Neutrinos (Theory)

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E. Witten (2000): for neutrino masses, the considerations have always been qualitative, and, despite some interesting attempts, there has never been a convincing quantitative model of the neutrino masses.

ν
ν
ν

Part A: Neutrinos from new physics

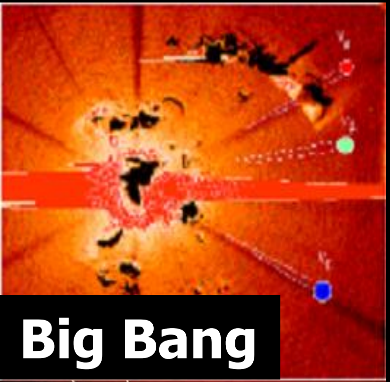
Part B: Neutrino mass from seesaw

Part C: Flavor mixing & oscillations

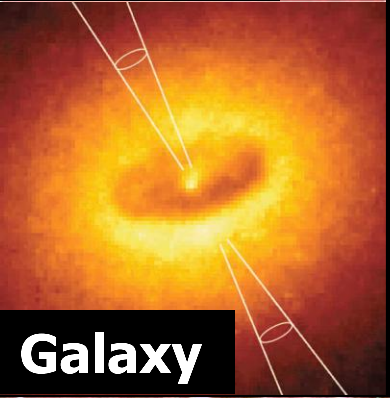
Part D: What is behind observation

@Lectures at the CTEQ summer school, Peking University, 11/7/2014

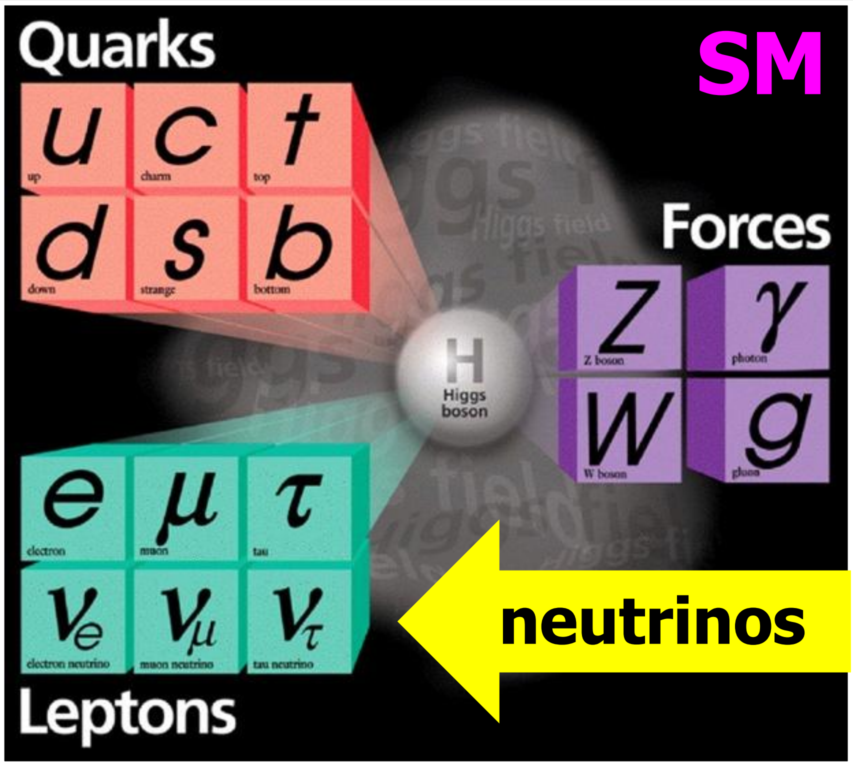
Neutrinos: a part of our everyday life!



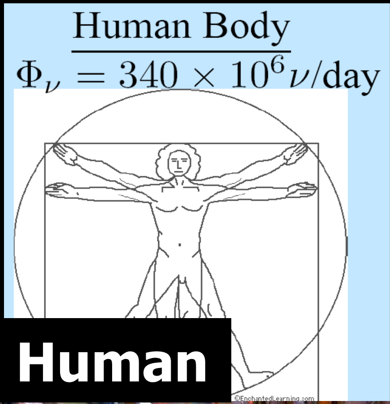
Big Bang



Galaxy



charge = 0
spin = ½
mass = 0
speed = c

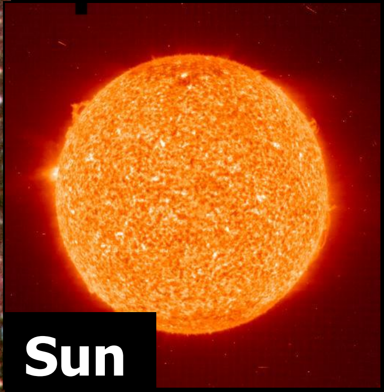


Human Body

$\Phi_\nu = 340 \times 10^6 \nu/\mathrm{day}$

Human

Supernova



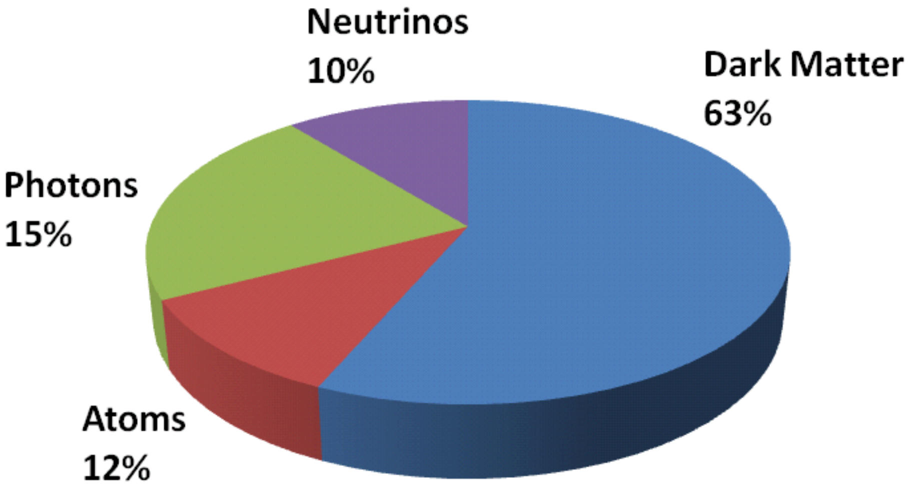
Sun

Earth

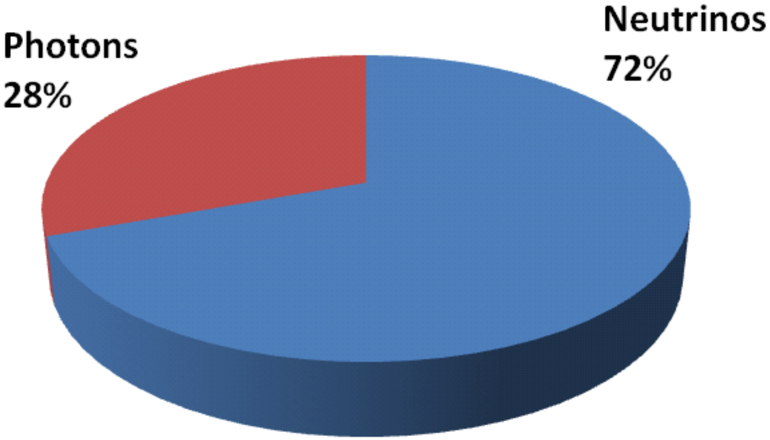
Reactor

Accelerator

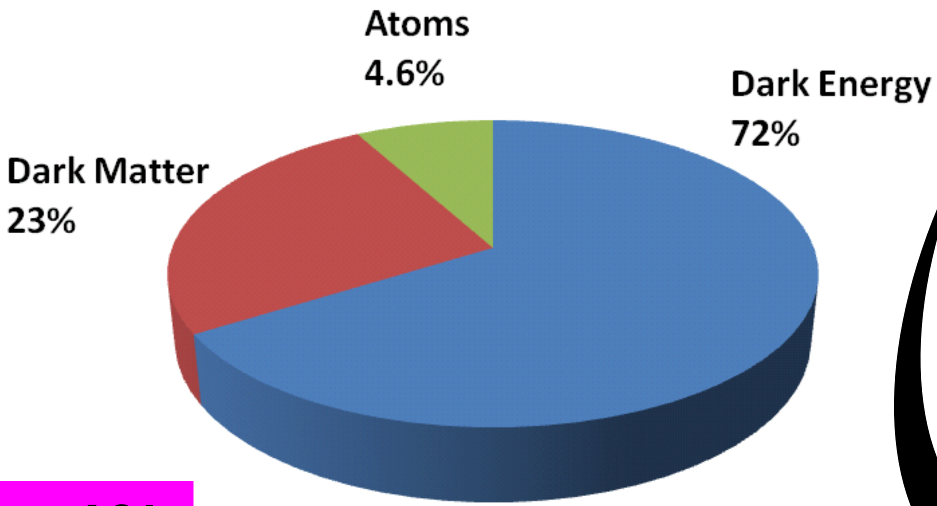
NEUTRINOS in the UNIVERSE



photon decoupling
t = 380 000 years



neutrino decoupling
t = 1 second



< 1%

Today
t = 13.7 billion years

A witness and participant in the evolution of the Universe

ν ν ν

The known and unknowns

Neutrino flavors can oscillate!

Finite neutrino mass/flavor mixing

the absolute ν mass scale?

ν mass hierarchy? (JUNO)

the Dirac/Majorana nature?

the CP-violating phases?

how many species? …

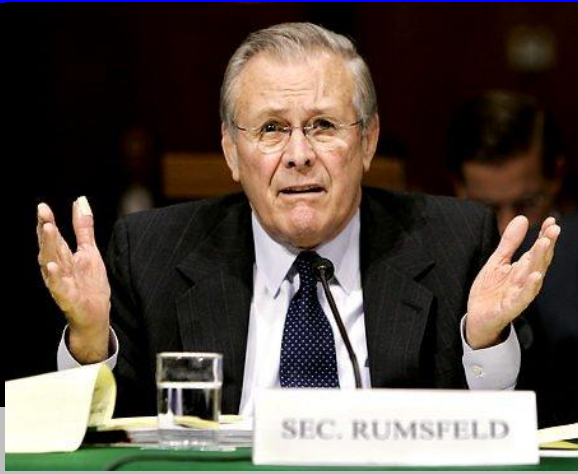
cosmic ν background?

supernova & stellar ν's?

ultrahigh-energy cosmic ν's?

keV warm dark matter?

cosmic baryon asymmetry?



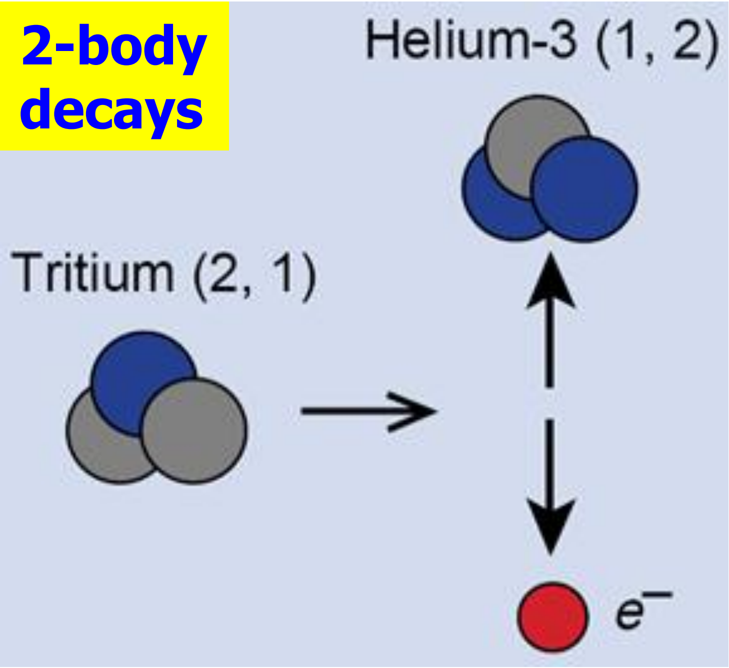
As we know
There are known knowns
There are things we know we know

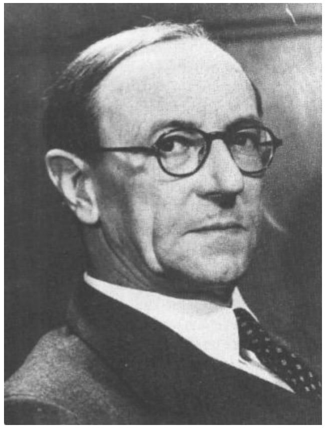
We also know
There are known unknowns
That is to say
We know there are some things we
don't know

