Lectures on parton showers and matrix elements

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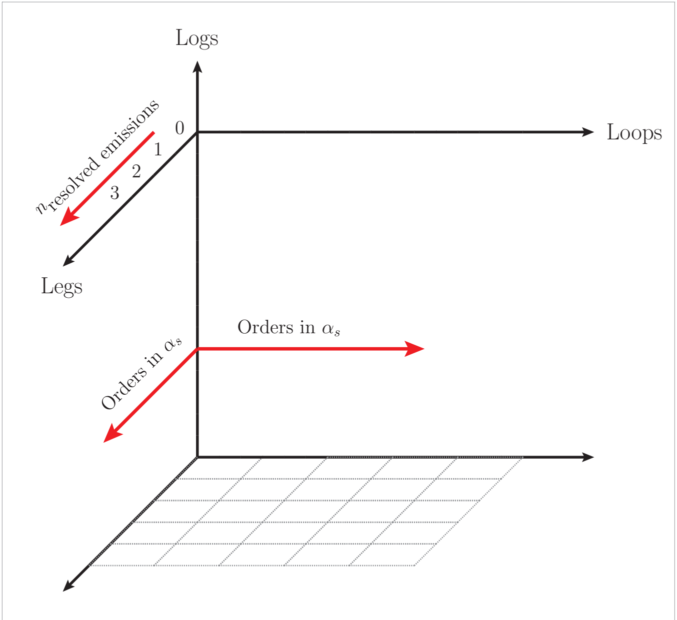
Lecture II: Improving parton showers with fixed-order calculations

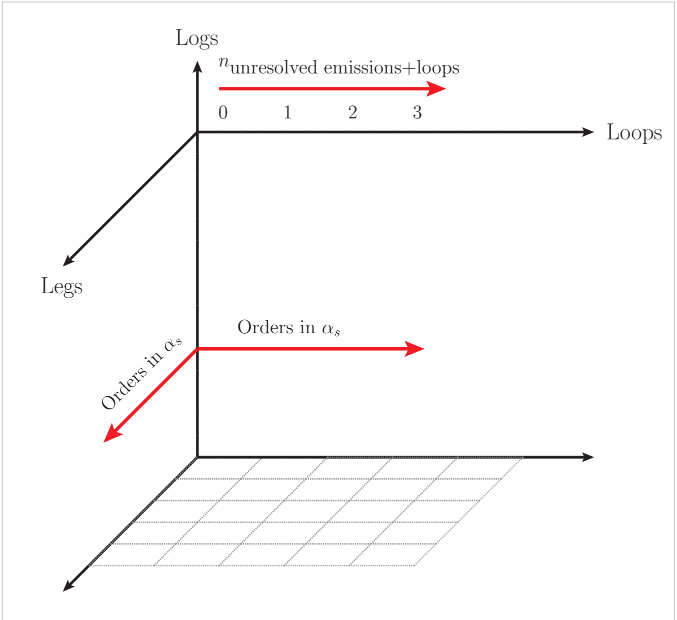
Recap of last lecture

- QCD scattering cross sections factorise.
- The factorisation can be cast into a probabilistic form suitable for a numerical implementation.
- Parton showers tell us how the inclusive cross section is sliced up into exclusive objects, where exclusive means a fixed number of resolved jets.
- Exclusive cross sections are defined through no-emission probabilities.
- All cross sections can be writen as a polynomial of logarithms.
- This log-structure can be illustrated on figures.

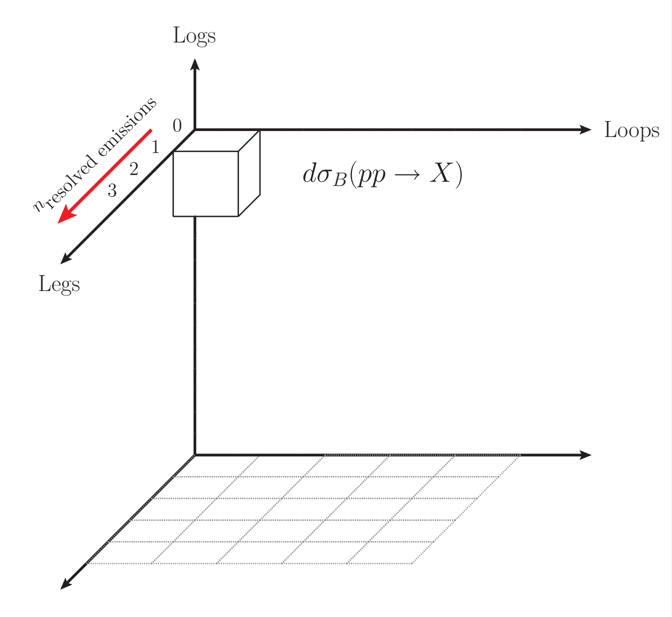
Recap: α_s orders are split into legs



Recap: α_s orders are split into legs and loops



Recap: *n*-leg MEs fill towers



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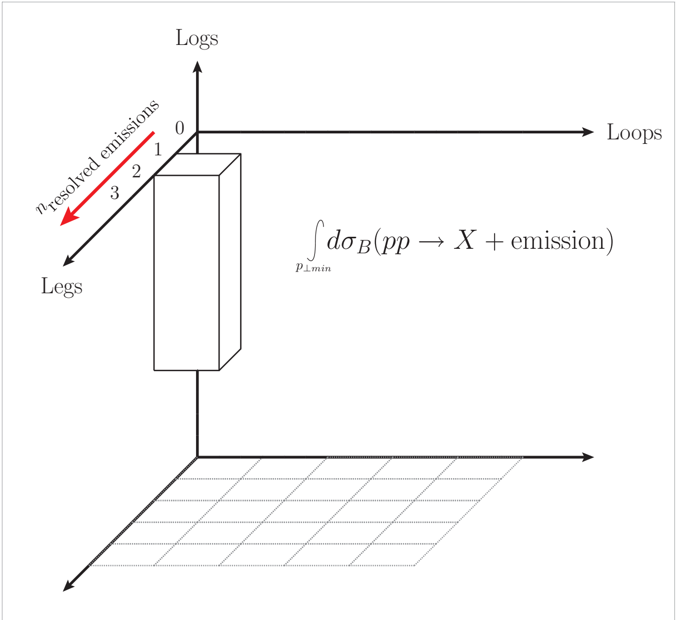
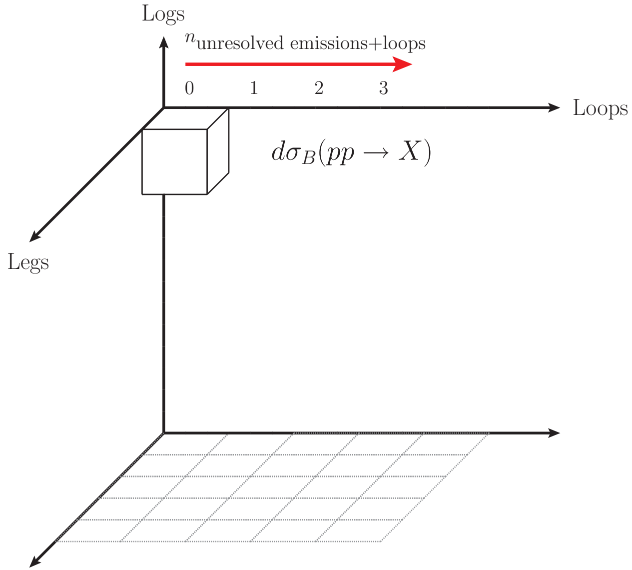


Recap: n-loop corrections fill towers



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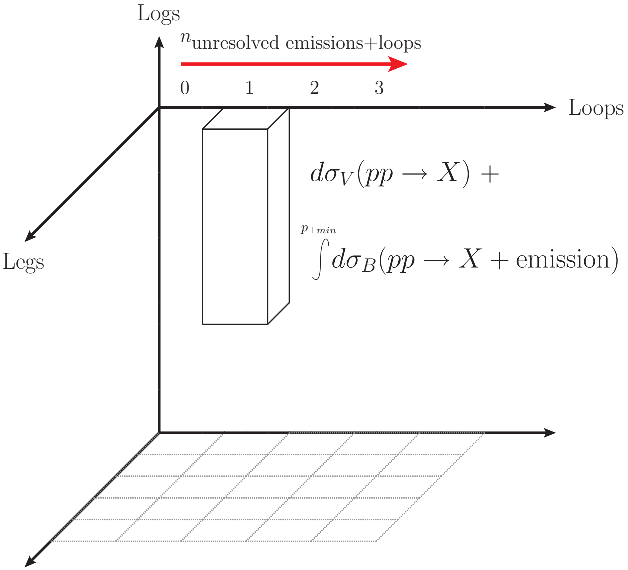
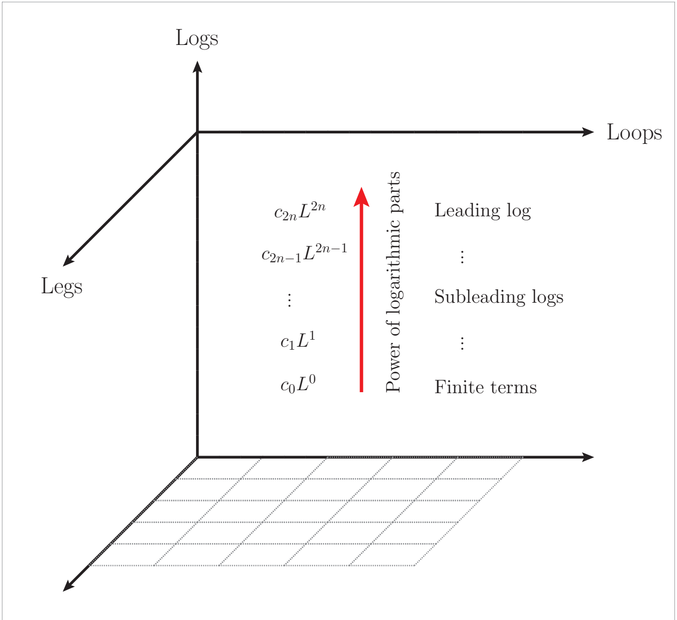


Recap: Towers are composed of logs



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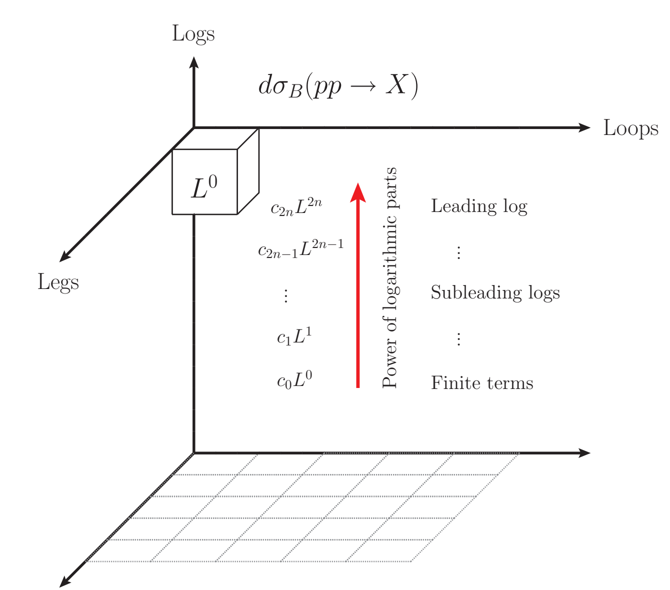

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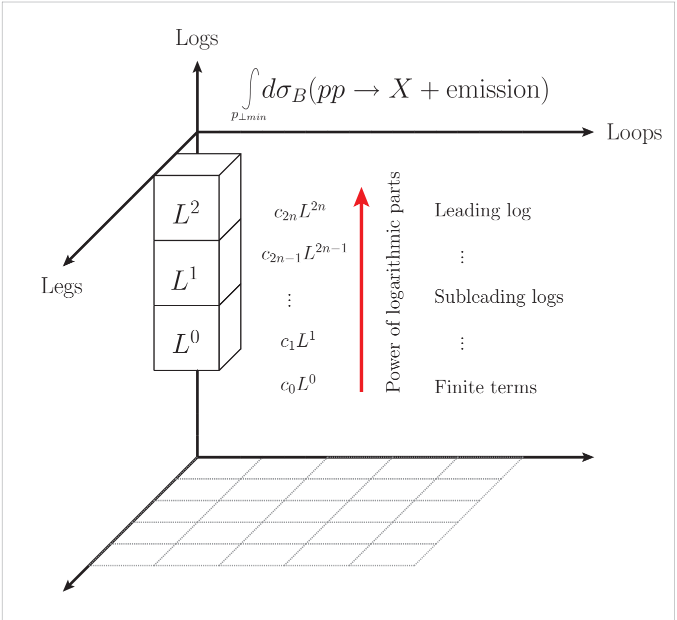
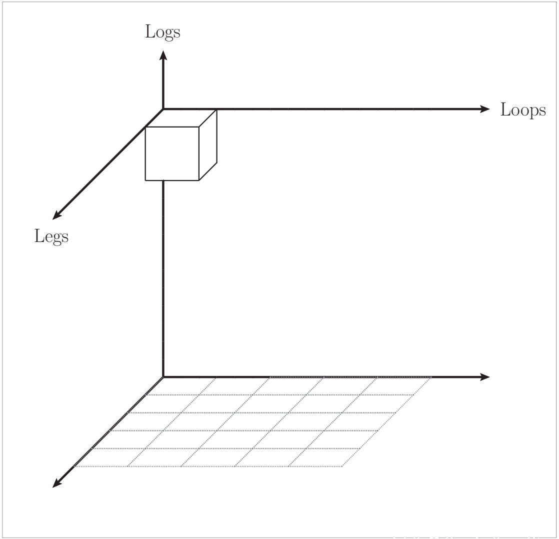
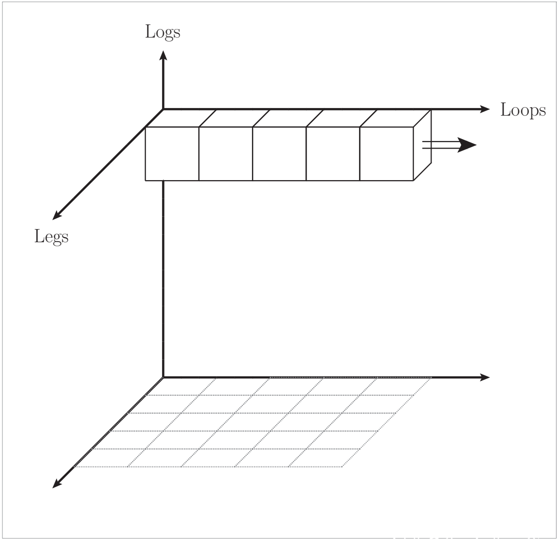


Recap: PS fixed order input



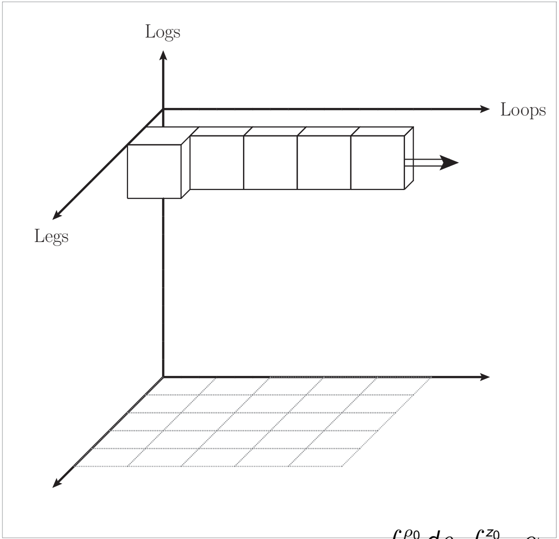
$d\sigma_B(pp \to X)$ $\mathcal{O}_0$

Recap: PS resums LL rows into no-emission probabilities (no PS emission)



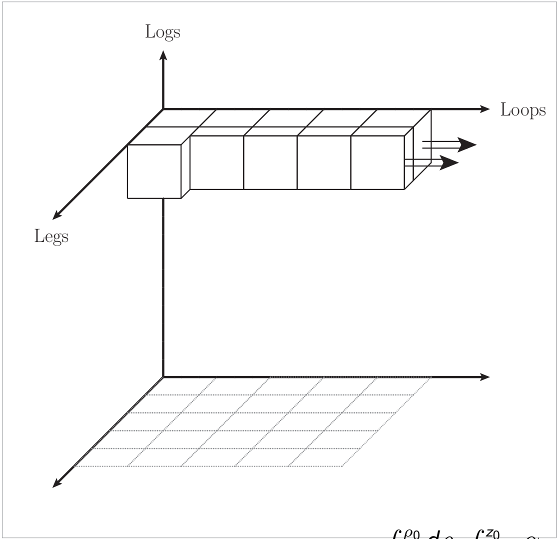
$d\sigma_B(pp \to X) \otimes \Pi_0(\rho_0, \rho_{min})$ $\mathcal{O}_0$

