages to allow for a consistent QCD interpretation of the measurements:
 - Ex: APPLGRID interface to aMC@NLO, SHERPA, JetPHOX, FEWZ vs DYNNLO (@NNLO), FEWZ vs SANC (EW NLO), FEWZ vs MCFM (NLO), aMC@NLO vs MCFM, HATHOR vs TOP++.. Etc..

Importance of correlation information



 \rightarrow By providing also correlations (b) allows for more flexibility to theory (T) to describe data (D)

- For s/d: most constraints comes from the Z data, however addition of the W data brings further constraint due to common sources of correlations cancelling out
 - Equivalent to taking ratio of measurements
- Similarly for 2.76 vs 7 TeV jets measurements:
 - Impressive error reductions
- Future measurements can benefit from careful identification of common correlated sources





Future ep colliders

LHeC (ep collider to complement LHC at CERN), EIC

cle Physics	Electron Ion Collider: The Next QCD Frontier
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BOP Publishing	

LHeC kinematic plane

LHeC (ep collider to complement LHC at CERN), EIC



- LHeC sensitivity extends to x=10⁻⁶
- Much increased luminosity for EIC and LHeC colliders compared to HERA.

 10^{-3}

-4

10

-2

10

-1

1 x

10

10 -5

-6

10

-7

10

Motivation for LHeC

• What HERA could/did not do:

Test of the isospin symmetry (u-d) with eD Investigation of the q-g dynamics in nuclei Verification of saturation prediction at low x Measurement of the strange quark distribution Discovery of Higgs in WW fusion in CC Study of top quark distribution in the proton Precise measurement of FL

Resolving d/u question at large Bjorken x Determination of gluon distribution at hi/lo x High precision measurement of αs no deuterons no time for eA too low c.o.m energy too low Luminosity too low cross section too low c.o.m energy too short running time with low energy runs too low Luminosity too small range overall not precise enough

HEP needs a TeV energy scale machine with 100 times higher luminosity than HERA to develop DIS physics further and to complement the physics at the LHC. The Large Hadron Collider p and A beams offer a unique opportunity to build a second ep and first eA collider at the energy frontier.

(M. Klein)

Summary

DIS provided us with most valuable information on the proton substracture

Low x and high x are exploited at HERA (the only ep collider) or at JLAB (fixed target experiments)

... Many more valuable measurements are already available, but not covered in this talk ...

PDFs still limit our knowledge of cross sections whether SM or BSM.

- Standard Model LHC measurements can themselves contribute to PDF discrimination and PDF improvement:
 - ▶ LHC data suggest that the light quark sea is flavour symmetric:
 - ♦ W,Z inclusive cross check against W+c
 - > Exploiting different energy beams for inclusive jets brings forward sensitivity to the gluon PDFs.
 - ♦ gluon PDF can also be improved through Photon-jet measurements [see P. Lenzi]
 - Top measurement is becoming a valuable player in the impact on PDFs (and alphas)

... Many more valuable measurements are already available, but not covered in this talk ...

More precision measurements from LHC to come from Run I and in future from Run 2

LHeC can represent a natural extension to LHC by providing an accurate and complete PDF set



Add to that: Quantum Chromodynamics: High energy experiments and theory: G. Dissertori, I. Knowles, M. Schmelling CTEQ lectures





HERAFitter QCD platform



Heritage of HERA transferred to LHC:

Open Source QCD Framework freely available at https://www.herafitter.org



vN scattering (fixed target)



Structure functions from vN data



Can determine alphas without gluon!



- Fit for $F_2^{avg}(x,Q^2)$ input model for
 - $R_L(x,Q^2)$ [L.W.Whitlow et.al. Phys.Lett. B250(1990)]
 - $\Delta x F_3(x, Q^2)$ [R.Thorne and R.Roberts, Phys.Lett. B 421 (1998)]
- Fit for $xF_3^{avg}(x,Q^2)$ no inputs required

- Theory curves corrected for:
 - Target Mass(H.Georgi& H.D.Politzer, Phys.Rev D14 1829)
 - Nuclear Effects:
 - correction measured in charged-lepton experiments from nuclear targets
 - standard way: apply the same correct. to neutrino scattering

