Lectures on parton showers and matrix elements

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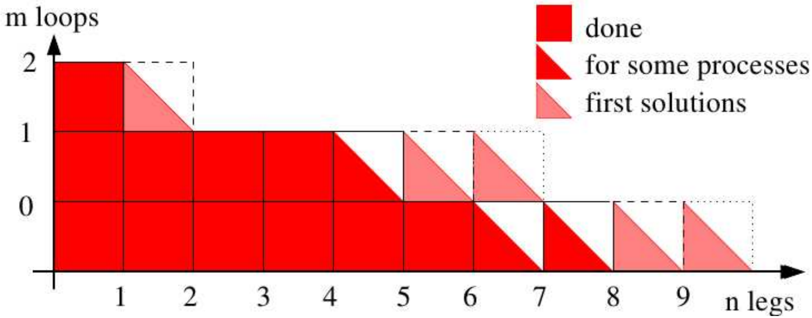
Lecture I: Parton showers

Outline

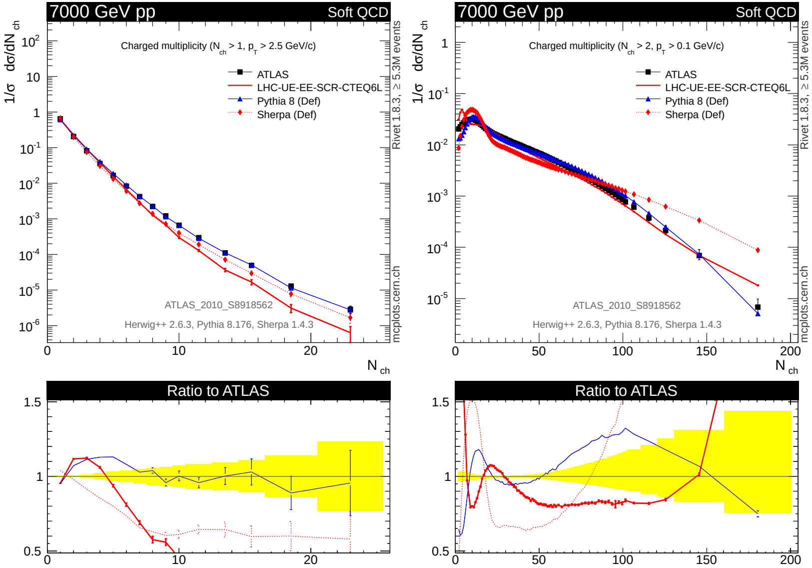
1. Introduction:
   - Why we need all-order (all-leg) approximations.
   - Factorisation
   - Logs and probabilities
2. Probabilities
   - The Sudakov form factor as probability
   - No-emission probabilities
   - Taking probabilities seriously
   - Parton shower basics
3. Parton shower details
   - Initial state radiation
   - Ordering the evolution
   - The soft limit
   - Momentum conservation and all-order improvements

Fixed-order calculations quickly become very complicated.



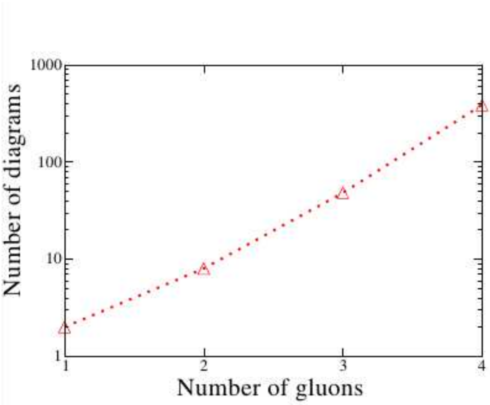
Ten-leg calculations are impressive. . . but multiplicities in the LHC detectors are much higher.

Why fixed-order calculations are not enough



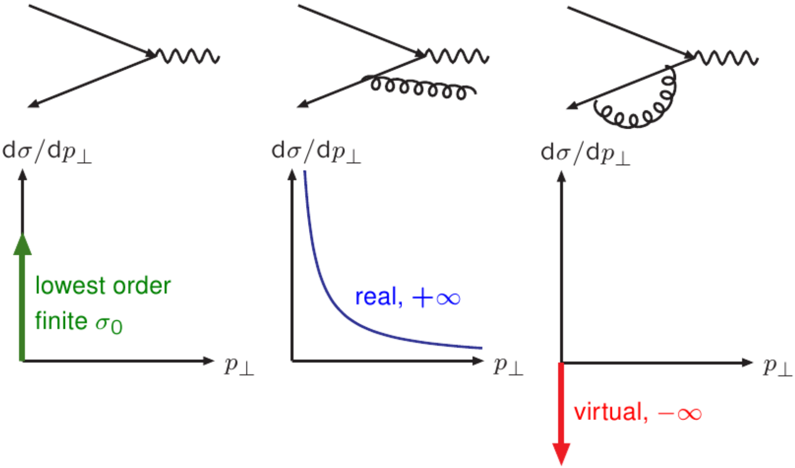
Why fixed-order calculations are not enough

The complexity of multi-gluon amplitudes grows factorially - see $e^+e^- \to q\bar{q} + ng$)

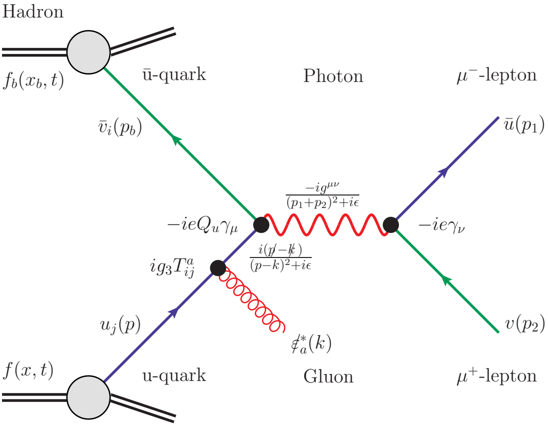


...and even after many clever tricks, this is too slow.

Fixed-order calculations become complex because of infrared singularities.



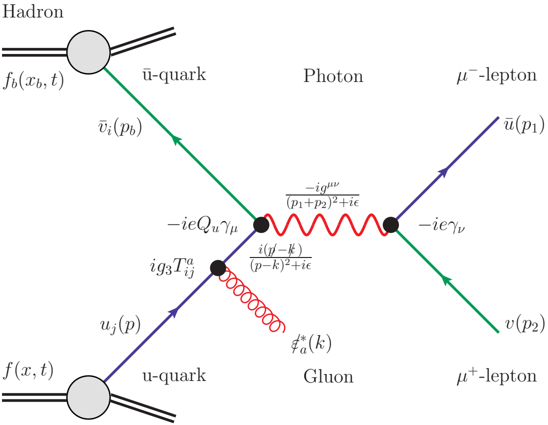
...these also make the predictions unreliable for some observables.



The hadronic cross section is

$$d\sigma(\mathrm{pp} \to \mu^+\mu^-\mathrm{g} + X) = dx dx_b f(x,t) f_b(x_b,t) d\hat{\sigma} \quad , \quad d\hat{\sigma} = \frac{\left|\mathcal{M}(\mathrm{u\bar{u}} \to \mu^+\mu^-\mathrm{g})\right|^2 d\Phi_{n+1}}{4\sqrt{(pp_b)^2}}$$

Factorisation: Divide and conquer

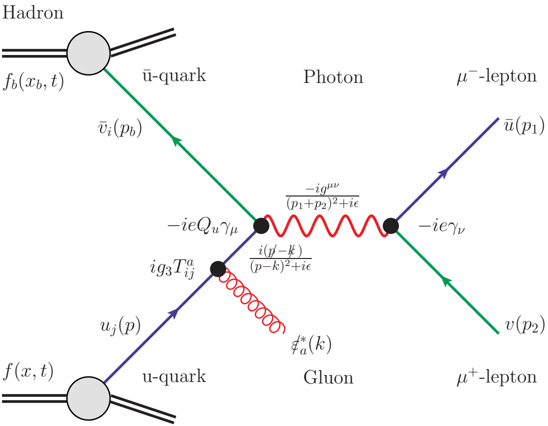


$E_{(p-k)} \approx zE_p$ and small gluon $p_\perp \Rightarrow$ Interal quark almost on-shell. Then:

$$\frac{i(\not p - \not k)}{(p-k)^2} \approx \frac{u(p_a)\bar{u}(p_a)}{p_a^2} \quad , \quad d\Phi_{n+1} \approx d\Phi_n \frac{d\phi dz dp_\perp^2}{4(2\pi)^3(1-z)} \quad , \quad \frac{1}{4\sqrt{(pp_b)^2}} \approx \frac{z}{4\sqrt{(p_ap_b)^2}}$$

$\Longrightarrow$ Matrix element, phase space integration and flux factors factorise!

Factorisation: Divide and conquer



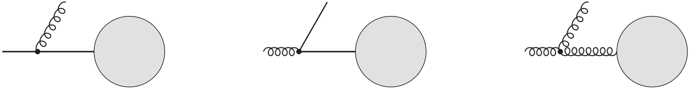
Matrix element, phase space integration and flux factors factorise:

$$d\sigma(\mathrm{pp} \to \mu^+\mu^-\mathrm{g} + X) = d\sigma(\mathrm{pp} \to \mu^+\mu^- + X) \int \frac{dp_\perp^2}{p_\perp^2} \frac{dz}{z} \frac{\alpha_s}{2\pi} C_F \frac{f(\frac{x_a}{z}, t)}{f_a(x_a, t)} \frac{1+z^2}{1-z}$$

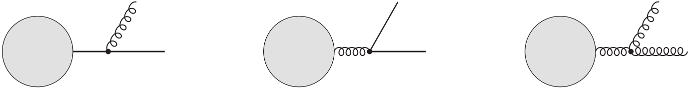
Every cross section containing an additional collinear gluon can be factorised as

$$d\sigma(\mathrm{pp} \to Y + \mathrm{g} + X) = d\sigma(\mathrm{pp} \to Y + X) \int \frac{dp_\perp^2}{p_\perp^2} \frac{dz}{z} \frac{\alpha_s}{2\pi} \frac{f(\frac{x_a}{z}, t)}{f_a(x_a, t)} P(z)$$

with the splitting kernels $P(z)$



$$P_{qq} = C_F \frac{1+z^2}{1-z} \qquad P_{qg} = T_R\left[z^2 + (1-z)^2\right] \qquad P_{gg} = C_A \frac{(1-z(1-z))^2}{z(1-z)}$$



This is independent of the process $\mathrm{pp} \to Y + X$!

$\Longrightarrow$ Multi-parton cross sections can be approximated by "dressing up" low-multiplicity results with many collinear partons!

The splitting kernels

... are independent of the "hard" scattering;

... have a probabilistic interpretation:

aa

$$\int_{p_{\perp\,min}^2}^{p_{\perp\,max}^2} \frac{dp_\perp^2}{p_\perp^2} \int_{z_{min}}^{z_{max}} dz \frac{\alpha_s}{2\pi} P(z) \equiv$$

Probability of emitting a gluon with momentum fraction $1 - z \in [z_{min}, z_{max}]$ and transverse momentum $p_\perp \in [p_{\perp\,min}, p_\perp\,max]$.

