Vector Boson and Direct Photon Production

Lecture 1

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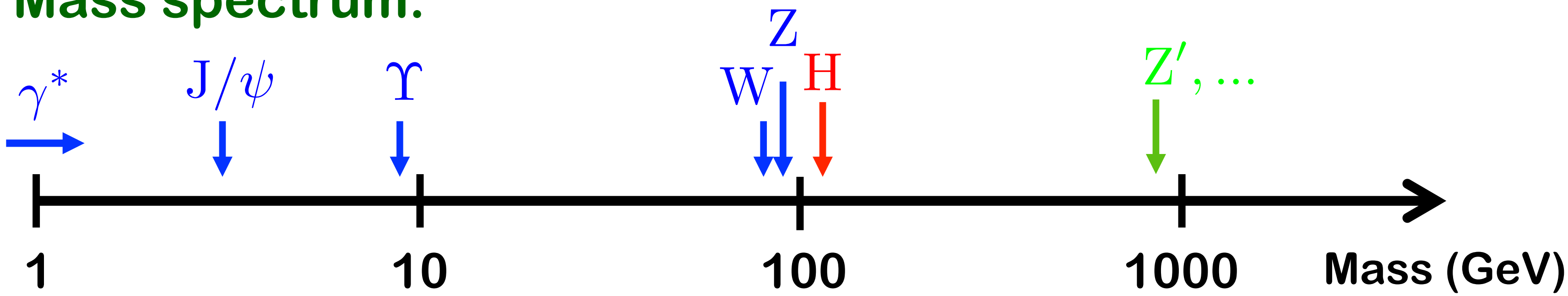
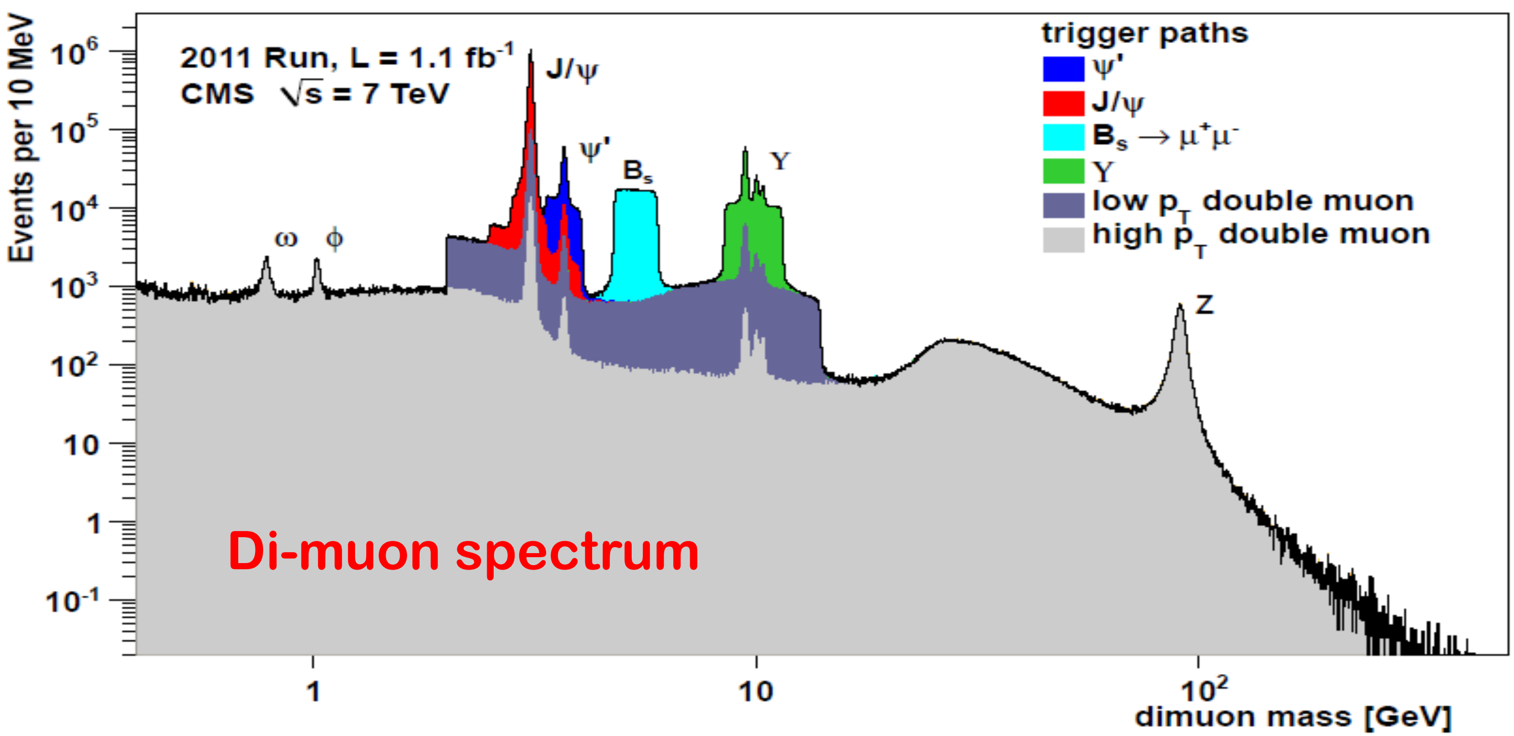
Vector Bosons

- ❑ **Mass spectrum:**



- ❑ **Real data:**



Outline of the two lectures

❑ **Lecture one:**

- ✧ Basics of vector bosons
- ✧ Drell-Yan like production process
- ✧ Cross section with a single hard scale – precision
- ✧ Cross section with two different scales – resummation

❑ **Lecture two:**

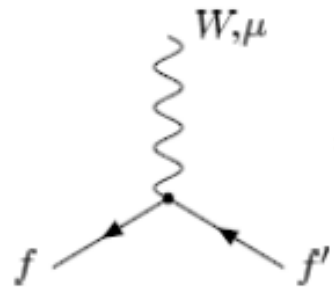
- ✧ Photon production at high pT – direct vs fragmentation
- ✧ Isolation cut – the need and its complication
- ✧ Photons from fixed target to collider energies
- ✧ Multi-boson associated production at collider energies

Basics of vector bosons

❑ Electro-weak gauge bosons (physical states):

$W^{\pm}$ boson:

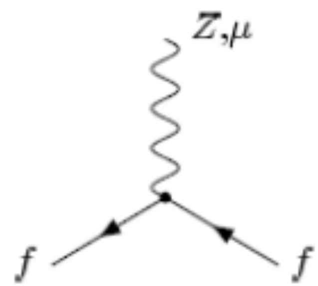
M_W = 80.4 GeV
g_2 = g_w – weak coupling
$V_{ff'}$ – CMK matrox for quarks
couples only to left-handed fermions



$$= \frac{-ig_2}{2\sqrt{2}}\gamma^\mu(1-\gamma_5)V_{ff'}$$

Z^0 boson:

M_Z = 91.2 GeV
$\cos\theta_W$ – weak mixing angle
couples to both left- and right-hand fermions



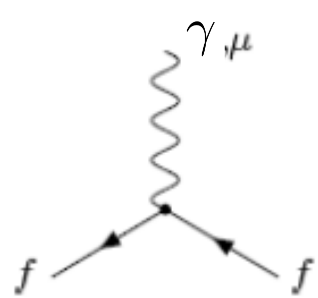
$$= \frac{-ig_2}{\cos\theta_W}\gamma^\mu(g_V^f + g_A^f\gamma_5)$$

$$g_A^f = -\frac{1}{2}T_3^f$$

$$g_V^f = \frac{1}{2}T_3^f - \sin\theta_W{}^2 Q_f$$

γ – photon:

M_γ = 0
e – electro-charge
Q_f – fraction in electro-charge

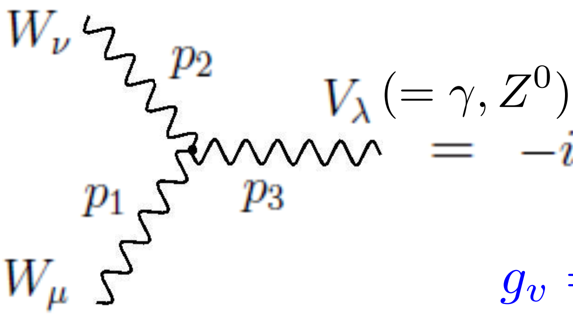


$$= -i\,e\,Q_f\gamma^\mu$$

Basics of vector bosons

❑ More interactions …

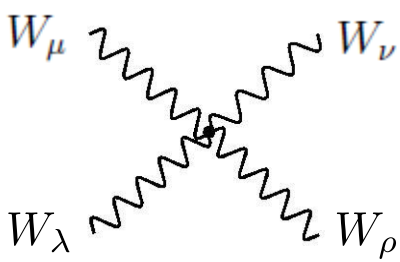
Triple gauge boson interactions:



$$= \quad -ig_V\left[(p_1-p_2)_\lambda g_{\mu\nu} + (p_2-p_3)_\mu g_{\nu\lambda} + (p_3-p_1)_\nu g_{\lambda\mu}\right]$$

$$g_v = g_2 \sin(\theta_W) \text{ for } \gamma, \text{ and } = g_2\cos(\theta_W) \text{ for } Z^0$$

Four point interactions:



$$= \quad ig_2^2(2g_{\mu\nu}g_{\lambda\rho} - g_{\mu\lambda}g_{\nu\rho} - g_{\mu\rho}g_{\lambda\nu})$$