Recap: PS fills layers of LL loop corrections (one PS emission)



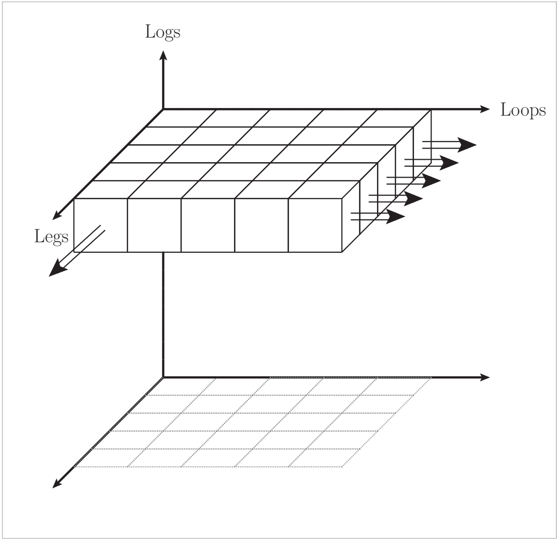
$$d\sigma_B(pp \to X) \otimes \int_{\rho_{min}}^{\rho_0} \frac{d\rho}{\rho} \int_{z_1}^{z_0} dz \frac{\alpha_s}{2\pi} P(z)\ \Pi_0(\rho_0, \rho)\ \mathcal{O}_1$$

Recap: PS fills layers of LL loop corrections (no or one PS emission)



$$d\sigma_B(pp \to X) \otimes \Pi_0(\rho_0, \rho_{min})\ \mathcal{O}_0 \ + \ d\sigma_B(pp \to X) \otimes \int_{\rho_{min}}^{\rho_0} \frac{d\rho}{\rho} \int_{z_1}^{z_0} dz \frac{\alpha_s}{2\pi} P(z)\ \Pi_0(\rho_0, \rho)\ \mathcal{O}_1$$

Recap: PS fills layers of LL loop corrections (sum of all PS results)



$$\sigma_{\text{0 or more jets}} = \sigma_{\text{exactly 0 jets}} + \sigma_{\text{exactly 1 jet}} + \sigma_{\text{exactly 2 jets}} + \ldots + \sigma_{\text{n or more jets}}$$

Recap of last lecture

- QCD scattering cross sections factorise.
- The factorisation can be cast into a probabilistic form suitable for a numerical implementation.
- Parton showers tell us how the inclusive cross section is sliced up into exclusive objects, where exclusive means a fixed number of resolved jets.
- Exclusive cross sections are defined through no-emission probabilities.
- All cross sections can be writen as a polynomial of logarithms.
- This log-structure can be illustrated on figures.

Systematic improvements of modern showers are possible due to local energy-momentum conservation.
$\Longrightarrow$ Systematic improvements are the topic of this lecture!

Improvement schemes

- Matrix element corrections.
  - Oldest scheme
  - Usage in HERWIG(++) and PYTHIA(8) slightly different.
  - Very hard to iterate.
- Matrix element matching.
  - Used ideas from ME corrections.
  - Typically combined with NLO corrections.
  - Very hard to iterate.
- Matrix element merging.
  - Slice phase space in two, use ME for hard jets, PS for soft jets.
  - Introduces resolution criterion.
  - Very easy to iterate.

We will use B_n for the tree-level n-parton differential cross section, and $\widetilde{\mathrm{B}}_n$ or $\overline{\mathrm{B}}_n$ for NLO cross sections that are differential in n-parton phase space.

Matrix element corrections

Remember how we constructed the parton shower:

- Find a factorizing approximation.
- Cast the factorising functions into probabilities.
- Choose branchings probabilistically.

Idea: Find new probabilities that add to the full ME!

For this, we need an overestimate for the double-differential partonic cross section $P_{\text{full-ME}}$, and find a corrective probability $P_{\text{ME-correction}}$, so that

$$P_{\text{full-ME}} \equiv \sum P_{\text{new}} = \sum P_{\text{shower}} * P_{\text{ME-correction},i} \qquad \text{with}$$

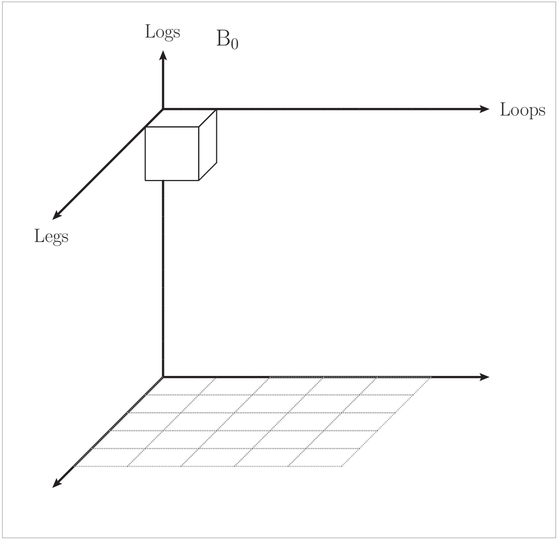
$$P_{\text{shower}} = \sum_{i \,\in\, [\text{possible PS splittings}]} P_{PS,i} \quad , \quad P_{\text{ME-correction},i} = \frac{\mathcal{P}_i P_{\text{full-ME}}}{P_{\text{shower}}} \quad \text{and} \quad \sum_i \mathcal{P}_i = 1$$

Then we can use two steps to correct an emission to the full ME result:

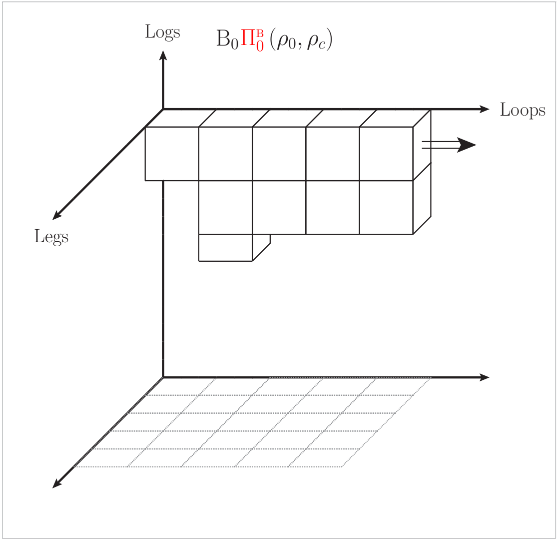
1. Choose a branching according to $P_{PS,i}$
2. Accept with probability $P_{\text{ME-correction},i}$

Summed over all possibilities, this gives the full ME ("Veto algorithm").

ME corrections: Start from lowest order cross section.

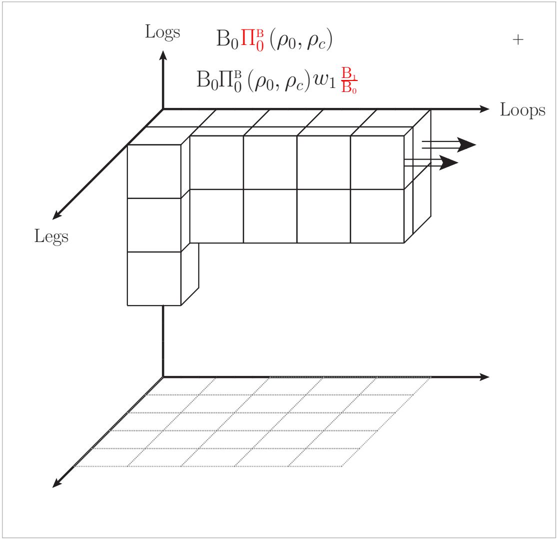


ME corrections: Produce no emissions according to new probability



where $\quad \Pi_0^{\mathrm{B}}(\rho_0,\rho_c) = \exp\left(-\int d\hat{\Phi}\frac{\mathrm{B}_1}{\mathrm{B}_0}\Theta(\rho(\hat{\Phi})-\rho_c)\right) = \exp\left(-\int d\hat{\Phi} P_{\mathrm{new}}\Theta(\rho(\hat{\Phi})-\rho_c)\right)$

ME corrections: Generate emissions according to new probability



This reproduces the full 1-parton radiation pattern, and is finite!

Matrix element corrections

Pro

- Rather natural within parton shower.
- Full ME (incl. interferences) gets exponentiated, not only approximation!
- Very efficient.

Contra

- Difficult to find overestimates, projectors and corrective weights.
- Exponentiation extends over full phase space (need to integrate the 1-parton ME over full phase space).
- Difficult to iterate, since ME-correction for $n+1$-partons has to divide out n-parton ME.

Subtleties

- The hardest emission has to be corrected, not only the first emission.
- Need to use "soft" and "hard" corrections if PS does not cover phase space: Add full ME in the gaps (hard), ME corrections for every "hardest emission" in the evolution (soft).

⇒ Unfortunately usual attitude: Process dependent, tricky to achieve generality.

Note: VINCIA iterates MEC's for $e^+e^- \to$ jets, and also aims for pp collisions.

NLO matching

NLO matching does not solve MEC problems, but uses the lessons to

Achieve NLO for inclusive +0-jet, and LO for inclusive +1-jet observables

To get there, remember that the NLO cross section is

$$\begin{aligned} \mathrm{B_{NLO}} &= [\mathrm{B}_n + \mathrm{V}_n + \mathrm{I}_n]\,\mathcal{O}_0 + \int d\Phi_{\mathrm{rad}}\,(\mathrm{B}_{n+1}\mathcal{O}_1 - \mathrm{D}_{n+1}\mathcal{O}_0) \\ &= [\mathrm{B}_n + \mathrm{V}_n + \mathrm{I}_n]\,\mathcal{O}_0 + \int d\Phi_{\mathrm{rad}}\,(\mathrm{S}_{n+1}\mathcal{O}_0 - \mathrm{D}_{n+1}\mathcal{O}_0) \\ &\quad + \int d\Phi_{\mathrm{rad}}\,(\mathrm{S}_{n+1}\mathcal{O}_1 - \mathrm{S}_{n+1}\mathcal{O}_0) + \int d\Phi_{\mathrm{rad}}\,(\mathrm{B}_{n+1}\mathcal{O}_1 - \mathrm{S}_{n+1}\mathcal{O}_1) \end{aligned}$$

where S_{n+1} are approximate virtual/real PS corrections.

Red term is the $\mathcal{O}(\alpha_s)$ part of a shower from B_n. $\Rightarrow$ For now discard from $\mathrm{B_{NLO}}$.

Thus, we have the seed cross section

$$\widehat{\mathrm{B}}_{\mathrm{NLO}} = \left[\mathrm{B}_n + \mathrm{V}_n + \mathrm{I}_n + \int d\Phi_{\mathrm{rad}}\,(\mathrm{S}_{n+1} - \mathrm{D}_{n+1})\right]\mathcal{O}_0 + \int d\Phi_{\mathrm{rad}}\,(\mathrm{B}_{n+1} - \mathrm{S}_{n+1})\,\mathcal{O}_1$$

This is not the NLO result. . . but showering the $\mathcal{O}_0$-part will restore this!

$\Longrightarrow$ NLO +PS accuracy!

POWHEG

We have found that NLO +PS is possible if we start from the seed cross section

$$\widehat{\mathrm{B}}_{\mathrm{NLO}} \quad = \quad \left[\mathrm{B}_n + \mathrm{V}_n + \mathrm{I}_n + \int d\Phi_{\mathrm{rad}}\left(\mathrm{S}_{n+1} - \mathrm{D}_{n+1}\right)\right]\mathcal{O}_0 + \int d\Phi_{\mathrm{rad}}\left(\mathrm{B}_{n+1} - \mathrm{S}_{n+1}\right)\mathcal{O}_1$$

where S_{n+1} is the PS approximation of the $n+1$-jet rate.

$\Longrightarrow$ The NLO matching only depends on the first PS step!

The first step can be done externally. Using $\mathrm{S}_{n+1} = \mathrm{B}_{n+1}$, i.e. a MEC for the first splitting, we find

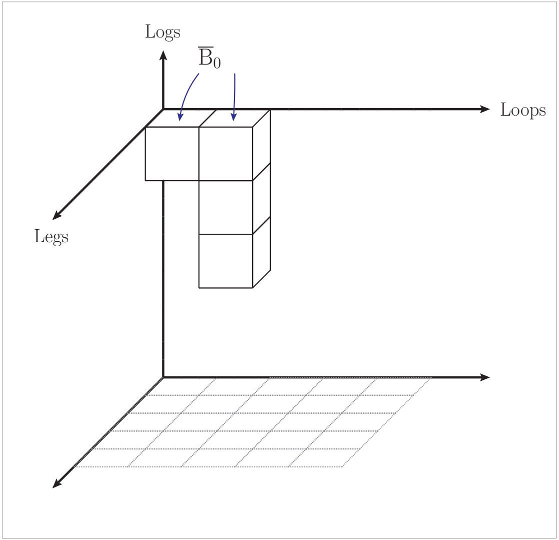
$$\widehat{\mathrm{B}}_{\mathrm{NLO}} \quad = \quad \left[\mathrm{B}_n + \mathrm{V}_n + \mathrm{I}_n + \int d\Phi_{\mathrm{rad}}\left(\mathrm{B}_{n+1} - \mathrm{D}_{n+1}\right)\right]\mathcal{O}_0 \;=\; \overline{\mathrm{B}}_n$$

$\Longrightarrow$ Seed cross section is simply the inclusive NLO result. This is POWHEG.

Roughly, POWHEG combines an ME correction with an NLO weight.

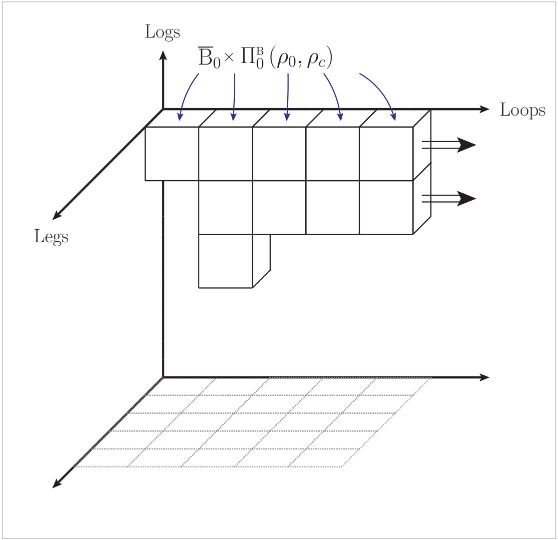
POWHEG-BOX is an ME generator that provides NLO inputs for parton showers. One (ME corrected) emission is done by POWHEG-BOX, other emissions have to be filled in by PS.

POWHEG illustration



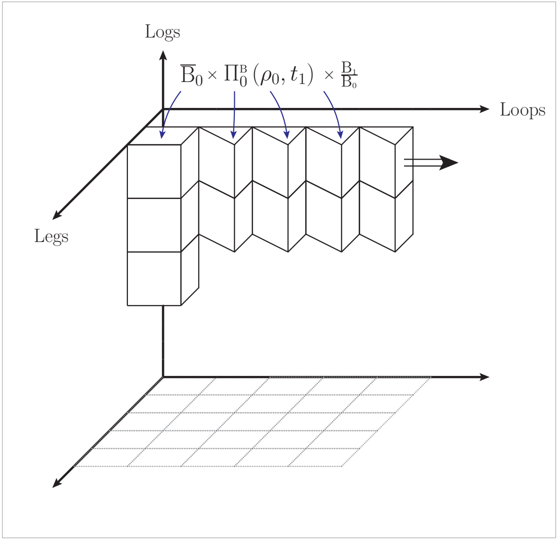
Shower from the seed cross section

POWHEG illustration



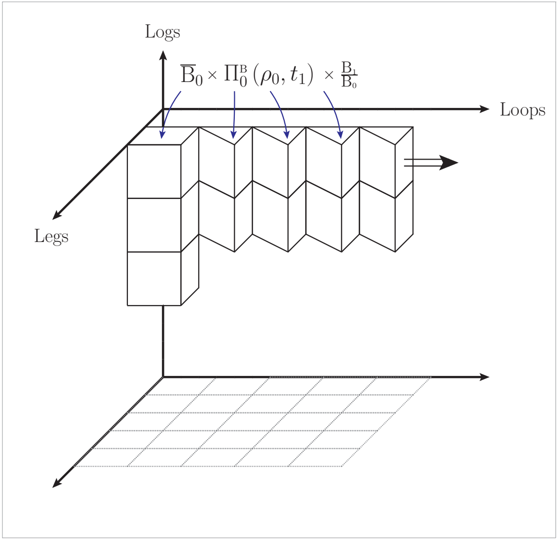
Shower from the seed cross section can give no emission,

POWHEG illustration



Shower from the seed cross section can give no emission, or one emission.