FL from vN data



pQCD and DGLAP formalism

 The kinematic region of data must sometimes be adjusted in order to stay in the validity region of the pQCD, where DGLAP evolution equations are valid



- 1. $Q^2 \text{ cut} : Q^2 > \text{few GeV}^2 \text{ so that}$ perturbative QCD is applicable- $\alpha_s(Q^2)$ small
- 2. W² cut: to avoid higher twist termsusual formalism is leading twist
- x cut: to avoid regions where ln(1/x) resummation (BFKL) and non-linear effects may be necessary

Low x phenomenology with Dipole Models Eur.Phys.I.C71 (2011)

- At low x and Q² the virtual photon-proton scattering can be described using the color dipole model:
 - Fluctuation of the photon into a quark-antiquark pair (dipole) interacting with proton
 - Dipole has built-in saturation assumption
- Following models have been considered:

• GBW dipole model: $\sigma(x, r^2) = \sigma_0 \left(1 - \exp[-\frac{r^2}{4R_0^2(x)}] \right) \qquad R_0^2(x) = \left(\frac{x}{x_0}\right)^{\lambda}$

Fitting parameters: σ_0 , λ , x_0 .

• IIM (CGC) dipole model: $\sigma(x, r^2) = \sigma_0 \begin{cases} N_0 \quad (\tau^2)^{\gamma_s + \frac{\ln(\tau)}{\kappa\lambda \ln(x)}} & \text{if } \tau \leq 1 \\ \left(1 - \exp[-a\ln^2(2b \ \tau)]\right) & \text{if } \tau > 1 \end{cases}$ $\tau = r/2R_0(x)$ Fitting parameters: σ_0, λ, x_0 .



B-SAT dipole model:

$$\sigma(\mathbf{x}, r^2) = \sigma_0 \left(1 - \exp\left[-\frac{\pi^2 r^2 \alpha_s(\mu^2) \mathbf{x} g(\mathbf{x}, \mu^2)}{3\sigma_0} \right] \right)$$
$$\mathbf{x} g(\mathbf{x}, \mathbf{Q}_0^2) = \mathbf{A}_g \mathbf{x}^{-\lambda_g} (1 - \mathbf{x})^{5.6}$$
$$\mu^2 = \frac{C}{r^2} + \mu_0^2$$
Fitting parameters: \mathbf{A}_g , λ_g , \mathbf{Q}_0 .
Fixed parameters: $\sigma_0 = 23.8$ (mb), $C = 1.0$, $\mu_0^2 = 4.0$.

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Stellenbosch, South Africa -



Schemes for Heavy Quarks

Heavy Quarks introduce an additional scale: Q², M_H

$$F = \sum_{a} \int rac{dy}{y} f_N^a(x,\mu_F) \otimes \omega_{h/a}\left(rac{x}{y},rac{Q}{\mu_F},rac{M_H}{\mu_F},lpha_S(\mu_R)
ight) + O\left(rac{\Lambda_{QCD}}{Q}
ight)$$

▶ 3 regions of interests are observed:

- $Q^2 << M_H^2 \longrightarrow$ decoupling
 - $Q^2 \sim M_H^2 \longrightarrow \text{transition region}$
 - $Q^2 >> M_H^2 \longrightarrow$ heavy quarks treated as light quarks

• Fixed Flavour Number Scheme (FFNS)

- it sums over fixed number of active parton flavours: u, d, s
- heavy quarks only in the final state, massive: nf=3 for charm, nf=4 for bottom
- Returns an efficient organisation of the perturbative series at M_H²~Q²
 - □ at Q²>>M_H² large logarithms spoil the perturbative series

Zero-Mass Variable Flavour Number Scheme (ZM-VFNS)

- introduces a PDF for heavy quarks and the number of active flavours increases by one unit when a heavy quark threshold is crossed
- heavy quarks omitted entirely below M_{H²} and included as massless partons above thresholds

o General-Mass Variable Flavour Number Scheme (GM-VFNS)

Interpolating scheme combining the best feature from FFNS and VFNS with matching scale of order M_H² for each of the heavy quark thresholds: Voica Radescu| DESY (CTEQ 2013 DIS 44