Also, note

$$\frac{dp_\perp^2}{p_\perp^2} = \frac{dQ^2}{Q^2} = \frac{d\Theta^2}{\Theta^2} = \frac{d\rho}{\rho} \quad (\text{for} \quad \rho = f(z)p_\perp^2)$$

$\Longrightarrow$ Many variables can be used to characterise the collinear limit!

...and note that we've put the z-range $[z_{min}, z_{max}]$. The lower limit z_{min} comes from the constraint $\frac{x_a}{z} < 1$, the upper limit when conserving 4-momentum.

Emission probabilities

Integrating the splitting probability, we get

$$\begin{aligned}\int_{p^2_{\perp min}}^{p^2_{\perp max}} \frac{dp^2_\perp}{p^2_\perp} \int_{z_{min}}^{z_{max}} dz \frac{\alpha_s}{2\pi} P(z) &\approx \int_{p^2_{\perp min}}^{p^2_{\perp max}} \frac{dp^2_\perp}{p^2_\perp} \int_{z_{min}}^{z_{max}} dz \frac{\alpha_s}{2\pi} \frac{2C_{F/A}}{(1-z)} \\ &\approx \alpha_s \ln\left(\frac{p^2_{\perp max}}{p^2_{\perp min}}\right) \ln\left(\frac{z_{max}}{z_{min}}\right)\end{aligned}$$

More generally, we can write

$$d\sigma(\mathrm{pp} \to Y + \mathrm{g} + X) = d\sigma(\mathrm{pp} \to Y + X) \otimes \left(\alpha_s c_2 L^2 + \alpha_s c_1 L + \alpha_s c_0\right)$$

with $L = \ln\left(Q^2/p^2_{\perp min}\right)$, $Q^2 = \mathcal{O}(p^2_{\perp max})$, $p^2_{\perp min} = \mathcal{O}(\Lambda_{\mathrm{QCD}})$.

Even more generally

$$\begin{aligned}d\sigma(\mathrm{pp} \to Y + n\mathrm{g}) &= d\sigma(\mathrm{pp} \to Y) \otimes \alpha_s^n \left(c_{2n} L^{2n} + c_{2n-1} L^{2n-1} + \cdots + c_0\right) \\ d\sigma(\mathrm{pp} \to Y + n\mathrm{g}) &\approx d\sigma(\mathrm{pp} \to Y) \alpha_s^n c_{2n} L^{2n}\end{aligned}$$

$\Rightarrow$ Multi-parton cross sections can be approximated by leading (double) log.
$\Rightarrow$ Comes from "dressing" low-multiplicity states with many collinear partons!

Logarithms

We found

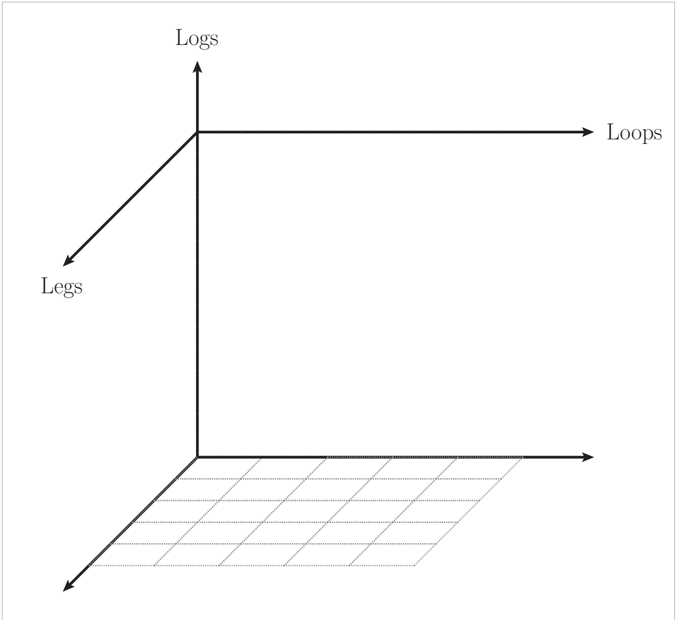
$$d\sigma(\text{pp} \to Y + n\text{g}) = d\sigma(\text{pp} \to Y) \otimes \alpha_s^n \left(c_{2n} L^{2n} + c_{2n-1} L^{2n-1} + \cdots + c_0 \right)$$

and

$$d\sigma(\text{pp} \to Y + n\text{g}) \approx d\sigma(\text{pp} \to Y) \alpha_s^n c_{2n} L^{2n}$$

We can illustrate this logarithmic structure with a "legs-and-logs" plot.

Symbolic figures for QCD calculations: α_s-orders fill L-shapes



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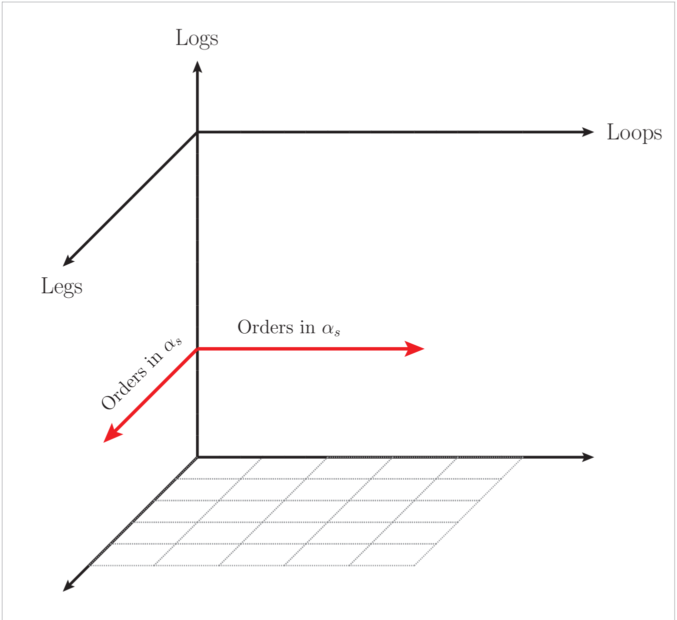

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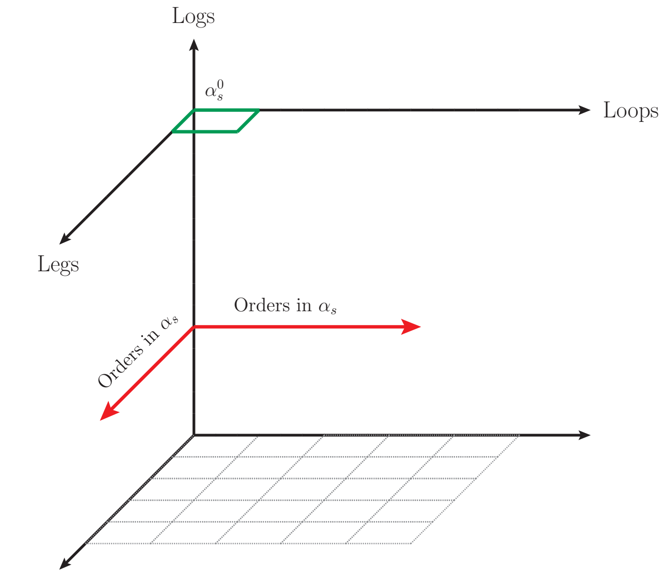

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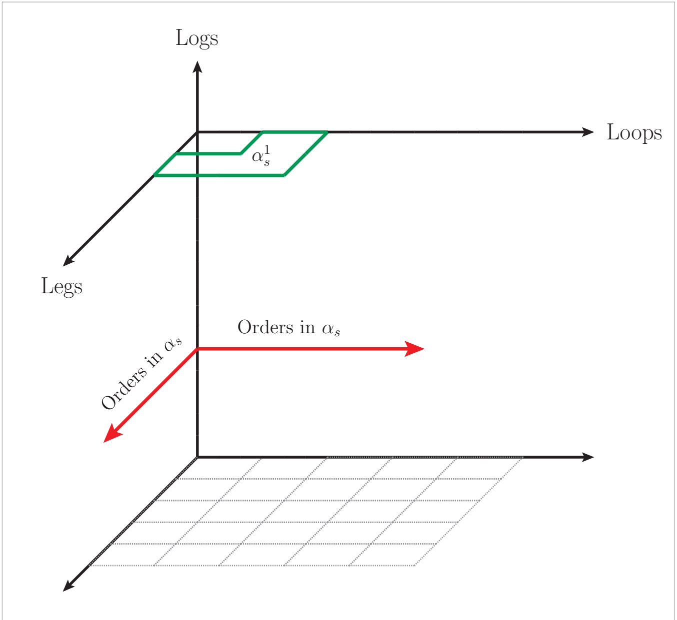

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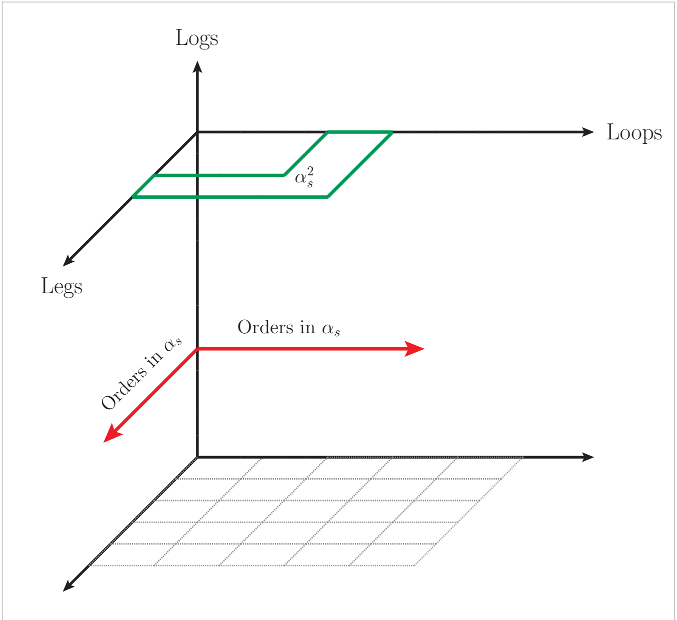
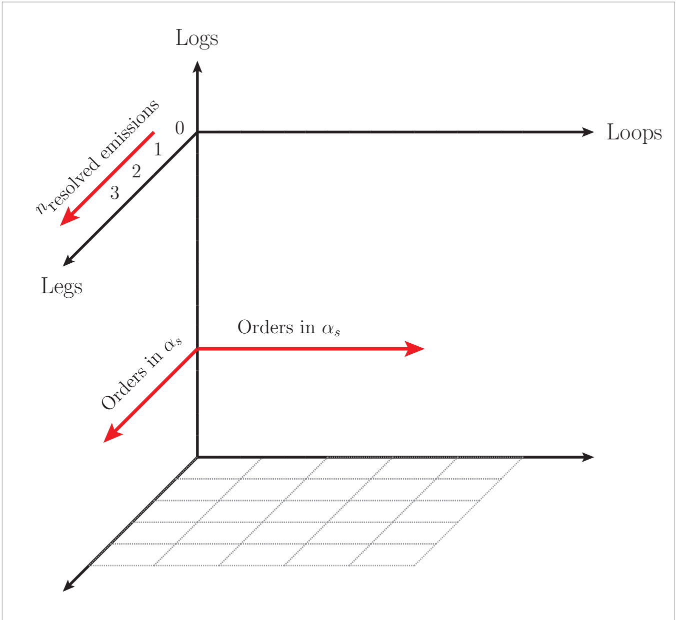