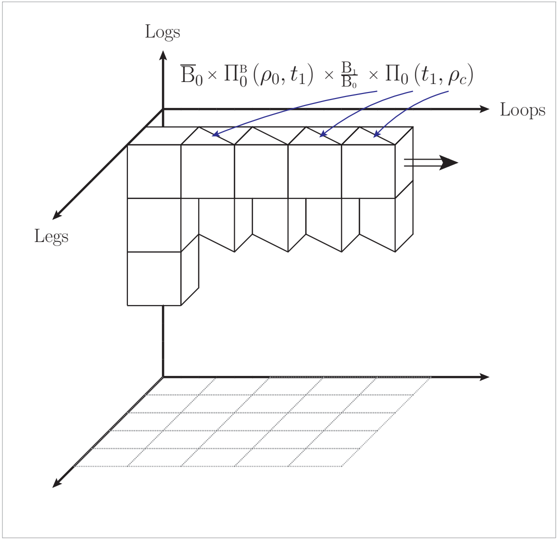
POWHEG illustration



Shower from the seed cross section can give no emission, or one emission.

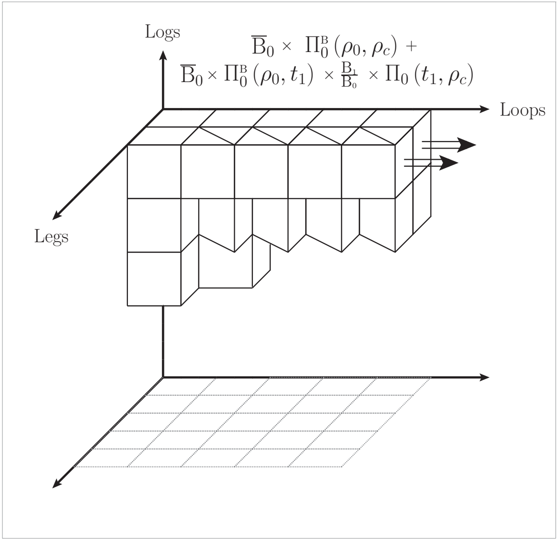
The hardness of the emission is defined differently from parton shower.

POWHEG illustration



The shower needs to be attached to this intermediate result, without introducing overlaps $\Rightarrow$ Truncated, vetoed shower necessary.

POWHEG illustration



The sum of all parts gives an NLO +PS simulation

POWHEG

Pro

- Inherits pros from ME correction.
- Full ME (incl. interferences) gets exponentiated, not only approximation!
- Mostly positive weights!

Contra

- Inherits cons from ME correction.
- Exponentiation extends over full phase space (need to integrate the 1-parton ME over full phase space).
- Difficult to iterate.

Subtleties

- Interface can be very subtle, nearly invalidating the PS independence. Format issues.
- Truncated, vetoed shower necessary.
- Can be redefined to consist of "soft" and "hard" corrections, by using $\mathrm{S}_{n+1} = \mathrm{B}_{n+1} F(\Phi)$ instead, at cost of introducing parameters.

MC@NLO

We have found that NLO +PS is possible if we start from the seed cross section

$$\widehat{\mathrm{B}}_{\mathrm{NLO}} = \left[\mathrm{B}_n + \mathrm{V}_n + \mathrm{I}_n + \int d\Phi_{\mathrm{rad}} \left(\mathrm{S}_{n+1} - \mathrm{D}_{n+1}\right)\right] \mathcal{O}_0 + \int d\Phi_{\mathrm{rad}} \left(\mathrm{B}_{n+1} - \mathrm{S}_{n+1}\right) \mathcal{O}_1$$

where S_{n+1} is the PS approximation of the $n+1$-jet rate.

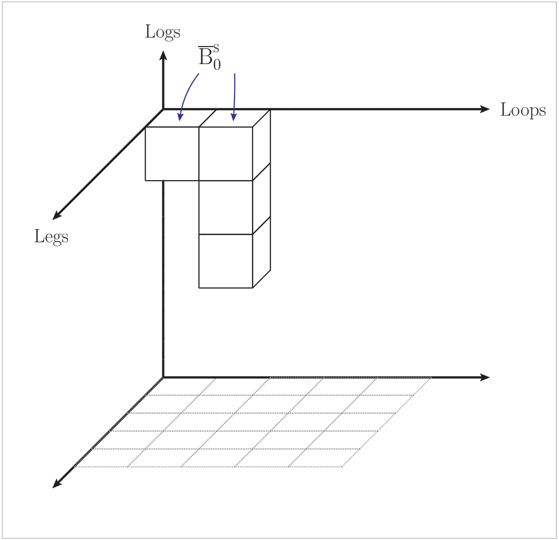
$\Longrightarrow$ The NLO matching only depends on the first PS step!

It is possible to keep $\mathrm{S}_{n+1} = \mathrm{B}_n \otimes \mathrm{K}\Theta(\mu_Q - \rho)$, where the Θ-function limits the subtraction to the PS phase space, and keep

$$\begin{aligned} \overline{\mathrm{B}}_n^{\mathrm{S}} &= \left[\mathrm{B}_n + \mathrm{V}_n + \mathrm{I}_n + \int d\Phi_{\mathrm{rad}} \left(\mathrm{B}_n \otimes \mathrm{K}\Theta(\mu_Q - \rho) - \mathrm{D}_{n+1}\right)\right] \mathcal{O}_0 && \mathbb{S}\text{-events} \\ \overline{\mathrm{B}}_n^{\mathrm{H}} &= \int d\Phi_{\mathrm{rad}} \left(\mathrm{B}_{n+1} - \mathrm{B}_n \otimes \mathrm{K}\Theta(\mu_Q - \rho)\right) \mathcal{O}_1 && \mathbb{H}\text{-events} \end{aligned}$$

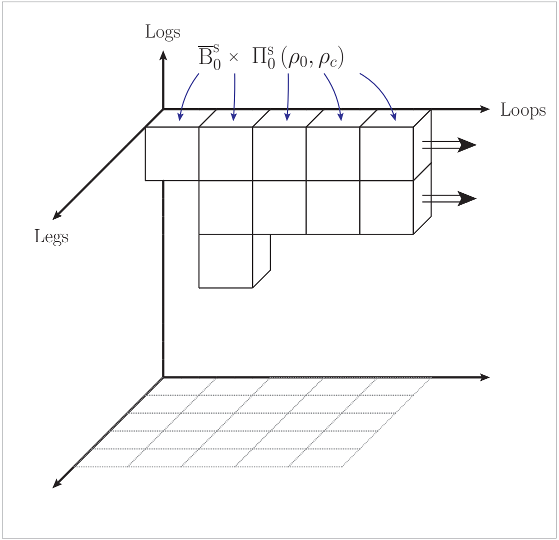
This emphasises the PS as an NLO subtraction. The matching now has soft $\mathbb{S}$-events and hard $\mathbb{H}$-events. $\mathbb{H}$-events are a non-logarithmic correction.

MC@NLO illustration



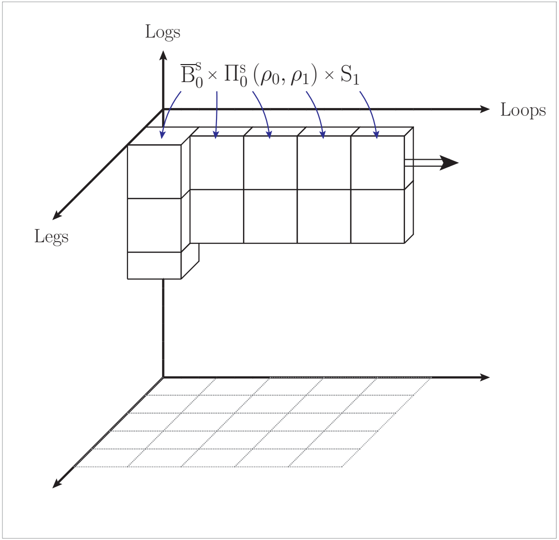
The shower off $\mathbb{S}$-events

MC@NLO illustration

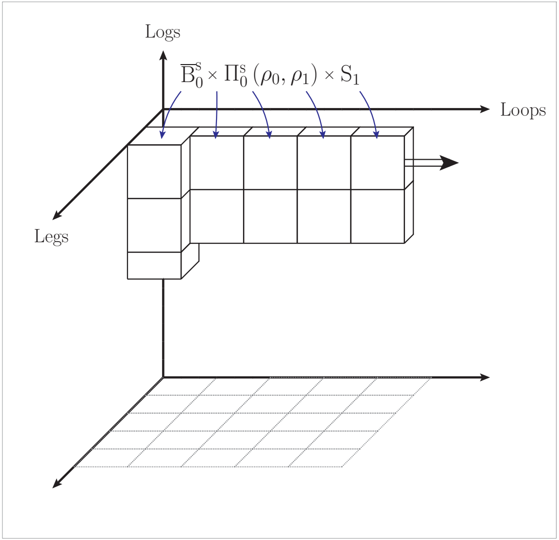


The shower off $\mathbb{S}$-events can give no emission

MC@NLO illustration

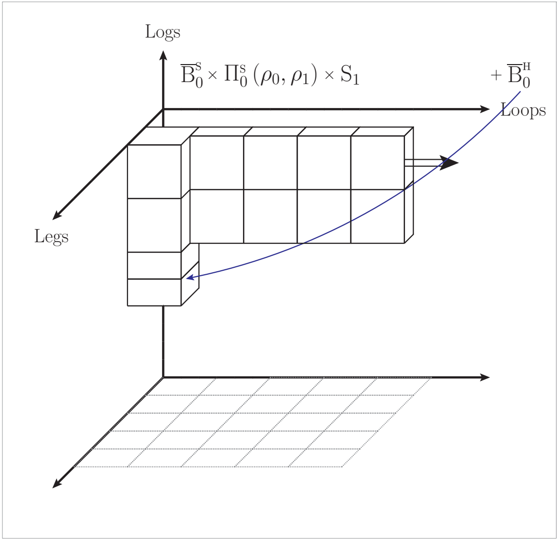


The shower off $\mathbb{S}$-events can give no emission, or one emission.



The shower off $\mathbb{S}$-events can give no emission, or one emission.

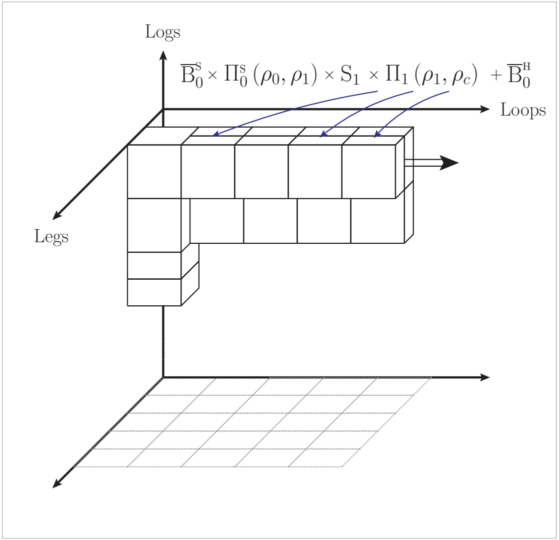
The emission is directly from PS $\Rightarrow$ Continuation obvious.



The shower off $\mathbb{S}$-events can give no emission, or one emission.

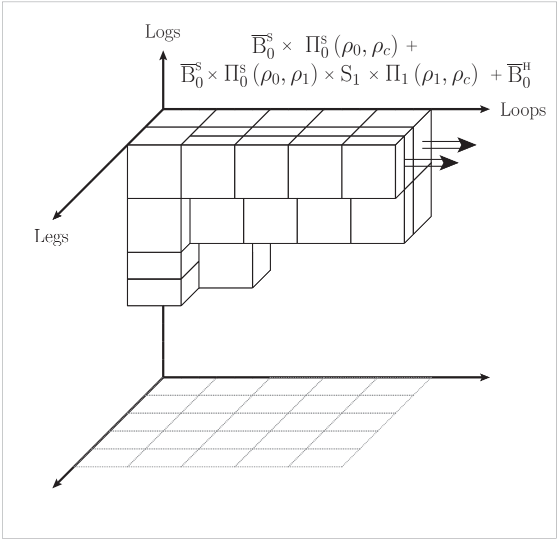
Now add the hard remainder $\mathbb{H}$-events.

MC@NLO illustration



The shower needs to be attached to this intermediate result, which is easy for $\mathbb{S}$-events, less clear for $\mathbb{H}$-events.

MC@NLO illustration



The sum of all parts gives an NLO +PS simulation

MC@NLO

Pro

- Interface to PS very easy.
- Very controlled change of resummation!
- No new shower necessary.

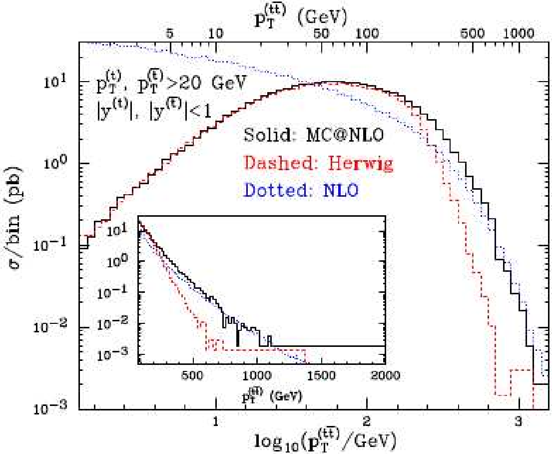
Contra

- $\mathbb{S}$-events alone, or $\mathbb{H}$-events alone are not necessarily positive.
- No clear prescription how to handle/shower $\mathbb{H}$-events.
- Difficult to iterate.

Subtleties

- PS needs to be a full NLO subtraction (requires colour-correct first emissions), or instead use $\mathsf{S}_{n+1} \approx \mathsf{B}_n \otimes \mathsf{K}\Theta(\mu_Q - \rho)$
- If PS is a full NLO subtraction, need to treat anti-probabilistic weights (see e.g. SHERPA, HERWIG++).

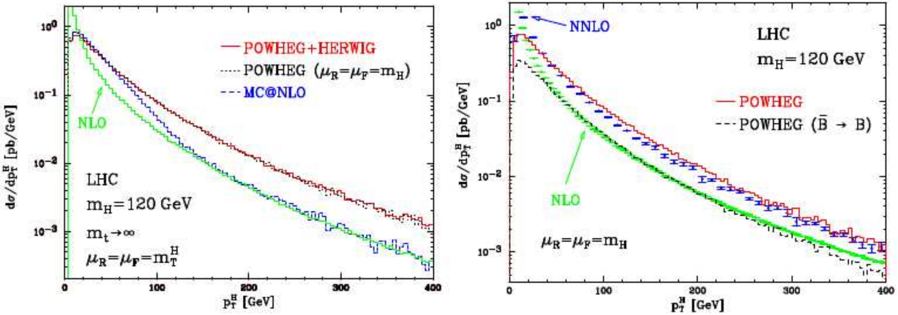
NLO matching results and comparisons



$p_\perp$ of $t\bar{t}$-system at a 14 TeV LHC for $t\bar{t}$-MC@NLO.

PS no-emission probability regulates the divergence. Hard tail given by fixed-order.

Question: When is this observable NLO accurate?



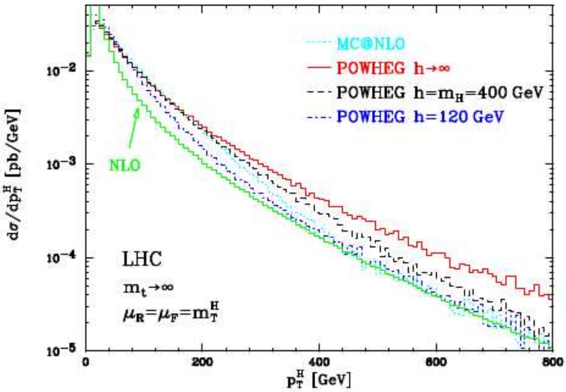
$p_\perp$ of Higgs boson at a 14 TeV LHC for $gg \to H$-POWHEG and $gg \to H$-MC@NLO.

PS no-emission probability regulates the divergence.

What happens in the tail?

Question: Is this observable NLO accurate?

NLO matching results and comparisons



$p_\perp$ of Higgs boson at a 14 TeV LHC for $gg \to H$-POWHEG.

Variations: Use a different PS kernel $\mathsf{S}_{n+1} = \mathsf{B}_{n+1} F(\Phi)$ in POWHEG.