Monte Carlo Method

[benchmark exercise with NNPDF - PDF4LHC Interim Report arXiv:1101:0536]

- Method consists in preparing replicas of data sets allowing the central values of the cross sections to fluctuate within their systematic and statistical uncertainties taking into account all point to point correlations
 - Shift central values randomly within the <u>uncorrelated errors</u> assuming Gauss distribution of the errors:

 $\sigma_i = \sigma_i (1 + \delta_i^{uncorr} RAND_i)$

• Shift central values with the same probability of the corresponding <u>correlated systematic</u> shift assuming Gauss distribution of the errors:

$$\sigma_i = \sigma_i (1 + \delta_i^{uncorr} RAND_i + \sum_{i=1}^{N_{sys}} \delta_{ij}^{corr} RAND_j)$$

- Preparation of the data is repeated for in time's (in-100)
 - For each MC replica, NLO QCD fit is performed to extract the N PDF sets
- Errors on the PDFs are estimated from the RMS of the spread of the N curves corresponding to the N individual extracted PDFs

Experimental Uncertainties:

Results of the MC method

- Standard error estimation of PDFs relies on the assumption that all errors follow Gauss statistics
- MC method can provide an independent cross check of it
 - Hessian Method and MC method give the same results in the linear error propagation approximation
- MC method allows to test various assumptions for error distributions
 - some systematic uncertainties follow Log-Normal distribution (i.e., lumi, detector acceptance, ...)



 Similar results to Gauss distributions when using Log-Normal assumptions



W Charge Asymmetry

- The interplay between the flavour asymmetries can be enhanced via ratio measurements:
 - W-asymmetry $A_w = [\sigma(W^+) \sigma(W^-)] / [\sigma(W^+) + \sigma(W^-)] = (u_v d_v) / (u_v + d_v + 2 \text{ qbar}) \text{ at } x1 = x2$



- CMS measures directly the electron asymmetry data from 2011 and clearly disfavour MSTW2008:
 - MSTW have addressed this in more recent versions of their PDFs [see J. Rojo's talk].
- ATLAS differential measurements of W⁺ and W⁻ (combined muon and electron) based on 2010 data translated into charge asymmetry AI as long as proper treatment of correlations are accounted for.
- LHCb extends the measurement (muon channel) to forward region and provides a comparison with various predictions (interesting region where distribution changes sign due to V-A structure)

NOTE: Selection criteria are optimized for each experiment Voica Radescu | DESY | CTEQ 2013 DIS

Impact of HERA on the LHC predictions

• Impressive precision of HERAPDF sea and gluon is relevant for W, Z production at the LHC:



this assumes that DGLAP worked which was confirmed at HERA

- Inclusion of HERA data shows tremendous improvement on the predictions for W and Z production at the central rapidity:
 - such an improvement is due to improvement in the low-x gluon and sea:
 - remember that at the LHC the W, Z bosons are made mostly of sea-sea partons at low x and the the scale of M_Z² the sea is driven by the gluon.

Impact of F₂ charm on W,Z cross sections

F₂ charm data helps constrain charm-quark by studying m_c -choice in variable flavor number schemes
 Eur. Phys. J. C 73:2311 (2013), [arXiv:1211.1182]



- Large spread of the total cross section predictions at the LHC for W+, W-, Z:
 - The spread is reduced significantly when predictions are evaluated at the m_c determined from F₂ charm

W+c sensitivity to Strange from CMS

CMS preliminary

 $W \rightarrow 1v$

Question: would other measurements confirm ATLAS favour of sbar=dbar?

CMS has released a preliminary W+c measurement directly sensitive to strange:



agreement with NNPDF2.3 (Coll):

has large strangeness



√s = 7 TeV

Data, 5.0 fb

MSTW08

Strange quark from W, Z measurements at ATLAS

- Strange quark is not so well constrained:
 - Neutrino dimuon data favours suppressed strange
- At LHC, Z cross sections together with y_z shape may provide a constraint on s-quark density and it can be cross checked by W+charm data.
 - The results for NNLO fits to inclusive W, Z differential data with free and fixed s:
 - For W+ and W- there is little difference, helps to fix the normalisation.
 - For Z, the cross section is increased and the shape is modified.