But there are unknown unknowns
The ones we don't know
We don't know

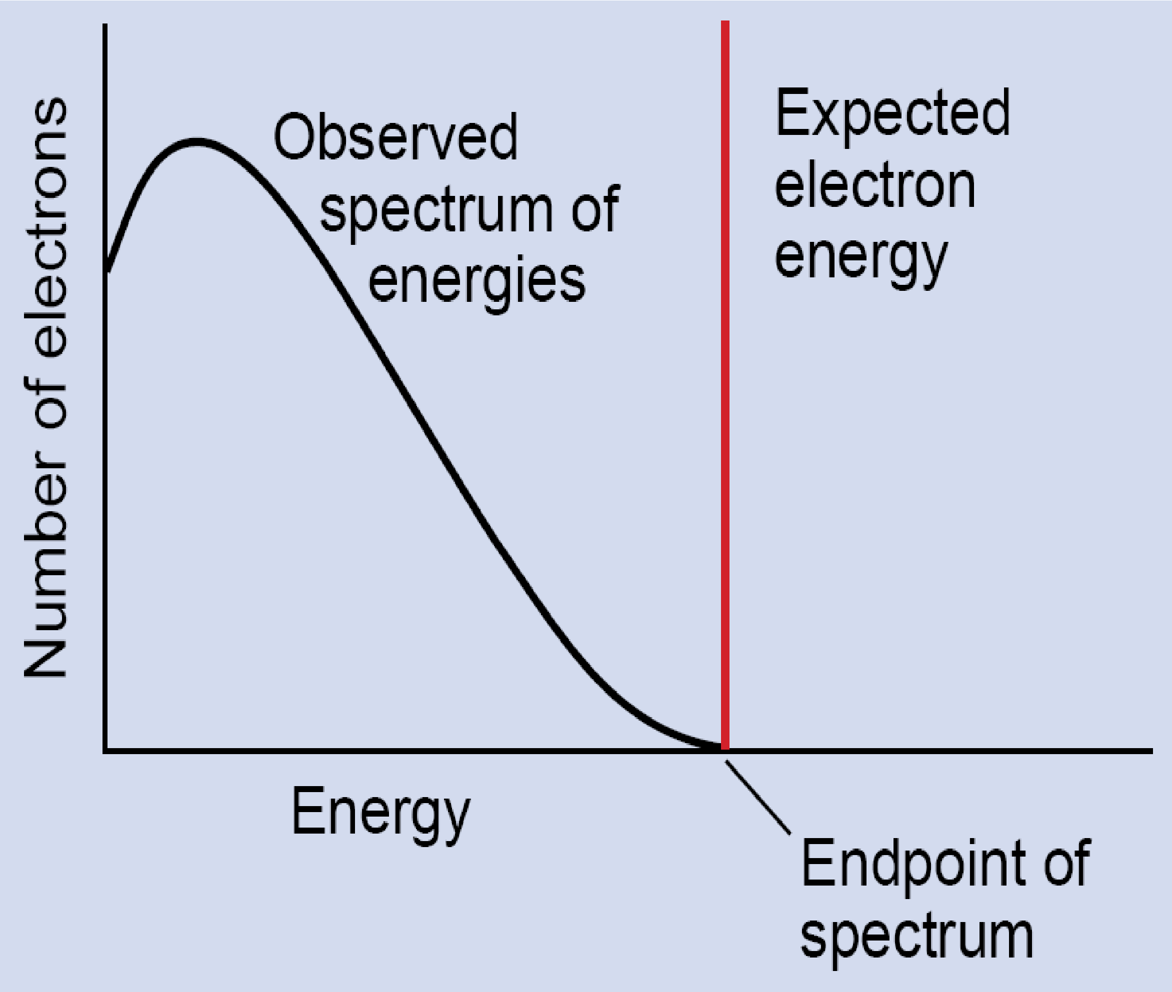
Beta decay in 1930

2-body decays





Energy crisis = New physics!

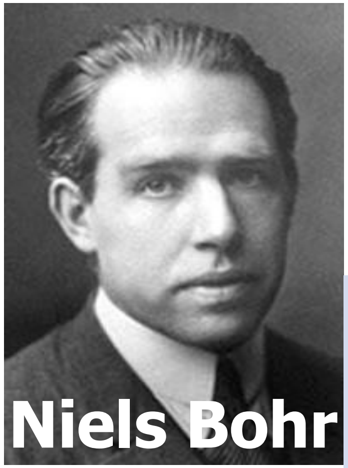


J. Chadwick 1914/C. Ellis 1920-1927

What to do?

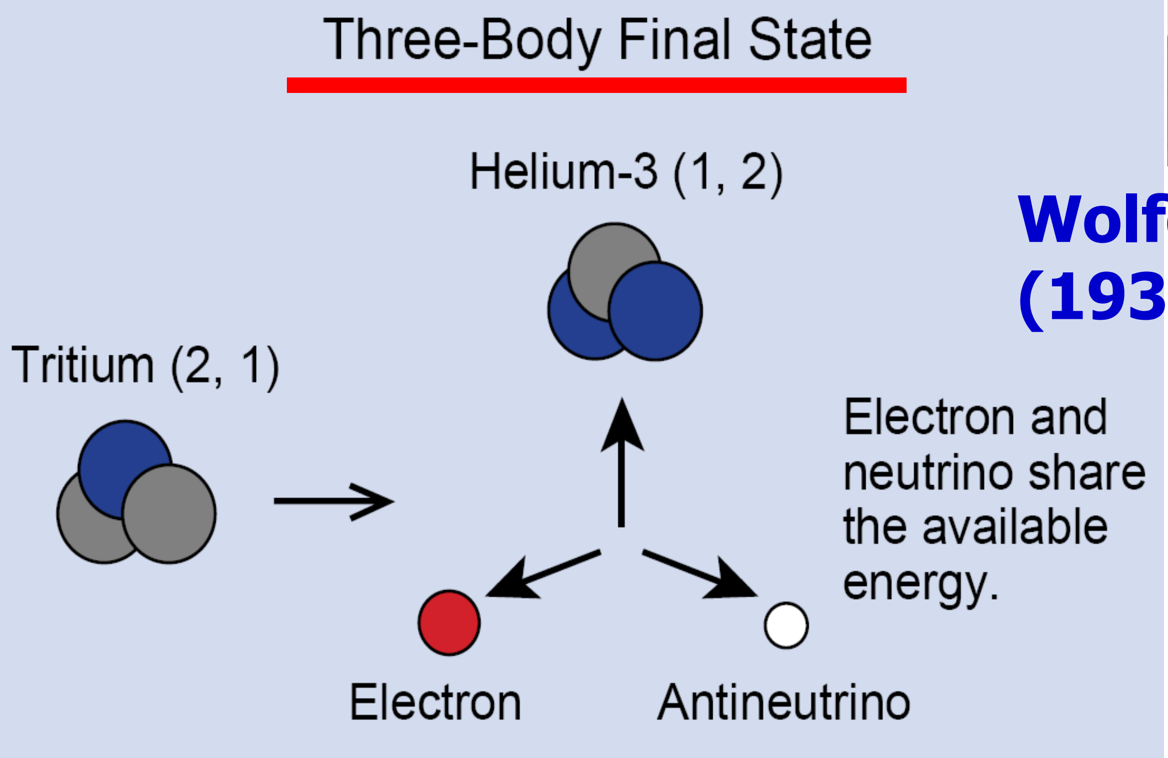
Part A

Two ways out?


Niels Bohr

- ♣ giving up sth
- ♣ adding in sth


Wolfgang Pauli (1930)



Dears, what attitude do you take towards new physics ????

亲们，你们遇到新物理（实验结果和理论预言不同）时，是啥态度?

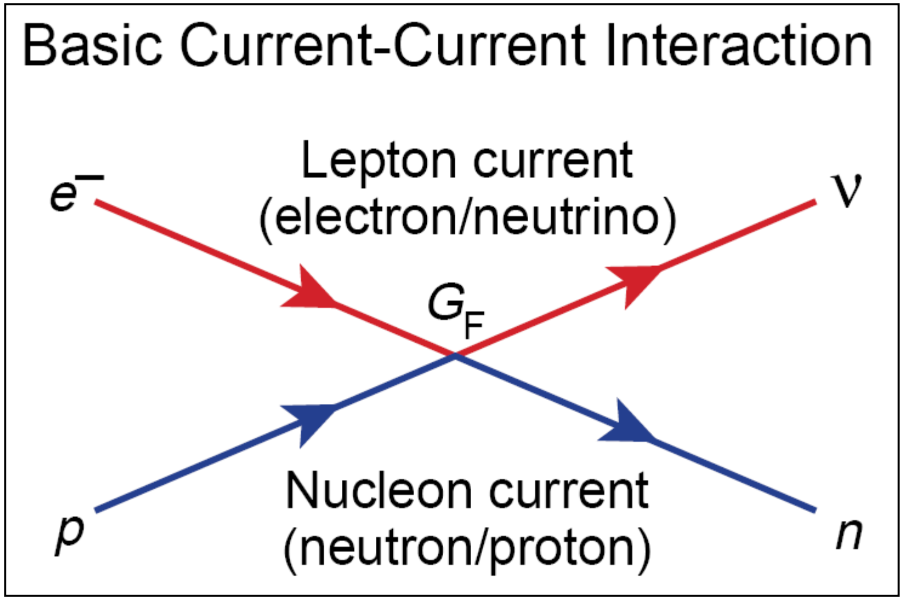
Fermi's theory

Enrico Fermi assumed a new force for β decay by combining 3 new concepts:

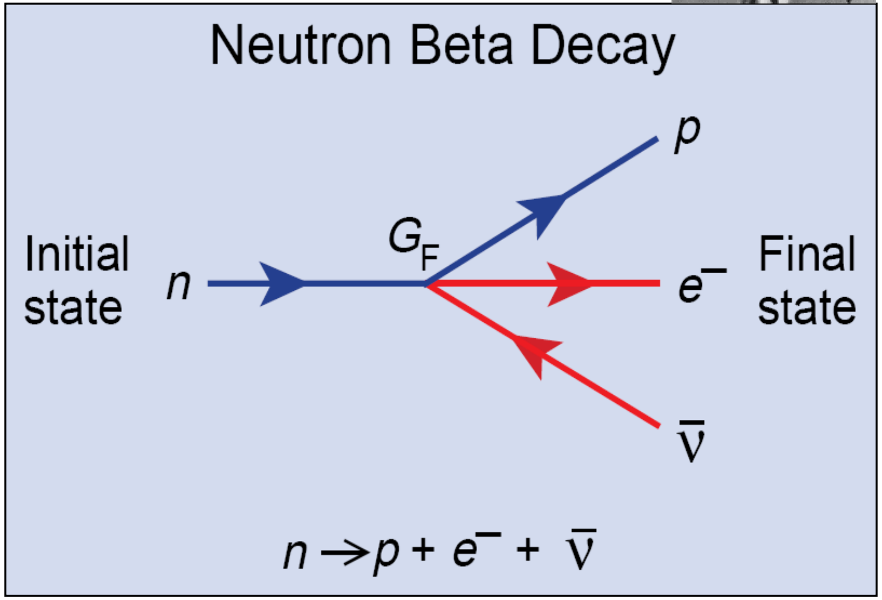
I will be remembered for this paper.

------ Fermi in Italian Alps, Christmas 1933

- ★ **Pauli's idea: neutrinos**
- ★ **Dirac's idea: creation of particles**
- ★ **Heisenberg's idea: isospin symmetry**



Basic Current-Current Interaction

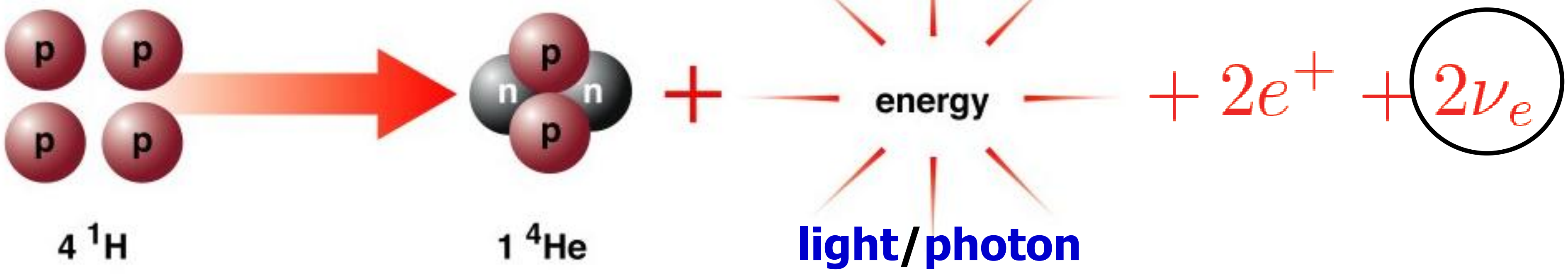


Neutron Beta Decay

Why the sun shines?