More WWVV – type four point interactions …

❑ Heavy quarkonia: $\mathrm{J}/\psi, \Upsilon, \ldots$

Not covered in these two lectures …

Basics of vector bosons

- ❑ **Large production cross sections:**

  Compare to the rate of BSM signals

- ❑ **Theoretically,**

  Production – well understood

  Calibration – new calculations

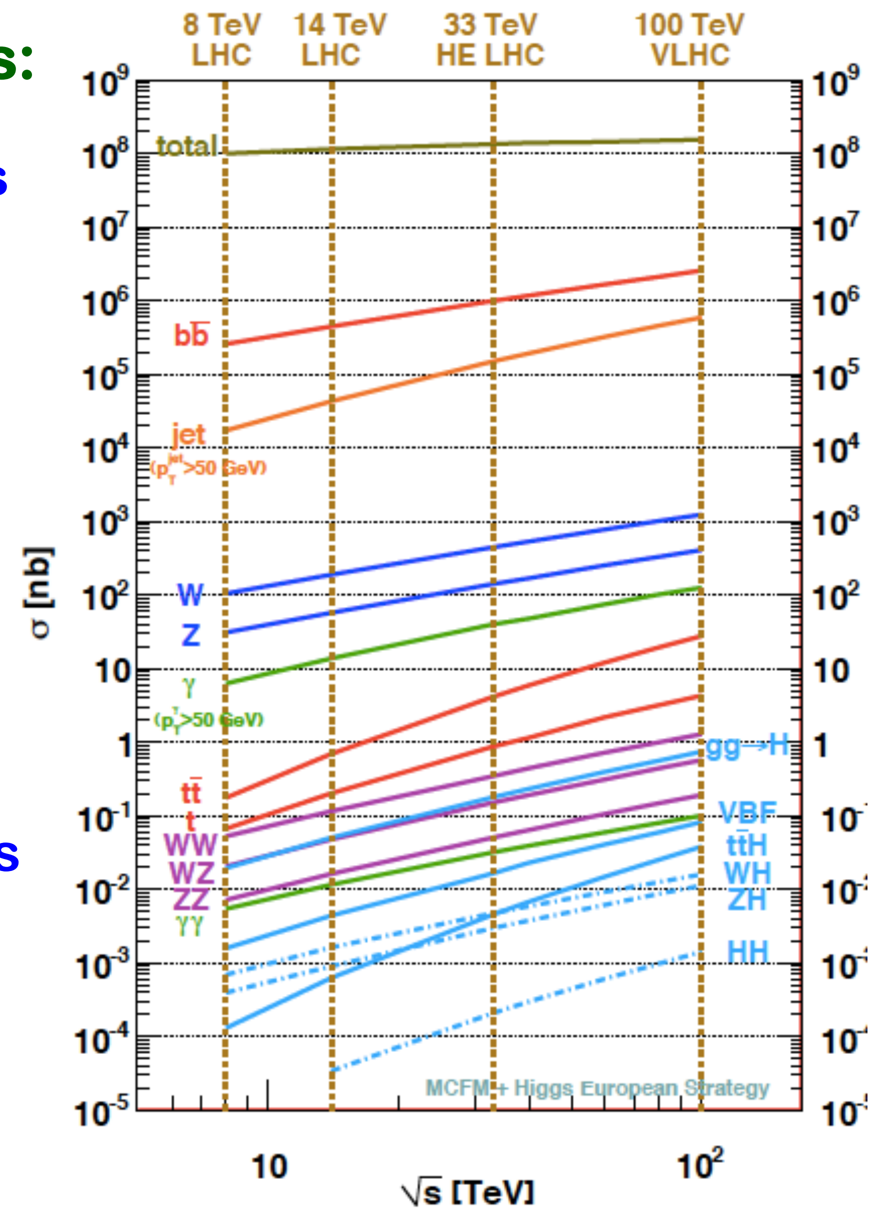
- ❑ **Experimentally,**

  Decay to a massive lepton pair

  – clean & well measured final states

  Leptons, missing energy (+ jets)

  – crucial backgrounds for new physics



Drell-Yan process

Christenson et al. 1970

❑ **Hadronic production of a massive lepton pair:**

First experiment:

$$p + U \rightarrow \mu^+\mu^-(Q) + X$$ @ BNL

["famous" Lederman experiment]

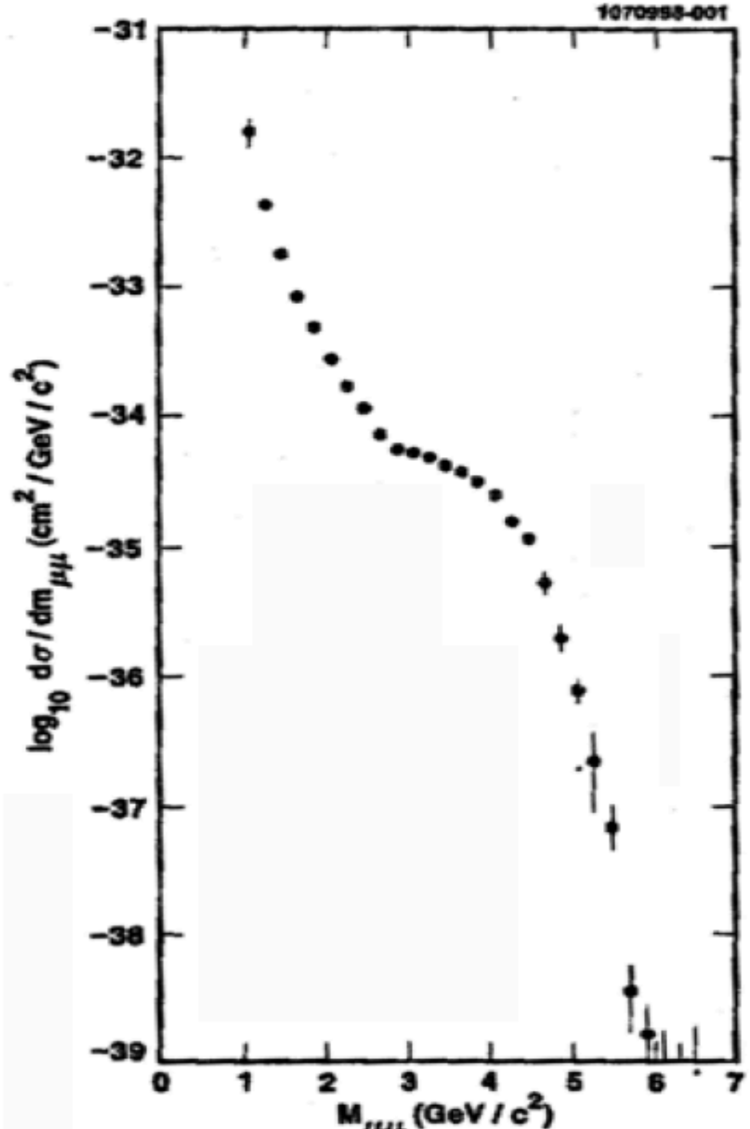
❑ **Two features:**

✧ A shoulder-like structure near Q = 3 GeV

[Discovery of J/Ψ in 1974]

✧ A rapid fall-off of cross section as Q increases

[Drell-Yan mechanism in 1970]



Drell-Yan process

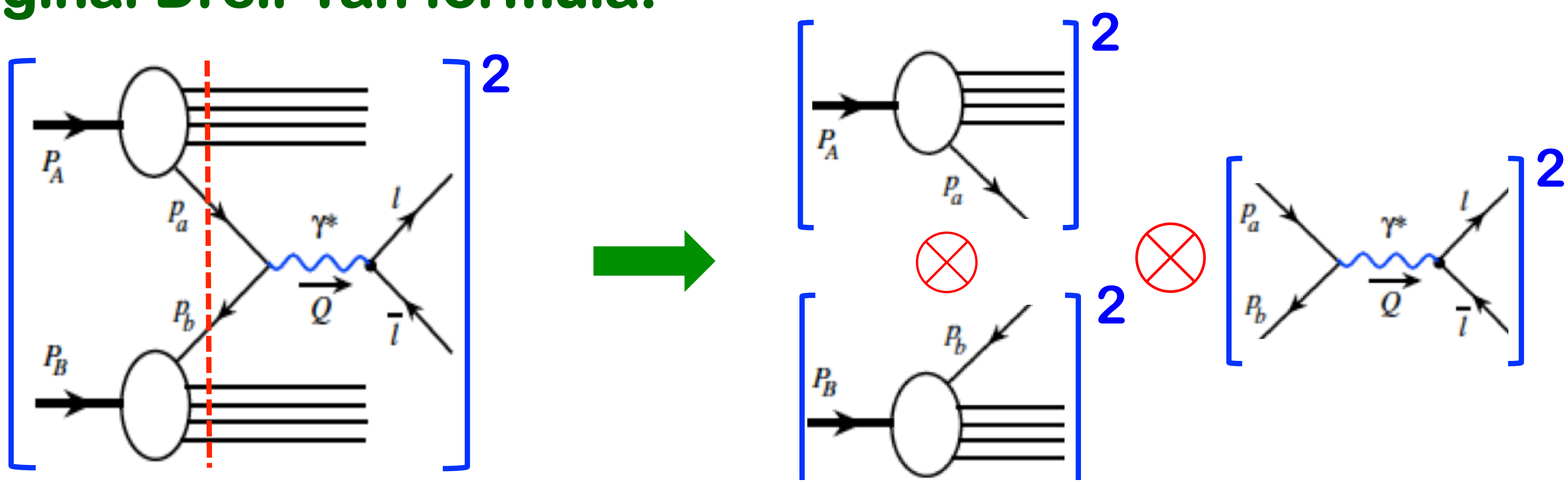
❑ Drell-Yan mechanism:

S.D. Drell and T.-M. Yan
Phys. Rev. Lett. 25, 316 (1970)

$$A(P_A) + B(P_B) \to \gamma^*(q)[\to l\bar{l}(q)] + X \quad \text{with} \quad q^2 \equiv Q^2 \gg \Lambda^2_{\rm QCD} \sim 1/\text{fm}^2$$

Lepton pair – from decay of a virtual photon, or in general,
a massive boson, e.g., W, Z, H^0, … (called Drell-Yan like processes)

❑ Original Drell-Yan formula:



$$\frac{d\sigma_{A+B\to l\bar{l}+X}}{dQ^2 dy} = \frac{4\pi\alpha_{em}^2}{3Q^4} \sum_{p,\bar{p}} x_A \phi_{p/A}(x_A)\, x_B \phi_{\bar{p}/B}(x_B)$$

No color yet!

Rapidity: $y = \frac{1}{2}\ln(x_A/x_B)$

$$x_A = \frac{Q}{\sqrt{S}} e^{y} \qquad x_B = \frac{Q}{\sqrt{S}} e^{-y}$$

Right shape – But – not normalization

Drell-Yan process

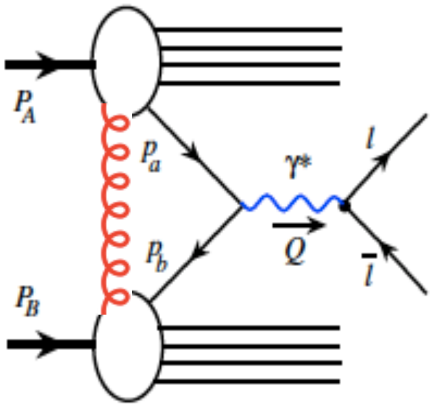
- **Significance:**
  - First example of a calculable hadron-hadron process in the context of the parton model

    Very nontrivial in QCD!