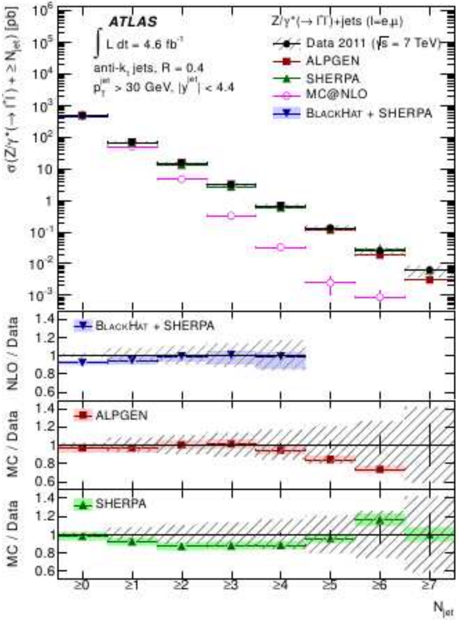
$\Rightarrow$ This is a very big "higher-order" effect!

Number of anti-$k_\perp$ jets in Z+jets events in ATLAS.

Zero-jet bin is NLO accurate, one-jet bin is leading order.

NLO matched calculation cannot describe high jet multiplicities.

$\Rightarrow$ No single NLO matched calculation will describe this data.



NLO matching

NLO matching can be obtained by showering the seed cross section

$$\widehat{\mathrm{B}}_{\mathrm{NLO}} = \left[\mathrm{B}_n + \mathrm{V}_n + \mathrm{I}_n + \int d\Phi_{\mathrm{rad}}\,(\mathrm{S}_{n+1} - \mathrm{D}_{n+1})\right]\mathcal{O}_0 + \int d\Phi_{\mathrm{rad}}\,(\mathrm{B}_{n+1} - \mathrm{S}_{n+1})\,\mathcal{O}_1$$

NLO matching methods differ in the choice of S_{n+1}:

POWHEG uses $\mathrm{S}_{n+1} = \mathrm{B}_{n+1}$ or $\mathrm{S}_{n+1} = \mathrm{B}_{n+1}F(\Phi)$

MC@NLO uses $\mathrm{S}_{n+1} = \mathrm{B}_n \otimes \mathrm{K}\Theta(\mu_Q - \rho)$

Pro

- Promotes the PS for one process to NLO accuracy!

Contra

- **New calculation needed whenever obervable depends on another jet!**
- Multiple matched calculations cannot be combined without major work.

Subtleties

- Interface to PS.
- Treatment of real-emission events.

Let's look at the process $pp \to e^+e^-$. Then

Inclusive observable $\equiv$ Observable only depends on e^+e^- momenta.
Example: Rapidity of e^+e^- pair
p_T of e^+e^- pair for $p_T = 0$ GeV
p_T of e^+ for $p_T \lesssim 45$ GeV

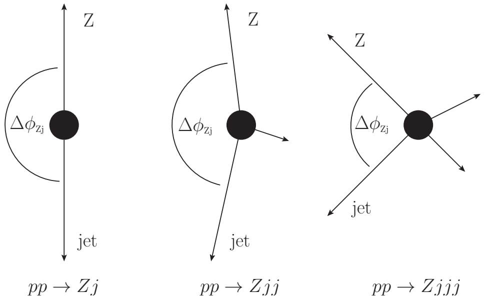
Exclusive observable $\equiv$ Any observable that depends on e^+e^- and other momenta.
Example: p_T of e^+e^- pair for $p_T > 0$ GeV
p_T of e^+ for $p_T \gtrsim 45$ GeV
Rate of events with no jet

So is it easy to decide if an observable is either?

Tricky observables

Consider the azimuthal angle $\Delta\phi_{\mathrm{Zj}}$ between the Z-boson and the hardtest jet in $pp \to$Z+jets events.

- Need at least $pp \to$ Zj for non-zero value.
- $\Delta\phi_{\mathrm{Zj}} = \pi$ for $pp \to$ Zj.
- Need at least two jets for $\Delta\phi_{\mathrm{Zj}} < \pi$
- Need at least three jets for $\Delta\phi_{\mathrm{Zj}} < \frac{2}{3}\pi$, since hard jet needs to be balanced by two softer jets!



To describe the full spectrum with at least LO accuracy, we need Zj, Zjj and Zjjj. If we want to do a fixed-order calcculation for that, we need Zj@NNLO.

$\Longrightarrow$ Many emissions needed to describe the whole distribution.
$\Longrightarrow$ Short-cut: Multileg merging.

The ME+PS merging problem

Goal: Get an accurate prediction of multijet observables (e.g. $\Delta\phi_{\mathrm{Zj}},\ n_{\mathrm{jets}}$)
Idea: Combine predictions for arbitrary many jets into a single calculation!

Problems:

- Cross sections in fixed-order perturbation theory are inclusive by definition $\Rightarrow$ Overlap:

$$\sigma(pp \to X) \supset \sigma(pp \to X + gluon)$$

- Fixed-order predictions break down for collinear or soft partons.
- PS gives sensible result in the collinear or soft regions, but breaks down for (many) well-separated jets.
- Adding PS and fixed-order again gives overlap, since the PS reproduces the leading-log approximation of the cross section!

Solutions:

- Remove overlap of FO cross sections by making them exclusive.
- Restrict which parton shower emissions are allowed.

For now, a simplification:

- Use only real emission corrections. "Cut away" the singularities with a phase-space cut t_{MS}. $t_{\text{MS}} \sim \min\{\text{all possible jet separations}\}$ works.
- This approximation is called a *tree-level* calculation, and t_{MS} is called *merging scale* cut.

Tree-level merging

For now, a simplification:

- Use only real emission corrections. "Cut away" the singularities with a phase-space cut t_{MS}. $t_{\text{MS}} \sim \min\{\text{all possible jet separations}\}$ works.
- This approximation is called a *tree-level* calculation, and t_{MS} is called *merging scale* cut.

What we want to achieve is

- Emissions above t_{MS} described by (exclusive) tree-level calculations.
  ... that should lead to a good description of high $p_\perp$ data.
- Emissions below t_{MS} described by the PS.
  ... because the PS gets soft/collinear partons right.

Watch out: Dependence on the arbitrary parameter t_{MS} should be small!

Making fixed-order calculations additive

To make fixed-order calculations exclusive (i.e. additive), remember that the PS generates exclusive cross sections

$$\sigma_{0\ or\ more\ jets} = \underbrace{\sigma_{exactly\ 0\ jets}}_{\text{exclusive due to Sudakov factor}} + \underbrace{\sigma_{exactly\ 1\ jet}}_{\text{exclusive due to Sudakov factors}} + \underbrace{\sigma_{2\ or\ more\ jets}}_{\text{inclusive}}$$

by multiplying PS Sudakov factors.

$\Rightarrow$ Convert the inclusive states of the ME calculation into *exclusive* states by multiplying PS no-emission probabilities.

Different choices how to produce PS no-emission probabilities give different schemes:

- MLM: Approximate no-emission probabilities by veto on jets.
- CKKW: Analytic Sudakov factors as no-emission probabilities.
- CKKW-L: PS no-emission probabilities directly from PS trial showers (similar in METS).

Minimising the dependence on t_{MS}

After making the tree-level matrix elements exclusive, we are allowed to add the calculations.

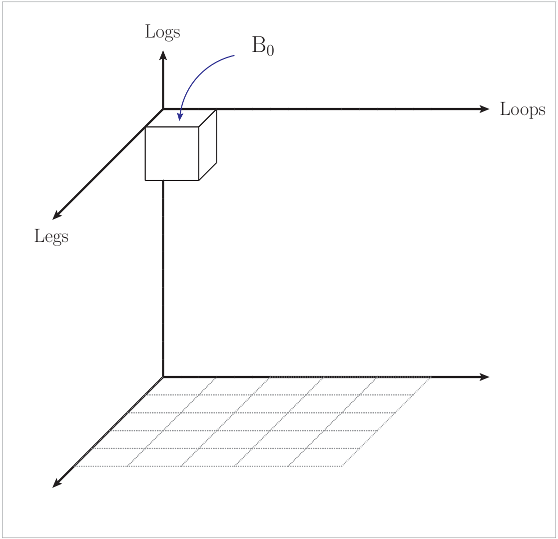
But we're missing soft/collinear emissions, i.e. emissions below t_{MS}.

These can be produced by parton showering.
Example: To get a state with a hard and a soft emission, start the PS on an exclusive one-jet tree-level calculation, and veto the event if the PS produced an emission $> t_{\text{MS}}$.

But remember: PS emissions use running α_s (PDFs) to capture higher orders!
$\Rightarrow$ So far, running α_s (PDFs) below t_{MS}, fixed values above t_{MS}
$\Rightarrow$ Remove mismatch by using running α_s (PDFs) also in tree-level calculations.

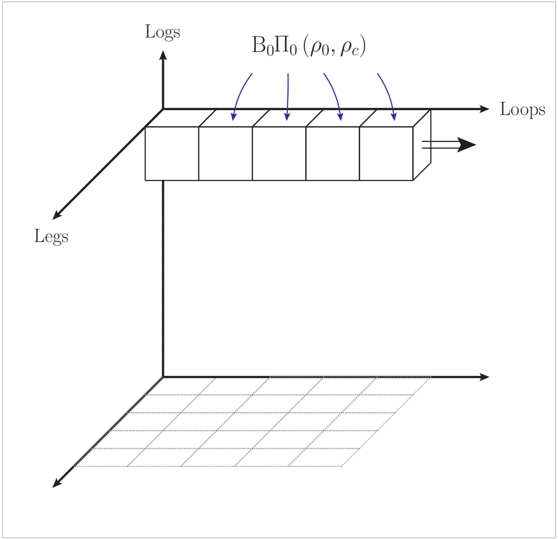
$\Rightarrow$ Matrix element + parton shower merging.
Let's look at an example.

ME+PS merging example



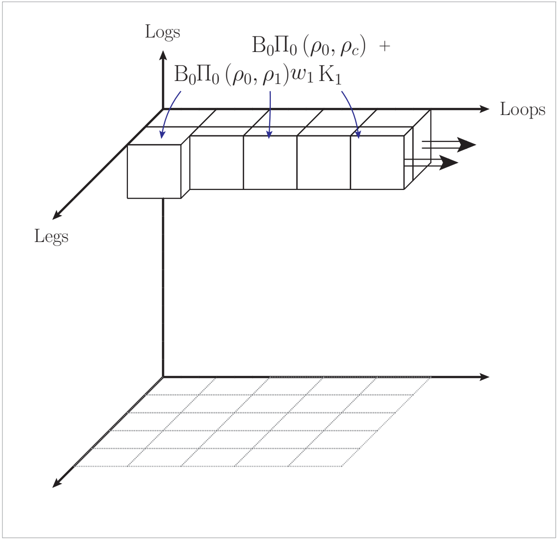
"Normal" shower from the 0-emission cross section can

ME+PS merging example



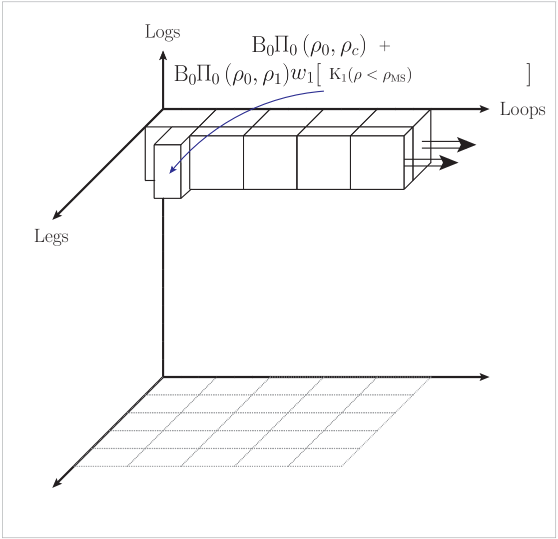
"Normal" shower from the 0-emission cross section can give no emission,

ME+PS merging example



"Normal" shower from the 0-emission cross section can give no emission, or one emission.

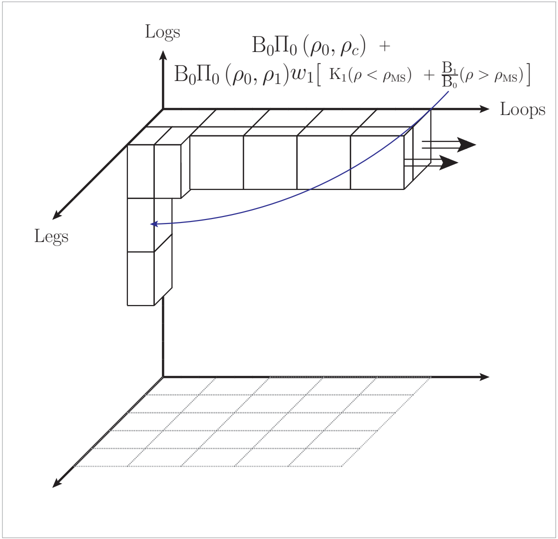
ME+PS merging example



"Normal" shower from the 0-emission cross section can give no emission, or one emission.

Veto all events with $\rho_{\text{emission}} > \rho_{\text{MS}}$.

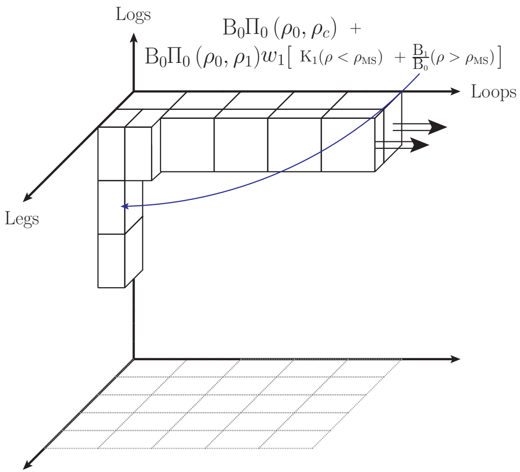
ME+PS merging example



"Normal" shower from the 0-emission cross section can give no emission, or one emission.

Veto all events with $\rho_{\text{emission}} > \rho_{\text{MS}}$. Add the reweighted 1-emission ME above ρ_{MS}.

ME+PS merging example



$\Longrightarrow$ **ME+PS merging**

"Normal" shower from the 0-emission cross section can give no emission, or one emission.

Veto all events with $\rho_{\text{emission}} > \rho_{\mathrm{MS}}$. Add the reweighted 1-emission ME above ρ_{MS}.

Merging algorithms step-by-step

We have defined a ME+PS merging by

1. Regularise MEs with t_{MS} cut.
2. Make MEs exclusive by multiplying PS no-emission probabilities $\Pi_i(\rho_i, \rho_{i+1})$.
3. Reweight MEs with factors w_i to include α_s and PDF running.
4. Shower these inputs.
   Veto if the PS produced a “hard” event.
5. Add up all processed phase space points.

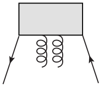
Note: To calculate the necessary no-emission probabilities $\Pi_i(\rho_i, \rho_{i+1})$ and α_s+PDF weights w_i, we need to define the scales $\rho_0, \rho_1, \ldots, \rho_n$.

This information can be extracted by constructing a parton shower history for each tree-level phase space point.

PS histories not only define the ordering of emissions (i.e. the scale sequence $\rho_0, \rho_1, \ldots, \rho_n$) but also complete, physical intermediate states.
Complete int. states can be used for trial showers. . . and much more.

Parton shower histories

Construction of PS histories for input phase space points is crucial in ME+PS merging.