$$4p \to {}^{4}\mathrm{He} + 2e^{+} + 2\nu_e + 26.73\ \mathrm{MeV}$$

neutrino

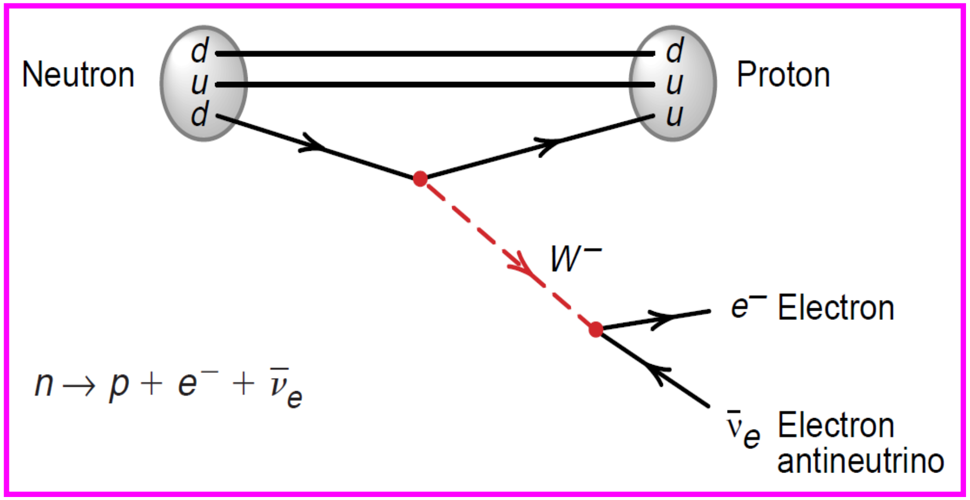


$$+\,2e^{+} + 2\nu_e$$

light/photon

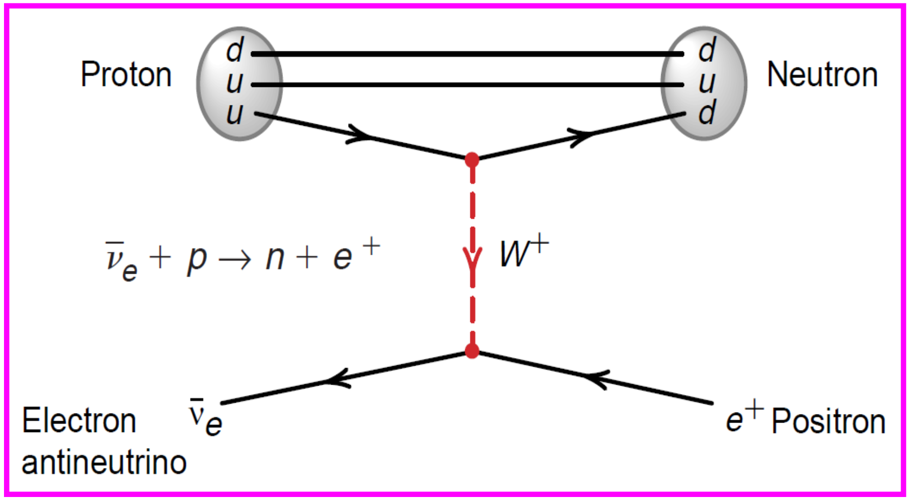
Hans Bethe (1939), George Gamow & Mario Schoenberg (1940, 1941)

The beta decay



$$n \to p + e^{-} + \bar{\nu}_e$$

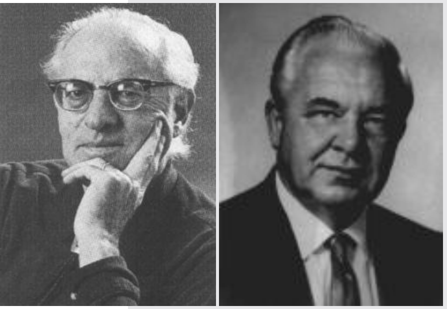
The inverse beta decay

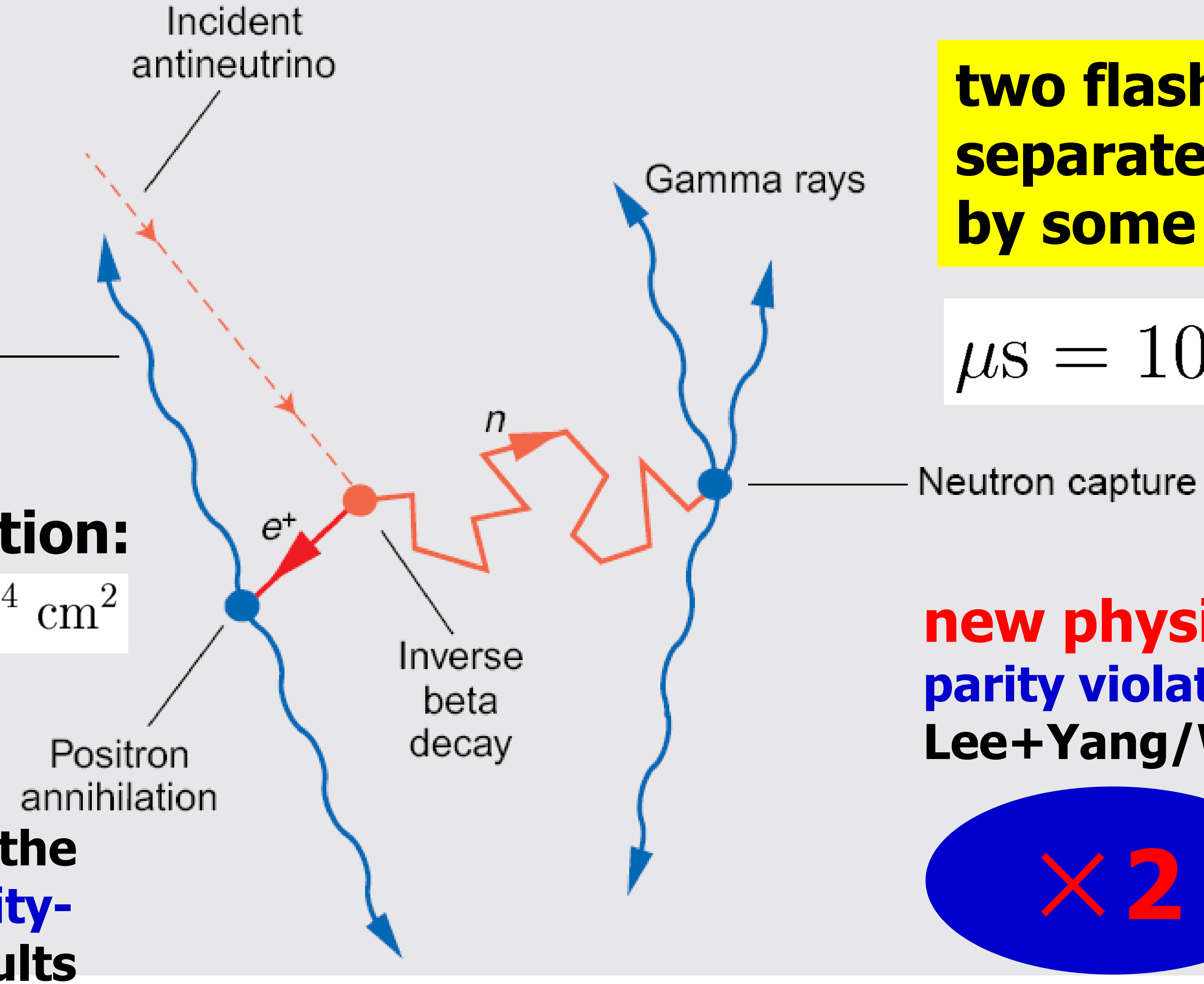


$$\bar{\nu}_e + p \to n + e^{+}$$

Neutrinos in 1956

Frederick Reines & Clyde Cowan detected reactor anti-ν's.





two flashes separated by some μs

$$\mu\text{s} = 10^{-6}\text{s}$$

their observation:

$$\sigma(\overline{\nu}_e p) \sim 6 \times 10^{-44}\ \text{cm}^2$$

↓

consistent with the theoretical (parity-conserving) results

new physics:

parity violation

Lee+Yang/Wu.

×2

Many things oscillate

1956: Discovery of electron antineutrino (C.L. Cowan *et al*)

1957: Postulation of neutrino-antineutrino oscillation (B. Pontecorvo)

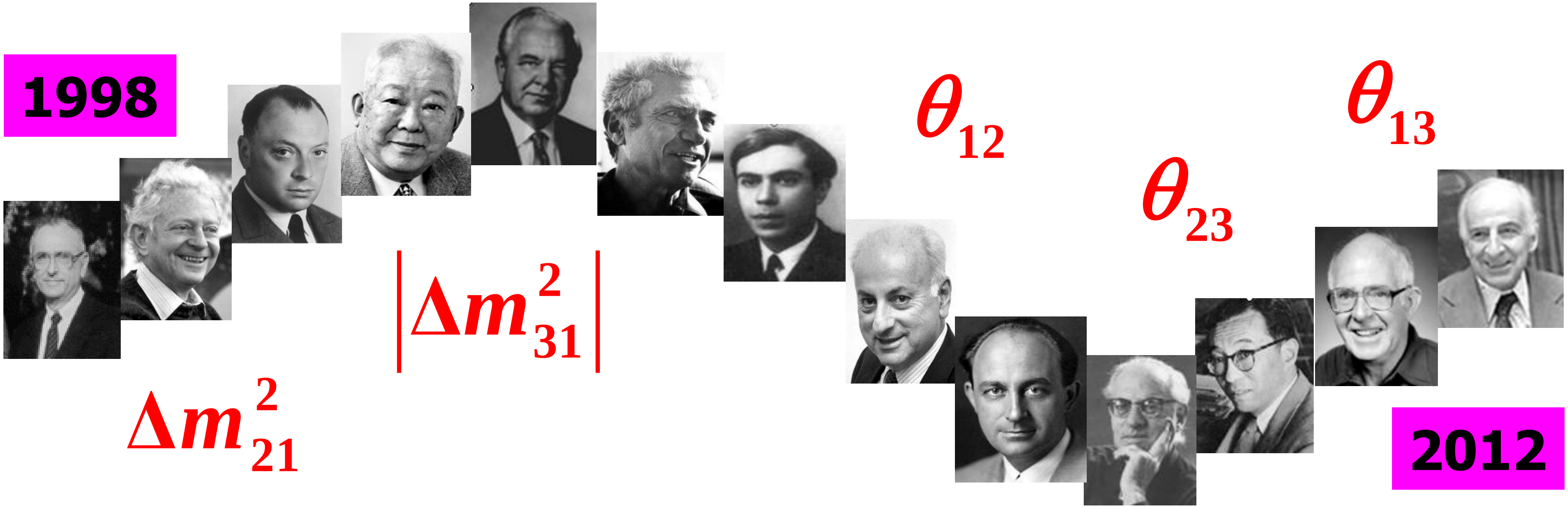
1962: Discovery of muon neutrino (G. Danby *et al*)

1962: Postulation of neutrino conversion (Z. Maki *et al*)

1968: Discovery of solar neutrino oscillation (R. Davis *et al*)

1987: Discovery of supernova neutrinos (K. Hirata *et al*)

2000: Discovery of tau neutrino (K. Kodama *et al*)



1998

Δm^2_{21}

$|\Delta m^2_{31}|$

θ_{12}

θ_{23}

θ_{13}

2012

What is mass?

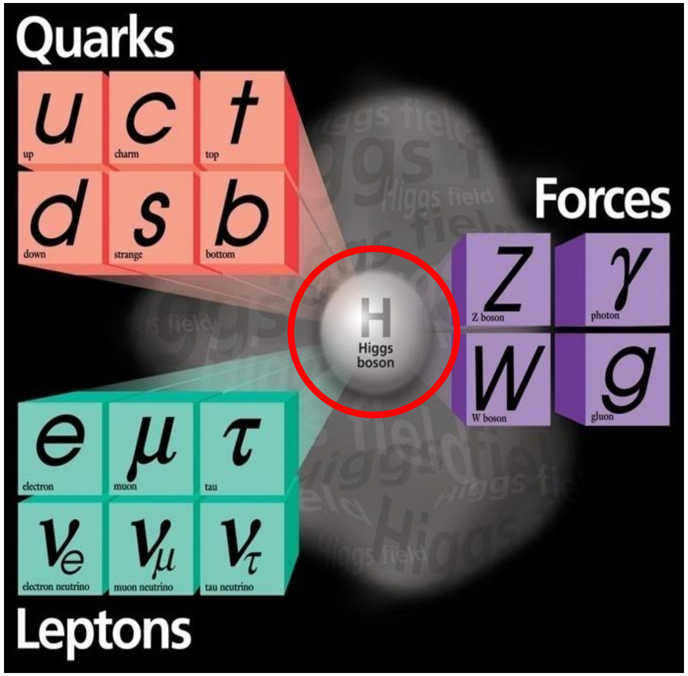
Mass is the inertial energy of a particle existing at rest.

- A **massless** particle has no way to exist at rest. It must always move at the speed of light.
- A **massive** fermion (lepton or quark) must exist in both the left- and right-handed states.

The **Brout-Englert-Higgs** mechanism is responsible for the origin of W / Z and fermion masses in the SM.

$$L_{\mathrm{SM}} = L(f, G) + \underline{L(f, H)} + \underline{L(G, H)} + L(G) - V(H)$$

All the **bosons** were discovered in **Europe**, and most of the **fermions** were discovered in **America**.