  - QCD factorization:
    Structure of the parton model calculation preserved in the presence of QCD corrections

  - Precision SM measurements – one of the early calculations of higher order QCD corrections

  - Important roles in searches for new physics



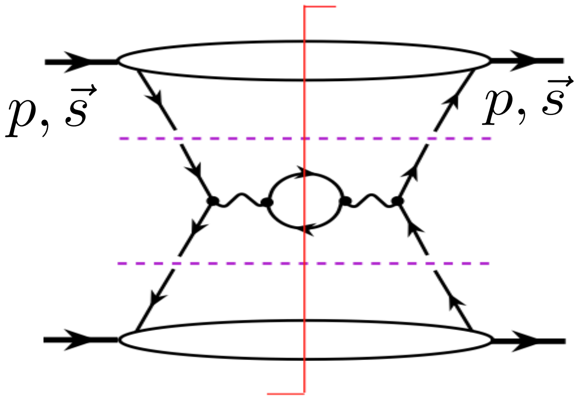
- **Prediction – Normalized angular distribution:**

$$\frac{dN}{d\Omega} = \left(\frac{d\sigma}{d^4q}\right)^{-1} \frac{d\sigma}{d^4q\,d\Omega} = \frac{3}{4\pi}\left(\frac{1}{\lambda+3}\right)\left[1 + \lambda\cos^2\theta + \mu\sin(2\theta)\cos\phi + \frac{\nu}{2}\sin^2\theta\cos(2\phi)\right]$$

  - Lam-Tung relation: $1 - \lambda - 2\nu = 0$ Transversely polarized photon!

Drell-Yan process in QCD

- **Spin decomposition – cut diagram notation:**



$\Leftarrow$ all γ structure: $\gamma^\alpha,\ \gamma^\alpha\gamma^5,\ \sigma^{\alpha\beta}(\text{or } \gamma^5\sigma^{\alpha\beta}),\ I,\ \gamma^5$

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- **Factorized cross section:**

$$\sigma(Q,\vec{s}) \pm \sigma(Q,-\vec{s}) \propto \langle p,\vec{s}|\mathcal{O}(\psi,A^\mu)|p,\vec{s}\rangle \pm \langle p,-\vec{s}|\mathcal{O}(\psi,A^\mu)|p,-\vec{s}\rangle$$

- **Parity-Time reversal invariance:**

$$\langle p,-\vec{s}|\mathcal{O}(\psi,A^\mu)|p,-\vec{s}\rangle = \langle p,\vec{s}|\mathcal{P}\mathcal{T}\mathcal{O}^\dagger(\psi,A^\mu)\mathcal{T}^{-1}\mathcal{P}^{-1}|p,\vec{s}\rangle$$

- **Good operators:**

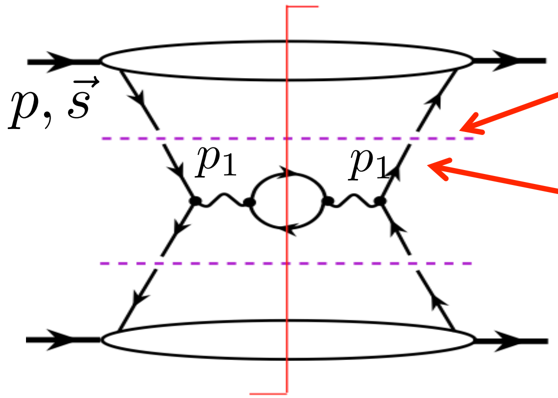
$$\langle p,\vec{s}|\mathcal{P}\mathcal{T}\mathcal{O}^\dagger(\psi,A^\mu)\mathcal{T}^{-1}\mathcal{P}^{-1}|p,\vec{s}\rangle = \pm\langle p,\vec{s}|\mathcal{O}(\psi,A^\mu)|p,\vec{s}\rangle$$

"+" for spin-averaged cross section $\longrightarrow$ **PDFs:**

$$\langle p,\vec{s}|\overline{\psi}(0)\gamma^+\psi(y^-)|p,\vec{s}\rangle\,, \qquad \langle p,\vec{s}|F^{+i}(0)F^{+j}|p,\vec{s}\rangle(-g_{ij})$$

Drell-Yan process in QCD

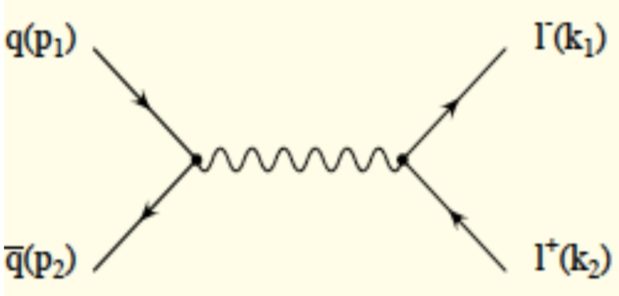
❑ Spin-averaged cross section – Lowest order:



$$\frac{1}{2p^+}\gamma^+\,\delta(x - p_1^+/p^+)dx$$

$$\frac{1}{2}\gamma\cdot p = \frac{1}{2}\sum_s u_s(p)\overline{u}_s(p)$$

$$\hat{s} = (p_1+p_2)^2 = Q^2$$



❑ Lowest order partonic cross section:

$$\overline{\Sigma}\,|M|^2 = \frac{e_q^2 e^4}{\hat{s}^2}\left[\frac{1}{2}\right]\left[\frac{1}{2}\right] 3\left\{\frac{1}{3}\right\}\left\{\frac{1}{3}\right\}\mathrm{Tr}[\not{p}_1\gamma^\nu\not{p}_2\gamma^\mu]\mathrm{Tr}[\not{k}_1\gamma_\nu\not{k}_2\gamma_\mu] = \left\{\frac{1}{3}\right\} e_q^2 e^4(1+\cos^2\theta)$$

$$PS^{(2)} = \frac{d^2k_1}{(2\pi)^3 2E_1}\frac{d^2k_2}{(2\pi)^3 2E_2}(2\pi)^4\delta^4(p_1+p_2-k_1-k_2) = \frac{1}{16\pi}d\cos(\theta)$$

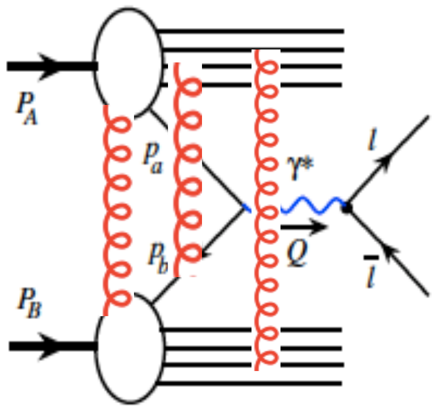
$$\sigma(q\bar{q}\to l^+l^-) = \left\{\frac{1}{3}\right\}\frac{4\pi\alpha^2}{3\,\hat{s}}e_q^2 \equiv \sigma_0$$

❑ Drell-Yan cross section:

$$\frac{d\sigma}{dQ^2dy} = \Sigma_q\int dx_A\,dx_B\,\phi_{q/A}(x_A)\phi_{\bar{q}/B}(x_B)\left[\left\{\frac{1}{3}\right\}\frac{4\pi\alpha^2}{3\,\hat{s}}e_q^2\right]\delta(Q^2-\hat{s})\,\delta(y-\frac{1}{2}\ln(\frac{x_A}{x_B}))$$

Drell-Yan process in QCD

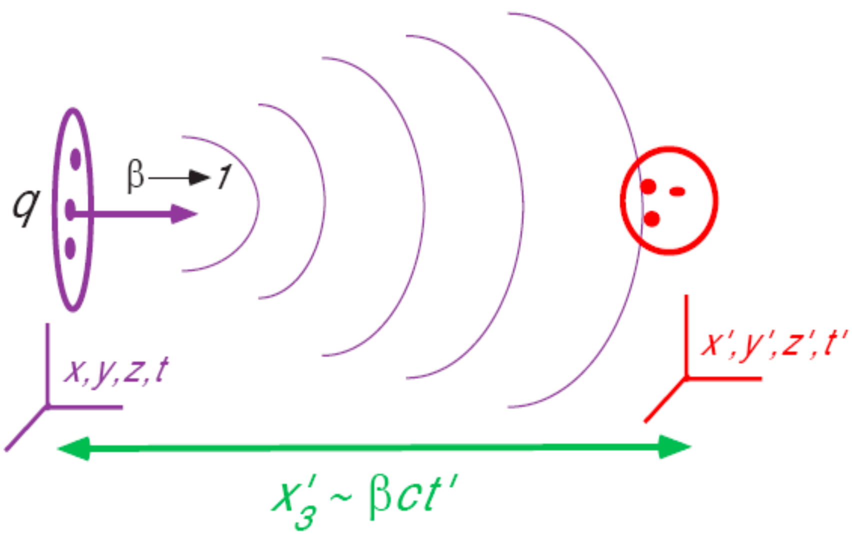
❑ Beyond the lowest order:



- ✧ Soft-gluon interaction takes place all the time
- ✧ Long-range gluon interaction before the hard collision

Break the Universality of PDFs
Loss the predictive power

❑ Factorization – power suppression of soft gluon interaction:



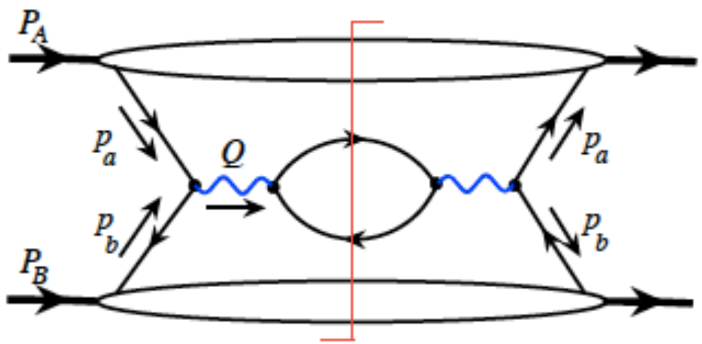
x-Frame	x'-Frame
$A^-(x) = \frac{e}{\lvert\vec{x}\rvert}$	$A'^-(x') = \frac{e\gamma(1+\beta)}{(x_T^2+\gamma^2\Delta^2)^{1/2}}$
	$\Longrightarrow 1$ "not contracted!"
$E_3(x) = \frac{e}{\lvert\vec{x}\rvert^2}$	$E_3(x') = \frac{-e\gamma\Delta}{(x_T^2+\gamma^2\Delta^2)^{3/2}}$
	$\Longrightarrow \frac{1}{\gamma^2}$ "strongly contracted!"