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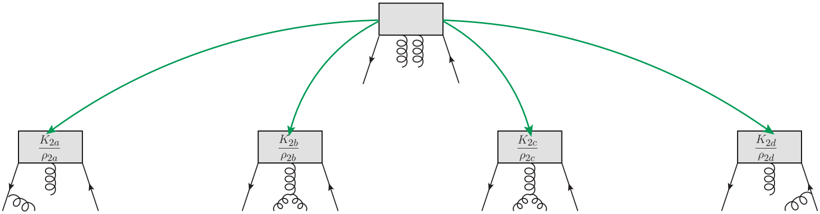

Parton shower histories

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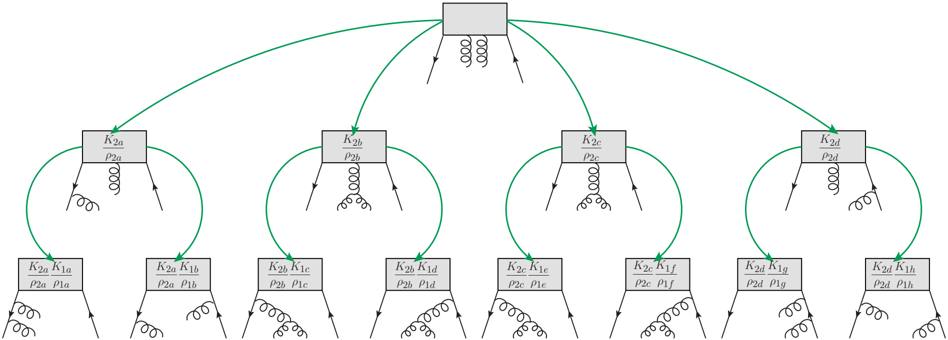
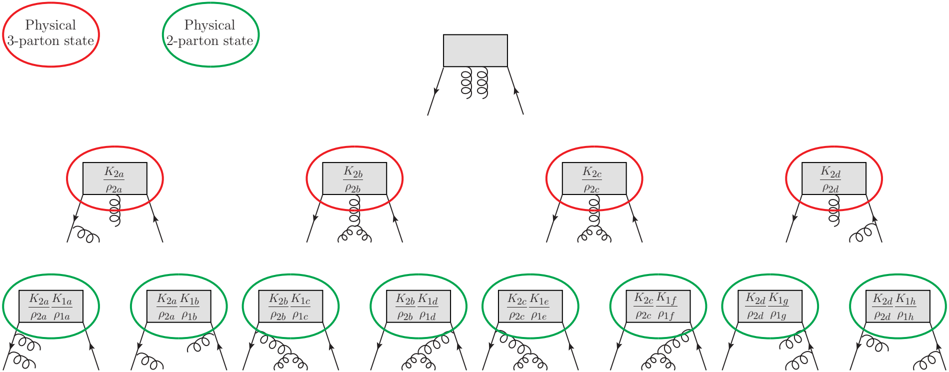


Different merging algorithms choose a PS history differently:

◇ CKKW only constructs the scales of one history, with the $k_\perp$ clustering algorithm.

Parton shower histories

Construction of PS histories for input phase space points is crucial in ME+PS merging.

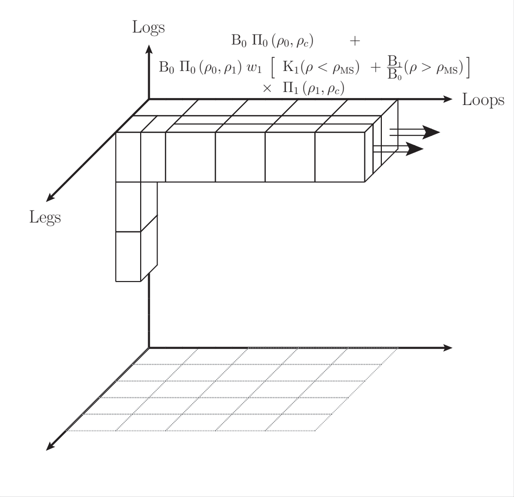


Different merging algorithms choose a PS history differently:

- ◇ METS chooses full intermediate states probabilistically at each step.
- ◇ CKKW-L constructs all histories, chooses path of full int. states probabilistically.

Physical intermediate states $S_{\text{n-jet}}$ allow trial showers: Run PS on $S_{\text{n-jet}}$.
If $\rho_{emission} > \rho_{n+1}$, veto $\Longrightarrow$ Generated no-emission probability

Multileg merging can be iterated!

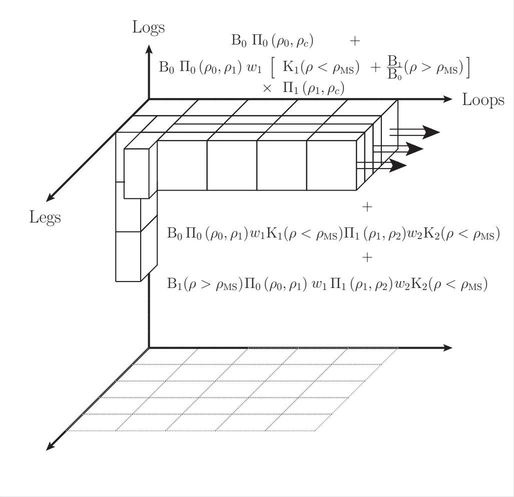


Previous zero+one leg merging result.

Now also veto all events with $\rho_{\text{emission}} > \rho_{\mathrm{MS}}$ when showering 1-emission MEs

. . . which can produce one hard + no soft jet

Multileg merging can be iterated!



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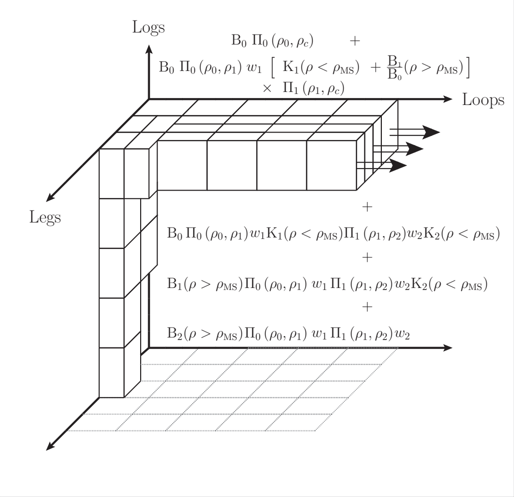


Previous zero+one leg merging result.

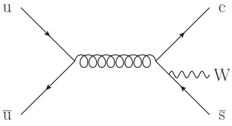
Now also veto all events with $\rho_{\text{emission}} > \rho_{\mathrm{MS}}$ when showering 1-emission MEs

. . . which can produce one hard + no soft jet, or one hard + one soft jet.

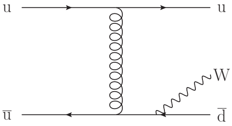
Then add the reweighted ME for two hard jets. Iterate.

Merging questions: New processes

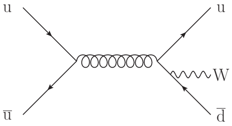
Now we can claim NLO accuracy, but. . .



New Born configuration



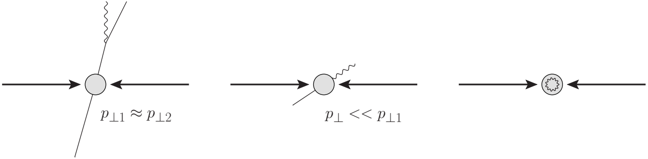
Standard shower history



???

- . . . what do we do with new Born states? What's a new Born state?
- How do we attach the QCD resummation (Sudakovs, α_s scales. . .)?
- If these are "weak corrections" to dijet states, should we merge multiple weak emissions?
  $\Longrightarrow$ Resum weak $\ln\left(\frac{\hat{s}}{M_B}\right)$ logs?

. . . and the trouble with weak bosons continues:



If a QCD-like history is enforced on this state, it will often be unordered. We cannot currently treat the resummation of unordered shower splittings, and don't have guidelines for choosing α_S scales!

Merging questions: Unordered states

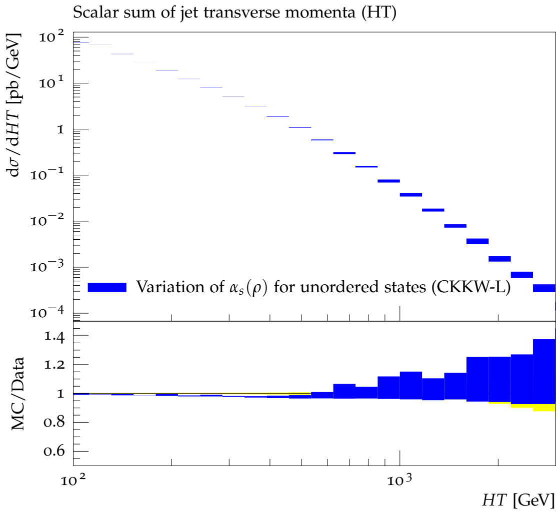
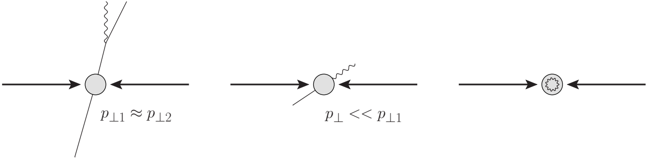


Figure: H_T in CKKW-L merging for Z+jets events @ 100 TeV

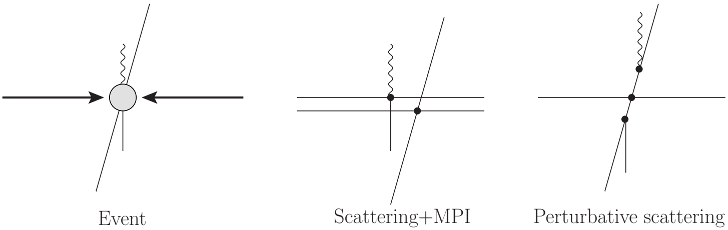
...and the trouble with weak bosons continues:



If a QCD-like history is enforced on this state, it will often be unordered. We cannot currently treat the resummation of unordered shower splittings, and don't have guidelines for choosing α_S scales!

$\Longrightarrow$ Need unordered shower emissions to improve this.

Merging questions: Competition with MPI



Assume we understand weak showers and sub-leading QCD logs. We still only model the competition between MPI and perturbative QCD!

At LHC, jets from MPI are relatively soft. $\Rightarrow$ Small effects.
At 100 TeV, MPI jets can be relatively hard. $\Rightarrow$ Competition must be understood!

- ◇ Can we simply only look at jets with large $p_\perp$, i.e ignore competition?
- ◇ Do we need to ME-correct MPI jets?
- ◇ Do we need weak bosons from MPI?

Multileg merging

Merging methods differ in the choice of

...with which no-emission probability to make MEs exclusive.

...how to decide on a sequence of states used in reweighting.

Pro

- Process independent.
- Combine multiple tree-level cross section with each other and with PS resummation.
- Good prediction for exclusive observables.

Contra

- Not NLO (yet, see later)
- **Changes inclusive cross sections.**

Subtleties

- Treatment of non-shower like configurations.
- Non-shower type configurations might (depending on the scheme) require truncated showers.

Bug vs. Feature in ME+PS

The ME includes terms that are not compensated by the PS approximate virtual corrections (i.e. no-emission probabilities).
These terms from the ME are what we need to describe multiple hard jets!

But if we simply add samples, the "improvements" will degrade the inclusive cross section: σ_{inc} will contain $\ln(t_{\mathrm{MS}})$ terms.

INCLUSIVE CROSS SECTIONS DO NOT KNOW ABOUT (CUTS ON) HIGHER MULTIPLICITIES. INCLUSIVE IS INCLUSIVE!

Traditional approach: Don't use a too small value for the merging scale.
$\rightarrow$ Uncancelled terms numerically not important.

New approach[1]:
Use a (PS) unitarity inspired approach exactly cancel the dependence of the inclusive cross section on t_{MS}.

[1] JHEP1302(2013)094 (Leif Lönnblad, SP), JHEP1308(2013)114 (Simon Plätzer)

We can use parton shower unitarity to rewrite CKKW-L as

$$
\begin{aligned}
\langle\mathcal{O}\rangle \quad &= \mathsf{B}_0 \Pi_{S_{+0}}(\rho_0, \rho_{\mathrm{MS}}) \mathcal{O}(S_{+0j}) \\
&+ \int \mathsf{B}_1 \Theta\left(t\left(S_{+1}\right) - t_{\mathsf{MS}}\right) w_f^0 w_{\alpha_\mathrm{s}}^0 \Pi_{S_{+0}}(\rho_0, \rho_1) \mathcal{O}(S_{+1j})
\end{aligned}
$$

Unitarised merging

We can use parton shower unitarity to rewrite CKKW-L as

$$\langle \mathcal{O} \rangle \quad = \mathrm{B}_0 - \int d\rho \; w_f^0 w_{\alpha_s}^0 \mathrm{B}_0 \mathrm{K}_0(\rho) \Pi_{S_{+0}}(\rho_0, \rho) \Theta\left(t\left(S_{+1}\right) - t_{\mathrm{MS}}\right) \mathcal{O}(S_{+0j})$$

$$+ \int \mathrm{B}_1 \Theta\left(t\left(S_{+1}\right) - t_{\mathrm{MS}}\right) w_f^0 w_{\alpha_s}^0 \Pi_{S_{+0}}(\rho_0, \rho_1) \mathcal{O}(S_{+1j})$$

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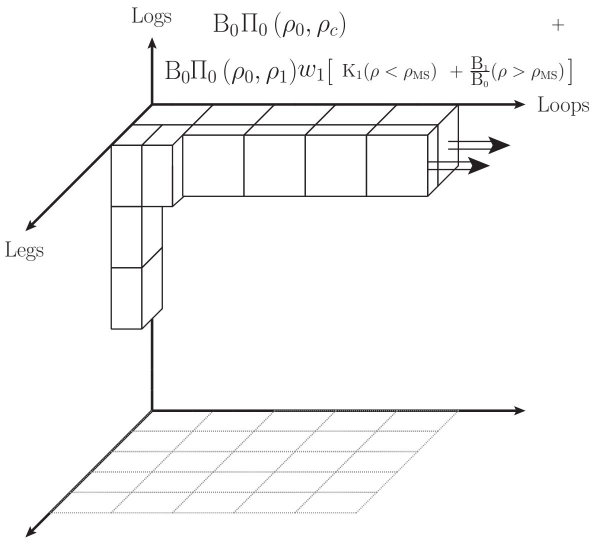
and replace

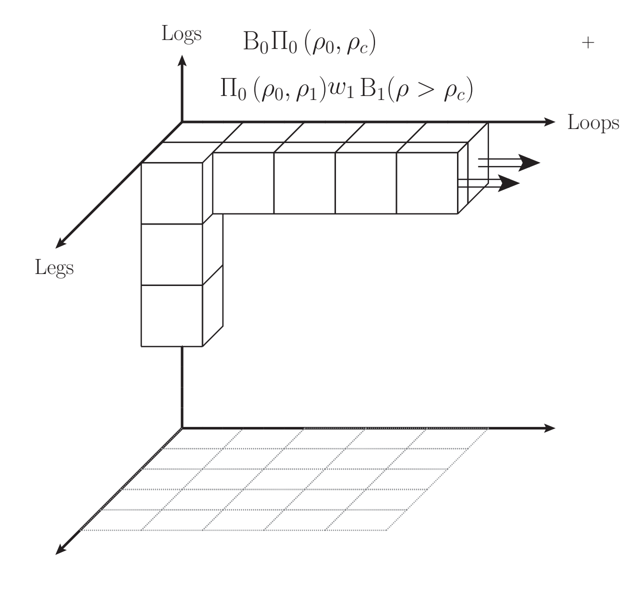
$$\begin{aligned}\langle\mathcal{O}\rangle = \mathrm{B}_0 &- \int d\rho\ w_f^0 w_{\alpha_s}^0 \mathrm{B}_1 \Pi_{S_{+0}}(\rho_0,\rho) \Theta\left(t\left(S_{+1}\right) - t_{\mathrm{MS}}\right) \mathcal{O}(S_{+0j}) \\ &+ \int \mathrm{B}_1 \Theta\left(t\left(S_{+1}\right) - t_{\mathrm{MS}}\right) w_f^0 w_{\alpha_s}^0 \Pi_{S_{+0}}(\rho_0,\rho_1) \mathcal{O}(S_{+1j})\end{aligned}$$

$\Longrightarrow$ UMEPS!