Symbolic figures for QCD calculations: Tree-level terms fill towers



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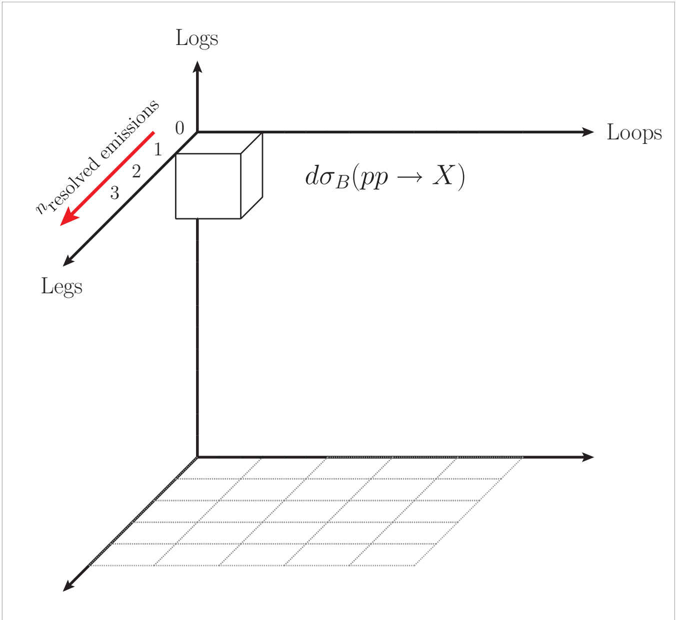

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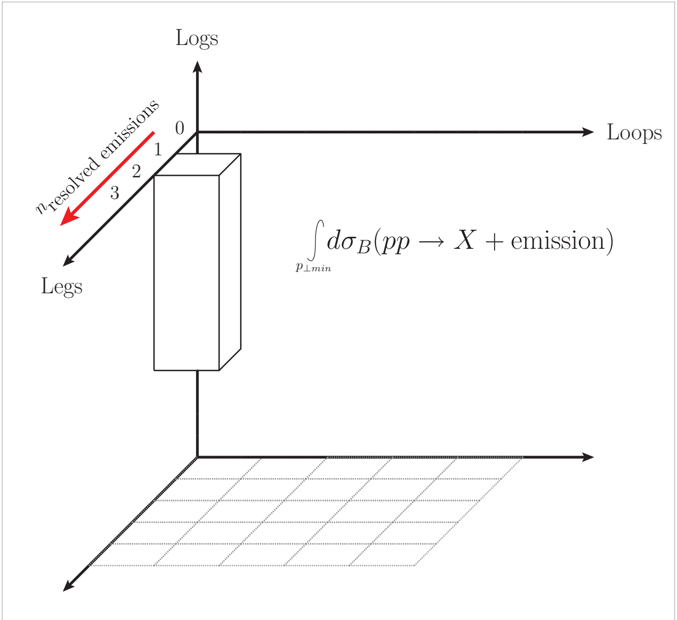
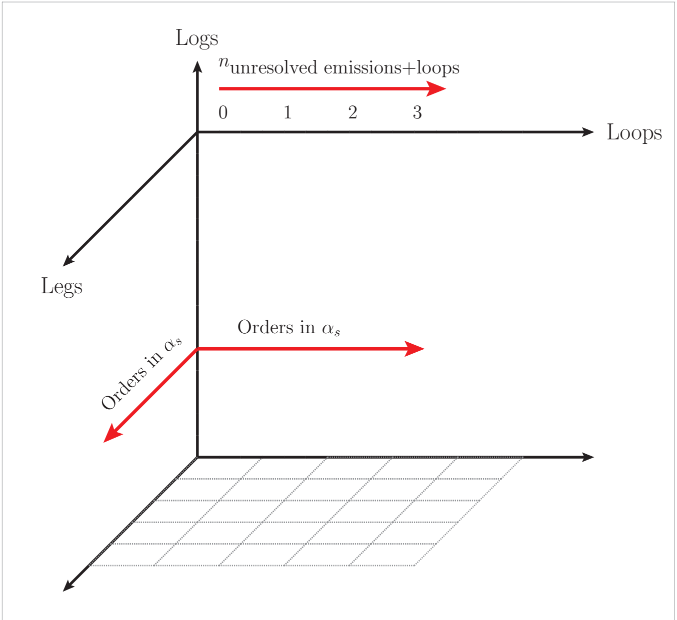


Symbolic figures for QCD calculations: Virtual corrections fill towers



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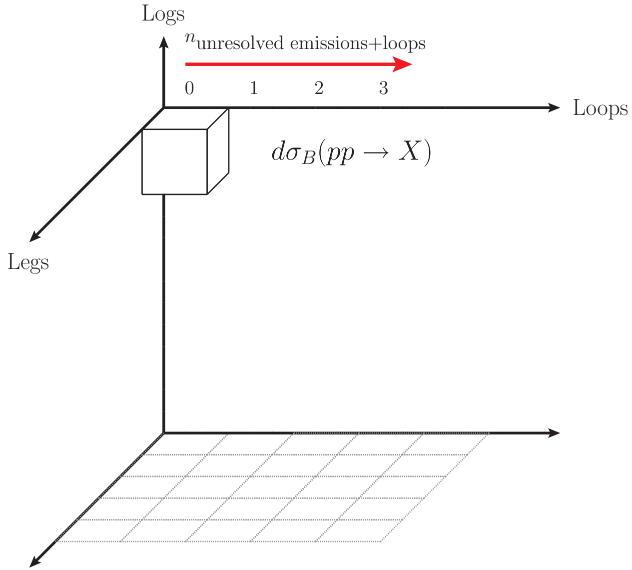

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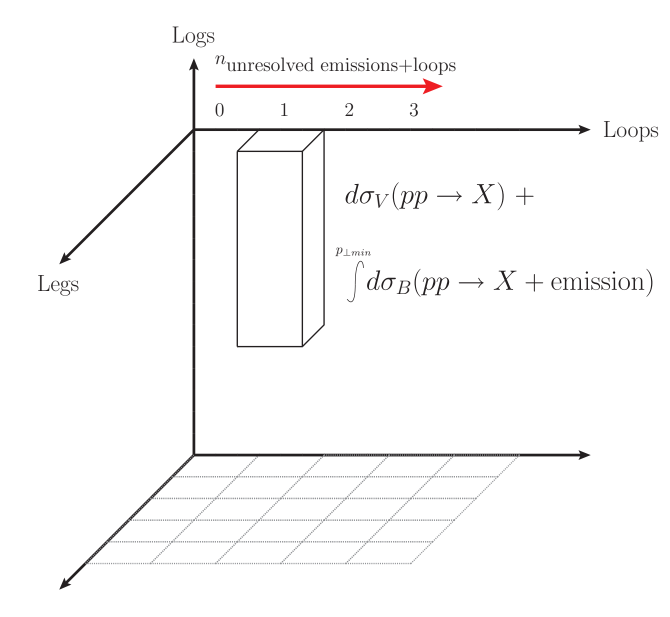
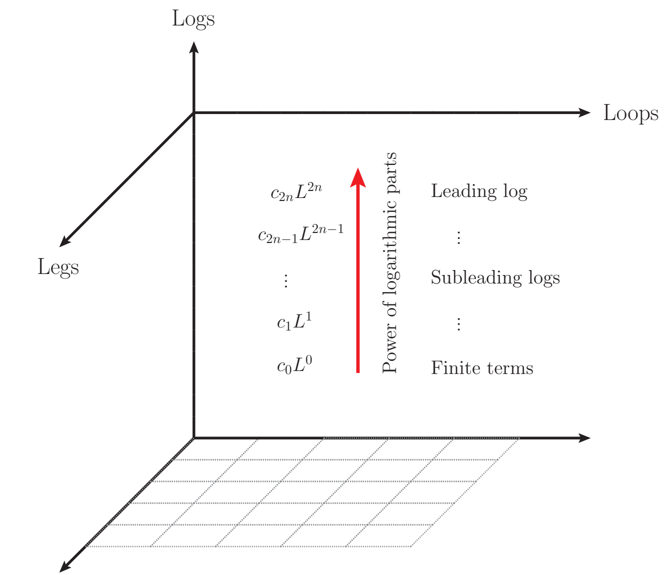


Symbolic figures for QCD calculations: Towers are composed of logs



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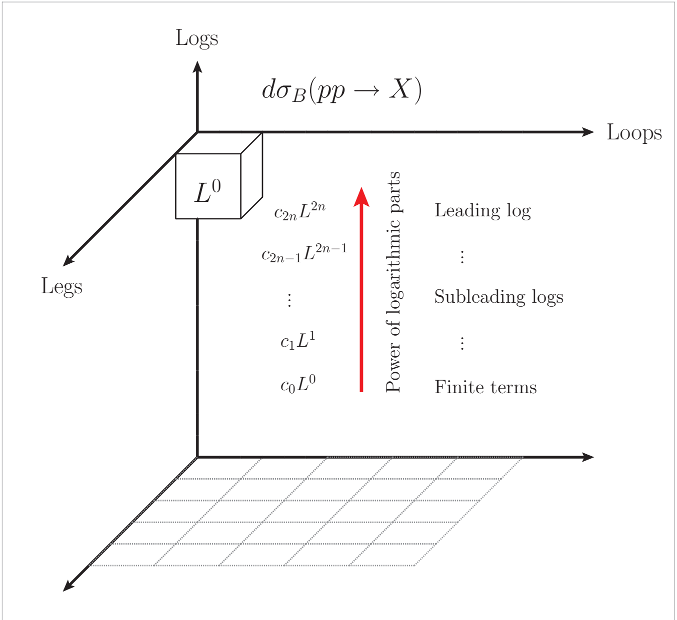

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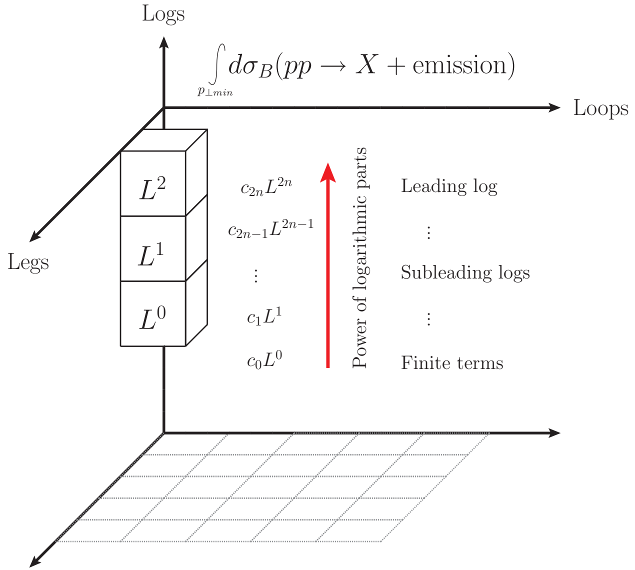