Drell-Yan process in QCD

Collins, Soper, Sterman, 1988

- **Factorization – approximation:**
  - ✧ Suppression of quantum interference between short-distance (1/Q) and long-distance (fm ~ 1/Λ_{QCD}) physics

  ➡ Need "long-lived" active parton states linking the two



$$\int d^4p_a \frac{1}{p_a^2 + i\varepsilon} \frac{1}{p_a^2 - i\varepsilon} \to \infty$$

Perturbatively pinched at $p_a^2 = 0$

➡ Active parton is effectively on-shell for the hard collision

  - ✧ Maintain the universality of PDFs:

    Long-range soft gluon interaction has to be power suppressed

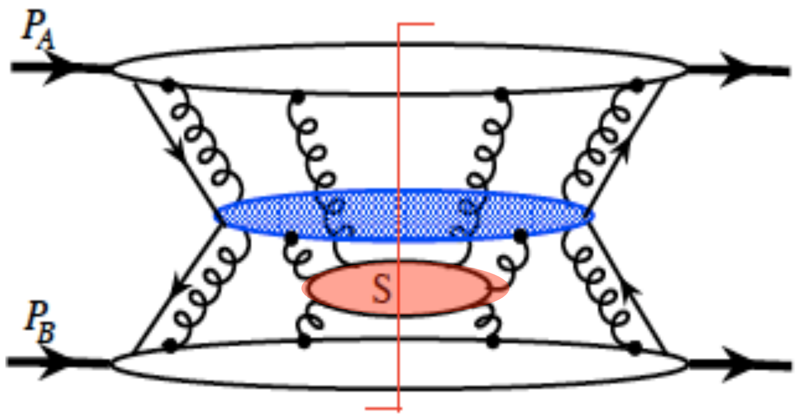
  - ✧ Infrared safe of partonic parts:

    Cancelation of IR behavior
    Absorb all CO divergences into PDFs

$$\begin{aligned}
\text{on-shell:} \quad & p_a^2,\ p_b^2 \ll Q^2; \\
\text{collinear:} \quad & p_{aT}^2,\ p_{bT}^2 \ll Q^2; \\
\text{higher-power:} \quad & p_a^- \ll q^-; \quad \text{and} \\
& p_b^+ \ll q^+
\end{aligned}$$

Drell-Yan process in QCD

❑ Leading singular integration regions (pinch surface):



Hard: all lines off-shell by Q

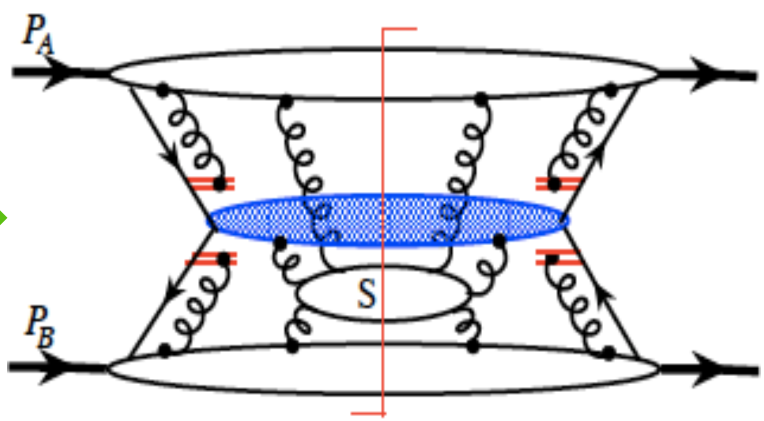
Collinear:
✧ lines collinear to A and B
✧ One "physical parton" per hadron

Soft: all components are soft

❑ Collinear gluons:

✧ Collinear gluons have the polarization vector: $\epsilon^\mu \sim k^\mu$

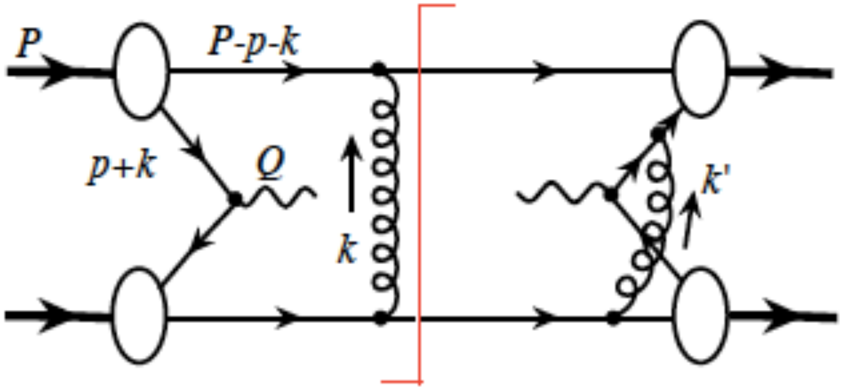
✧ The sum of the effect can be represented by the eikonal lines,



which are needed to make the PDFs gauge invariant!

Drell-Yan process in QCD

❑ Trouble with soft gluons:



$$(xp+k)^2 + i\epsilon \propto k^- + i\epsilon$$
$$((1-x)p-k)^2 + i\epsilon \propto k^- - i\epsilon$$

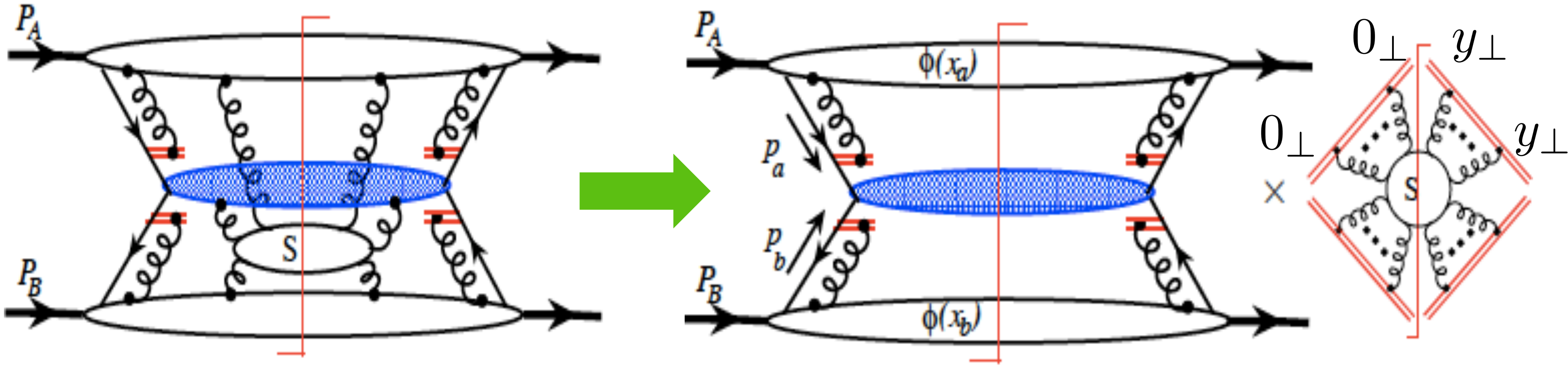
✧ **Soft gluon exchanged between a spectator quark of hadron B and the active quark of hadron A could rotate the quark's color and keep it from annihilating with the antiquark of hadron B**

✧ **The soft gluon approximations (with the eikonal lines) need $k^\pm$ not too small. But, $k^\pm$ could be trapped in "too small" region due to the pinch from spectator interaction:** $k^\pm \sim M^2/Q \ll k_\perp \sim M$

Need to show that soft-gluon interactions are power suppressed

Drell-Yan process in QCD

❑ **Most difficult part of factorization:**



✧ Sum over all final states to remove all poles in one-half plane

– no more pinch poles

✧ Deform the $k^\pm$ integration out of the trapped soft region

✧ Eikonal approximation ⟶ soft gluons to eikonal lines

– gauge links

✧ Collinear factorization: Unitarity ⟶ soft factor = 1

All identified leading integration regions are factorizable!

Factorized Drell-Yan cross section

- **TMD factorization ($q_\perp \ll Q$):**

$$\frac{d\sigma_{AB}}{d^4q} = \sigma_0 \int d^2k_{a\perp} d^2k_{b\perp} d^2k_{s\perp} \delta^2(q_\perp - k_{a\perp} - k_{b\perp} - k_{s\perp}) \mathcal{F}_{a/A}(x_A, k_{a\perp}) \mathcal{F}_{b/B}(x_B, k_{b\perp}) \mathcal{S}(k_{s\perp})$$

$$+\mathcal{O}(q_\perp/Q) \qquad x_A = \frac{Q}{\sqrt{s}} e^{y} \qquad x_B = \frac{Q}{\sqrt{s}} e^{-y}$$

The soft factor, $\mathcal{S}$, is universal, could be absorbed into the definition of TMD parton distribution

- **Collinear factorization ($q_\perp \sim Q$):**

$$\frac{d\sigma_{AB}}{d^4q} = \int dx_a\, f_{a/A}(x_a, \mu) \int dx_b\, f_{b/B}(x_b, \mu)\, \frac{d\hat{\sigma}_{ab}}{d^4q}(x_a, x_b, \alpha_s(\mu), \mu) + \mathcal{O}(1/Q)$$

- **Spin dependence:**

The factorization arguments are independent of the spin states of the colliding hadrons

→ same formula with polarized PDFs for γ^*,W/Z, H^0...