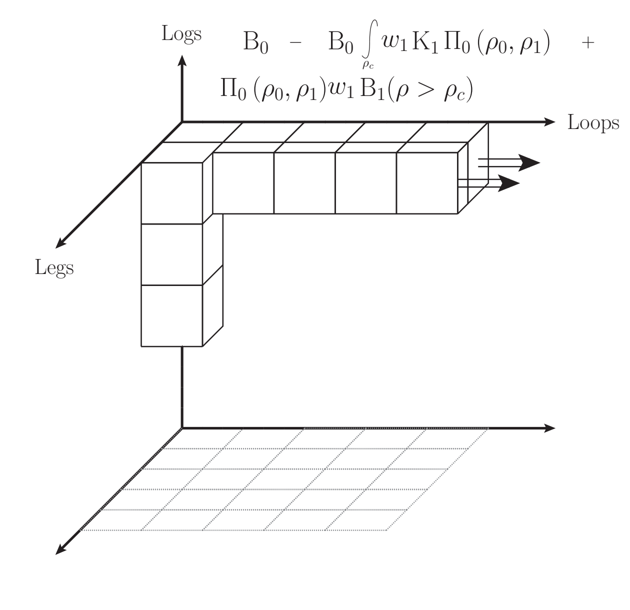
ME+PS, merging zero and one-emission MEs. . . again



ME+PS, put $t_{\mathsf{MS}} \to$ PS cut-off ρ_c for simplicity



ME+PS, cross section changes because $B_1 \neq B_0 K_0$



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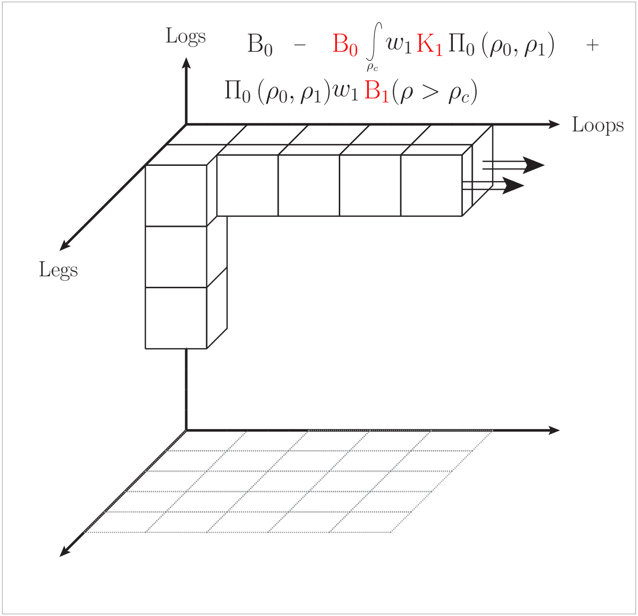
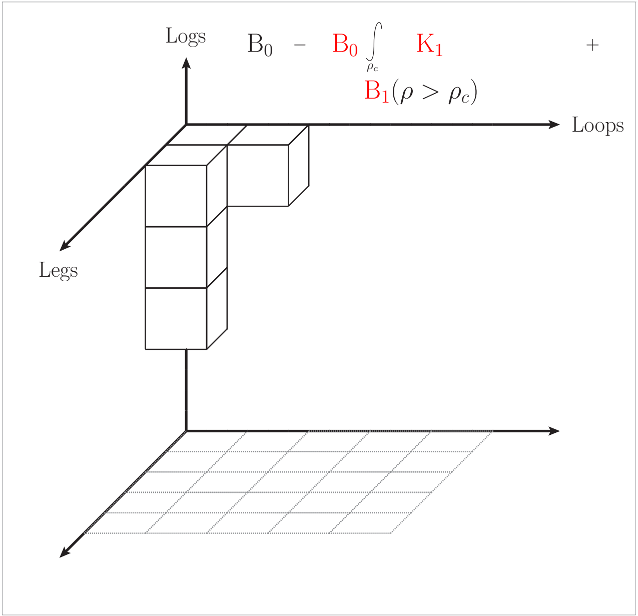
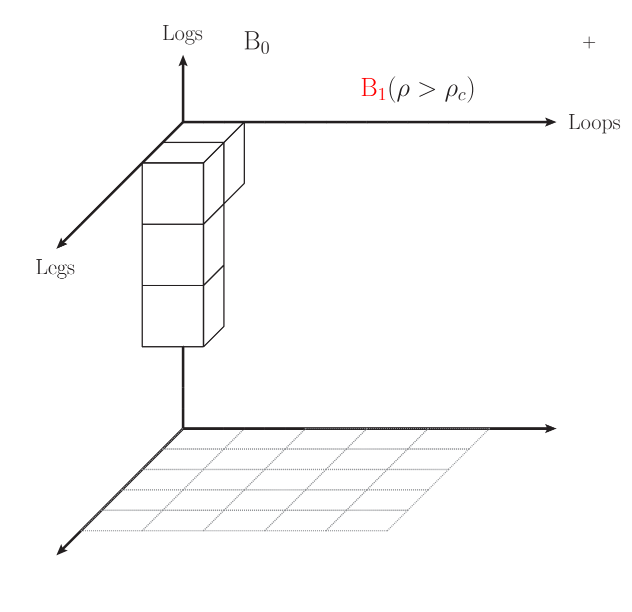


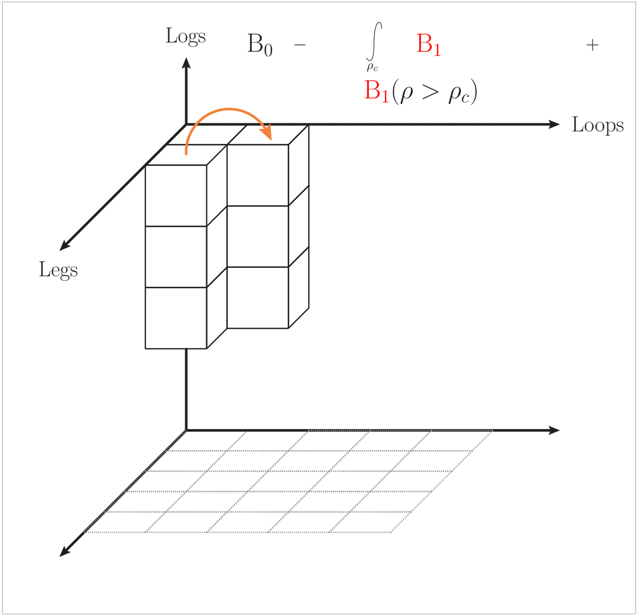
ME+PS, cross section changes because virtual cannot cancel real correction!



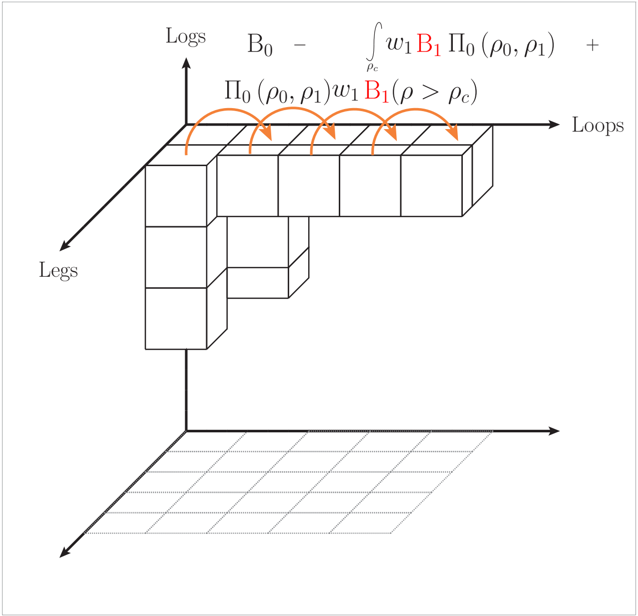
Forget the approximate PS virtual corrections!



Add new approximate virtual corrections by integrating real corrections! (LoopSim)



This also works when integrating reweighted exclusive real corrections! (UMEPS)



Unitarised ME+PS merging (UMEPS)

This sketch can directly be extended to the case when we have

$\widehat{\mathrm{B}}_2 =$ LO cross section, weighted with w_f, w_{α_s} and Π's

$\int \widehat{\mathrm{B}}_{n\to m} =$ integrated LO cross section, weighted with w_f, w_{α_s} and Π's.

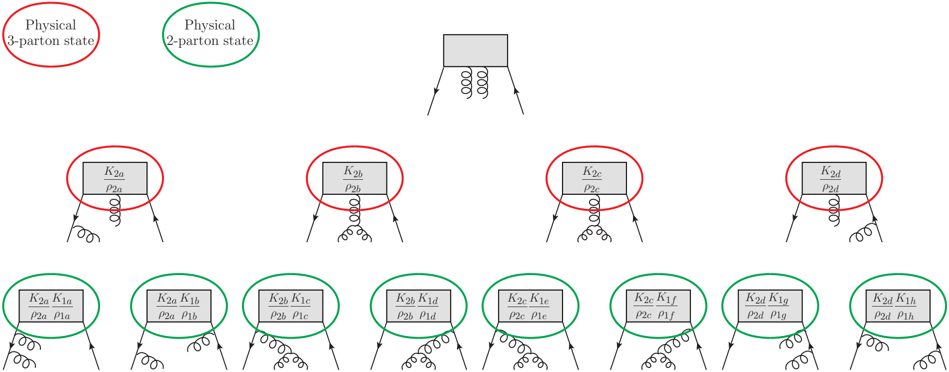
For example two-jet merging:

$$
\begin{aligned}
\langle \mathcal{O} \rangle \;=\; \int d\phi_0 \Bigg\{ & \mathcal{O}(S_{+0j}) \left[\mathrm{B}_0 - \int \widehat{\mathrm{B}}_{1\to 0} - \int \widehat{\mathrm{B}}_{2\to 0} \right] \\
& + \int \mathcal{O}(S_{+1j}) \left[\widehat{\mathrm{B}}_1 - \int \widehat{\mathrm{B}}_{2\to 1} \right] \\
& + \int\int \mathcal{O}(S_{+2j})\, \widehat{\mathrm{B}}_2 \Bigg\}
\end{aligned}
$$

Integrated configurations are available anyway since we need them to perform the reweighting with no-emission probabilities!

$\Rightarrow$ Do integration simply by replacing input state $S_{\text{n-jet}}$ by $S_{\text{n-1-jet}}$.

Unitarised ME+PS merging (UMEPS)



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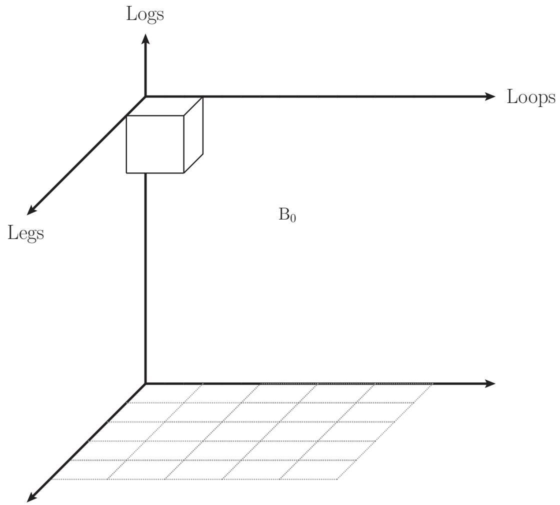
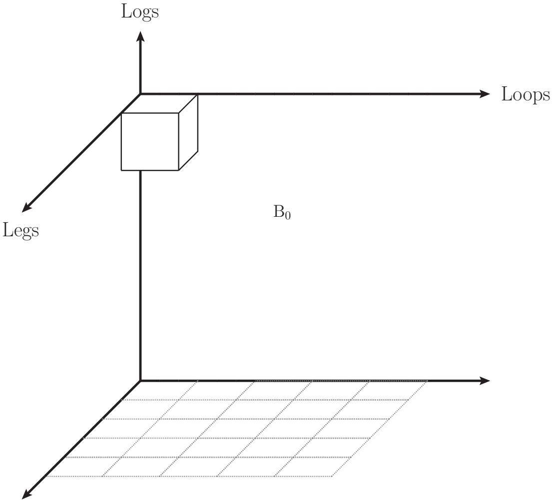


Start from the 0-parton ME

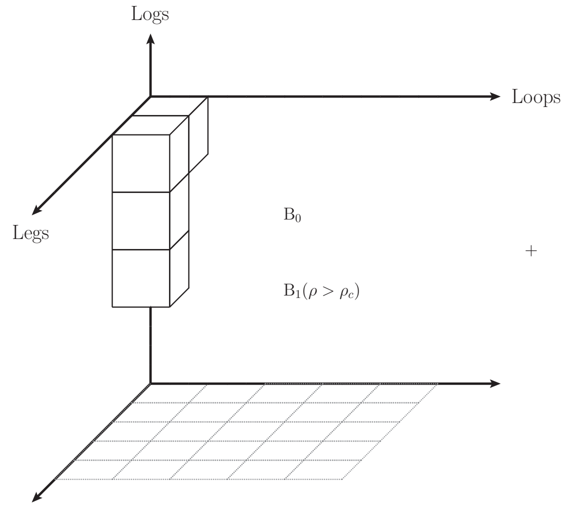
UMEPS step-by-step: 0-jet inclusive ✓



Start from the 0-parton ME

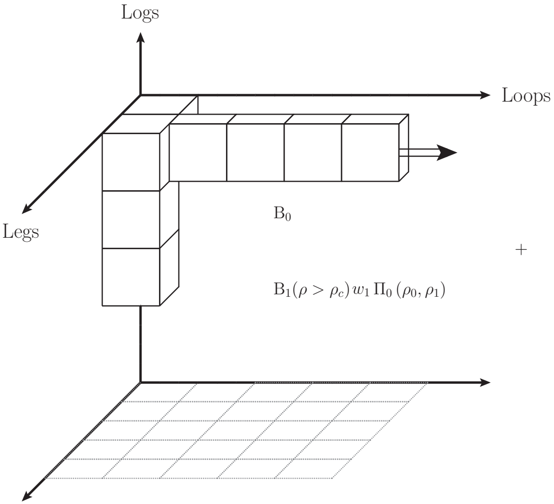
…and do nothing above t_{MS}.

UMEPS step-by-step: 0-jet inclusive ✗, 1-jet inclusive ✓



Then start from the 1-parton ME

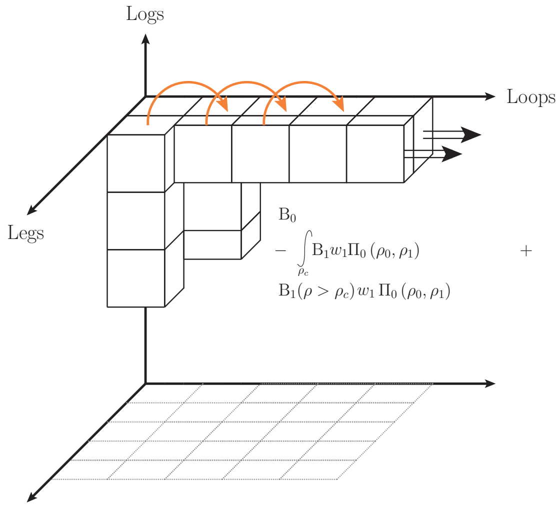
UMEPS step-by-step: 0-jet inclusive ✗, 1-jet inclusive ✓



Then start from the 1-parton ME

. . . and multiply no-emission probabilities and α_s (PDF) weights.

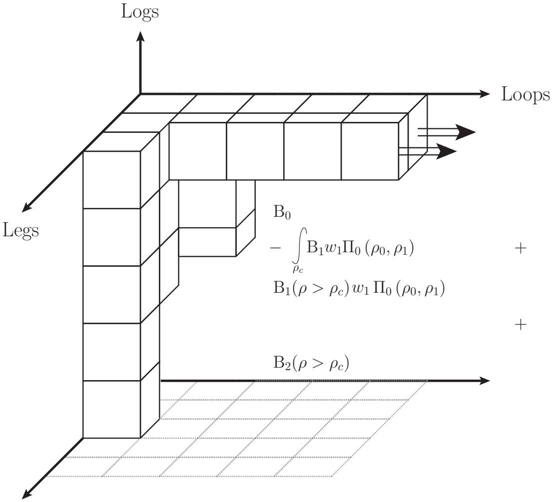
UMEPS step-by-step: 0-jet inclusive ✓, 1-jet inclusive ✓



Now restore the 0-jet inclusive cross section.

... by subtracting the integrated reweighted 1-jet cross section.

UMEPS step-by-step: 0-jet inclusive ✗, 1-jet inclusive ✗, 2-jet inclusive ✓



B_0

$- \int_{\rho_c} \mathrm{B}_1 w_1 \Pi_0 (\rho_0, \rho_1)$ $+$

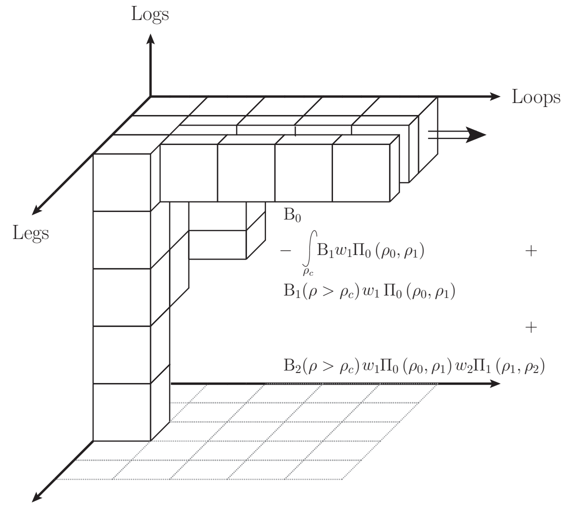
$\mathrm{B}_1(\rho > \rho_c) w_1 \Pi_0 (\rho_0, \rho_1)$

$+$

$\mathrm{B}_2(\rho > \rho_c)$

Then start from the 2-parton ME

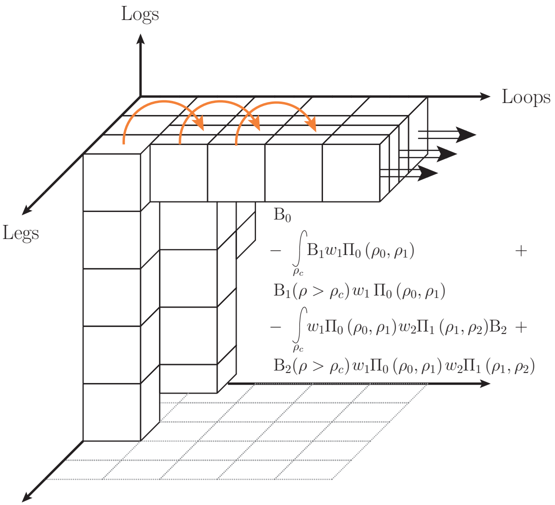
UMEPS step-by-step: 0-jet inclusive $\mathcal{X}$, 1-jet inclusive $\mathcal{X}$, 2-jet inclusive ✓



Then start from the 2-parton ME

. . . and multiply no-emission probabilities and α_s (PDF) weights.