We found

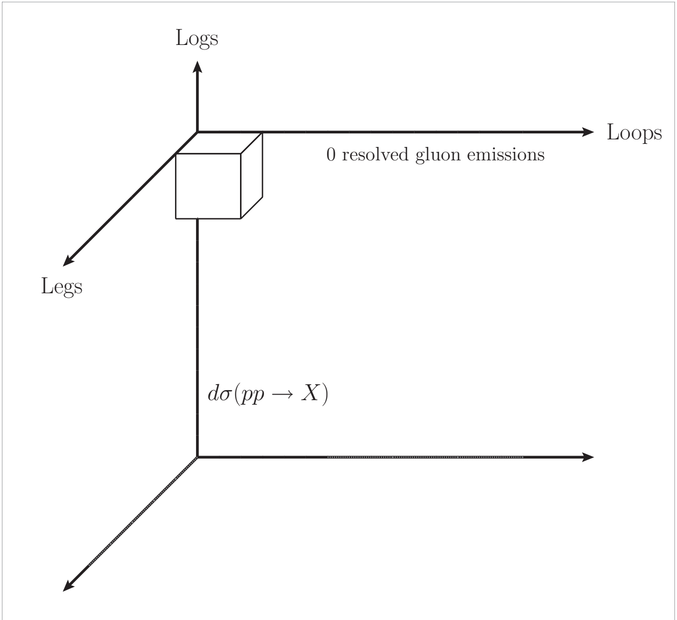
$$d\sigma(\text{pp} \to Y + n\text{g}) = d\sigma(\text{pp} \to Y + (n-1)\text{g}) \otimes \alpha_s^n \left(c_{2n} L^{2n} + c_{2n-1} L^{2n-1} + \cdots + c_0 \right)$$

and

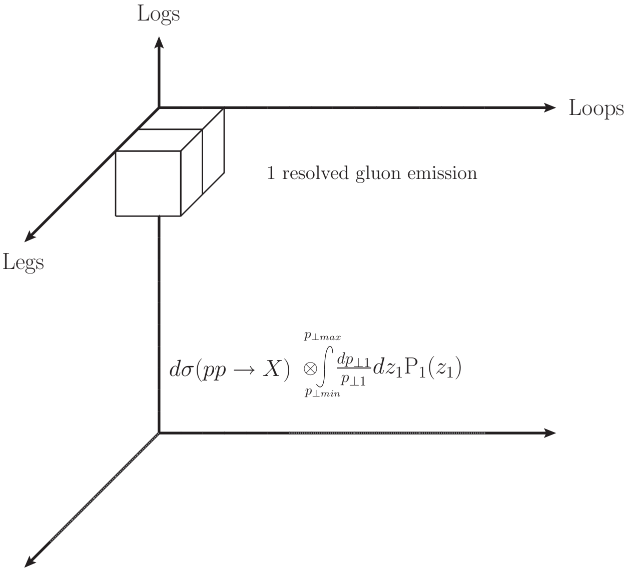
$$d\sigma(\text{pp} \to Y + n\text{g}) \approx d\sigma(\text{pp} \to Y) \alpha_s^n c_{2n} L^{2n}$$

We can illustrate this logarithmic structure with a "legs-and-logs" plot.

Iterating the collinear approximation: Hard process

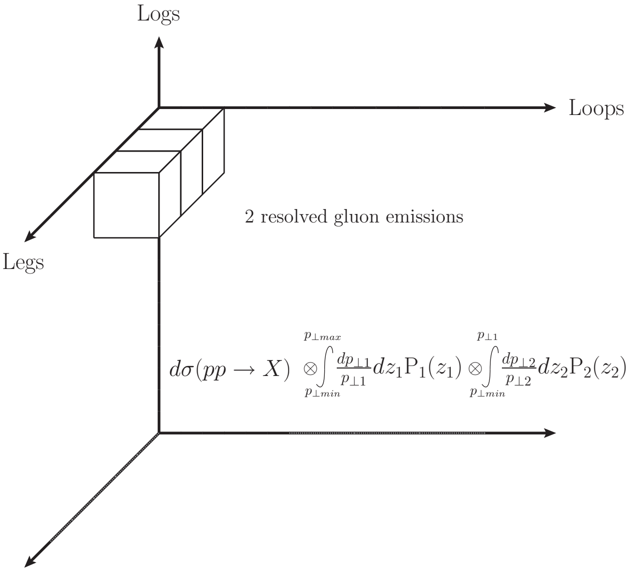


Iterating the collinear approximation: One emission



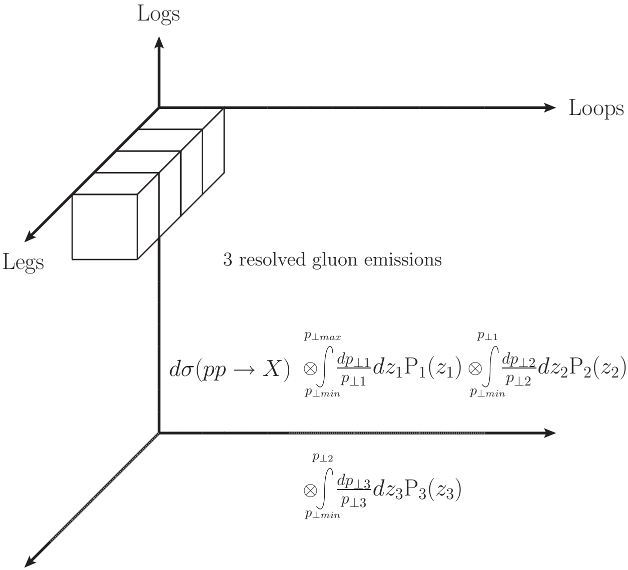
$$d\sigma(pp \to X) \otimes \int_{p_{\perp min}}^{p_{\perp max}} \frac{dp_{\perp 1}}{p_{\perp 1}} dz_1 \mathrm{P}_1(z_1)$$

Iterating the collinear approximation: Two emissions



$$d\sigma(pp \to X) \otimes \int_{p_{\perp min}}^{p_{\perp max}} \frac{dp_{\perp 1}}{p_{\perp 1}} dz_1 \mathrm{P}_1(z_1) \otimes \int_{p_{\perp min}}^{p_{\perp 1}} \frac{dp_{\perp 2}}{p_{\perp 2}} dz_2 \mathrm{P}_2(z_2)$$

Iterating the collinear approximation: Three emissions

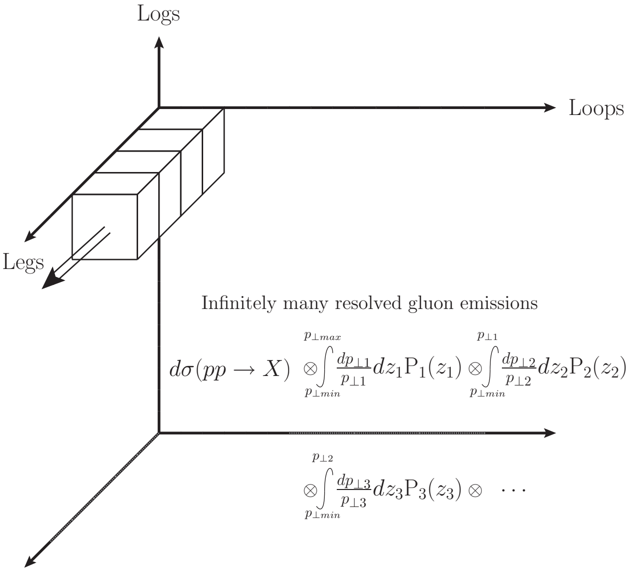


3 resolved gluon emissions

$$d\sigma(pp \to X) \;\otimes \int_{p_{\perp min}}^{p_{\perp max}} \frac{dp_{\perp 1}}{p_{\perp 1}} dz_1 \mathrm{P}_1(z_1) \otimes \int_{p_{\perp min}}^{p_{\perp 1}} \frac{dp_{\perp 2}}{p_{\perp 2}} dz_2 \mathrm{P}_2(z_2)$$

$$\otimes \int_{p_{\perp min}}^{p_{\perp 2}} \frac{dp_{\perp 3}}{p_{\perp 3}} dz_3 \mathrm{P}_3(z_3)$$

Iterating the collinear approximation: Infinitely many emissions



Infinitely many resolved gluon emissions

$$d\sigma(pp \to X) \; \otimes \int_{p_{\perp min}}^{p_{\perp max}} \frac{dp_{\perp 1}}{p_{\perp 1}} dz_1 \mathrm{P}_1(z_1) \otimes \int_{p_{\perp min}}^{p_{\perp 1}} \frac{dp_{\perp 2}}{p_{\perp 2}} dz_2 \mathrm{P}_2(z_2)$$

$$\otimes \int_{p_{\perp min}}^{p_{\perp 2}} \frac{dp_{\perp 3}}{p_{\perp 3}} dz_3 \mathrm{P}_3(z_3) \otimes \;\; \cdots$$

Comments on iterating the collinear approximation

So far, we had $\quad d\sigma(\mathrm{pp} \to Y + n\mathrm{g}) \approx d\sigma(\mathrm{pp} \to Y)\alpha_s^n c_{2n} L^{2n}$.

- (Multiple) gluon emission give the largest contribution to this multi-parton cross section.
- Other QCD splittings (e.g. $g \to q\bar{q}$) also give very important contributions to the cross section. These are sub-leading, but should be included in a sensible prediction.
- The dominant contributions to the inclusive cross section are produced by ordered emissions, where any ordering

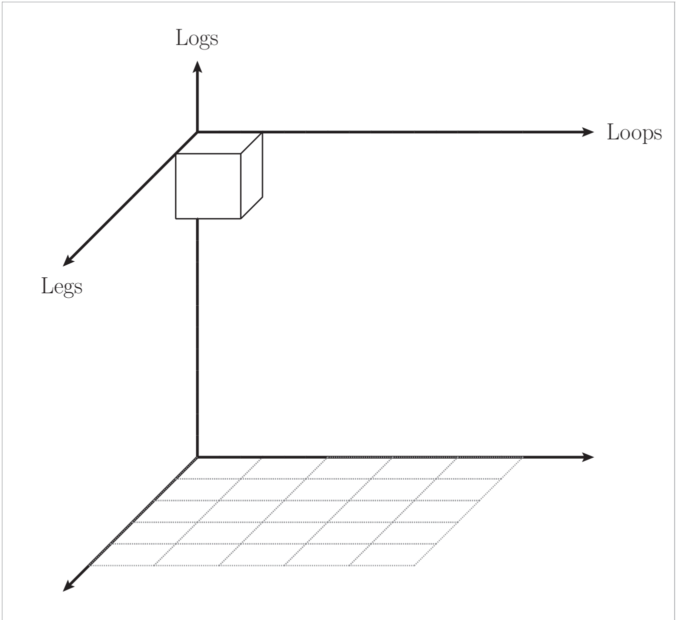
  $$\rho_0 > \rho_1 > \rho_2 > \dots$$

  is allowed if $\frac{d\rho}{\rho} = \frac{dp_\perp^2}{p_\perp^2}$. The many ways of choosing an ordering are one major difference between parton shower Monte Carlo's (see later).
- A sensible prediction respects energy-momentum conservation.