Cross section with a single hard scale

❑ **Partonic hard parts:**

$$\hat{\sigma}(\alpha_s, \mu_F, \mu_R) = [\alpha_s(\mu_R)]^{n_\alpha} \left[\hat{\sigma}^{(0)} + \frac{\alpha_s}{2\pi} \hat{\sigma}^{(1)}(\mu_F, \mu_R) + \left(\frac{\alpha_s}{2\pi}\right)^2 \hat{\sigma}^{(2)}(\mu_F, \mu_R) + \cdots \right]$$

LO NLO NNLO

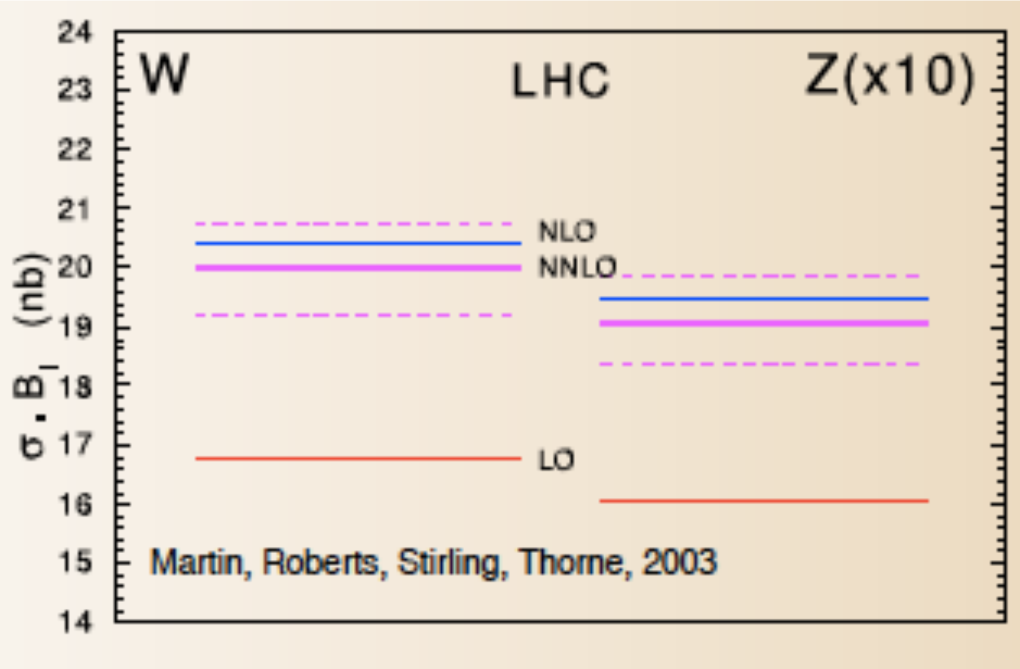
❑ **NNLO total x-section** $\sigma(AB \to W, Z)$:

(Hamberg, van Neerven, Matsuura; Harlander, Kilgore 1991)

✧ **Scale dependence:**

a few percent

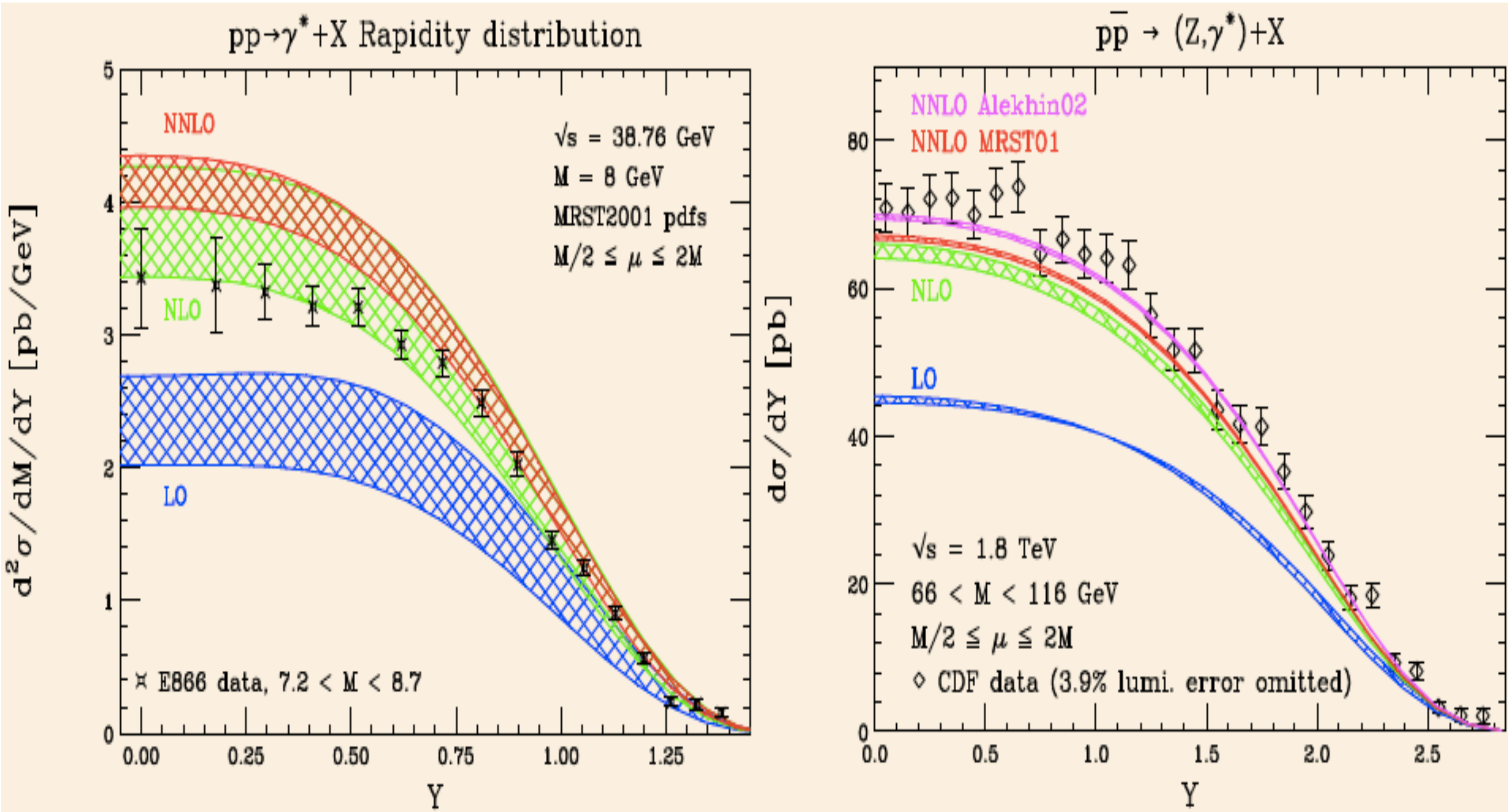
✧ **NNLO K-factor is about 0.98 for LHC data, 1.04 for Tevatron data**



Cross section with a single hard scale

❑ **NNLO differential x-section:**

Anastasiou, Dixon, Melnikov, Petriello, 2003-05



Cross section with a single hard scale

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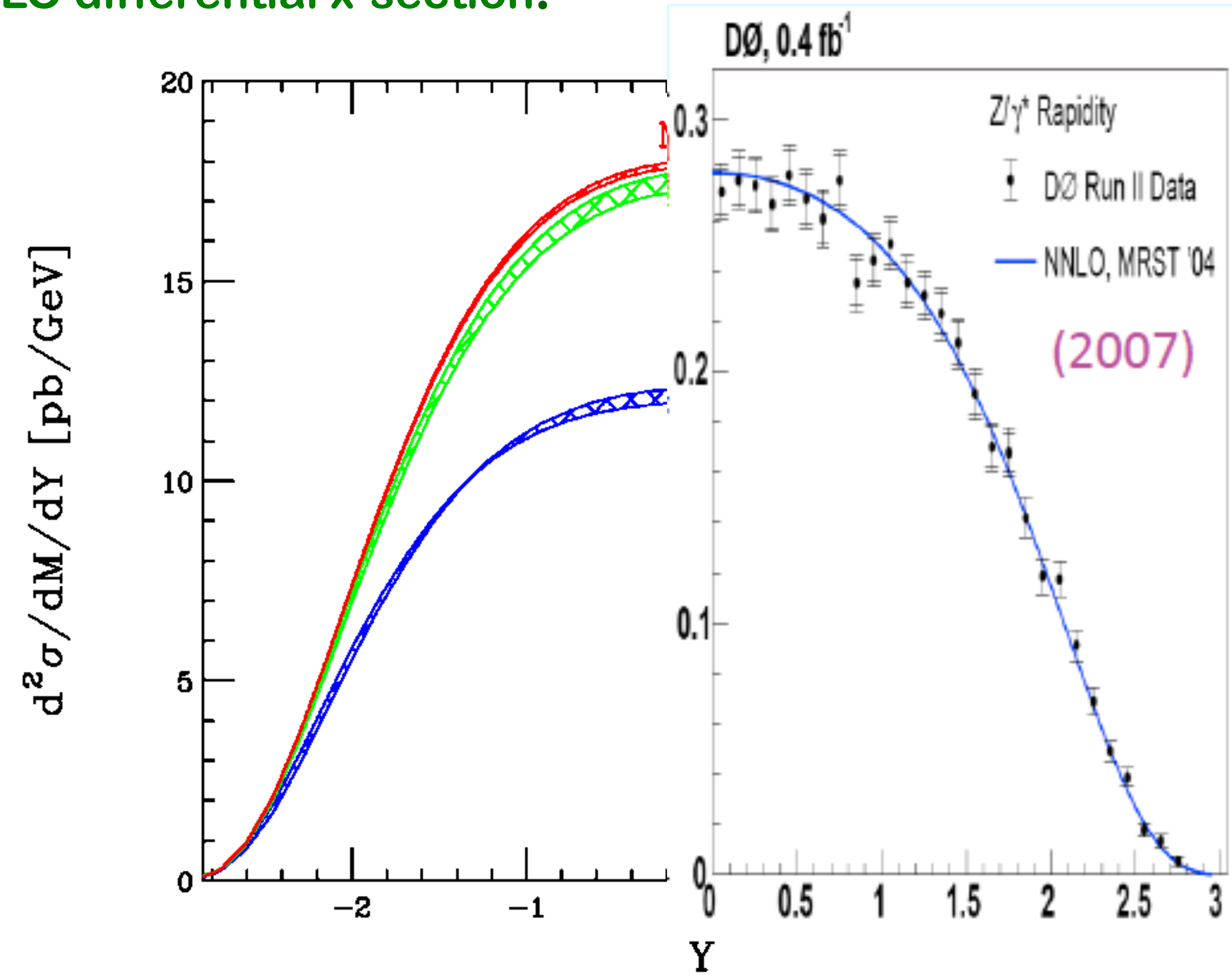
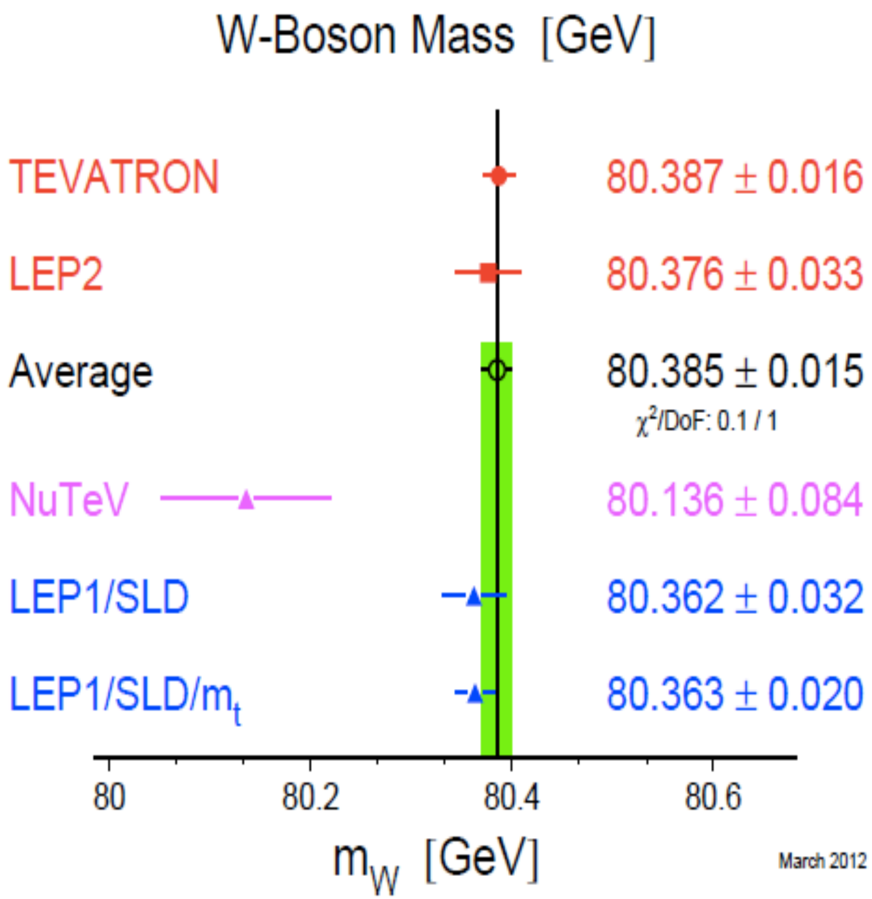
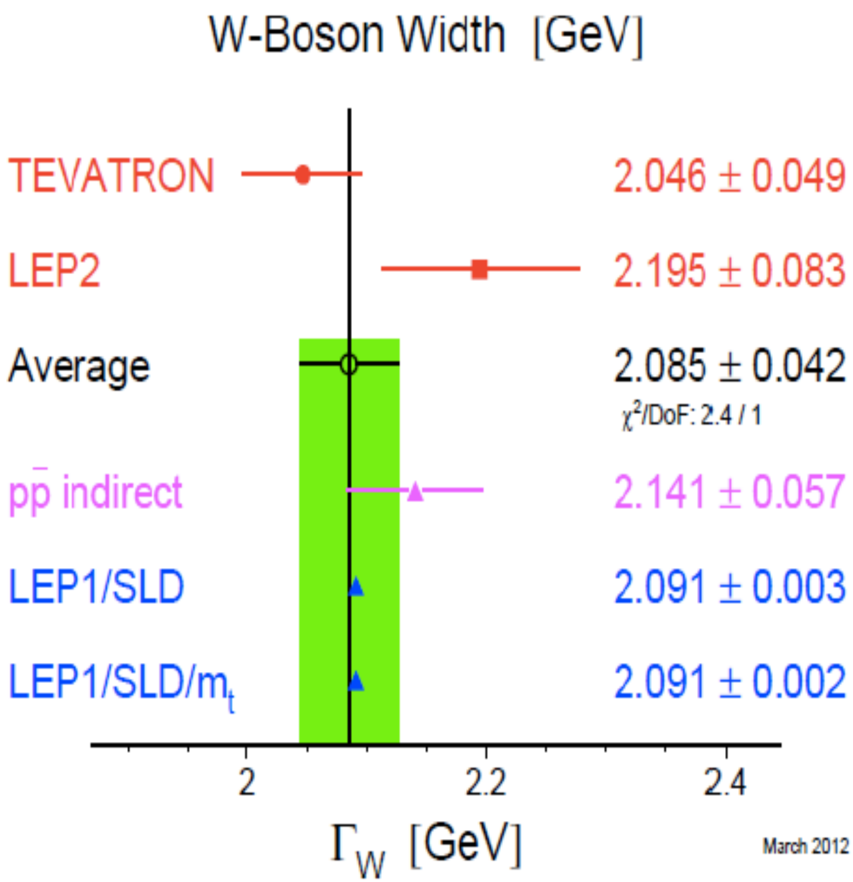