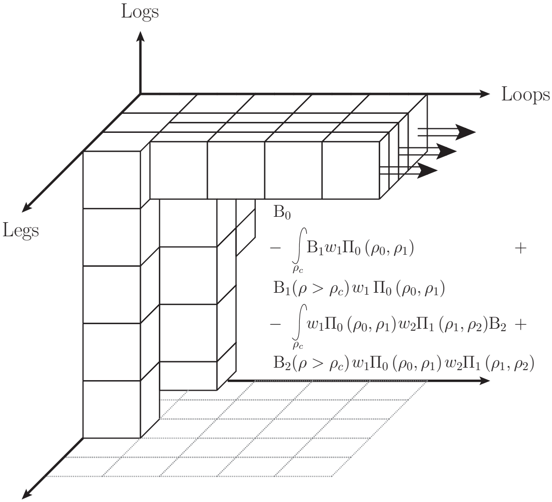
UMEPS step-by-step: 0-jet inclusive ✓, 1-jet inclusive ✓, 2-jet inclusive ✓



Now restore the 0-jet and 1-jet inclusive cross sections

. . . by subtracting the integrated reweighted 2-jet cross section.

UMEPS step-by-step: 0-jet inclusive ✓, 1-jet inclusive ✓, 2-jet inclusive ✓



. . . and continue further, adding and subtracting. . .

Unitarised paradigm, summary

Pro

- Inherits Pros from multileg merging.
- Does not change any of the inclusive cross sections by having better approximate $\mathcal{O}(\alpha_s^{+1})$ corrections.

Contra

- Not NLO (yet, see later)
- Subtraction means counter events with negative weight.

Subtleties

- Inherited from multileg merging.

Matching vs. Merging

Matrix element matching:

+Next-to-leading order accurate.

+Improved description of "first" Sudakov.

−Only possible one process at a time.

−Multiple jets always given by PS.

Matrix element merging:

+Process independent method.

+Valid for any number of additional partons.

−Only a leading-order method.

However, for data description, we need more:

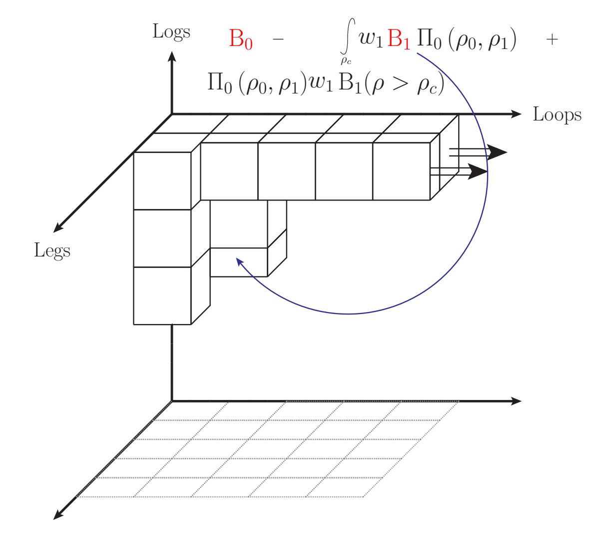
$p_{\perp Z}$ is both a 0- and a 1-jet observable.

$H_T, \Delta\phi_{\mathrm{Zj}}, n_{\mathrm{jets}}$ are "tricky" jet observables.

⇒ To describe these with small uncertainties, combine NLO calculations!

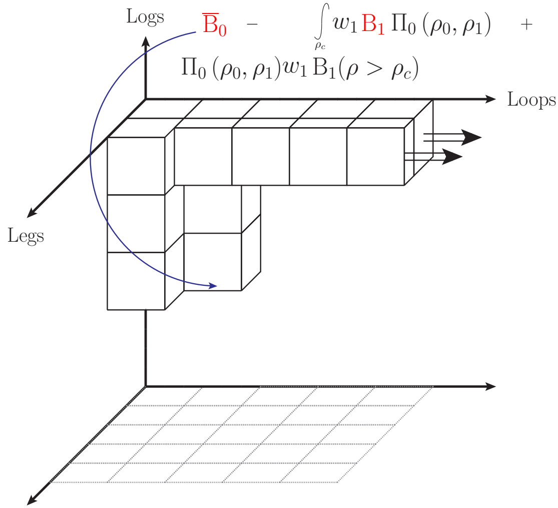
⇒ **NLO merging**

Intermediate step: MENLOPS



Leading-order merging includes the real corrections to +0-jet production, but has only approximate virtual corrections.

Intermediate step: MENLOPS



Replace the lowest multiplicity with the NLO result $\overline{\mathrm{B}}_0$.

$\Rightarrow$ +0-jet @ NLO, high multiplicities still given by tree-level MEs.

NLO merging: Strategy

Any leading-order method **X** only ever contains approximate virtual corrections.

We want to use the full NLO multijet results whenever possible, e.g. have

- NLO accuracy for inclusive $W + 0$ jet observables
- NLO accuracy for inclusive $W + 1$ jet observables
- NLO accuracy for inclusive $W + 2$ jet observables

. . . all at the same time. And the method should be process-independent.

NLO merging: Strategy

Any leading-order method **X** only ever contains approximate virtual corrections.

We want to use the full NLO multijet results whenever possible, e.g. have

NLO accuracy for inclusive W + 0 jet observables
NLO accuracy for inclusive W + 1 jet observables
NLO accuracy for inclusive W + 2 jet observables

. . . all at the same time. And the method should be process-independent.

To do NLO multi-jet merging for your preferred LO scheme **X**, do:

- ◇ Subtract approximate **X** $\mathcal{O}(\alpha_s)$-terms, add multiple NLO calculations.
- ◇ Make sure fixed-order calculations do not overlap by cutting, vetoing events, and/or vetoing emissions.
- ◇ Adjust higher orders to suit other needs.

$\Rightarrow$ **X**@NLO

The meaning of "NLO " will become clear below.

NLO merging schemes

FxFx[1]: Combine MC@NLO's by MLM jet matching@NLO
Pro: Probably fewest counter events.
Con: Restricted t_{MS} range. Accuracy unclear.

MEPS@NLO[2]: Combine MC@NLO's by METS@NLO
Pro: Improved Sudakovs.
Con: Restricted t_{MS} range.

UNLOPS[3]: Combine MC@NLO's or POWHEG's by UMEPS @NLO
Pro: Unitarity by approximate NNLO terms.
Con: Naively, many counter events.

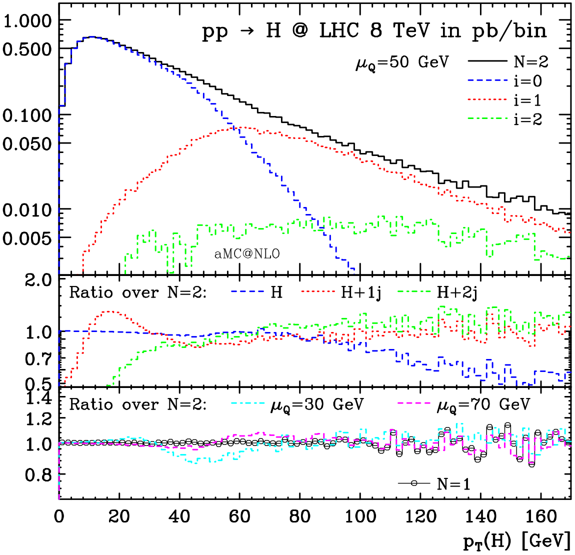
MiNLO[4]: Get zero-jet NLO by reweighted one-jet POWHEG after integration
Pro: Improved resummation, unitary.
Con: Process-dependent, only two NLO's can be combined.

[1] JHEP1212(2012)061 (Frixione, Frederix), [2] JHEP1304(2013)027 (Höche, Krauss, Schönherr, Siegert)

[3] JHEP1303(2013)166 (Lönnblad, SP), JHEP1308(2013)114 (Plätzer), [4] JHEP1305(2013)082 (Hamilton, Nason, Oleari, Zanderighi)

- Start from MC@NLO calculations.
- Reweight with CKKW-type α_s-running, Sudakov factors (or suppression functions)
- Remove double-counted $\mathcal{O}(\alpha_s^{+1})$-terms
- Match "matrix element jets" to "shower jets" (instead of matching "matrix element partons" to "shower jets")

FxFx plots

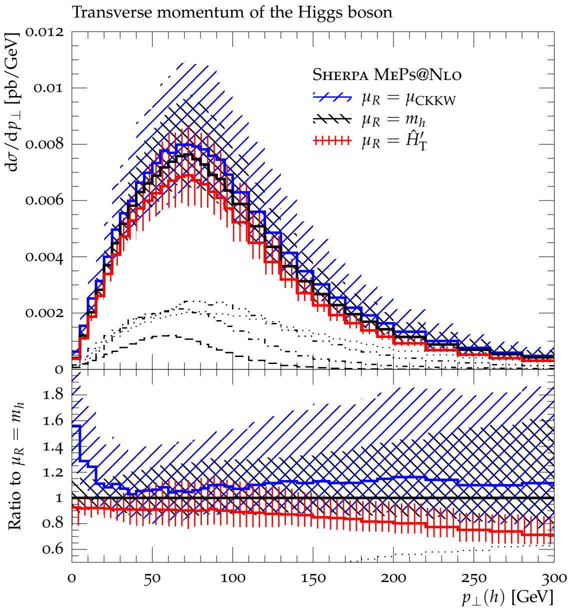


pp → H @ LHC 8 TeV in pb/bin
μ_Q=50 GeV
N=2
i=0
i=1
i=2
aMC@NLO
Ratio over N=2:
H
H+1j
H+2j
Ratio over N=2:
μ_Q=30 GeV
μ_Q=70 GeV
N=1
1.000
0.500
0.100
0.050
0.010
0.005
2.0
1.0
0.7
0.5
1.4
1.2
1.0
0.8
0
20
40
60
80
100
120
140
160
p_T(H) [GeV]

Merging MC@NLO calculations with MEPS@NLO

- Start from S-MC@NLO calculations.
- Disallow real-emission states above t_{MS}.
- Reweight with CKKW-type α_s/PDF-running, carefully preserving NLO accuracy by subtractions
- Reweight with $\mathcal{O}(\alpha_s^{+1})$-subtracted PS Sudakov factors (generated by "forgetful" shower)
- Reweight with $\mathcal{O}(\alpha_s^{+1})$-subtracted MC@NLO Sudakov factors
- When iterating, do not veto hard real emissions for highest multiplicity, and do not subtract the S-MC@NLO Sudakov

MEPS@NLO plots



UMEPS is a leading-order method, i.e. it contains only approximate virtual corrections.

We want to use the full NLO results whenever possible.

UMEPS is a leading-order method, i.e. it contains only approximate virtual corrections.

We want to use the full NLO results whenever possible.

Basic idea: Do NLO multi-jet merging for UMEPS:

- ◇ Subtract approximate UMEPS $\mathcal{O}(\alpha_\mathrm{s})$-terms, add back full NLO.
- ◇ To preserve the inclusive (NLO) cross section, add approximate NNLO.

⇒ UNLOPS[1].

For UNLOPS merging, we need exclusive NLO inputs:

$$\widetilde{\mathsf{B}}_n = \mathsf{B}_n + \mathsf{V}_n + \mathsf{I}_{n+1|n} + \int d\Phi_{\mathrm{rad}} \left(\mathsf{B}_{n+1|n}\Theta\left(\rho_{\mathrm{MS}} - t\left(S_{+n+1}, \rho\right)\right) - \mathsf{D}_{n+1|n}\right)$$

We can get these e.g. from POWHEG-BOX or MC@NLO output.

[1] JHEP1303(2013)166 (Leif Lönnblad, SP), also in JHEP1308(2013)114 (Simon Plätzer)

The UNLOPS method

Start with UMEPS:

$$\langle \mathcal{O} \rangle = \int d\phi_0 \Bigg\{ \mathcal{O}(S_{+0j}) \Bigg(\mathrm{B}_0 + \quad - \int \widehat{\mathrm{B}}_{1\to 0} \quad - \int \widehat{\mathrm{B}}_{2\to 0} \Bigg)$$

$$+ \int \mathcal{O}(S_{+1j}) \Bigg(\quad \widehat{\mathrm{B}}_1 \quad - \int \widehat{\mathrm{B}}_{2\to 1} \quad \Bigg) + \int\!\!\int \mathcal{O}(S_{+2j}) \widehat{\mathrm{B}}_2 \Bigg\}$$

The UNLOPS method

Remove all unwanted $\mathcal{O}(\alpha_{\mathrm{s}}^{n})$- and $\mathcal{O}(\alpha_{\mathrm{s}}^{n+1})$-terms:

$$\langle\mathcal{O}\rangle = \int d\phi_0 \Bigg\{ \mathcal{O}(S_{+0j}) \Bigg(- \left[\int \widehat{\mathrm{B}}_{1\to 0} \right]_{-1,2} - \int \widehat{\mathrm{B}}_{2\to 0} \Bigg) + \int \mathcal{O}(S_{+1j}) \Bigg(\left[\widehat{\mathrm{B}}_1 \right]_{-1,2} - \left[\int \widehat{\mathrm{B}}_{2\to 1} \right]_{-2} \Bigg) + \int\!\!\int \mathcal{O}(S_{+2j}) \widehat{\mathrm{B}}_2 \Bigg\}$$

The UNLOPS method

Add full NLO results:

$$\langle \mathcal{O} \rangle = \int d\phi_0 \Bigg\{ \mathcal{O}(S_{+0j}) \Bigg(\widetilde{\mathrm{B}}_0 - \left[\int \widehat{\mathrm{B}}_{1\to 0} \right]_{-1,2} - \int \widehat{\mathrm{B}}_{2\to 0} \Bigg)$$

$$+ \int \mathcal{O}(S_{+1j}) \left(\widetilde{\mathrm{B}}_1 + \left[\widehat{\mathrm{B}}_1 \right]_{-1,2} - \left[\int \widehat{\mathrm{B}}_{2\to 1} \right]_{-2} \right) + \int\!\!\int \mathcal{O}(S_{+2j}) \widehat{\mathrm{B}}_2 \Bigg\}$$

Unitarise:

$$\langle\mathcal{O}\rangle = \int d\phi_0 \Bigg\{ \mathcal{O}(S_{+0j}) \Bigg(\widetilde{\mathrm{B}}_0 - \int_s \widetilde{\mathrm{B}}_{1\to 0} + \int_s \mathrm{B}_{1\to 0} - \left[\int \widehat{\mathrm{B}}_{1\to 0} \right]_{-1,2} - \int_s \mathrm{B}^{\uparrow}_{2\to 0} - \int \widehat{\mathrm{B}}_{2\to 0} \Bigg)$$

$$+ \int \mathcal{O}(S_{+1j}) \left(\widetilde{\mathrm{B}}_1 + \left[\widehat{\mathrm{B}}_1 \right]_{-1,2} - \left[\int \widehat{\mathrm{B}}_{2\to 1} \right]_{-2} \right) + \int\!\!\int \mathcal{O}(S_{+2j}) \widehat{\mathrm{B}}_2 \Bigg\}$$

The UNLOPS method

UNLOPS merging of zero and one parton at NLO:

$$\langle \mathcal{O} \rangle = \int d\phi_0 \Bigg\{ \mathcal{O}(S_{+0j}) \Bigg(\widetilde{\mathrm{B}}_0 - \int_s \widetilde{\mathrm{B}}_{1\to 0} + \int_s \mathrm{B}_{1\to 0} - \Bigg[\int \widehat{\mathrm{B}}_{1\to 0} \Bigg]_{-1,2} - \int_s \mathrm{B}^{\uparrow}_{2\to 0} - \int \widehat{\mathrm{B}}_{2\to 0} \Bigg)$$

$$+ \int \mathcal{O}(S_{+1j}) \Bigg(\widetilde{\mathrm{B}}_1 + \Big[\widehat{\mathrm{B}}_1 \Big]_{-1,2} - \Bigg[\int \widehat{\mathrm{B}}_{2\to 1} \Bigg]_{-2} \Bigg) + \int\!\!\int \mathcal{O}(S_{+2j}) \widehat{\mathrm{B}}_2 \Bigg\}$$