Higgs: Yukawa interaction

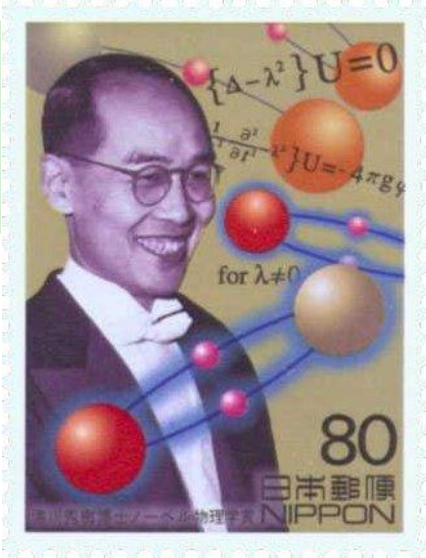
force	strength	range	mediator	mass
strong	1	$10^{-15}\,\mathrm{m}$	gluon/π	$\sim 10^{2}\,\mathrm{MeV}$
EM	$1/137$	∞	photon	$=0$
weak	10^{-6}	$10^{-18}\,\mathrm{m}$	W/Z/H	$\sim 10^{2}\,\mathrm{GeV}$
gravitation	6×10^{-39}	∞	graviton	$=0$

Yukawa relation for the mediator's mass ***M*** and the force's range ***R*** :

$$\boxed{M \simeq \frac{200\mathrm{MeV}\times10^{-15}\,\mathrm{m}}{R}}$$

$$\boxed{L_{\mathrm{SM}} = \underline{L(f,G)+L(f,H)}+L(G,H)+L(G)-V(H)}$$

Fermion masses, flavor mixing, CP violation



In the SM

All ν's are massless due to the model's simple structure:

---- SU(2)×U(1) gauge symmetry and Lorentz invariance:

Fundamentals of a quantum field theory

---- Economical particle content:

No right-handed neutrino; only a single Higgs doublet

---- Mandatory renormalizability:

No dimension ≥ 5 operator (*B*-*L* conserved in the SM)

Neutrinos are massless in the SM: Natural or not?

YES: It's toooooooo light and almost left-handed;

NO: No fundamental symmetry/conservation law.

Part B

Weinberg operator

Way 1: to relax the requirement of renormalizability (S. Weinberg 79)

$$\boxed{\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} + \frac{\mathcal{L}_{\rm d=5}}{\Lambda} + \frac{\mathcal{L}_{\rm d=6}}{\Lambda^2} + \cdots}$$

In the SM, the lowest-dimension operator that violates lepton/baryon number is unique:

$$\boxed{\frac{1}{M}HHLL}$$

neutrino mass

Seesaw: $m_{1,2,3} \sim \langle H \rangle^2 / M$

$$m_{1,2,3} < 1\,\mathrm{eV} \;\Rightarrow\; M > 10^{13}\,\mathrm{GeV}$$

$$\boxed{\frac{1}{M^2}QQQL}$$

proton decay

Example : $p \to \pi^0 + e^+$

$$\tau_p > 10^{33}\,\text{years} \;\Rightarrow\; M > 10^{15}\,\mathrm{GeV}$$

Neutrino masses/proton decays: windows onto physics at high scales

Part B

Dirac mass term

Way 2: to add **3 right-handed** neutrinos & demand a **(B - L) symmetry**

A pure Dirac mass term

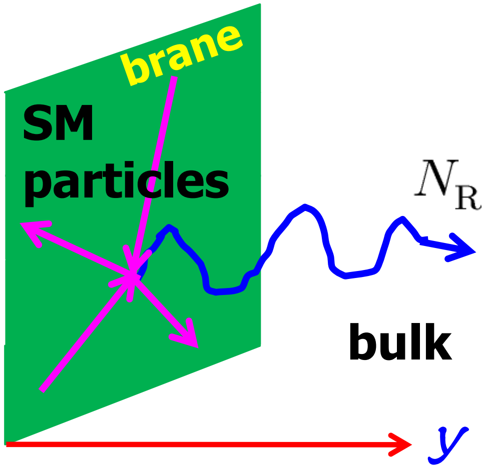
$$-\mathcal{L}_{\rm lepton} = \overline{l_{\rm L}} Y_l H E_{\rm R} + \overline{l_{\rm L}} Y_\nu \tilde{H} N_{\rm R} + {\rm h.c.}$$

$$\begin{aligned} M_l &= \frac{v}{\sqrt{2}} Y_l \\ M_\nu &= \frac{v}{\sqrt{2}} Y_\nu \end{aligned}$$

NOT convincing
Everything not forbidden is compulsory!

The hierarchy problem: $y_i / y_e = m_i / m_e \lesssim 0.5\ {\rm eV} / 0.5\ {\rm MeV} \sim 10^{-6}$

A very speculative way out: the smallness of Dirac masses is ascribed to the assumption that *N*_R have access to an extra spatial dimension (Dienes, Dudas, Gherghetta 98; Arkani-Hamed, Dimopoulos, Dvali, March-Russell 98) :



The wavefunction of *N*_R spreads out over the extra dimension *y*, giving rise to a suppressed Yukawa interaction at *y* = 0.

$$\left[\overline{l_{\rm L}} Y_\nu \tilde{H} N_{\rm R}\right]_{y=0} \sim \boxed{\frac{1}{\sqrt{L}}} \left[\overline{l_{\rm L}} Y_\nu \tilde{H} N_{\rm R}\right]_{y=L}$$

$\Rightarrow \Lambda_{\rm String} / \Lambda_{\rm Planck} \sim 10^{-12}$

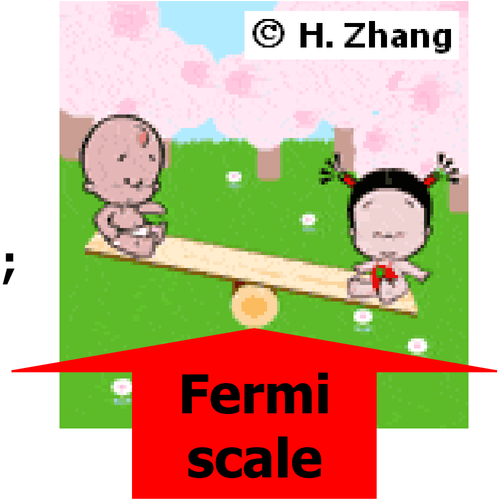
(e.g., King 08)

Part B

Majorana masses

Seesaw: add **new heavy degrees of freedom** and allow (**B-L**) violation:

Seesaw—A Footnote Idea:
H. Fritzsch, **M. Gell-Mann**,
P. Minkowski, **PLB 59 (1975) 256**



Type-1: SM + 3 right-handed neutrinos (**Minkowski 77; Yanagida 79; Glashow 79; Gell-Mann, Ramond, Slanski 79; Mohapatra, Senjanovic 79**)

$$-\mathcal{L}_{\rm lepton} = \overline{l_{\rm L}} Y_l H E_{\rm R} + \overline{l_{\rm L}} Y_\nu \tilde{H} N_{\rm R} + \frac{1}{2} \overline{N^c_{\rm R}} M_{\rm R} N_{\rm R} + {\rm h.c.}$$

Type-2: SM + 1 Higgs triplet (**Konetschny, Kummer 77; Magg, Wetterich 80; Schechter, Valle 80; Cheng, Li 80; Lazarides et al 80; Mohapatra, Senjanovic 80**)

$$-\mathcal{L}_{\rm lepton} = \overline{l_{\rm L}} Y_l H E_{\rm R} + \frac{1}{2} \overline{l_{\rm L}} Y_\Delta \Delta i\sigma_2 l^c_{\rm L} - \lambda_\Delta M_\Delta H^T i\sigma_2 \Delta H + {\rm h.c.}$$

variations

Type-3: SM + 3 triplet fermions (**Foot, Lew, He, Joshi 89**)

$$-\mathcal{L}_{\rm lepton} = \overline{l_{\rm L}} Y_l H E_{\rm R} + \overline{l_{\rm L}} \sqrt{2} Y_\Sigma \Sigma^c \tilde{H} + \frac{1}{2} {\rm Tr}\left(\overline{\Sigma} M_\Sigma \Sigma^c\right) + {\rm h.c.}$$

combinations

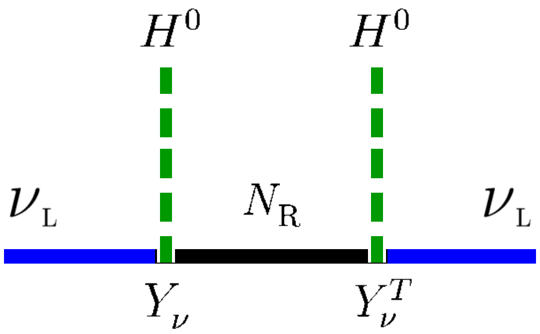
Seesaw formula

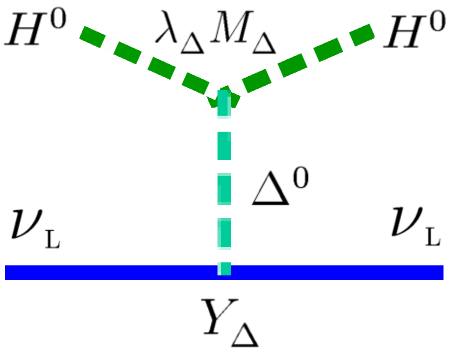
Weinberg operator: **the unique dimension-five operator of ν-masses after integrating out the heavy degrees of freedom.**

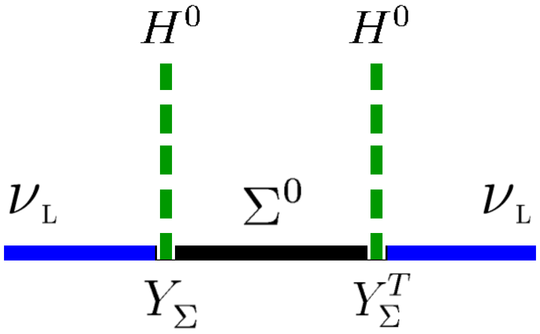
$$\frac{\mathcal{L}_{\rm d=5}}{\Lambda} = \begin{cases} \frac{1}{2}\left(Y_\nu M_{\rm R}^{-1} Y_\nu^T\right)_{\alpha\beta} \overline{l_{\alpha \rm L}} \tilde{H} \tilde{H}^T l^c_{\beta \rm L} + {\rm h.c.} \\ -\frac{\lambda_\Delta}{M_\Delta}\left(Y_\Delta\right)_{\alpha\beta} \overline{l_{\alpha \rm L}} \tilde{H} \tilde{H}^T l^c_{\beta \rm L} + {\rm h.c.} \\ \frac{1}{2}\left(Y_\Sigma M_\Sigma^{-1} Y_\Sigma^T\right)_{\alpha\beta} \overline{l_{\alpha \rm L}} \tilde{H} \tilde{H}^T l^c_{\beta \rm L} + {\rm h.c.} \end{cases}$$

$$M_\nu = \begin{cases} -\frac{1}{2} Y_\nu \frac{v^2}{M_{\rm R}} Y_\nu^T & (\text{Type 1}) \\ \lambda_\Delta Y_\Delta \frac{v^2}{M_\Delta} & (\text{Type 2}) \\ -\frac{1}{2} Y_\Sigma \frac{v^2}{M_\Sigma} Y_\Sigma^T & (\text{Type 3}) \end{cases}$$

After SSB, a Majorana mass term is $-\mathcal{L}_{\rm mass} = \frac{1}{2}\overline{\nu_{\rm L}} M_\nu \nu^c_{\rm L} + {\rm h.c.}$ $\langle \tilde{H} \rangle = v/\sqrt{2}$







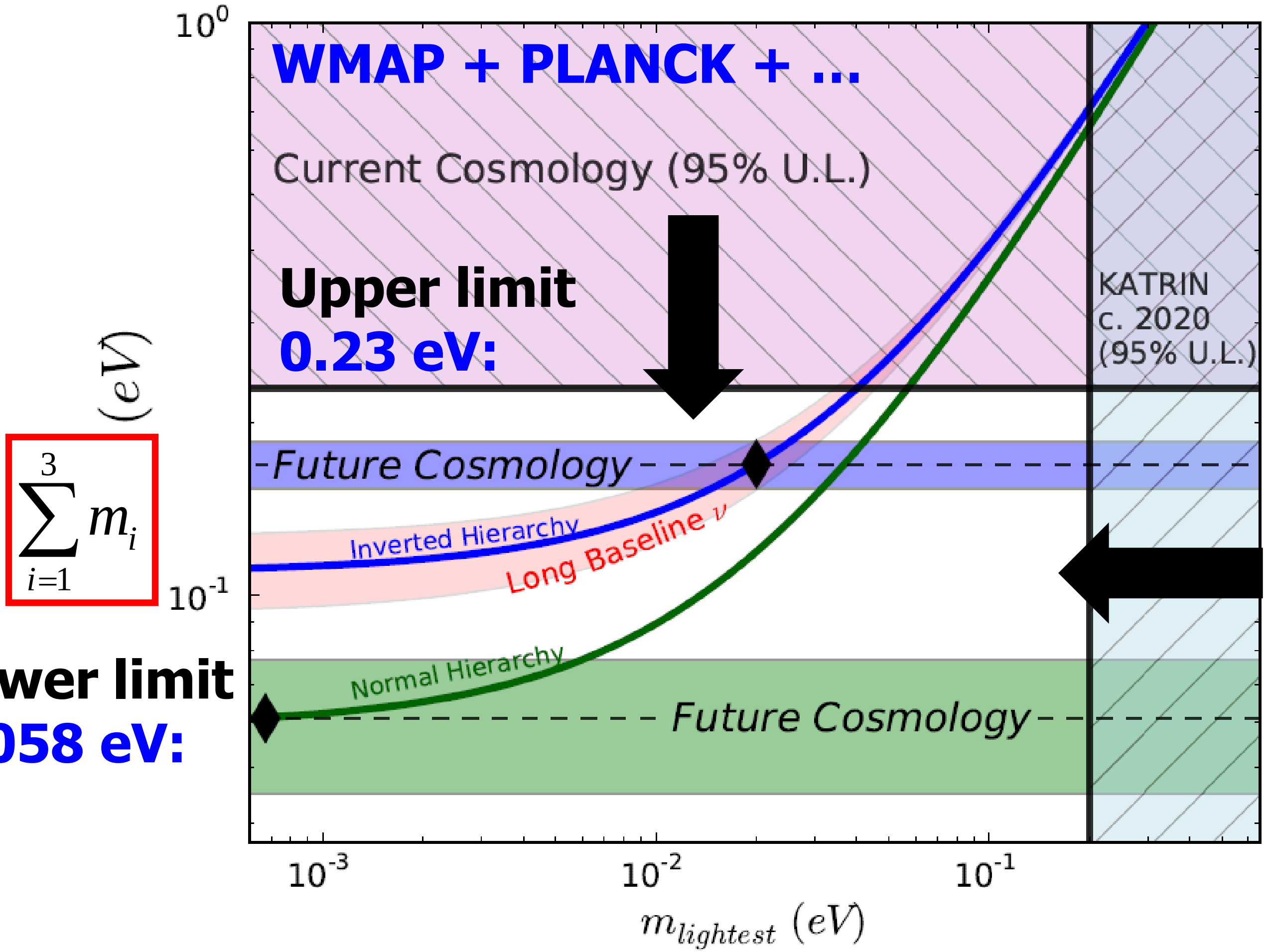
Light neutrino masses

Three ways: the β decay, the 0νββ decay, and cosmology (CMB + LSS).

$$\langle m \rangle_e^2 = \sum_{i=1}^{3} m_i^2 \left| U_{ei} \right|^2$$

$$\left| \langle m \rangle_{ee} \right| = \left| \sum_{i=1}^{3} m_i U_{ei}^2 \right|$$

$$\sum_{i=1}^{3} m_i$$



WMAP + PLANCK + …

Upper limit
0.23 eV:

Lower limit
0.058 eV:

mass scale
≤ O(0.1) eV

Why so tiny?