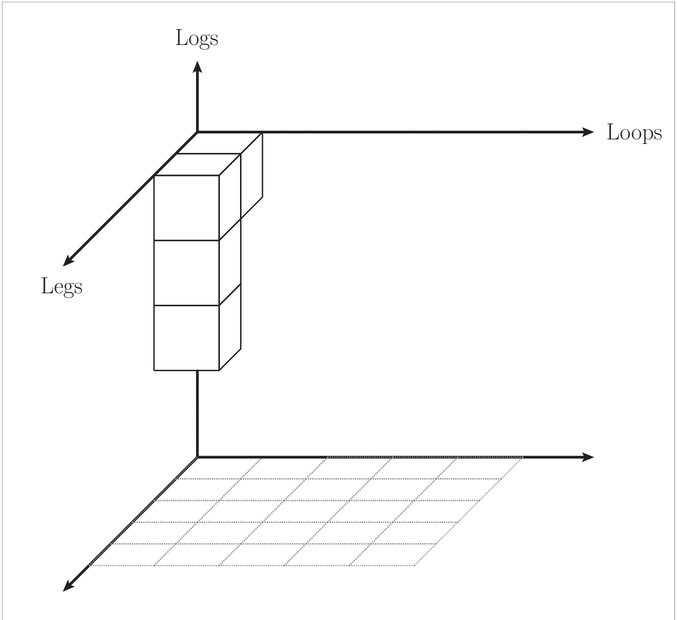
Note that $\quad d\sigma(\mathrm{pp} \to Y)\alpha_s^n c_{2n} L^{2n} \quad$ is divergent as $\quad p_{\perp min} \to 0$.

$\Longrightarrow$ To give a sensible approximation of the multi-parton cross section, we need to do more than just multiply splitting probabilities!

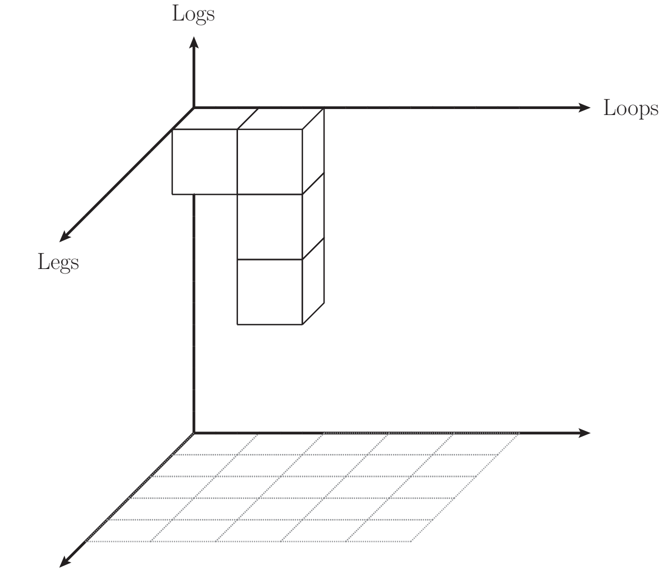
Remember the KLN theorem: Lowest order is finite



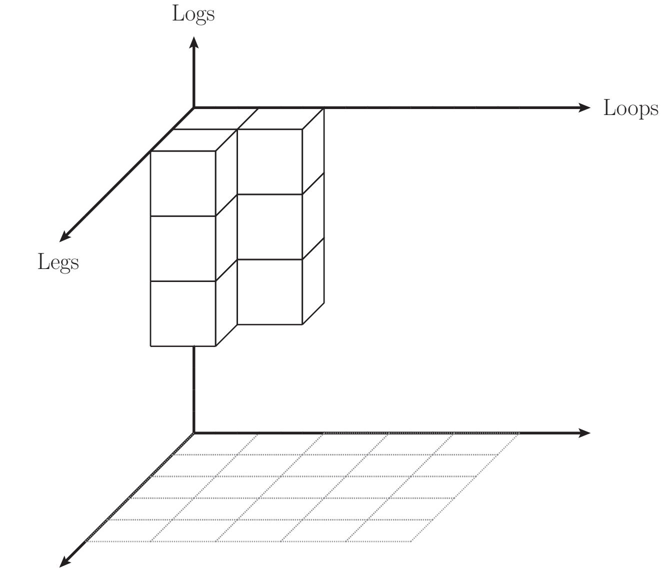
Remember the KLN theorem: Real emissions diverge



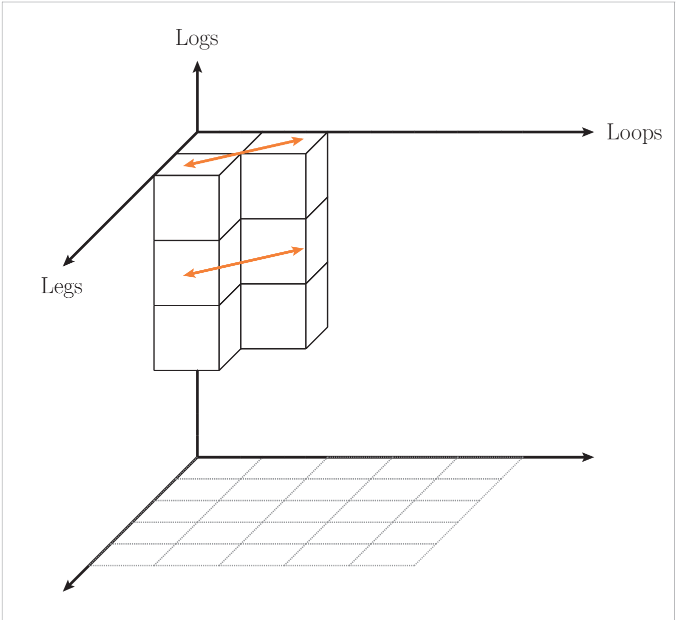
Remember the KLN theorem: Virtual corrections diverge



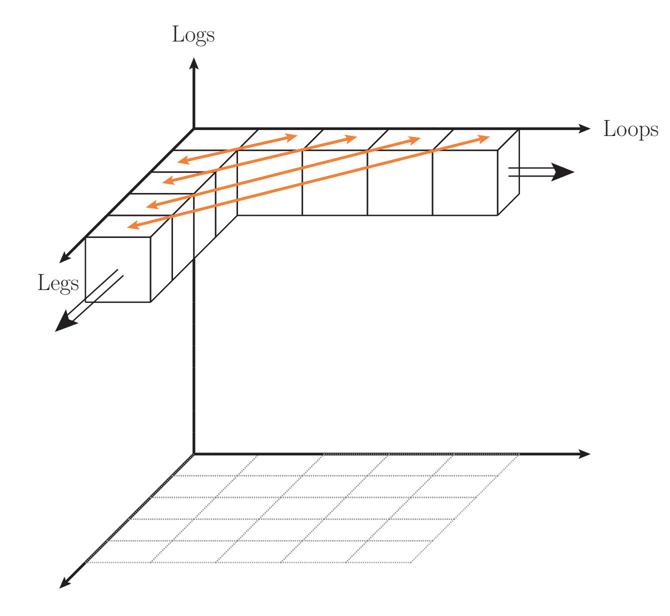
Remember the KLN theorem: Virtual + Real is finite . . .



Remember the KLN theorem: Virtual + Real is finite . . . because all logs cancel!



Can we cancel the product of splittings with all-order virtual corrections?



The Sudakov form factor

$\Longrightarrow$ To give a sensible approximation of the multi-parton cross section, we also need (approximate all-order) virtual corrections!

Approximate all-order virtual corrections form a Sudakov form factor

$$
\begin{aligned}
\Pi(\rho_0, \rho_{min}) &= \exp\left(-\int_{\rho_{min}}^{\rho_0} \frac{d\rho}{\rho} \int dz \frac{\alpha_s}{2\pi} P(z)\right) \\
&= 1 - \int_{\rho_{min}}^{\rho_0} \frac{d\rho_1}{\rho_1} \int_{z_{min}}^{z_0} dz_1 \frac{\alpha_s}{2\pi} P_1(z_1) \\
&\quad + \int_{\rho_{min}}^{\rho_0} \frac{d\rho_1}{\rho_1} \int dz_1 \frac{\alpha_s}{2\pi} P_1(z_1) \int_{\rho_{min}}^{\rho_1} \frac{d\rho_1}{\rho_1} \int dz_2 \frac{\alpha_s}{2\pi} P_2(z_2) \; + \ldots
\end{aligned}
$$

But how do we get there?

$\Longrightarrow$ Probabilities!

Taking probabilities seriously

We have already found:

$$\frac{\delta p_\perp^2}{p_\perp^2}\int_{z_1}^{z_0} dz \frac{\alpha_s}{2\pi}P(z) \equiv$$

Probability of an emission with $1-z \in [z_1, z_0]$ and $p_\perp^2$ in the range $[p_{\perp min}^2, p_{\perp min}^2 + \delta p_\perp^2]$.

Then the probability of no emission is

$$1 - \frac{\delta p_\perp^2}{p_\perp^2}\int_{z_1}^{z_0} dz \frac{\alpha_s}{2\pi}P(z)$$

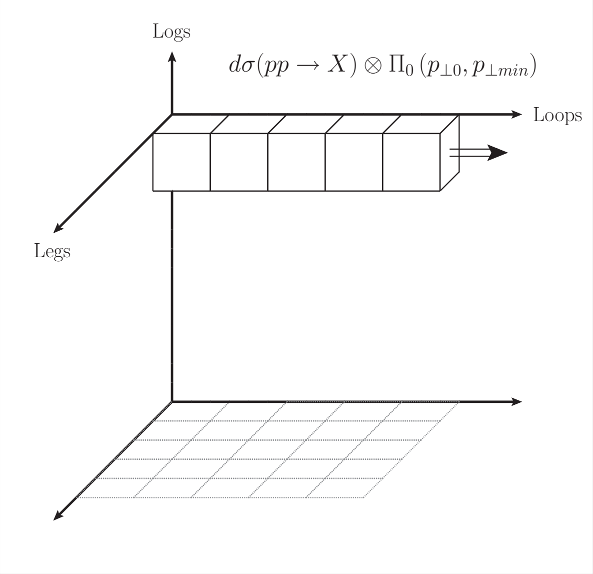
or, if $\delta p_\perp^2$ is divided into n parts, and the no-emission probabilities are independent

$$\left[1 - \frac{\delta p_\perp^2/n}{p_\perp^2}\int_{z_1}^{z_0} dz \frac{\alpha_s}{2\pi}P(z)\right]^n \underset{n\to\infty}{\to} \exp\left(-\int_{p_{\perp min}^2}^{p_{\perp min}^2+\delta p_\perp^2} \frac{dp_\perp^2}{p_\perp^2}\int_{z_1}^{z_0} dz \frac{\alpha_s}{2\pi}P(z)\right)$$

The Sudakov factor is the probability of no resolvable emission in the range $[p_{\perp min}^2, p_{\perp min}^2 + \delta p_\perp^2]$, where resolvable means $1-z \in [z_1, z_0]$.

The no-emission probability introduces all-order virtual corrections.

These do not change the number of legs $\Rightarrow$ Fill rows.



We have already found:

$$\int_{\rho_{min}}^{\rho_0} \frac{d\rho}{\rho} \int_{z_1}^{z_0} dz \frac{\alpha_s}{2\pi} P(z) \quad \equiv$$

Probability of a resolvable emission with $p_\perp^2$ in the range $[\rho_{min}, \rho_0^2]$.

$$\exp\left(- \int_{\rho_{min}}^{\rho_0} \frac{d\rho}{\rho} \int_{z_1}^{z_0} dz \frac{\alpha_s}{2\pi} P(z) \right) \equiv$$

Probability of no resolvable emission with $p_\perp^2$ in the range $[\rho_{min}, \rho_0]$.