Cross section with a single hard scale

- **W mass & width:**

Fernando Febres Cordero, CTEQ SS2012

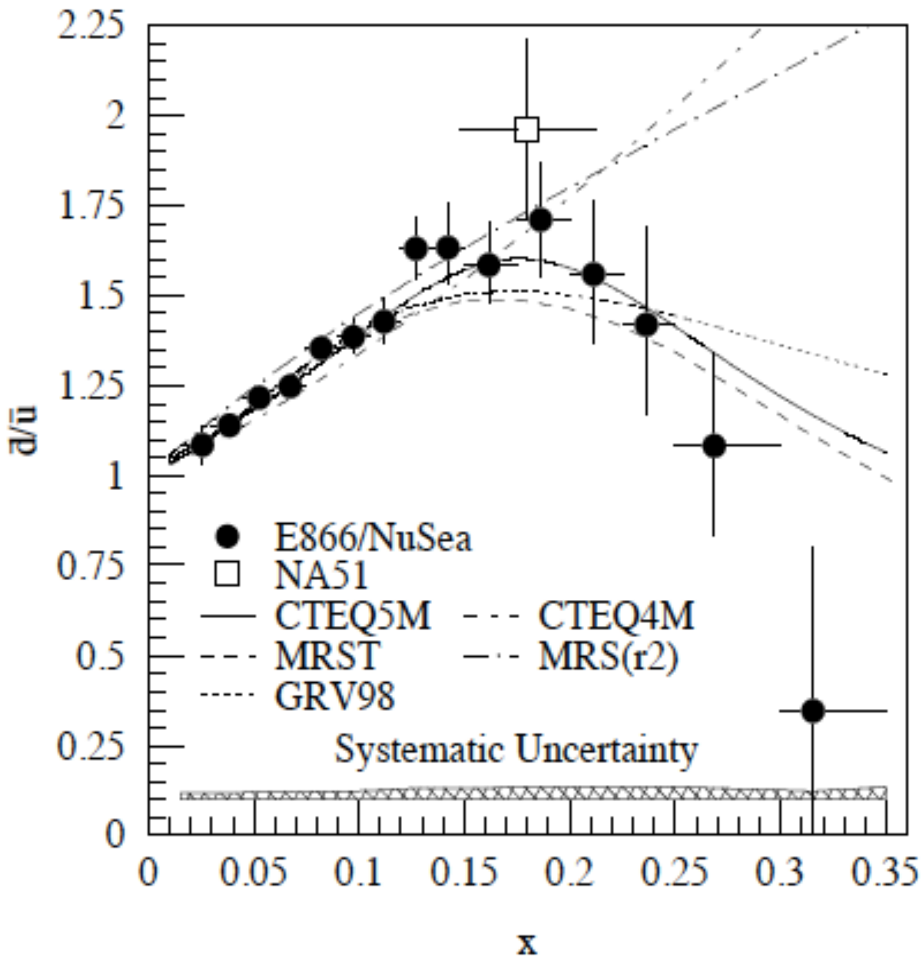


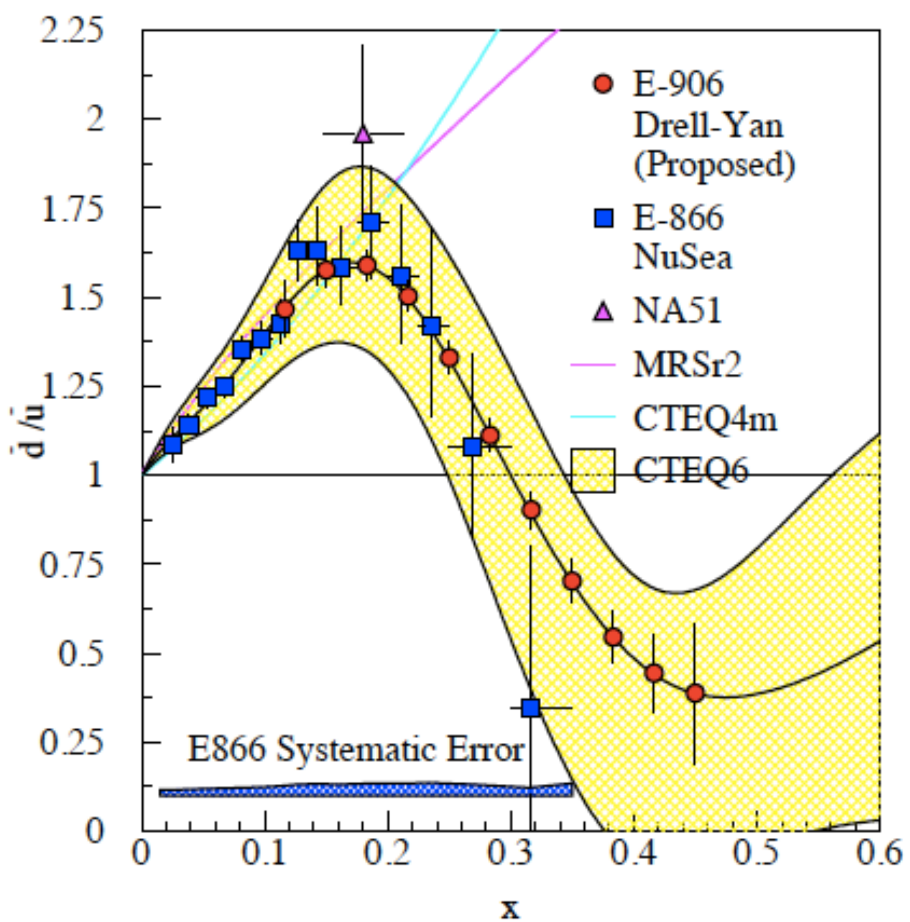


Cross section with a single hard scale

❑ **Flavor asymmetry of the sea:**

$$\sigma_{DY}(p+d)/2\sigma_{DY}(p+p) \simeq \left[1+\bar{d}(x)/\bar{u}(x)\right]/2$$