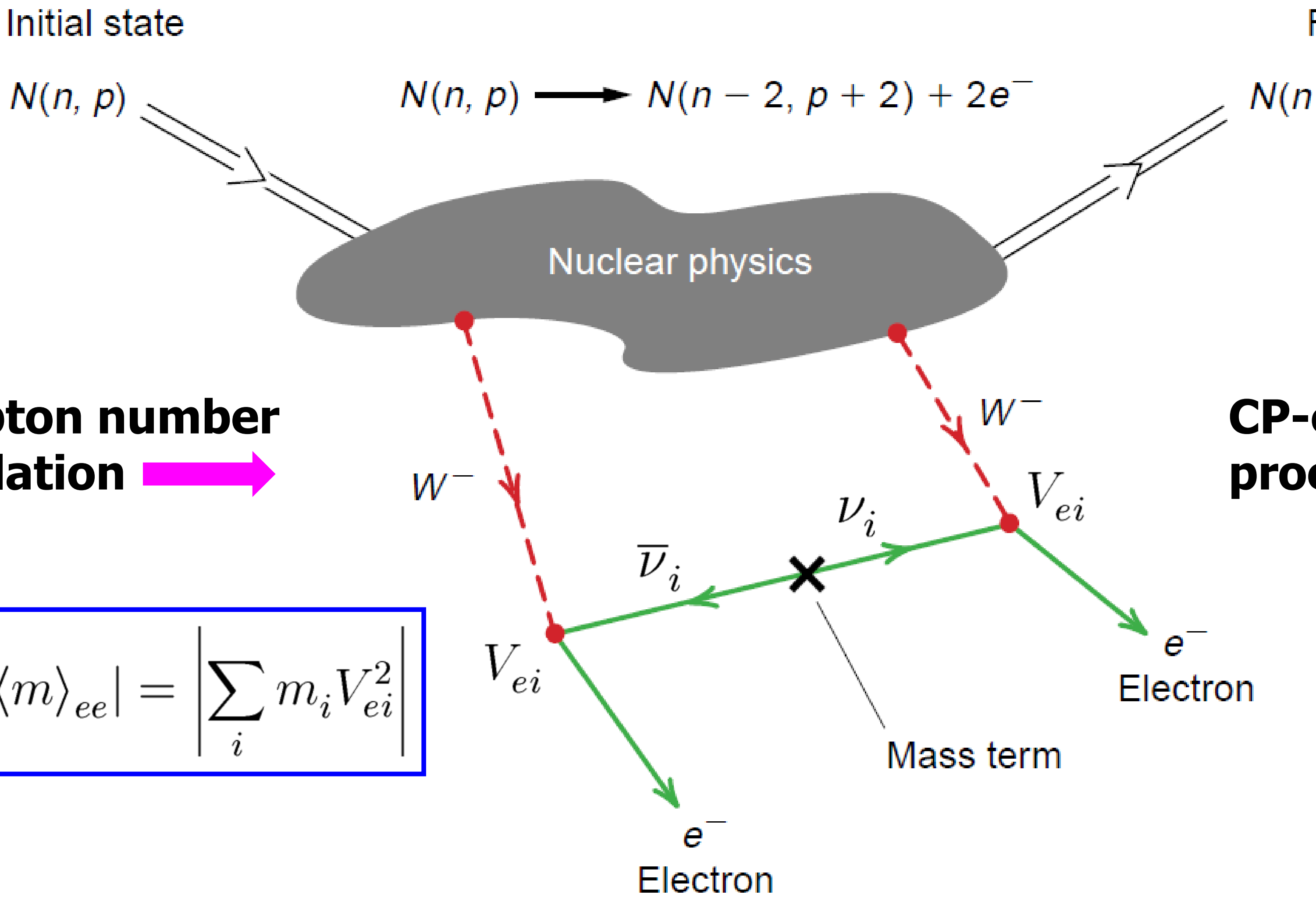
arXiv:1309.5383

Stage-4 CMB

$$\sigma\left(\sum m_\nu\right) = 16 \text{ meV}$$
$$\sigma\left(N_{\text{eff}}\right) = 0.020\,.$$

$0\nu\beta\beta$

The neutrinoless double beta decay can happen if massive neutrinos are the Majorana particles (W.H. Furry 1939):



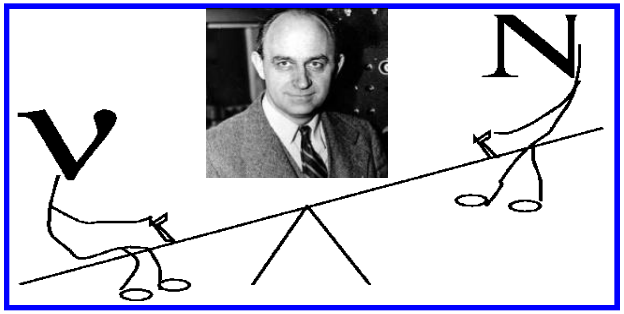
Lepton number violation ➡

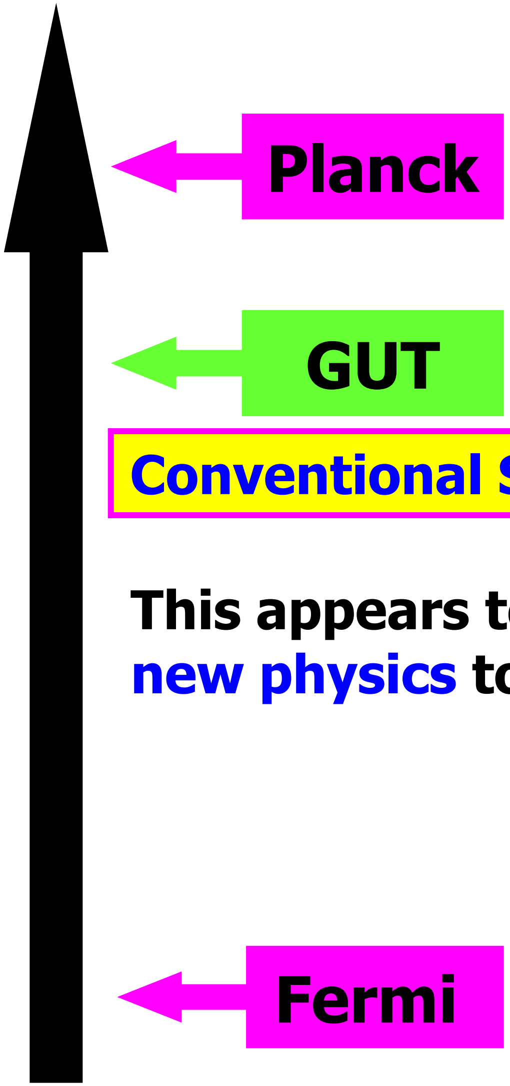
CP-conserving process ⬅

$$\boxed{|\langle m\rangle_{ee}| = \left|\sum_i m_i V_{ei}^2\right|}$$

Seesaw scale?

What is the energy scale at which the **seesaw** mechanism works?





Planck

GUT to unify strong, weak & electromagnetic forces

Conventional Seesaws: heavy degrees of freedom near **GUT**

This appears to be rather reasonable, since one often expects **new physics** to appear around a **fundamental** scale

Fermi

Naturalness √

Testability ×

Uniqueness ×

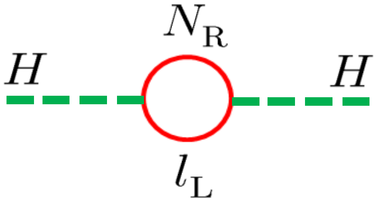
Hierarchy ×

Hierarchy problem

Seesaw-induced fine-tuning problem: the Higgs mass is very sensitive to quantum corrections from the heavy degrees of freedom in seesaw (Vissani 98; Casas et al 04; Abada et al 07)

Type 1:

$$\boxed{\delta m_H^2 = -\frac{y_i^2}{8\pi^2}\left(\Lambda^2 + M_i^2 \ln\frac{M_i^2}{\Lambda^2}\right)}$$

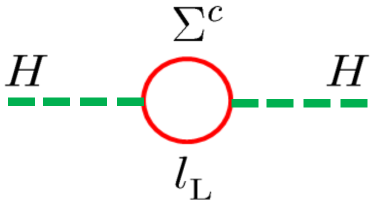


Type 2:

$$\boxed{\delta m_H^2 = \frac{3}{16\pi^2}\left[\lambda_3\left(\Lambda^2 + M_\Delta^2 \ln\frac{M_\Delta^2}{\Lambda^2}\right) + 4\lambda_\Delta^2 M_\Delta^2 \ln\frac{M_\Delta^2}{\Lambda^2}\right]}$$

Type 3:

$$\boxed{\delta m_H^2 = -\frac{3y_i^2}{8\pi^2}\left(\Lambda^2 + M_i^2 \ln\frac{M_i^2}{\Lambda^2}\right)}$$



here y_i & M_i are eigenvalues of Y_ν (or Y_Σ) & M_R (or M_Σ), respectively.

An illustration of fine-tuning

$$\boxed{M_i \sim \left[\frac{(2\pi v)^2|\delta m_H^2|}{m_i}\right]^{1/3} \sim \underline{10^7\mathrm{GeV}}\left[\frac{0.2\ \mathrm{eV}}{m_i}\right]^{1/3}\left[\frac{|\delta m_H^2|}{0.1\ \mathrm{TeV}^2}\right]^{1/3}}$$

Possible way out: (1) Supersymmetric seesaw? (2) TeV-scale seesaw?

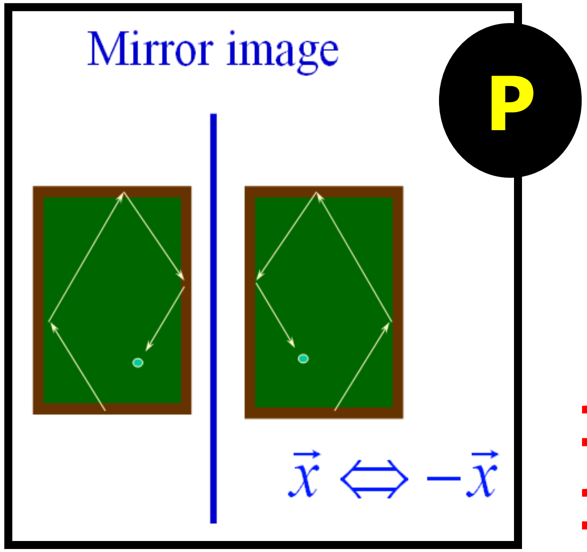
Flavor mixing + CPV

Flavor mixing: mismatch between **weak/flavor** eigenstates and **mass** eigenstates of fermions due to coexistence of **2** types of interactions.

Weak eigenstates: members of weak isospin doublets transforming into each other through the interaction with the ***W*** boson;

Mass eigenstates: states of definite masses that are created by the interaction with the Higgs boson (**Yukawa** interactions).

CP violation: **matter** and **antimatter**, or a reaction & its CP-conjugate process, are distinguishable --- coexistence of **2** types of interactions.