We can construct an all-legs and all-loops result with probabilities only!
$\Longrightarrow$ Ideal for numerical iteration with random numbers.
$\Longrightarrow$ Monte Carlo parton showers.

So far, we have been concerned with gluons. The same result is valid for photon emission, $g \to q\bar{q}$ branchings, $\gamma \to q\bar{q}$ branchings...
Branchings/emissions (in some limit) can be approximated by probabilities.

An algorithm to produce multiple emissions

0. Construct a state with no emissions (easy!).
1. Begin algorithm at a "largest $p_\perp$" ρ_{max} (evolution parameter).
2. Propose a new state with an emission at $\rho < \rho_{max}$.
3. Decide if the new state should be constructed according to the splitting function probability. If yes, construct the new state.
4. Set $\rho_{max} = \rho$. Start from 1. (possibly with a new input state).

An algorithm to produce multiple emissions

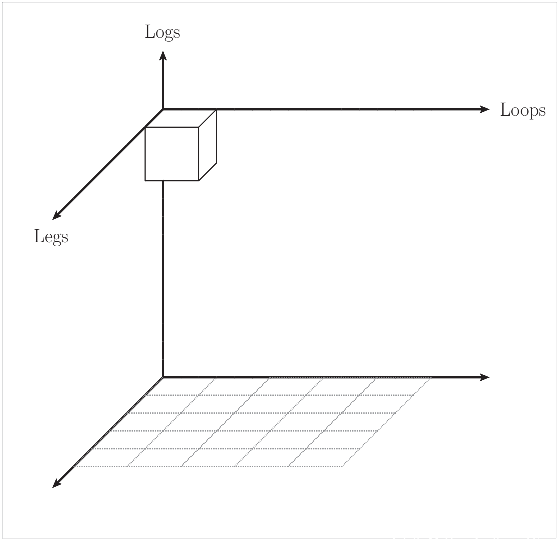
0. Construct a state with no emissions (easy!).
1. Begin algorithm at a "largest $p_\perp$" ρ_{max} (evolution parameter).
2. Propose a new state with an emission at $\rho < \rho_{max}$.
3. Decide if the new state should be constructed according to the splitting function probability. If yes, construct the new state.
4. Set $\rho_{max} = \rho$. Start from 1. (possibly with a new input state).

When the "$p_\perp$" is decreased by $\delta\rho$, there are two possibilities:

- ◇ The algorithm produced an emission.
- ◇ The algorithm did not produce an emission.

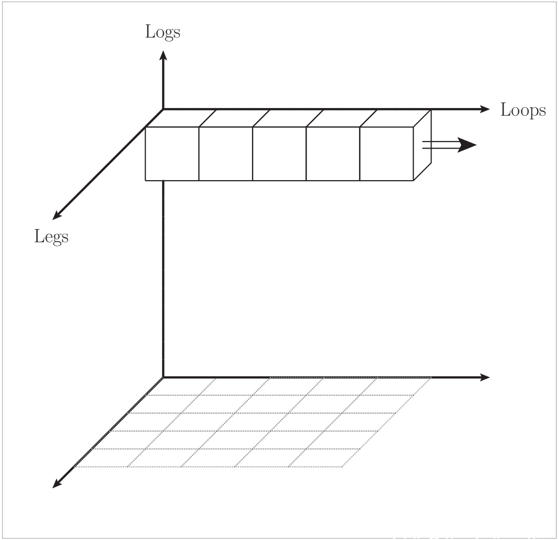
$$\begin{aligned}
&\text{P(No emission above } \rho_{min}) + \text{ P(One emission above } \rho_{min}) \\
&= \text{P(No emission above } \rho_{min}) + \text{ P(No emission above } \rho)\times \text{ P(One emission at } \rho) \\
&= d\sigma \otimes \Pi_0(\rho_0, \rho_{min})\, \mathcal{O}_0 \qquad + \; d\sigma \otimes \int_{\rho_{min}}^{\rho_0} \frac{d\rho}{\rho} \int_{z_1}^{z_0} dz \frac{\alpha_s}{2\pi} P(z)\, \Pi_0(\rho_0, \rho)\, \mathcal{O}_1
\end{aligned}$$

Parton shower: Fixed order input



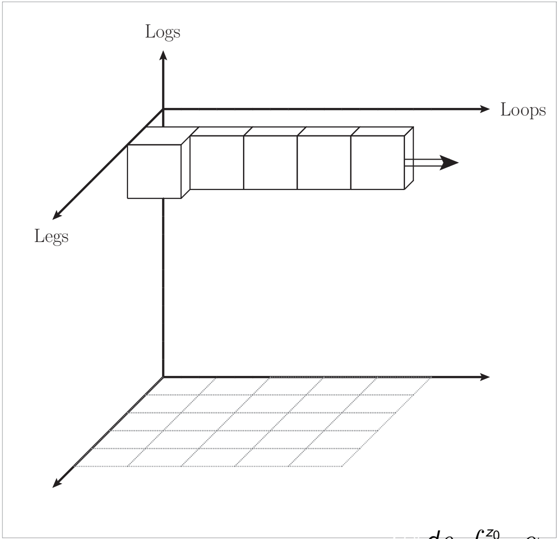
$d\sigma_B(pp \to X)$ $\mathcal{O}_0$

Parton shower: No emission



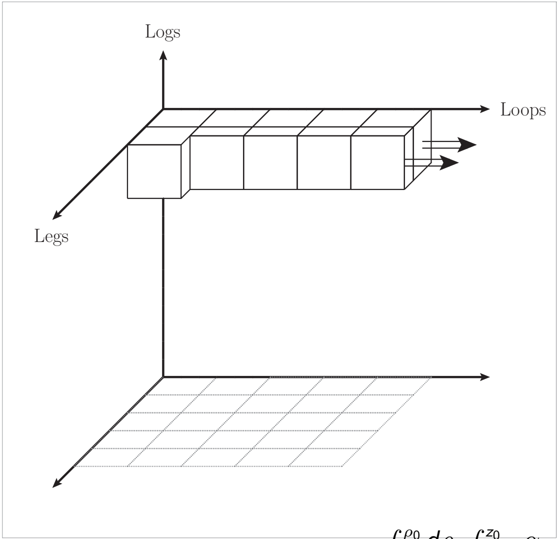
$d\sigma_B(pp \to X) \otimes \Pi_0(\rho_0, \rho_{min})$ $\mathcal{O}_0$

Parton shower: One emission at ρ



$$d\sigma_B(pp \to X) \otimes \quad \frac{d\rho}{\rho} \int_{z_1}^{z_0} dz \frac{\alpha_s}{2\pi} P(z)\ \Pi_0(\rho_0, \rho)\ \mathcal{O}_1$$

Parton shower: No or one emission



$$d\sigma_B(pp \to X) \otimes \Pi_0(\rho_0, \rho_{min})\ \mathcal{O}_0\ +\ d\sigma_B(pp \to X) \otimes \int_{\rho_{min}}^{\rho_0} \frac{d\rho}{\rho} \int_{z_1}^{z_0} dz \frac{\alpha_s}{2\pi} P(z)\ \Pi_0(\rho_0, \rho)\ \mathcal{O}_1$$

Parton shower cross sections

Each of these cross sections is finite because of Sudakov suppression:

$$d\sigma_B(pp \to X) \otimes \int_{\rho_{min}}^{\rho_0} \frac{d\rho}{\rho} \int_{z_1}^{z_0} dz \frac{\alpha_s}{2\pi} P(z)\, \Pi_0(\rho_0, \rho)\, \mathcal{O}_1 \underset{\rho_{min}\to 0}{\to} \text{finite}$$

Now remember that we derived the no-emission probability from

$$P_{\text{emission}} + P_{\text{no emission}} = 1$$

$\Longrightarrow$ The PS never changes the cross section, it only changes shapes.
This is called parton shower unitarity. More formally:

$$d\sigma_B(pp \to X)\, \Pi_0(\rho_0, \rho_{min})\, \mathcal{O}_0 \;+\; d\sigma_B(pp \to X) \int_{\rho_{min}}^{\rho_0} \frac{d\rho}{\rho} \int_{z_1}^{z_0} dz \frac{\alpha_s}{2\pi} P(z)\, \Pi_0(\rho_0, \rho)\, \mathcal{O}_1$$

$$= d\sigma_B(pp \to X) \left\{ \left[1 - \int_{\rho_{min}}^{\rho_0} \frac{d\rho}{\rho} \int_{z_1}^{z_0} dz \frac{\alpha_s}{2\pi} P(z)\, \Pi_0(\rho_0, \rho) \right] \mathcal{O}_0 + \int_{\rho_{min}}^{\rho_0} \frac{d\rho}{\rho} \int_{z_1}^{z_0} dz \frac{\alpha_s}{2\pi} P(z)\, \Pi_0(\rho_0, \rho)\, \mathcal{O}_1 \right\}$$

Unitarity means that parton showers define how the *inclusive* cross section is sliced up into *exclusive* cross sections:

$$\begin{aligned} \sigma_{0\ or\ more\ jets} &= \sigma_{exactly\ 0\ jets} + \sigma_{1\ or\ more\ jets} \\ &= \sigma_{exactly\ 0\ jets} + \sigma_{exactly\ 1\ jet} + \sigma_{2\ or\ more\ jets} \end{aligned}$$

Remember:

$$\Pi(\rho_0, \rho_1) = \exp\left(- \int_{\rho_1}^{\rho_0} \frac{d\rho}{\rho} \int_{z_1}^{z_0} dz \frac{\alpha_s}{2\pi} P(z) \right) \equiv$$

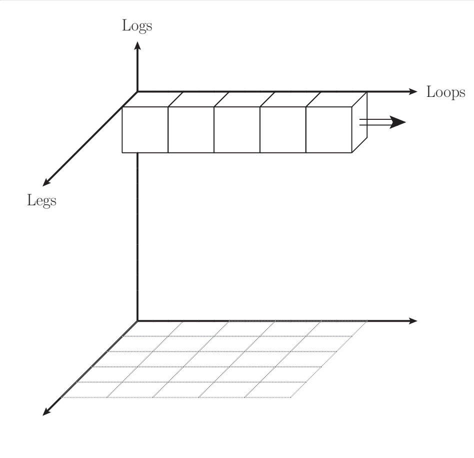
Probability of no resolvable emission with evolution scale in the range $[\rho_1, \rho_0]$.

$$\frac{d\rho}{\rho} \int_{z_1}^{z_0} dz \frac{\alpha_s}{2\pi} P(z) \Pi(\rho_0, \rho) \quad \equiv$$

Probability of a exactly one resolvable emission, with evolution scale ρ.

Now, let's have a look at the parton shower again!