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Iterate for the case of M different NLO calculations, and N tree-level calculations:

$$\langle \mathcal{O} \rangle = \sum_{m=0}^{M-1} \int d\phi_0 \int \cdots \int \mathcal{O}(S_{+mj}) \Bigg\{ \widetilde{\mathrm{B}}_m + \left[\widehat{\mathrm{B}}_m \right]_{-m,m+1} + \int_s \mathrm{B}_{m+1\to m}$$

$$- \sum_{i=m+1}^{M} \int_s \widetilde{\mathrm{B}}_{i\to m} - \sum_{i=m+1}^{M} \left[\int \widehat{\mathrm{B}}_{i\to m} \right]_{-i,i+1} - \sum_{i=m+1}^{M} \int_s \mathrm{B}^{\uparrow}_{i+1\to m} - \sum_{i=M+1}^{N} \int \widehat{\mathrm{B}}_{i\to m} \Bigg\}$$

$$+ \int d\phi_0 \int \cdots \int \mathcal{O}(S_{+Mj}) \Bigg\{ \widetilde{\mathrm{B}}_M + \left[\widehat{\mathrm{B}}_M \right]_{-M,M+1} - \left[\int \widehat{\mathrm{B}}_{M+1\to M} \right]_{-M} - \sum_{i=M+1}^{N} \int \widehat{\mathrm{B}}_{i+1\to M} \Bigg\}$$

$$+ \sum_{n=M+1}^{N} \int d\phi_0 \int \cdots \int \mathcal{O}(S_{+nj}) \Bigg\{ \widehat{\mathrm{B}}_n - \sum_{i=n+1}^{N} \int \widehat{\mathrm{B}}_{i\to n} \Bigg\}$$

The UNLOPS method

UNLOPS merging of zero and one parton at NLO:

$$\langle\mathcal{O}\rangle = \int d\phi_0 \Bigg\{ \mathcal{O}(S_{+0j}) \Bigg(\widetilde{\mathrm{B}}_0 - \int_s \widetilde{\mathrm{B}}_{1\to 0} + \int_s \mathrm{B}_{1\to 0} - \Bigg[\int \widehat{\mathrm{B}}_{1\to 0}\Bigg]_{-1,2} - \int_s \mathrm{B}^{\uparrow}_{2\to 0} - \int \widehat{\mathrm{B}}_{2\to 0} \Bigg)$$

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$$- \sum_{i=m+1}^{M} \int_s \widetilde{\mathrm{B}}_{i\to m} - \sum_{i=m+1}^{M} \Bigg[\int \widehat{\mathrm{B}}_{i\to m}\Bigg]_{-i,i+1} - \sum_{i=m+1}^{M} \int_s \mathrm{B}_{i+1\to m} - \sum_{i=M+1}^{N} \int \widehat{\mathrm{B}}_{i\to m} \Bigg\}$$

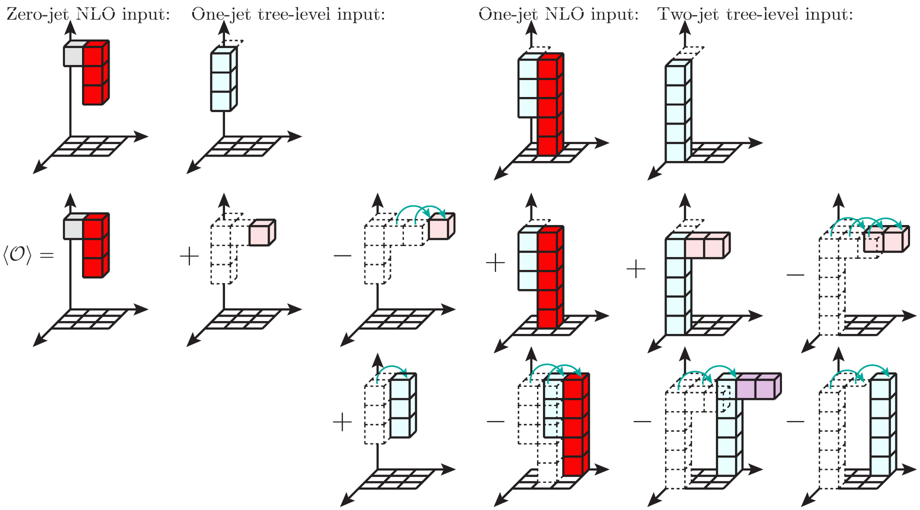
$$+ \int d\phi_0 \int \cdots \int \mathcal{O}(S_{+Mj}) \Bigg\{ \widetilde{\mathrm{B}}_M + \Big[\widehat{\mathrm{B}}_M\Big]_{-M,M+1} - \Bigg[\int \widehat{\mathrm{B}}_{M+1\to M}\Bigg]_{-M} - \sum_{i=M+1}^{N} \int \widehat{\mathrm{B}}_{i+1\to M} \Bigg\}$$

$$+ \sum_{n=M+1}^{N} \int d\phi_0 \int \cdots \int \mathcal{O}(S_{+nj}) \Bigg\{ \widehat{\mathrm{B}}_n - \sum_{i=n+1}^{N} \int \widehat{\mathrm{B}}_{i\to n} \Bigg\}$$

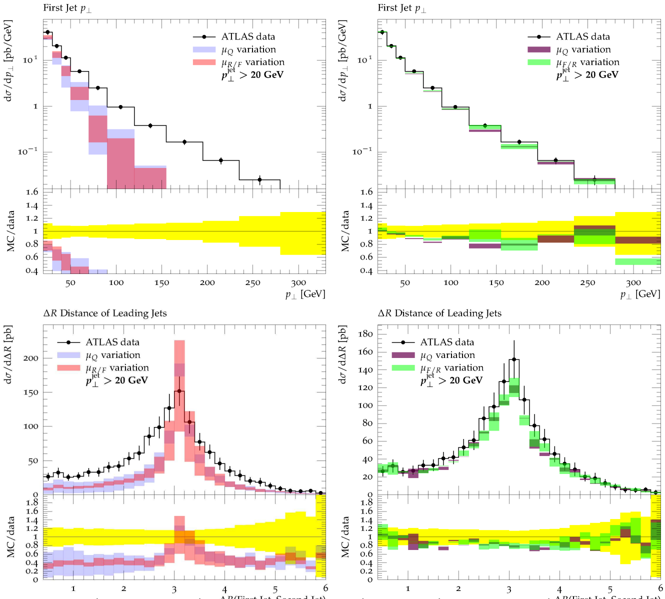
Inputs (B_n, $\widetilde{\mathrm{B}}_n$ or $\overline{\mathrm{B}}_n$) taken from external tools.

Merging done internally in PYTHIA 8.

Full-fledged example for UNLOPS merging



UNLOPS results (W+jets)



Inclusive sample containing (W + no resolved)@NLO, (W + one resolved)@NLO and (W + two resolved)@LO.

NLO merged results (H+jets)

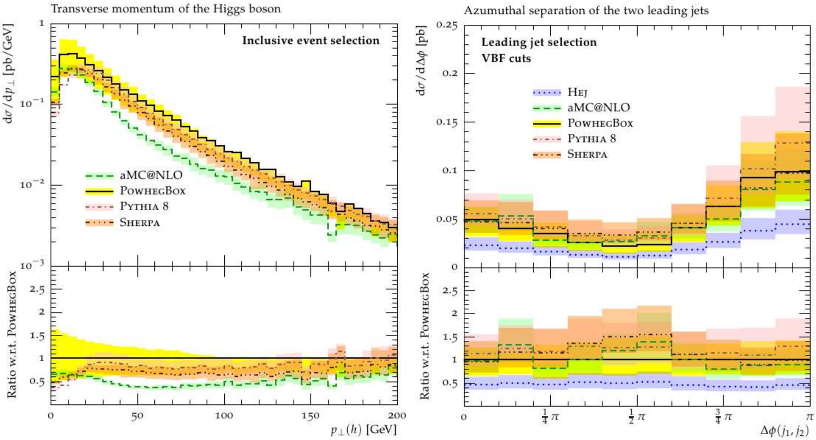


Figure: $p_{\perp,H}$ and $\Delta\phi_{12}$ for gg→H after merging (H+0)@NLO, (H+1)@NLO, (H+2)@NLO, (H+3)@LO, compared to other generators.

⇒ The generators come closer together if enough fixed-order matrix elements are employed. The uncertainties after cuts are still very large.

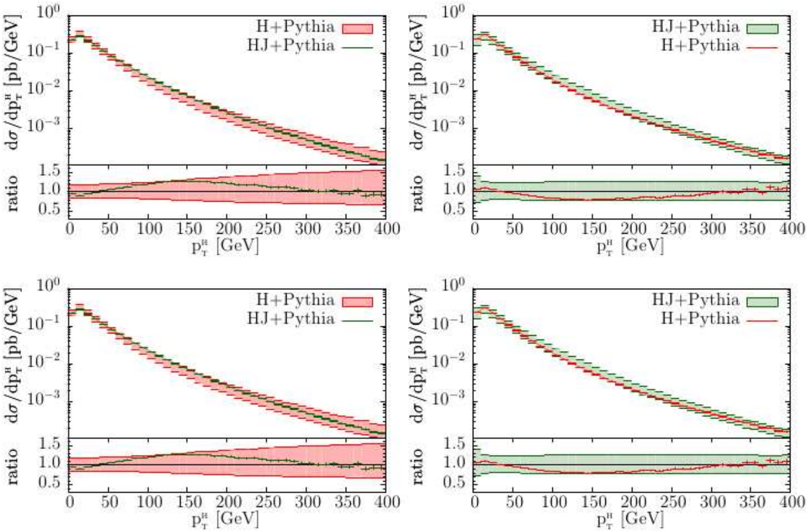
MiNLO is philosophically different from the other schemes. It emphasises the usage of accurate Sudakov factors.

- Begin with HJ-POWHEG
- Use CKKW-style running α_s, carefully keeping NLO accuracy.
- Reweight with analytic Sudakov factors.
- Choose these Sudakov factors so that $\int \text{HJ-POWHEG} \otimes \alpha_s\text{-weight} \otimes \text{Sudakovs} = \sigma^{\text{NLO}}_{\text{0-jet}} + \text{non-log } \mathcal{O}(\alpha_s^2)$ $\Longrightarrow$ Unitary scheme.

In the inclusive cross section, the improved analytical Sudakov factor cancels the logarithms in the 1-jet NLO calculation by exponentiating most terms of the calculation!

$\Longrightarrow$ Roughly, the analytical Sudakov roughly corresponds to a "1-jet@NLO-ME-corrected" no-emission probability - if that were possible.

MiNLO plots


H+Pythia
HJ+Pythia
HJ+Pythia
H+Pythia
H+Pythia
HJ+Pythia
HJ+Pythia
H+Pythia
dσ/dp_T^H [pb/GeV]
ratio
p_T^H [GeV]
10^0
10^-1
10^-2
10^-3
1.5
1.0
0.5
0
50
100
150
200
250
300
350
400

NLO merging methods have (mostly) been derived from LO schemes. Thus, we face many confusing acronyms.

Goal: Combine as many NLO calculations as are available into one inclusive calculation.

Pro

- Best Monte Carlo predictions for broad variety of processes at LHC.

Contra

- Not NNLO (yet, see later)
- All schemes contain counter events with negative weight.

Subtleties

- Inherited from the multileg merging scheme used to derive the method.
- All schemes differ in the treatment of yet higher orders.

Idea: Use a NLO merging scheme, assume that the 0-jet inclusive cross section after merging is $\sigma^{\text{NLO merged}} = \sigma_0^{\text{NLO}} = 1 + c_1\alpha_s$, and that we know $\sigma_0^{\text{NNLO}} = 1 + c_1\alpha_s + c_2\alpha_s^2$.

Then note

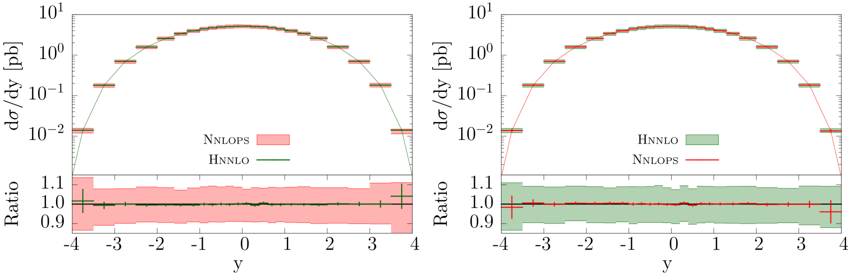
$$\frac{\sigma^{\text{NNLO}}}{\sigma^{\text{NLO merged}}}\sigma^{\text{NLO merged}} = (1 + c_2\alpha_s^2 + \mathcal{O}(\alpha_s^3))(1 + c_1\alpha_s) = \sigma^{\text{NNLO}} + \mathcal{O}(\alpha_s^3)$$

$\Rightarrow$ A unitary NLO merging scheme can easily be upgraded to NNLO!

MiNLO was upgraded (NNLO for Higgs) with a multiplicative K-factor.
$\Rightarrow$ POWHEG philosophy at NNLO

UNLOPS was upgraded (NNLO for Drell-Yan) by defining two classes of states - "0-jet exclusive" and "1-jet inclusive", and putting new NNLO only for "0-jet exclusive" states.
$\Rightarrow$ MC@NLO philosophy at NNLO

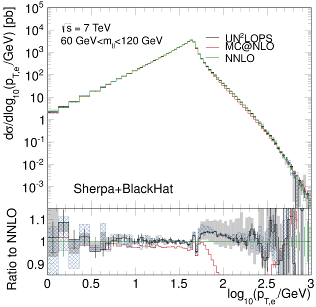
- NNLO with $\mu = m_H/2$, $\texttt{HJ-MiNLO}$ "core scale" m_H [NNLO from $\texttt{HNNLO}$, Catani,Grazzini]
- $(7_{\mathrm{Mi}} \times 3_{\mathrm{NN}})$ pts scale var. in $\texttt{NNLOPS}$, 7pts in NNLO

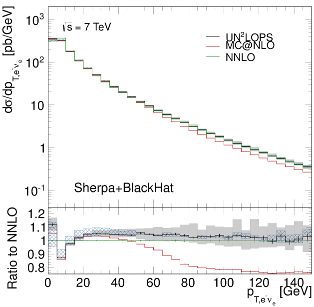


☞ Notice: band is 10%

[Until and including $\mathcal{O}(\alpha_S^4)$, PS effects don't affect y_H (first 2 emissions controlled properly at $\mathcal{O}(\alpha_S^4)$ by MiNLO+POWHEG)]

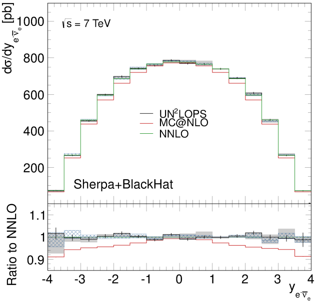
UN^{2}LOPS

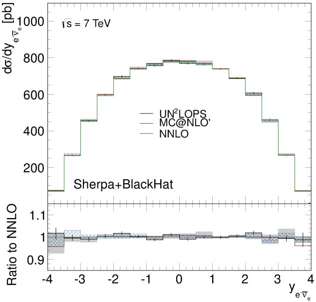




NLO calculated with NLO PDFs ↓

↓ NLO calculated with NNLO PDFs





Summary of MEPS lecture

- Parton showers can systematically improved with fixed-order calculations.
- Three major schools exist
  - Matrix element corrections: Oldest scheme, dating back to 80's. Available for simple processes in all parton showers. Iteratively used for e^+e^- in VINCIA (even at NLO).
  - Matrix element matching: "PS" used as extended subtraction for NLO calculations. Two schools: MC@NLO and POWHEG. Differences in exponentiation and in treatment of real corrections.
  - Matrix element merging: Emphasis on combining many multijet ME's. Make fixed-order calculations additive by making them exclusive through no-emission probabilities. Then minimise the impact of arbitrary slicing parameters. Three schools: MLM, CKKW(-L) and UMEPS. Differences in generation (approximation of) no-emission probabilities, and in the treatment of non-showerlike configurations. NLO merging: Combination of multiple NLO calculations. Take leading-order merging **X**, remove approximate $\mathcal{O}(\alpha_s)$ terms and add the full NLO. Inherits philosophy from LO merging scheme. NLO merging should be the workhorse for LHC Run II. NNLO matching: Brand new extension of NLO merging methods.

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MARIE CURIE ACTIONS

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Matrix element corrections
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UNLOPS = UMEPS@NLO
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MiNLO
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