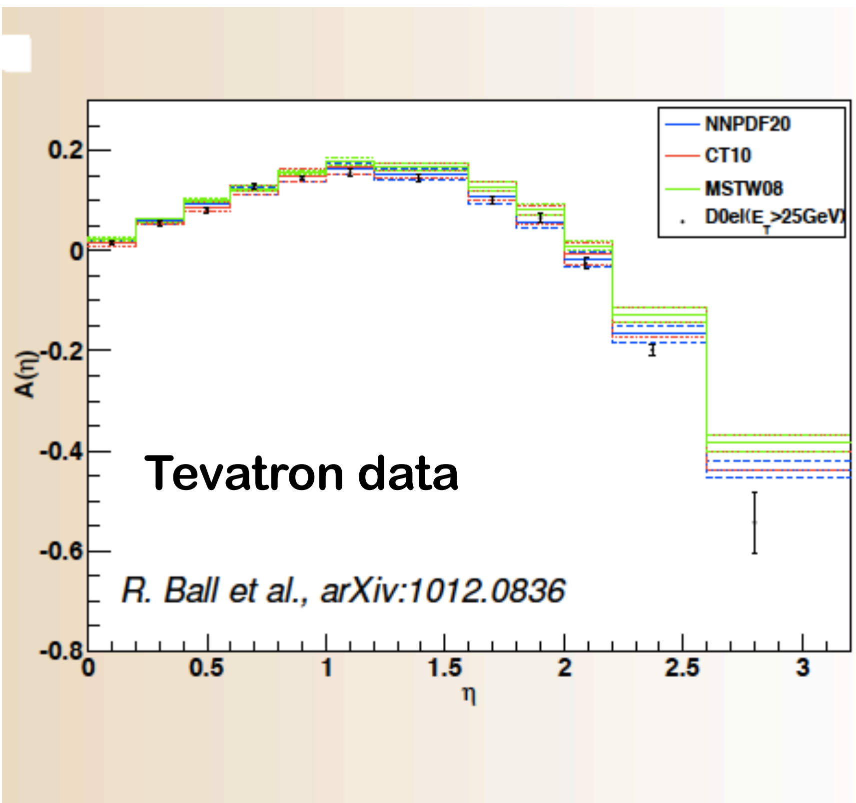


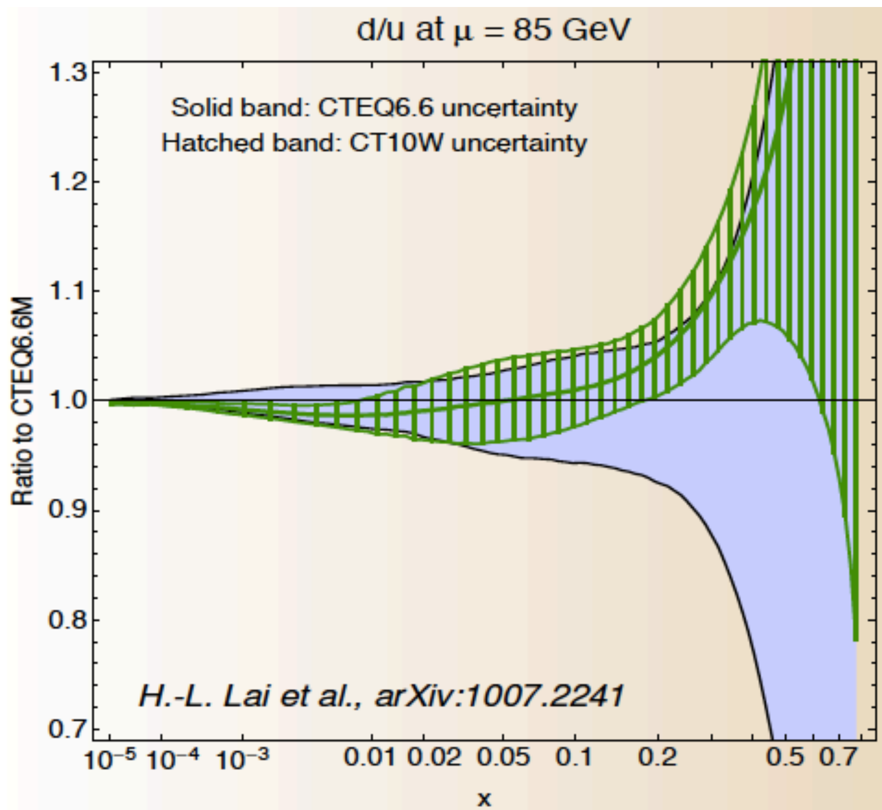


Cross section with a single hard scale

- **Charged lepton asymmetry:** $y \to y_{\max}$

$$A_{ch}(y_e) = \frac{d\sigma^{W^+}/dy_e - d\sigma^{W^-}/dy_e}{d\sigma^{W^+}/dy_e + d\sigma^{W^-}/dy_e} \longrightarrow \frac{d(x_B, M_W)/u(x_B, M_W) - d(x_A, M_W)/u(x_A, M_W)}{d(x_B, M_W)/u(x_B, M_W) + d(x_A, M_W)/u(x_A, M_W)}$$



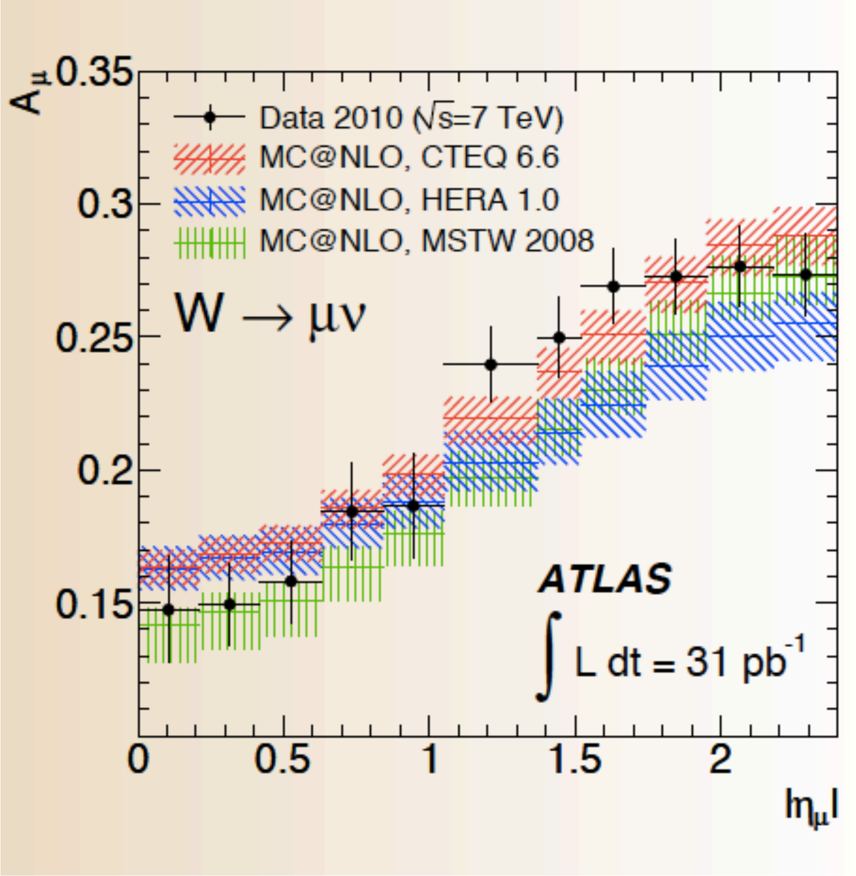


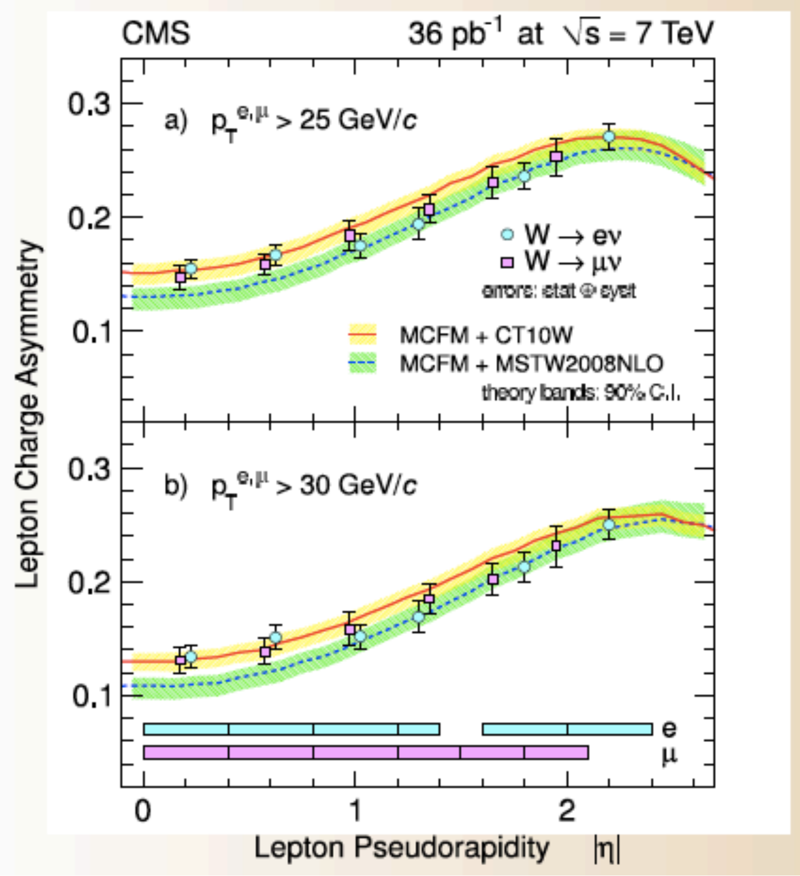
The A_{ch} data distinguish between the PDF models, reduce the PDF uncertainty

Cross section with a single hard scale

❑ **Charged lepton asymmetry:** $y \to y_{\max}$

$$A_{ch}(y_e) = \frac{d\sigma^{W^+}/dy_e - d\sigma^{W^-}/dy_e}{d\sigma^{W^+}/dy_e + d\sigma^{W^-}/dy_e} \longrightarrow \frac{d(x_B, M_W)/u(x_B, M_W) - d(x_A, M_W)/u(x_A, M_W)}{d(x_B, M_W)/u(x_B, M_W) + d(x_A, M_W)/u(x_A, M_W)}$$





Sensitive both to d/u at x > 0.1 and u/d at x ~ 0.01

Cross section with two hard scales

$$Q_1^2 \gg Q_2^2 \gg \Lambda_{\rm QCD}^2, \quad Q_1^2 \gg Q_2^2 \gtrsim \Lambda_{\rm QCD}^2$$