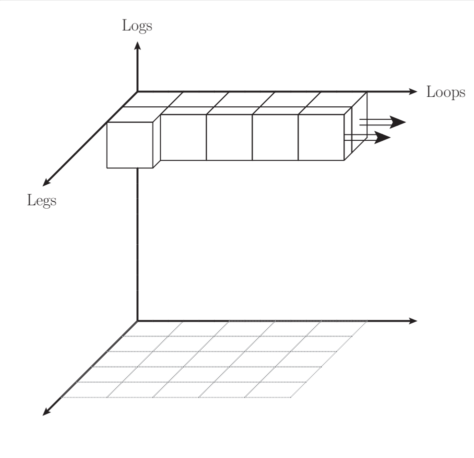
Parton shower example



PS generates

- no emission

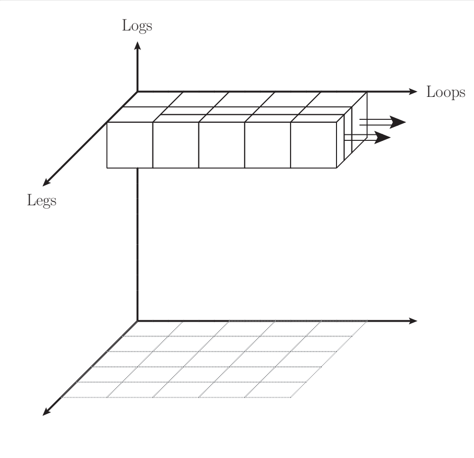
Parton shower example



PS generates

- no emission
- or one emission at ρ_1

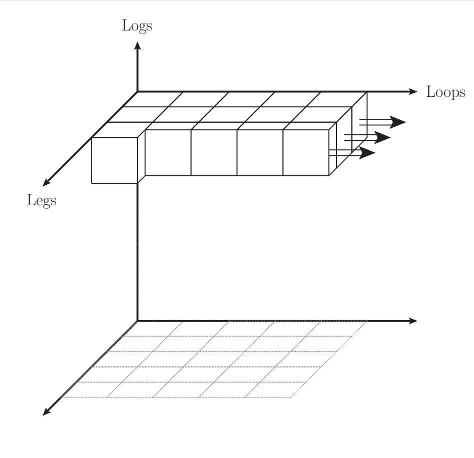
Parton shower example



PS generates

- no emission
- or one emission at ρ_1 and no further emission

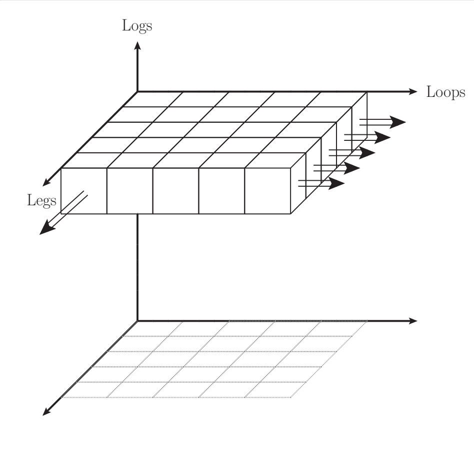
Parton shower example



PS generates

- no emission
- or one emission at ρ_1 and no further emission
- or one emission at ρ_1 and one at ρ_2

Parton shower example



PS generates

- no emission
- or one emission at ρ_1 and no further emission
- or one emission at ρ_1 and one at ρ_2
- and so on for arbitrary many emissions

We have quietly dropped PDFs before. Keeping the PDFs, we would have arrived at

No-emission probability:

$$\Pi(\rho_0, \rho_1) = \exp\left(-\int_{\rho_1}^{\rho_0} \frac{d\rho}{\rho} \int_{z_1}^{z_0} \frac{dz}{z} \frac{\alpha_s}{2\pi} \frac{f_1(\frac{x}{z}, \rho)}{f_0(x, \rho)} P(z)\right)$$

Probability of an emission with $x_{new} = \frac{x}{z}$ at evolution scale ρ:

$$\frac{d\rho}{\rho} \int_{z_1}^{z_0} \frac{dz}{z} \frac{\alpha_s}{2\pi} \frac{f_1(\frac{x}{z}, \rho)}{f_0(x, \rho)} P(z) \Pi(\rho_0, \rho)$$

Note

$$\frac{d \ln \Pi}{d \ln \rho} = \int_{z_1}^{z_0} \frac{dz}{z} \frac{\alpha_s}{2\pi} \frac{f_1(\frac{x}{z}, \rho)}{f_0(x, \rho)} P(z)$$

$\Rightarrow$ PDFs are crucial for radiating off an initial state parton.

Backward evolution

Remember: PDFs evolve according to the DGLAP equation, from small virtuality Q^2 to larger virtuality Q_0^2. PDFs are small at large Q_0^2.

Should parton showers do the same?

No, since it would be **very** unlikely to "hit" a resonance (i.e. a Higgs or Z-boson propagator) in a narrow virtuality window at large Q_0^2.
$\Longrightarrow$ Simulating high-scale physics would be nearly impossible!

$\Longrightarrow$ Instead, reformulate DGLAP to evolve from large Q_0^2 and small x to smaller Q^2 and larger x/z.
$\Longrightarrow$ Backwards evolution.

DGLAP	: Sums up all emissions by evolving from Q^2 to Q_0^2
Backward evolution:	Performs all emissions (that had previously been summed up) by evolving from Q_0^2 to Q^2.

Review

Achievements so far:

- Found a way to approximate (one of) the largest contributions to a n-parton cross section: the collinear approximation
  ... and devised a probabilistic algorithm to produce this result.
- The parton shower produces finite results by introducing all-order (resummed) virtual corrections.
- This extends the validity of perturbation theory even to small $p_\perp$.
- We know how to treat emissions off final and initial state partons.

To get there, we needed

- To derive emission and no-emission probabilities.
- Find a prescription for momentum conservation - otherwise, we cannot iterate the procedure.
- We had to define an evolution scale ρ to reproduce the dominant terms.

But...

- Momentum conservation can be implemented in many different ways.
- The evolution scale ρ can be defined freely, as long as $d\rho/\rho = dp_\perp^2/p_\perp^2$. This e.g. allows (relative) angle, virtuality, $p_\perp^2$...

Choosing an ordering variable: Double-counting and hardness

Backward evolution in the initial state means evolving from a "hard process" at large momentum transfer to smaller momentum transfers.

The hard process is the "starting point" of the radiation cascade.

We want to start from an "exact" result, i.e. a good description of the inclusive cross section with n partons, and produce approximate higher order corrections.

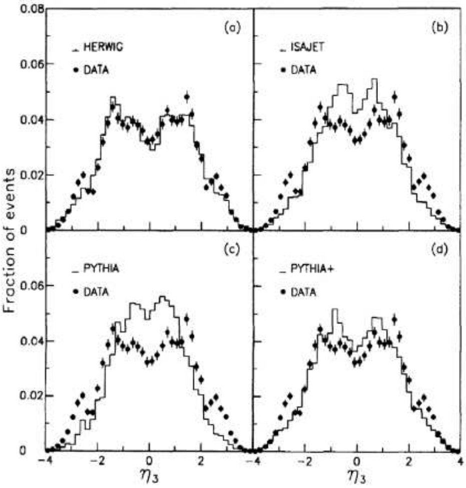
If the evolution scale is defined such that after some emissions, a "harder" process is generated, then the exact starting point is obscured, and we cannot do backward evolution.

$\Longrightarrow$ Initial state showers suggest to use a "hardness" ordering, i.e. where large momentum transfers happen early in the cascade (e.g. Q^2 or $p_\perp^2$).

Choosing an ordering variable: Is virtuality ordering safe?

Ordering:
PYTHIA: Virtuality,
HERWIG: Something else.

$\Longrightarrow$ Something is missing.



$\Longrightarrow$ Virtuality ordering did not capture the physics!
$\Longrightarrow$ Missing another important ingredient!

The soft limit and QM interference

When trying to find an approximation of additional gluon emissions, we found that the largest contribution to $Q_i(p_i + k) \to Q_i'(p_i) + g(k)$ arose from an on-shell propagator

$$u(p_i)\not{\varepsilon}\frac{(\not{p}_i + \not{k})}{(p_i + k)^2} = u(p_i)\frac{p_i\varepsilon}{2p_ik} = u(p_i)\frac{p_i\varepsilon}{(1-z)E_{Q_i}^2(1-\cos\Theta_{Q_ig})}$$

Apart from collinear divergence $\Theta_{Q_ig} \to 0$, there is also a soft divergence $z \to 1$.

$\Longrightarrow$ We were missing the soft piece before!

For $z \to 1$, already the amplitudes universally factorise. Thus, upon squaring

$$d\sigma_{n+1} = d\sigma_n \int \frac{dw}{w}\frac{d\Omega}{2\pi}\frac{\alpha_s}{2\pi}\sum_{ij} C_{ij}W_{ij}$$

$$\text{with} \quad W_{ij} = \frac{1-\cos\Theta_{Q_iQ_j}}{(1-\cos\Theta_{Q_ig})(1-\cos\Theta_{Q_jg})}$$

$\Longrightarrow$ QM interference between gluon emission off partons Q_i and Q_j!

How can soft emissions be independent?

Coherence in the soft limit

How can soft emissions be independent?
Let us write

$$W_{ij} = W_{ij}^1 + W_{ij}^2 \quad \text{with} \quad W_{ij}^i = \frac{1}{2}\left(W_{ij} + \frac{1}{(1-\cos\Theta_{Q_i g})} - \frac{1}{(1-\cos\Theta_{Q_j g})}\right)$$

Then, after integrating over the azimuthal angle, we get

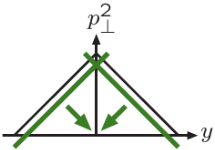
$$\int \frac{d\phi_{Q_i g}}{2\pi} W_{ij}^i = \begin{cases} \frac{1}{(1-\cos\Theta_{Q_i g})} & \text{for} \quad \Theta_{Q_i g} < \Theta_{Q_i Q_j} \\ 0 & \text{else} \end{cases}$$

Soft emissions are independent if ordered in emission angle!
Another (opening cone) argument shows: $p_\perp$-ordered final state emissions are okay as well.

HERWIG had angular ordering in the CDF plot. Color coherence necessary to describe data! But angle does not define hardness!

We found: Hardness ordering (Q^2, $p_\perp^2$) motivated by ISR, Θ ordering by soft limit. Both mutually exclusive!

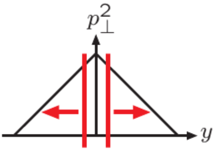
Q^2



Virtuality

- Defines hardness, as necessary in ISR.
- No coherence. Additional vetoes necessary.

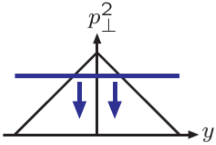
$E^2\Theta^2$



Angle

- Does not define hardness. Additional vetoes necessary.
- Coherence by construction.

$p_\perp^2$



$p_\perp$

- Defines hardness, as necessary in ISR.
- Coherence in FSR. ISR not clear.

Is it hopeless? No! $\Longrightarrow$ Dipole/antenna showers.

Dipoles / antennae

In the soft limit, we found

$$d\sigma_{n+1} = d\sigma_n \int \frac{dw}{w} \frac{d\Omega}{2\pi} \frac{\alpha_s}{2\pi} \sum_{ij} C_{ij} W_{ij}$$

and after writing

$$W_{ij} = W_{ij}^1 + W_{ij}^2 \quad \text{with} \quad W_{ij}^i = \frac{1}{2}\left(W_{ij} + \frac{1}{(1 - \cos\Theta_{Q_i g})} - \frac{1}{(1 - \cos\Theta_{Q_j g})} \right)$$

derived angular ordering.