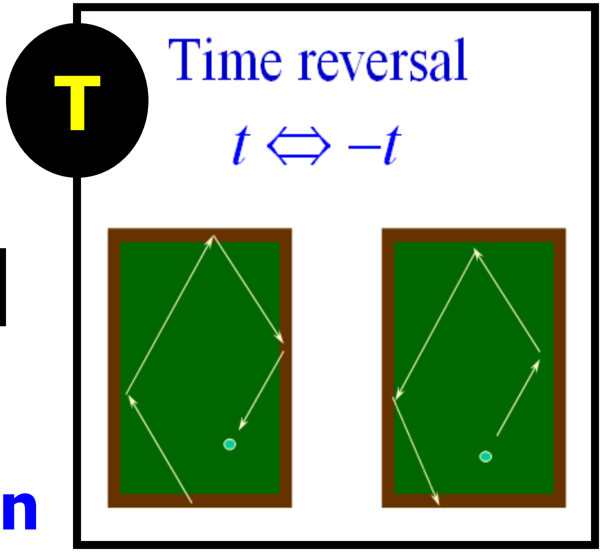


C

Charge-conjugation

1957: P violation

1964: CP violation



Part C

CKM + PMNS

The **Yukawa** interactions of all fermions are **formally invariant** under **CP** transformation if and only if the **Yukawa** coupling matrices are all real (Kobayashi and Maskawa, 1973)

$$L_{\mathrm{SM}} = L(f, G) + L(f, H) + L(G, H) + L(G) - V(H)$$

Nobel Prize 2008

If the **flavor states** are transformed into the **mass states**, the source of flavor mixing and **CP** violation will show up in the ***CC*** interactions:

quarks

$$\mathcal{L}_{\mathrm{cc}} = \frac{g}{\sqrt{2}} \overline{(u\ c\ t)_{\mathrm{L}}}\ \gamma^{\mu} U \begin{pmatrix} d \\ s \\ b \end{pmatrix}_{\mathrm{L}} W^{+}_{\mu} + \mathrm{h.c.}$$

leptons

$$\mathcal{L}_{\mathrm{cc}} = \frac{g}{\sqrt{2}} \overline{(e\ \mu\ \tau)_{\mathrm{L}}}\ \gamma^{\mu} V \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\mathrm{L}} W^{-}_{\mu} + \mathrm{h.c.}$$

Comment A: flavor mixing and **CP** violation can occur since fermions interact with both the **gauge bosons** and the **Higgs boson**.

Comment B: both the **CC** and Yukawa interactions have been verified.

Comment C: the **CKM** matrix ***U*** is unitary, the **PMNS** matrix ***V*** is too?

Part C

Physical phases

If neutrinos are **Dirac** particles, the phases ***x***, ***y*** and ***z*** can be removed. Then the neutrino mixing matrix is

Dirac neutrino mixing matrix

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

If neutrinos are **Majorana** particles, left- and right-handed fields are correlated. Hence only a common phase of three left-handed fields can be redefined (e.g., ***z* = 0**). Then

Majorana neutrino mixing matrix

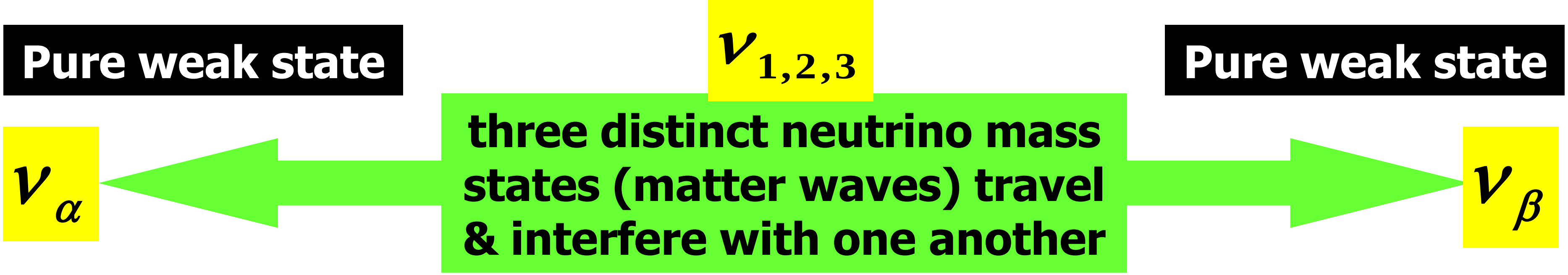
$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Part C

What is ν-oscillation?

Oscillation — a spontaneous periodic change from one neutrino flavor state to another, is a spectacular quantum phenomenon. It can occur as a natural consequence of neutrino mixing.

In a neutrino oscillation experiment, the neutrino beam is produced and detected via the weak **charged-current interactions**.



Pure weak state

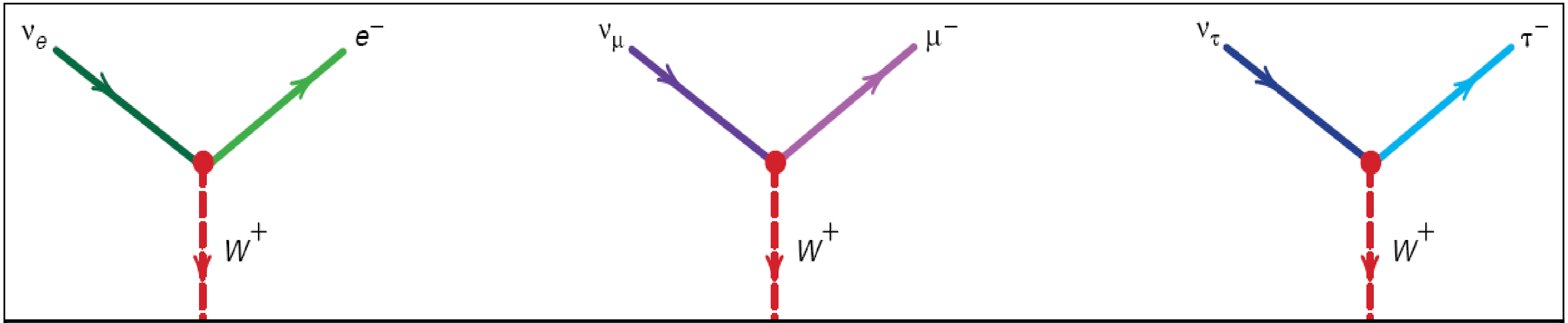
$\nu_{1,2,3}$

Pure weak state

ν_α

three distinct neutrino mass states (matter waves) travel & interfere with one another

ν_β

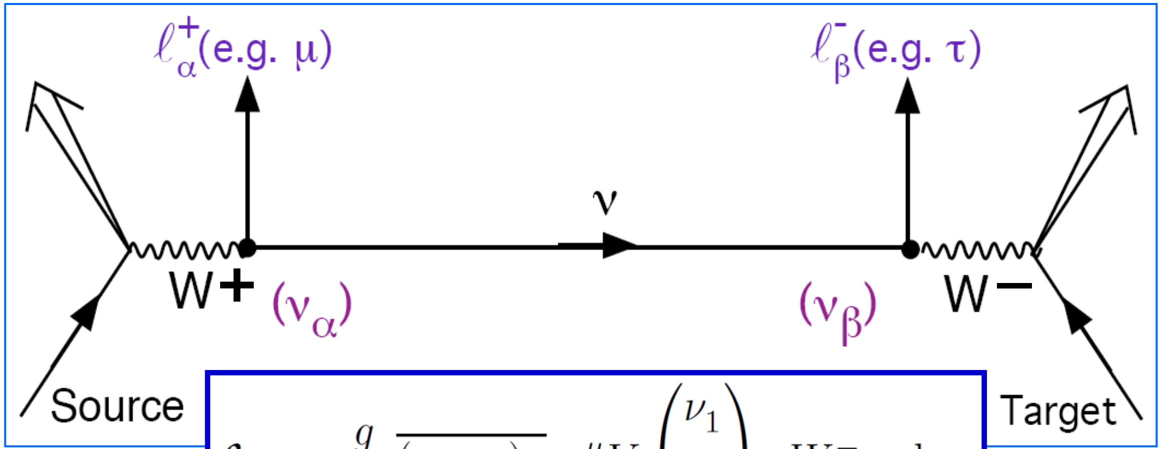


For example: $\overline{\nu_e}$ beam: β decay; ν_μ beam: π decay; ν_τ beam: D decay

Part C

3-flavor oscillation

Production and detection of a neutrino beam by CC weak interactions:



$$\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}}\,\overline{(e\ \mu\ \tau)_{\rm L}}\,\gamma^\mu V \begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}_{\rm L} W^-_\mu + {\rm h.c.}$$

$$|\nu_\alpha(0)\rangle = |\nu_\alpha\rangle = \sum_{i=1}^{3} \underline{V^*_{\alpha i}}|\nu_i\rangle$$

$$|\nu_\alpha(t)\rangle = \sum_{i=1}^{3} \underline{V^*_{\alpha i}}e^{-iE_it}|\nu_i\rangle$$

$\alpha, \beta, \gamma = e, \mu, \tau$

$i, j, k = 1,2,3$

$$A(\nu_\alpha \to \nu_\beta) = \langle\nu_\beta|\nu_\alpha(t)\rangle = \left(\sum_{j=1}^{3} V_{\beta j}\langle\nu_j|\right)\left(\sum_{i=1}^{3} V^*_{\alpha i}e^{-iE_it}|\nu_i\rangle\right) = \sum_{i=1}^{3} V^*_{\alpha i}V_{\beta i}e^{-iE_it}$$

$$P(\nu_\alpha \to \nu_\beta) = \left|\langle\nu_\beta|\nu_\alpha(t)\rangle\right|^2 = \left|\sum_{i=1}^{3} V^*_{\alpha i}V_{\beta i}e^{-iE_it}\right|^2$$

$$= \sum_{i=1}^{3}\left|V^*_{\alpha i}V_{\beta i}\right|^2 + 2\sum_{i<j}^{3} {\rm Re}\left[V^*_{\alpha i}V_{\beta i}V_{\alpha j}V^*_{\beta j}e^{i(E_j-E_i)t}\right]$$

CP violation

The final formula of 3-flavor oscillation probabilities with CP violation:

$$\begin{aligned} P\left(\nu_\alpha \to \nu_\beta\right) &= \delta_{\alpha\beta} - 4\sum_{i<j}^{3} \mathrm{Re}\left(V_{\alpha i}V_{\beta j}V^*_{\alpha j}V^*_{\beta i}\right)\sin^2\frac{\Delta m^2_{ji}L}{4E} \\ &\quad + 8\mathcal{J}\sum_{\gamma}\epsilon_{\alpha\beta\gamma}\sin\frac{\Delta m^2_{21}L}{4E}\sin\frac{\Delta m^2_{31}L}{4E}\sin\frac{\Delta m^2_{32}L}{4E} \end{aligned}$$

Under CPT invariance, CP- and T-violating asymmetries are identical:

$$\begin{aligned} P\left(\nu_\alpha \to \nu_\beta\right) - P\left(\overline{\nu}_\alpha \to \overline{\nu}_\beta\right) &= P\left(\nu_\alpha \to \nu_\beta\right) - P\left(\nu_\beta \to \nu_\alpha\right) \\ &= 16\mathcal{J}\sum_{\gamma}\epsilon_{\alpha\beta\gamma}\sin\frac{\Delta m^2_{21}L}{4E}\sin\frac{\Delta m^2_{31}L}{4E}\sin\frac{\Delta m^2_{32}L}{4E} \end{aligned}$$

Jarlskog invariant, a rephasing-invariant measure of CP / T violation:

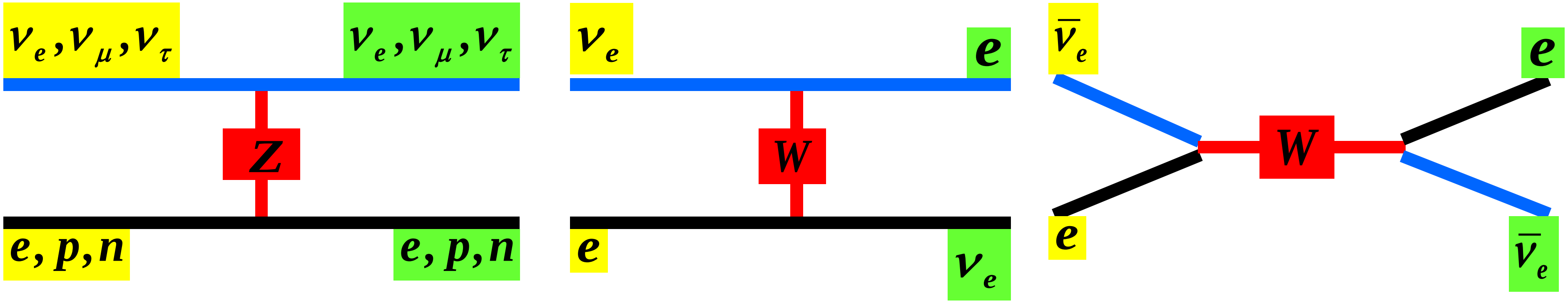
$$J = \sin\theta_{12}\cos\theta_{12}\sin\theta_{23}\cos\theta_{23}\sin\theta_{13}\cos^2\theta_{13}\sin\delta \le 1/6\sqrt{3} \approx 9.6\%$$

Part C

Matter effect?

When light travels through a medium, it sees a refractive index due to coherent forward scattering from the constituents of the medium.

A similar phenomenon applies to neutrino flavor states as they travel through matter. All flavor states see a common refractive index from NC forward scattering, and the electron (anti) neutrino sees an extra refractive index due to CC forward scattering in matter.