ATLAS result is the kinematic region probed by LHC data at $x^{0.01}$ and indicates a flavour symmetric sea with an enhanced strangeness, in agreement with the CT10 (s/d~0.75)

 It is above of MSTW08, ABKM09, NNPDF2.3 (s/d~0.5)

Strange quark content of the proton

Strange quark is not so well constrained:



- Flavour SU(3) symmetry: three light quark distributions are equal:
 - Strange quark density may be suppressed due to their larger mass as favored by Neutrino dimuon data.
 - Often it is assumed that s=sbar and rs=0.5 $r_s(x) = 0.5(s(x) + \bar{s}(x))/\bar{d}(x)$
- At LHC, Z cross sections together with y_z shape may provide a constraint on s-quark density and cross checked against its W+charm data.

Pushing Energy Frontier

LHC can provide with its multitude of new measurements



Factorisation schemes

 Basically factorisation scheme tells us which part are absorbed by PDFs and which by Hard Scattering coefficients:

$$F_2^{(Vh)}(x,Q^2) = \sum_{i=f,\bar{f},G} \int_0^1 d\xi C_2^{(Vi)}\left(\frac{x}{\xi},\frac{Q^2}{\mu^2},\frac{\mu_f^2}{\mu^2},\alpha_s(\mu^2)\right) \otimes f_{i/h}(\xi,\mu_f^2,\mu^2)$$

Hard scattering coefficients



- Two sort of extreme prescriptions are generally used:
 - ♦ MS bar scheme: absorb as little as possible into pdfs

$$F_2^{\overline{MS}}(x,Q^2) = x \sum e_q^2 \int \frac{dx_2}{x_2} q^{\overline{MS}}(x,Q^2) \left[\delta \left(1 - \frac{x}{x_2} \right) + \frac{\alpha_s}{2\pi} C^{\overline{MS}} \left(\frac{x}{x_2} \right) + \dots \right]$$

DIS scheme: absorb as much as possible into pdfs

$$F_2^{DIS}(x,Q^2) = x \sum e_q^2 q(x,Q^2)$$

Higher Twist

So far considered only "leading twist"



twist = dimension (spin) of operators in Operator Product Expansion (OPE)

 $F_2(x,Q^2) = \sum C_{2\,i} \otimes f_i + {\sf non-leading}$ power of Q

- Factorization theorem (Collins hep-ph/9709499):



n>0 higher twists non-leading powers ...

 NOT covered by factorization theorem... but contributions can be large ?!? Slide from Hannes Jung: <u>http://www.desy.de/~jung/qcd_and_mc_2012</u> Voice Radescul DESY

Scale dependence

- A hard scattering calculation introduces a renormalization scale to remove UV divergencies and a factorization scale that separates the hard from soft physics:
 - Scale dependence usually represent largest theoretical uncertainties
- Factorization and renormalization scales are both usually chosen to be the characteristic energy scale in the process.
 - > This was in the standard convention various logarithms in the calculations vanish.

$$F(x,Q^2) = \sum_i C_i(\mu_R,\mu_F,Q^2,y) \otimes f_i(z,\mu_F),$$

i - number of active flavours in the proton, what about heavy *c* and *b*?

 m_c =1.5 M_p , m_b =4.7 M_p

QCD analysis of the proton structure: treatment of heavy quarks essential

Different prescriptions how to treat Heavy Quarks in PDF Fits (HQ schemes):

- c, b massless particles in the proton
- c, b massive, produced in hard scattering

Scale dependence -- continuation

- > The physical structure functions should not depend on the choices made for muF and muR
- ▶ For calculations carried to the order NLO in alphas:

$$\mu_R \frac{\partial F}{\partial \mu_R} = 0 \text{ up to } O(\alpha_S^2);$$

$$\mu_F \frac{\partial F}{\partial \mu_F} = 0 \text{ up to } O(\alpha_S^2).$$

Then performing the expansion in NLO of the structure function:

$$\begin{split} F(x,Q^2) &= \sum_i \left[C_i^0(\mu_R,\mu_F,Q^2,y) + \alpha_S(\mu_R)C_i^1(\mu_R,\mu_F,Q^2,y) \right] \otimes f_i(z,\mu_F). \\ \mu_R \frac{\partial F}{\partial \mu_R} &= \sum_i \left[\underbrace{\mu_R \frac{\partial C_i^0}{\partial \mu_R}}_{O(\alpha_s^0)} + \underbrace{\mu_R \frac{\partial \alpha_S}{\partial \mu_R}C_i^1}_{O(\alpha_s^2)} + \underbrace{\alpha_S \frac{\partial C_i^1}{\partial \mu_R}}_{O(\alpha_s^1)} \right] \otimes f_i = 0 \Longrightarrow \alpha_S \frac{\partial C_i^1}{\partial \mu_R} = 0, \end{split}$$

DIS Cross Section

General Form for the Differential cross section:

$$\begin{aligned} \frac{d^2\sigma}{dxdQ^2} &= A^i \left\{ (1 - y - \frac{x^2 y^2 M^2}{Q^2}) F_2^i + y^2 x F_1^i \mp (y - \frac{y^2}{2}) x F_3^i \right\} & \text{A}^i: \text{ process dependent} \\ \eta_\gamma &= 1; \qquad \eta_{\gamma Z} = \left(\frac{G_F M_Z^2}{2\sqrt{2}\pi\alpha}\right) \left(\frac{Q^2}{Q^2 + M_Z^2}\right); \qquad \eta_Z = \eta_{\gamma Z}^2; \\ \eta_W &= \frac{1}{2} \left(\frac{G_F M_W^2}{4\pi\alpha} \frac{Q^2}{Q^2 + M_W^2}\right)^2, \end{aligned}$$

structure functions express the dependence of cross-section on the constituents of the nucleon

At HERA four processes give access to proton substructure: CC e+p, CC e-p, NC e+p, NCe-p

(convenient to introduce reduced cross section to better single out the effect of PDFs)

$$\begin{split} \tilde{\sigma}(x,Q^2) &= \frac{xQ^4}{2\pi\alpha^2} \frac{1}{Y_+} \frac{d^2\sigma}{dxdQ^2} = \tilde{F}_2(x,Q^2) \mp \frac{Y_-}{Y_+} \cdot x \tilde{F}_3(x,Q^2) - \frac{y^2}{Y_+} \cdot \tilde{F}_L(x,Q^2). \\ & Y_{\pm} = 1 \pm (1-y)^2, \end{split}$$
Voice Radescul DESY

Additional Constraints to PDFs: on gluon

- Sensitivity to gluon PDF arise from the coupled singlet-gluon QCD evolution
 - scaling violation:



quark helicity $\pm \frac{1}{2}$, $F_L = 0$

off-shell quarks may absorb longitudinal photons

- ightarrow Interesting case also for study physics at low Q2
 - heavy flavour schemes, evolutions, higher twists
- From the measurement of semi-inclusive structure function *Fc*
- From the inclusive jet cross section (for ep and pp collisions) :
 - \diamond Able to decorrelate gluon and strong coupling



Nuclear Corrections at high x

- The nuclear environment in which a struck quark is embedded could modify the x distribution of the quarks inside the nucleon
- To extract F2n/F2p ratio experimentaly from DIS measurements off the proton and deuteron, a smearing model to account for the Fermi-motion of the nucleons in the deuteron was taken into account [SLAC-MIT]
 - At low x: ratio close to $1 \rightarrow$ no valence
 - At high x: ratio close to $\frac{1}{4}$ \rightarrow high momentum partons in the proton are mainly up PDFs







Impact of LHeC on PDFs: zoom on high x

* Experimental uncertainties are shown at the starting scale $Q^2=1.9$ GeV²



Alphas from HERA

- Addition of the HERA Jet cross section data (NLOJet++/fastNLO) into the fits allows to constrain simultaneously alphas and gluon
- Comparison of the PDFs with free alphas fit with and without Jet data



Effect of the charm data

- Addition of the HERA combined F_2 charm data can help reduce model uncertainty of $m_c(1.35-1.65)$:
 - Inclusive data show low sensitivity, addition of the charm data have strong constraining power



Level of PDF agreement



Overall disagreement in W, Z cross sections was found ~8%

(Plots from G.Watt -68%CL)

Measurements at LHC sensitive to PDFs