But we could have directly used W_{ij} as splitting probability ($\equiv$ QCD antenna), or partitioned cleverly ($\equiv$ QCD dipole).

Both antennae and dipoles can be inferred from NLO subtraction methods. This means they come with a well-defined phase space mapping.

Energy-momentum conservation

We have stressed the importance of energy-momentum conservation, but not given a prescription.

NLO subtraction formalisms give a one-to-one correspondence

$$d\Phi_{n+1} = d\Phi_n d\hat{\Phi} = d\Phi_n J(\rho, z, \phi) d\rho dz \frac{d\phi}{2\pi}$$

which maps an on-shell n-particle phase space point unto an on-shell $n+1$-particle configuration. The $n+1$-particle is completely covered.

This can be achieved by

aborbing the "recoil" of a $1 \to 2$ splitting with a spectator (dipoles).

performing $2 \to 3$ splittings (antennae).

$\Rightarrow$ Modern showers are all built in this way!

Momentum conservation in each intermediate step is the major advantage compared to analytical tools. It also makes systematic step-by-step improvements possible ($\to$ next lecture).

Freedom in the recoil scheme is an uncertainty of exclusive prediction!

Running scales

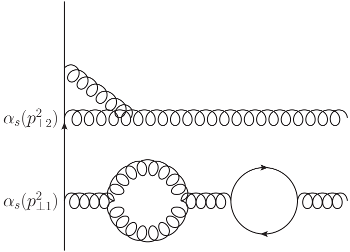
Until now, we have found:

- Parton showers generate the leading collinear logarithms. Angular ordering (or modern showers) include the soft limit as well.
- Local momentum conservation (formally beyond LL) is included.
- Initial state radiation requires PDF evaluations at dynamical scales (e.g. Q^2, $p_\perp^2$ of the branching).

Another important improvement is evaluation of α_s at dynamical scales $\alpha_s = \alpha_s(p_\perp^2)$.

This is known as Modified Leading Log Approximation. This resums dominant universal propagator corrections to all orders.

After this improvement, many more soft emissions are produced. The PS cut-off must ensure to avoid the Landau pole (e.g. $p_{\perp min} > \Lambda_{QCD}$).



Effects (almost) beyond the reach of parton showers

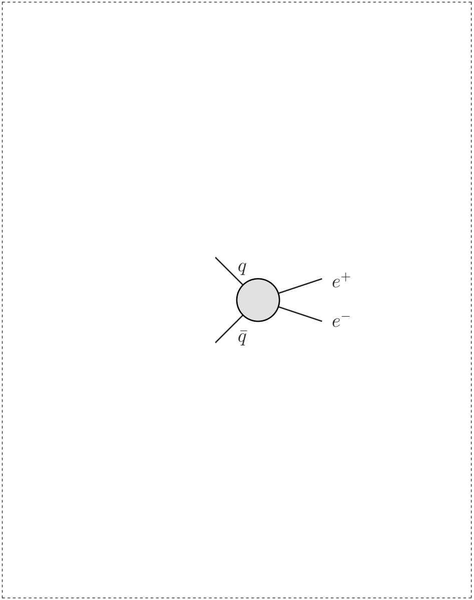
However, remember that

1. Any parton shower is only sensible in the collinear regime:
   Only very collimated parton cascades are reliably modelled.
2. Showers are always leading order correct for (very) inclusive observables.
3. Non-relativistic threshold effects are not included.
4. High-energy (i.e. low-x) enhancements $\ln(\hat{s}/\hat{t})$ are not included (exception CCFM, Müller dipole cascades).
5. Traditionally, showers only include QCD (and QED).

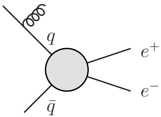
More developments still necessary!

Major industry of improvements for points 1. and 2. (Matrix element corrections, Merging, NLO Matching, NLO Merging, NNLO Matching). Steady progress on the other points.

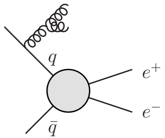
Event generation example...start from hard process



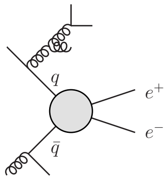
Event generation example. . . we know how to emit ISR gluons



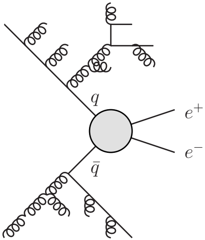
Event generation example. . . or FSR gluons



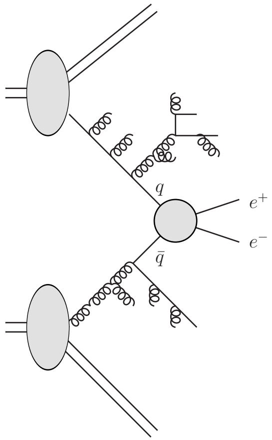
Event generation example. . . or split gluons into quarks



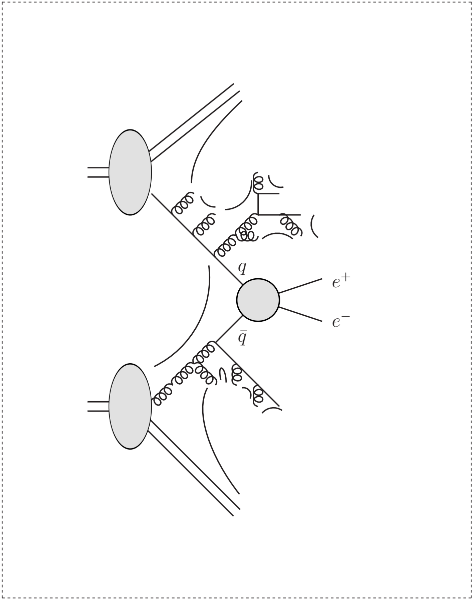
Event generation example. . . and how to do this arbitrarily often

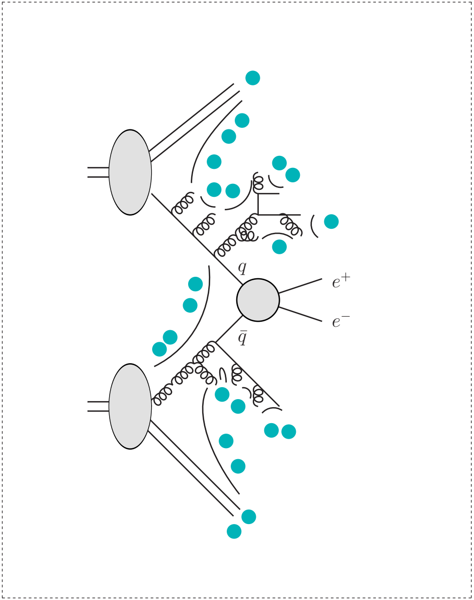


Event generation example. . . then add beam remnants

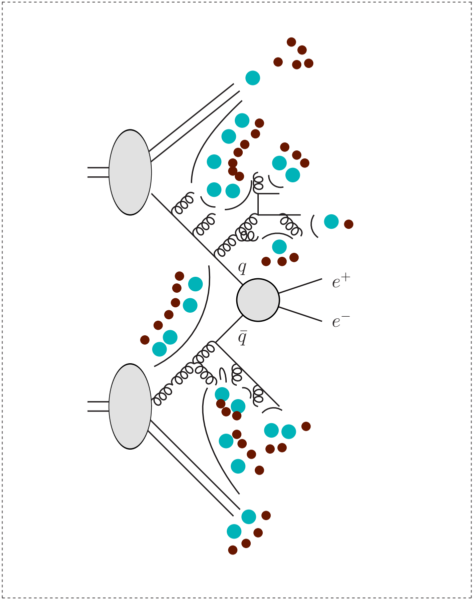


Event generation example. . . and form strings




q
e^+
e^-
$\bar{q}$

Event generation example. . . and decay the hadrons



Common event generator frameworks

Parton showers are usually part of event generator frameworks.
Commonly used event generators for LHC physics are

- **HERWIG++**: Improved angular ordered $\tilde{q}$ shower and $p_\perp$-ordered Catani-Seymour dipole shower.
- **PYTHIA 8**: $p_\perp$-ordered dipole shower based on DGLAP+ME-corrections, and VINCIA antenna shower as FSR plugin.
- **SHERPA** : $p_\perp$-ordered Catani-Seymour dipole shower, ANTS antenna shower

All three include QED radiation, EW effects, underlying event, diffractive modelling, hadronisation, higher-order improvements, hadron decays...

Public shower programs outside event generators include

- **HERWIRI** : Based on HERWIG 6, adding soft corrections.
- **DEDUCTOR**: Λ-ordered Nagy-Soper dipole shower
- Please let me know if I'm missing other public codes.

The use of HERWIG 6 and **PYTHIA 6** is discouraged.

- QCD scattering cross sections factorise in the soft and collinear limits.
- The factorisation is universal, and can be viewed as probabilistic.
- The existence of emission and no-emission probabilities makes all-order (all-legs) numerical implementations possible.
- Parton showers require an ordering criterion. Hardness and angle are well-motivated, but not without pitfalls.
- Almost all modern showers are based on antennae or dipoles.
- With the inclusion of soft effects, momentum conservation and running scales, many (all-order) refinements are already added.
- Some effects are beyond the parton shower approximation. But systematic enhancements are possible.

References

Introduction
Good references for event generators in general are:
MCnet report (Phys. Rept. 504 (2011) 145-233)
Many older lectures of MCNet (montecarlonet.org) and CTEQ schools.
Peter Skands' TASI lectures (arXiv:1207.2389)
Stefan Höche's TASI lectures (http://slac.stanford.edu/ shoeche/tasi14/ws/tasi.pdf)

Factorisation: Divide and conquer
The book: Collins, Perturbative Quantum Chromodynamics
Collins, Soper, Sterman (Nucl.Phys.B250(1985)199)

Backward evolution
The ISR paper: PLB 175 (1985) 321

Choosing an ordering variable: Is virtuality ordering safe?
Plot taken from CDF (PRD 50 (1994) 5562)

Dipoles / antennae
Ariadne (CPC 71 (1992) 15)
Catani, Seymour (Nucl.Phys.B485(1997)291)
Kosower antennae (Phys.Rev. D57 (1998) 5410)
Nagy, Soper (JHEP 0709 (2007) 114)
Vincia (Phys.Rev. D78 (2008) 014026)
Dinsdale, Ternick, Weinzierl (Phys.Rev. D76 (2007) 094003)
Sherpa CS (JHEP 0803 (2008) 038)
Sherpa ANTS (JHEP 0807 (2008) 040)
Herwig++ CS (JHEP 1101 (2011) 024)

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Amati et al. (Nucl.Phys. B173 (1980) 429)

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Herwig++ (JHEP 0312 (2003) 045,JHEP 1101 (2011) 024)
Pythia 8 (Comput.Phys.Commun. 178 (2008) 852-867)
Vincia (Phys.Rev. D78 (2008) 014026, Phys.Rev. D84 (2011) 054003, Phys.Lett. B718 (2013) 1345-1350)
Sherpa (JHEP 0803 (2008) 038, JHEP 0807 (2008) 040)
Other showers:
HERWIRI (Phys.Lett.B685(2010)283, Phys.Rev.D81(2010)076008, Phys.Lett.B719(2013)367, arXiv:1305.0023)
DEDUCTOR (JHEP 1406 (2014) 097, JHEP 1406 (2014) 178, JHEP 1406 (2014) 179)