Consequence of Mikheyev-Smirnov-Wolfenstein (MSW) matter effect:

- ♣ **Matter-modified oscillation behavior:**

$$\boxed{\Delta m_{ij}^2 + 2\sqrt{2}G_{\mathrm{F}}N_e E}$$

- ♣ **Fake CP-violating effect in oscillation.**

Experimental data

Quark mixing:

$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & e^{-i\delta} & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Experiments:

$\theta_{12} \rightarrow \theta_{23} \rightarrow \theta_{13} \rightarrow \delta$ new physics ?

~13° ~2° ~0.2° ~65° unitarity ?

turning point

Lepton mixing: (in general, we consider three Majorana neutrinos)

$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & e^{-i\delta} & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\theta_{23} \rightarrow \theta_{12} \rightarrow \theta_{13} \rightarrow \delta/\rho/\sigma$ new physics ?

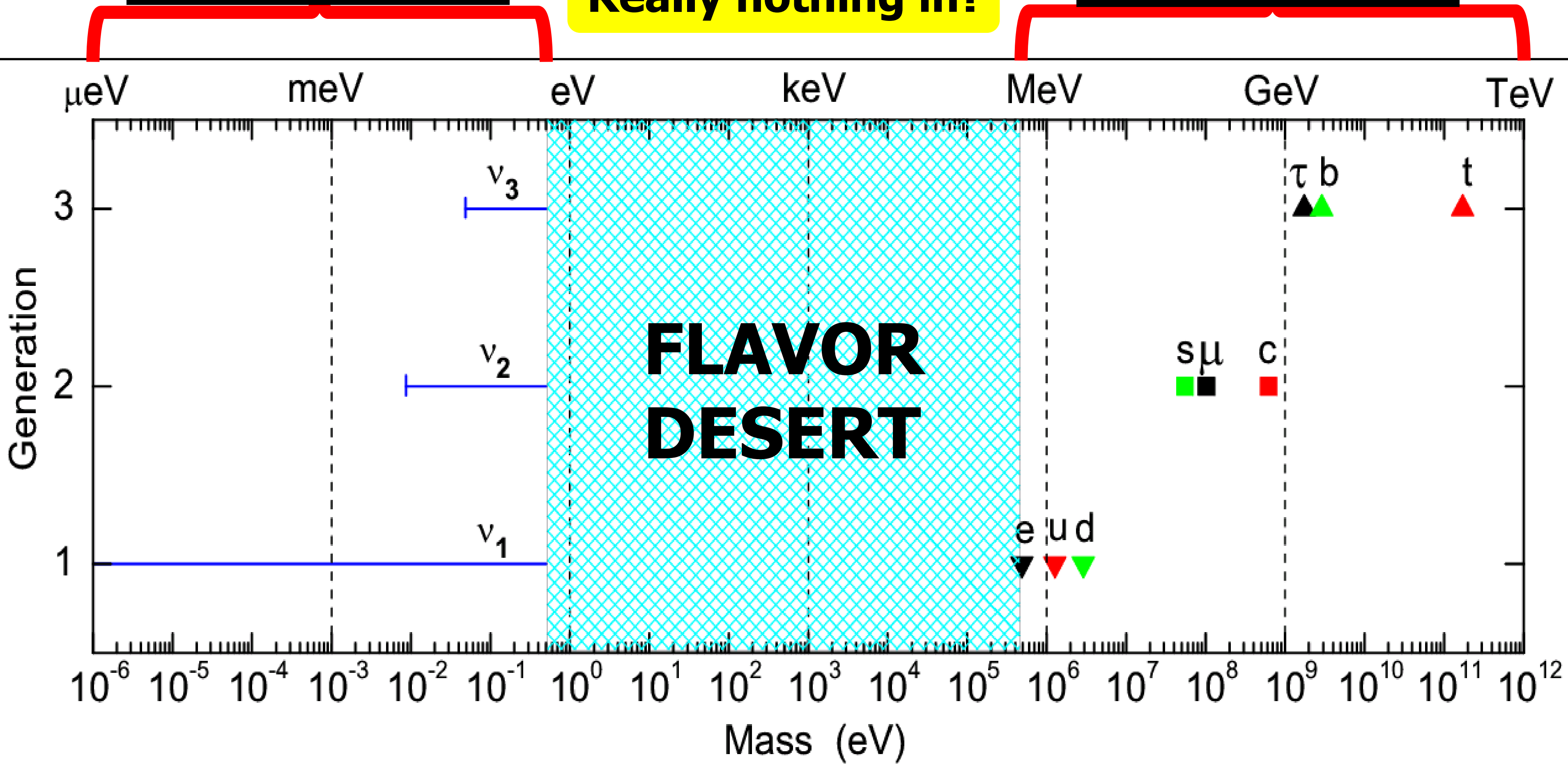
~45° ~34° ~9° ~? ? ? unitarity ?

Daya Bay

Part D

Flavor puzzles

Neutral **Really nothing in?** **Charged**



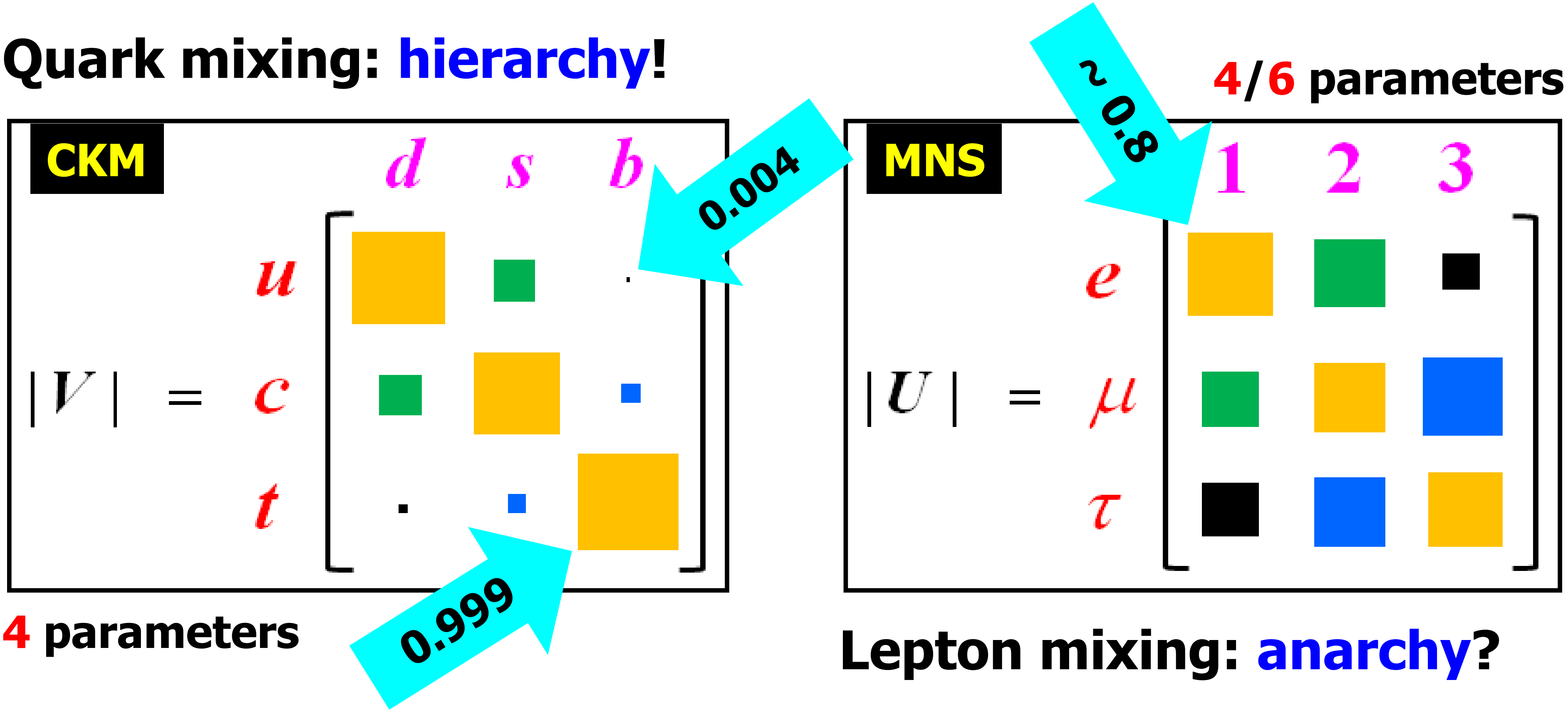
Flavor hierarchy + Flavor desert puzzles: 12 free (mass) parameters.

Flavor puzzles

$$-\mathcal{L}_{cc} = \frac{g}{\sqrt{2}}\left[\overline{(u\ \ c\ \ t)_L}\,\gamma^\mu\, \underset{\uparrow}{V} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L W^+_\mu + \overline{(e\ \ \mu\ \ \tau)_L}\,\gamma^\mu\, \underset{\uparrow}{U} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L W^-_\mu\right] + \text{h.c.}$$

CKM **PMNS**

Quark mixing: hierarchy!



4/6 parameters

4 parameters

Lepton mixing: anarchy?

Possible structures

Quark Mixing

large — large small

0 θ_{12} θ_{23} θ_{13}

CKM Matrix = Identity Matrix + Corrections

Experiments

Flavor Symmetry

CP Violation

Lepton Mixing

large small

θ_{12} θ_{23} θ_{13}

PMNS Matrix = Constant Term+Corrections

Experiments

Flavor Symmetry

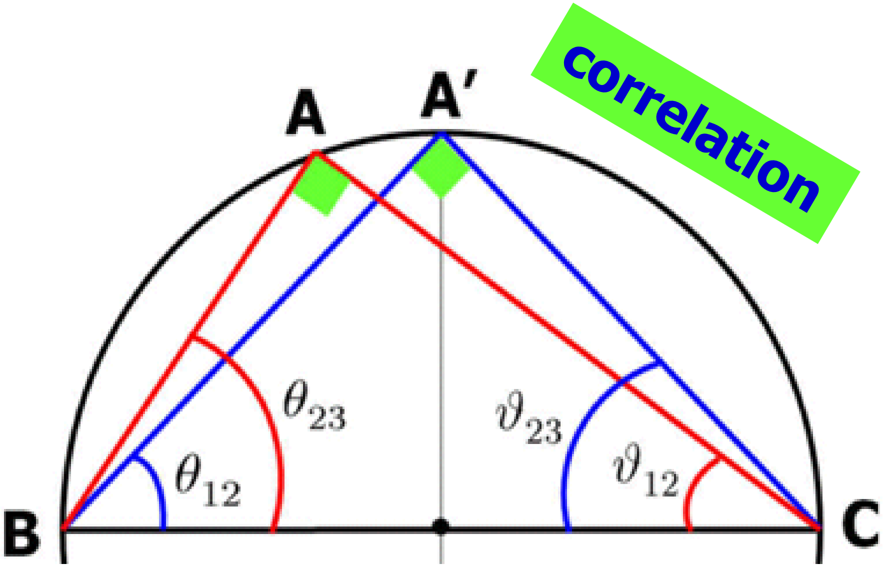
CP Violation

Typical examples

Democratic Mixing Pattern (96)

$$V = \begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ -\frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{3} \\ \frac{\sqrt{3}}{3} & -\frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} \end{pmatrix} \quad \Longrightarrow \quad V = \begin{pmatrix} \frac{\sqrt{6}}{3} & \frac{\sqrt{3}}{3} & 0 \\ -\frac{\sqrt{6}}{6} & \frac{\sqrt{3}}{3} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{6}}{6} & -\frac{\sqrt{3}}{3} & \frac{\sqrt{2}}{2} \end{pmatrix}$$

Tri-bimaximal Mixing Pattern (02)



$$\theta_{12} = \pi/4 \qquad \vartheta_{12} = \pi/4 - \theta_*$$

$$\theta_{23} = \pi/4 + \theta_* \qquad \vartheta_{23} = \pi/4$$

Democratic **Tri-bimaximal**

$$\theta_* = \arctan(\sqrt{2}) - \pi/4 = \pi/4 - \arctan(1/\sqrt{2}) \approx 9.7^\circ$$

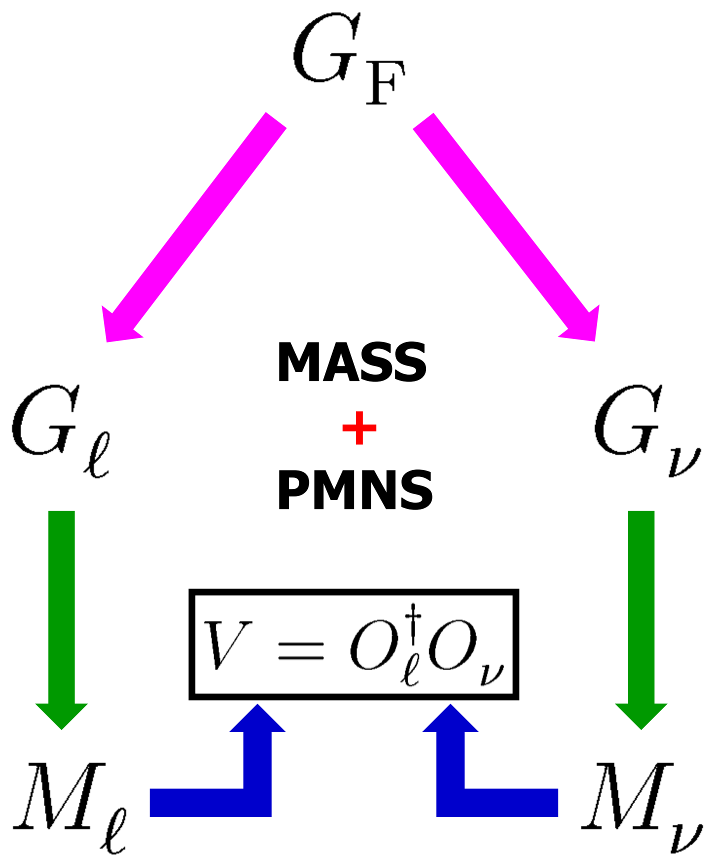
Part D

Flavor symmetries?

Some small discrete groups for model building (Altarelli, Feruglio 2010).

Too many possibilities!
Which one stands out?

Group	d	Irreducible representation
$D_3 \sim S_3$	6	$1, 1', 2$
D_4	8	$1_1, \ldots, 1_4, 2$
D_7	14	$1, 1', 2, 2', 2''$
A_4	12	$1, 1', 1'', 3$
$A_5 \sim PSL_2(5)$	60	$1, 3, 3', 4, 5$
T'	24	$1, 1', 1'', 2, 2', 2'', 3$
S_4	24	$1, 1', 2, 3, 3'$
$\Delta(27) \sim Z_3 \rtimes Z_3$	27	$1_1, 1_9, 3, \bar{3}$
$PSL_2(7)$	168	$1, 3, \bar{3}, 6, 7, 8$
$T_7 \sim Z_7 \rtimes Z_3$	21	$1, 1', \bar{1}', 3, \bar{3}$



MASS
+
PMNS

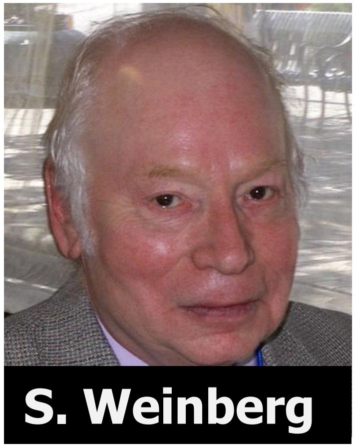
$$\boxed{V = O_\ell^\dagger O_\nu}$$

Texture zeros?

Flavor mixing angles depend on the fermion mass ratios?

$$\theta_{ij} = f\left(\frac{m_\alpha}{m_\beta}, \frac{m_k}{m_l}, \cdots\right)$$

$$M_{l,\nu} = \begin{pmatrix} 0 & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{pmatrix}$$



S. Weinberg

1977

2×2 3×3

Texture zeros

1978

H. Fritzsch

Example: 7 two-zero textures of the Majorana neutrino mass matrix allowed by current experimental data:

$$\mathbf{A_1}\begin{pmatrix} 0 & 0 & \times \\ 0 & \times & \times \\ \times & \times & \times \end{pmatrix} \quad \mathbf{B_1}\begin{pmatrix} \times & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{pmatrix} \quad \mathbf{B_3}\begin{pmatrix} \times & 0 & \times \\ 0 & 0 & \times \\ \times & \times & \times \end{pmatrix} \quad \mathbf{C}\begin{pmatrix} \times & \times & \times \\ \times & 0 & \times \\ \times & \times & 0 \end{pmatrix}$$

Frampton, Glashow, Marfatia; Xing, 2002

$$\mathbf{A_2}\begin{pmatrix} 0 & \times & 0 \\ \times & \times & \times \\ 0 & \times & \times \end{pmatrix} \quad \mathbf{B_2}\begin{pmatrix} \times & 0 & \times \\ 0 & \times & \times \\ \times & \times & 0 \end{pmatrix} \quad \mathbf{B_4}\begin{pmatrix} \times & \times & 0 \\ \times & \times & \times \\ 0 & \times & 0 \end{pmatrix}$$

- ◆ **1-zero case: less predictive**
- ◆ **2-zero case: survive**
- ◆ **3-zero case: excluded**

Part D

Uniqueness + Testability?

Flavor Symmetry

Texture zeros

Element correlations

GUT relations

They reduce the number of free parameters, and thus lead to predictions for 3 flavor mixing angles in terms of either the mass ratios or constant numbers.

Example (flavor symmetries)

$$M_{l,\nu} = \begin{pmatrix} 0 & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{pmatrix}$$

Dependent on mass ratios

Example (flavor symmetries)

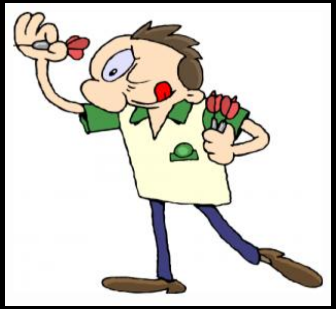
$$M_{\nu} = \begin{pmatrix} b+c & -b & -c \\ -b & a+b & -a \\ -c & -a & a+c \end{pmatrix}$$

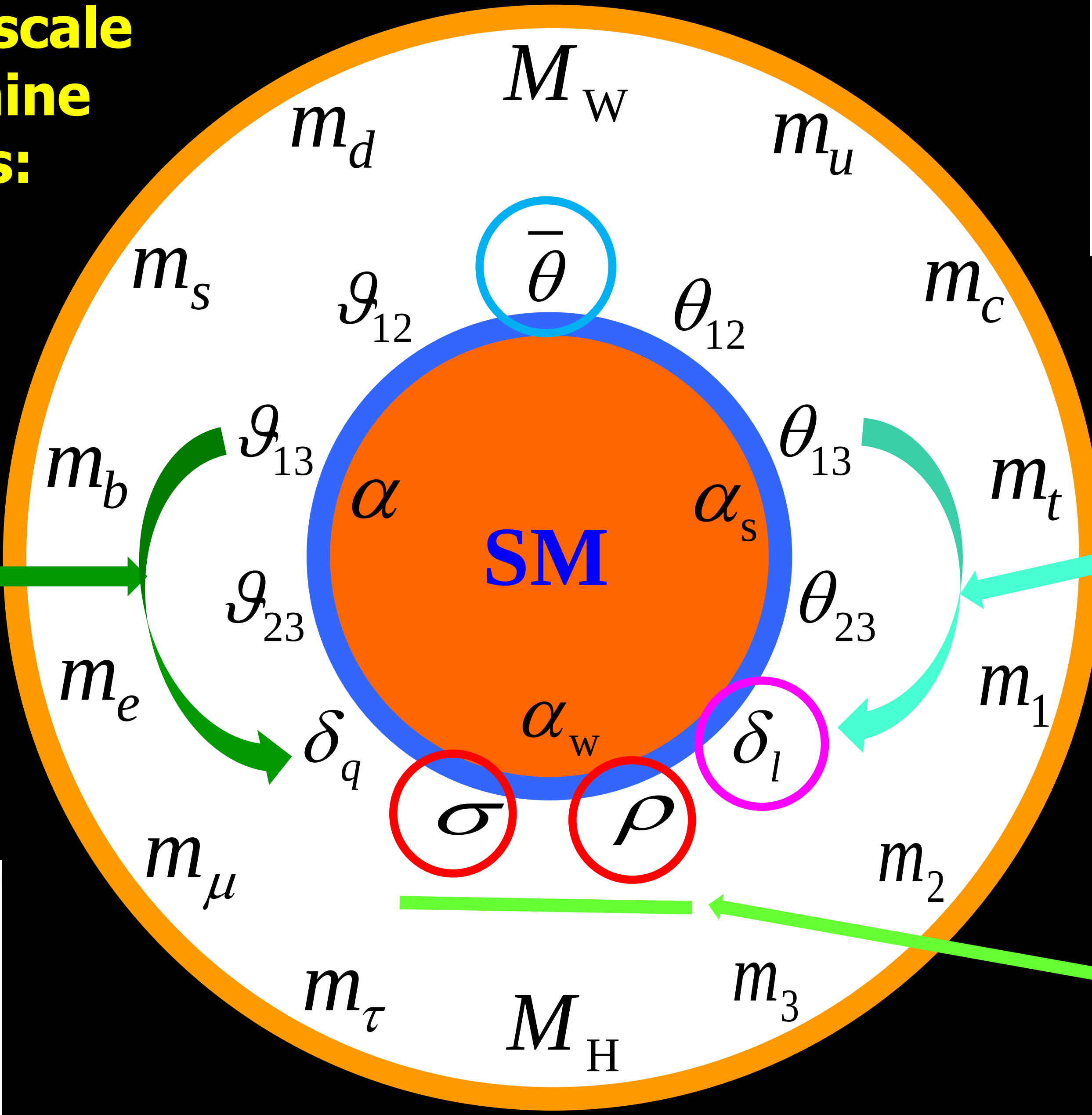
Dependent on simple numbers

PREDICTIONS

SM + neutrinos are left with CP-violating phases

The time scale to determine CP phases:



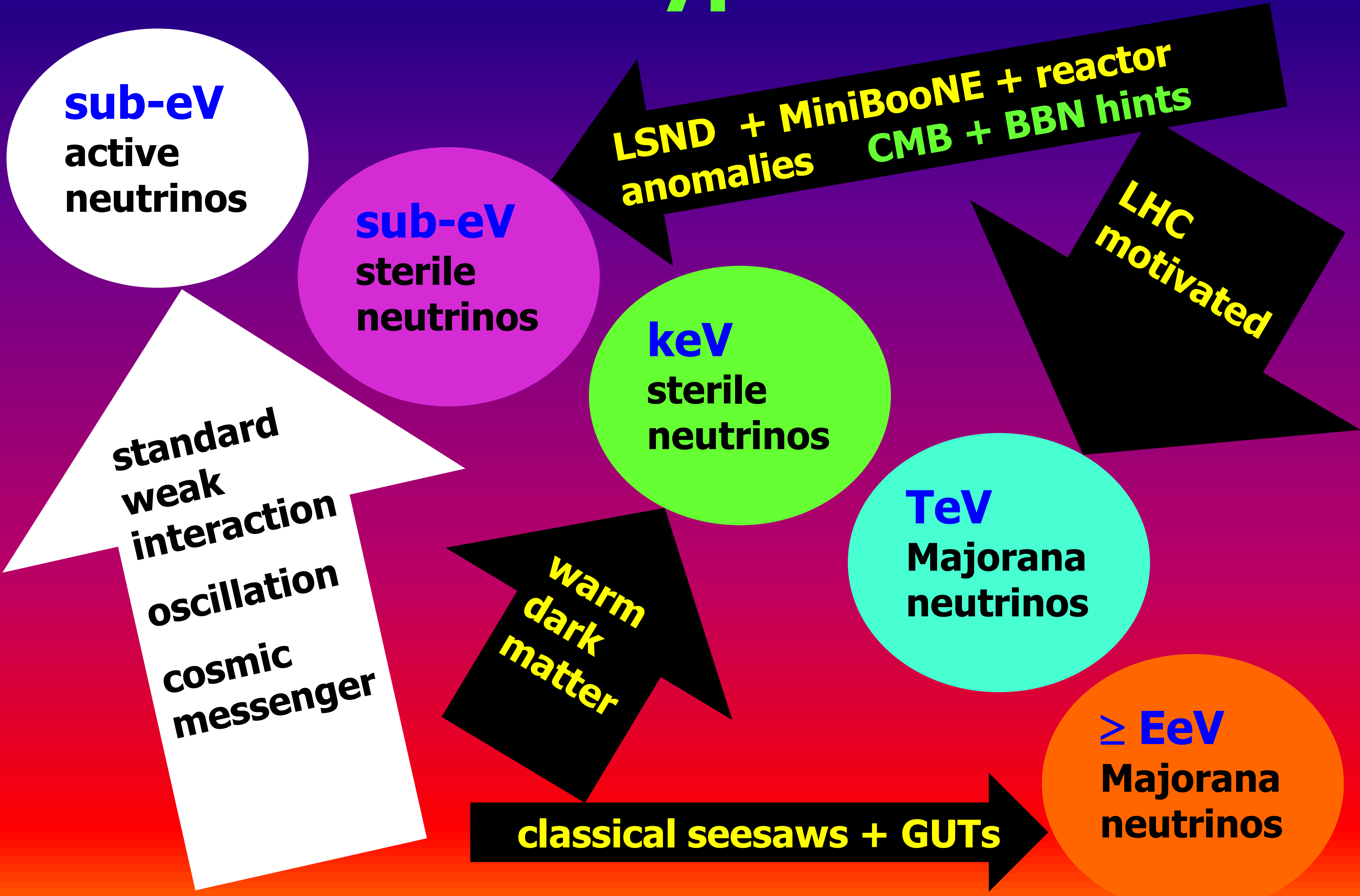


~20 yrs!

~20 yrs?

~60 yrs? or more?

Real + Hypothetical ν's



sub-eV
active
neutrinos

standard
weak
interaction

oscillation

cosmic
messenger

sub-eV
sterile
neutrinos

LSND + MiniBooNE + reactor
anomalies

CMB + BBN hints

keV
sterile
neutrinos

warm
dark
matter

LHC
motivated

TeV
Majorana
neutrinos

classical seesaws + GUTs

≥ EeV
Majorana
neutrinos

非常感谢您！　祝周末开心！