- **Large perturbative logarithms:**

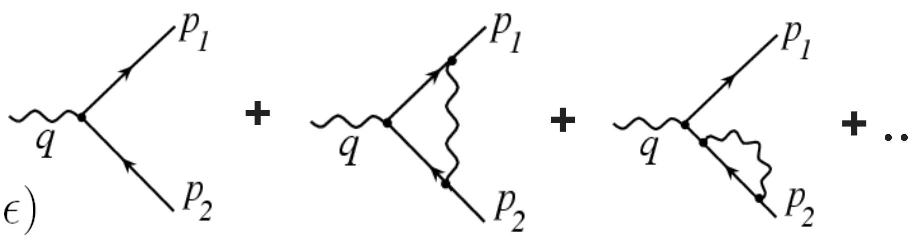
  $\alpha_s(\mu^2 = Q_1^2)$ **is small, But,** $\alpha_s(Q_1^2)\ln(Q_1^2/Q_2^2)$ **is not necessary small!**

- **Massless theory:**

  Two powers of large logs for each order in perturbation theory

  $$\alpha_s(Q_1^2)\ln^2(Q_1^2/Q_2^2)$$ **due to overlap of IR and CO regions**

- **Example – EM form factor:**



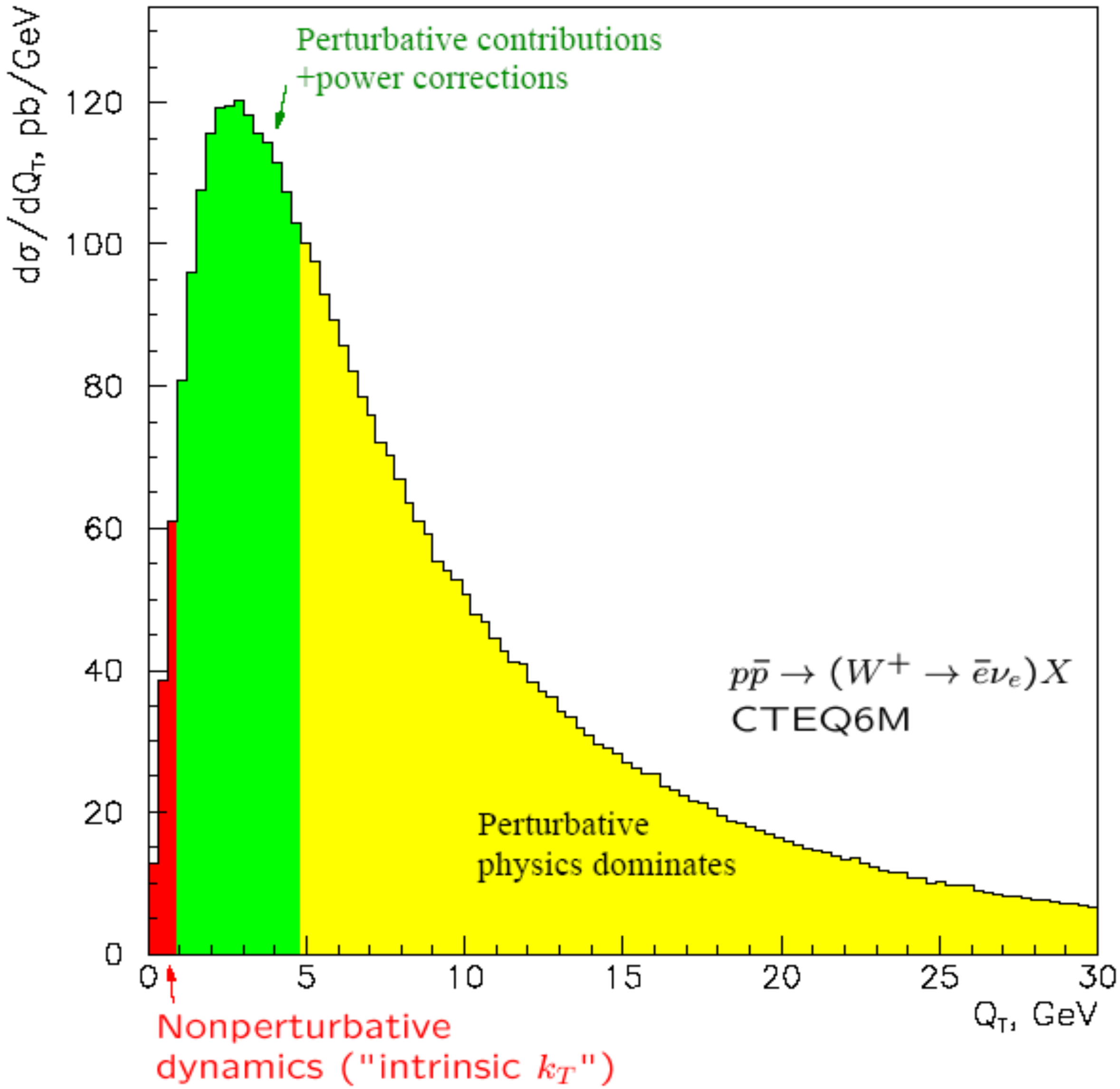
$$\Gamma_\mu(q^2,\epsilon) = -ie\mu^\epsilon\ \bar{u}\ (p_1)\gamma_\mu v(p_2)\ \rho(q^2,\epsilon)$$

$$\rho(q^2,\epsilon) = -\frac{\alpha_s}{2\pi}C_F\left(\frac{4\pi\mu^2}{-q^2-i\epsilon}\right)^\epsilon \frac{\Gamma^2(1-\epsilon)\Gamma(1+\epsilon)}{\Gamma(1-2\epsilon)}\left\{\frac{1}{(-\epsilon)^2}-\frac{3}{2(-\epsilon)}+4\right\}$$

$$= 1 - \frac{\alpha_s}{4\pi}C_F\ \ln^2(q^2/\mu^2)+\ldots$$ **Sudakov double logarithms**

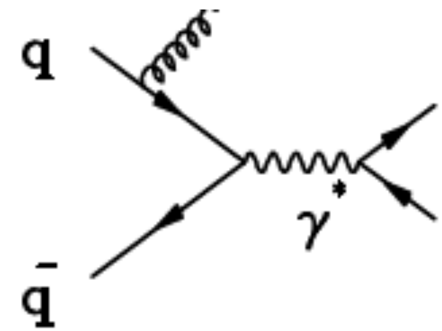
Common to all massless theories

Drell-Yan Q_T-distribution



Showing the different theoretical regions in momentum space

Drell-Yan type subprocess

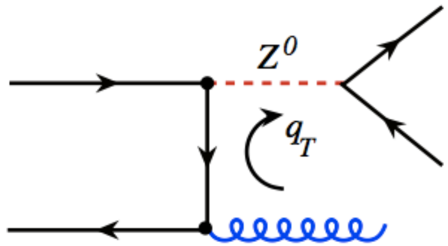


Photon can be replaced by W, Z, Higgs, etc.

Leading double log contribution

❑ LO Differential Q_T-distribution as $Q_T \to 0$:

$$\frac{d\sigma}{dy\,dQ_T^2}\bigg|_{\text{LO}} \approx \left(\frac{d\sigma}{dy}\right)_{\text{Born}} \times 2C_F\left(\frac{\alpha_s}{\pi}\right)\frac{\ln\left(Q^2/Q_T^2\right)}{Q_T^2} \Rightarrow \infty$$



$$\Longrightarrow \int_0^{Q^2} \frac{d\sigma}{dy\,dQ_T^2}\bigg|_{\text{real+virutal}} dQ_T^2 \approx \left(\frac{d\sigma}{dy}\right)_{\text{Born}} + O(\alpha_s) \qquad \text{with } Q^2 \approx M_Z^2$$

❑ Integrated Q_T-distribution:

$$\int_0^{Q_T^2} \frac{d\sigma}{dy\,dp_T^2}\bigg|_{\text{real+virutal}} dp_T^2 \equiv \left[\int_0^{Q^2} - \int_{Q_T^2}^{Q^2}\right] \frac{d\sigma}{dy\,dp_T^2}\bigg|_{\text{real+virutal}} dp_T^2$$

$$\approx \left(\frac{d\sigma}{dy}\right)_{\text{Born}} \times \left[1 - \int_{Q_T^2}^{Q^2} 2C_F \frac{\alpha_s}{\pi}\frac{\ln\left(Q^2/p_T^2\right)}{p_T^2} dp_T^2\right] = \left(\frac{d\sigma}{dy}\right)_{\text{Born}} \times \left[1 - C_F\frac{\alpha_s}{\pi}\ln^2\left(Q^2/Q_T^2\right)\right]$$

$$\approx \left(\frac{d\sigma}{dy}\right)_{\text{Born}} \times \exp\left[-C_F\frac{\alpha_s}{\pi}\ln^2\left(Q^2/Q_T^2\right)\right]$$

Effect of gluon emission

Assume this exponentiates

Resummed Q_T distribution

- Differentiate the integrated Q_T-distribution:

$$\frac{d\sigma}{dy\,dQ_T^2} \approx \left(\frac{d\sigma}{dy}\right)_{\text{Born}} \times 2C_F\left(\frac{\alpha_s}{\pi}\right)\frac{\ln\left(Q^2/Q_T^2\right)}{Q_T^2} \times \exp\left[-C_F\left(\frac{\alpha_s}{\pi}\right)\ln^2\left(Q^2/Q_T^2\right)\right] \Rightarrow 0$$

as $Q_T \to 0$

- Compare to the explicit LO calculation:

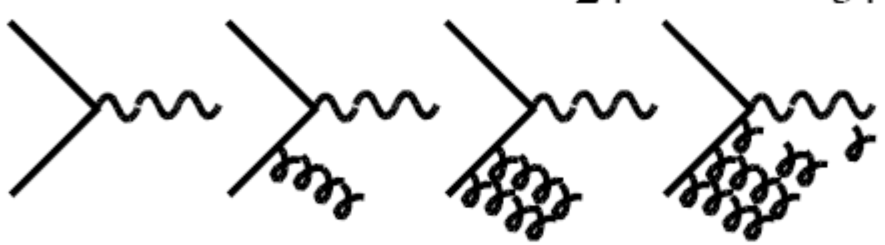
$$\left.\frac{d\sigma}{dy\,dQ_T^2}\right|_{\text{LO}} \approx \left(\frac{d\sigma}{dy}\right)_{\text{Born}} \times 2C_F\left(\frac{\alpha_s}{\pi}\right)\frac{\ln\left(Q^2/Q_T^2\right)}{Q_T^2} \Rightarrow \infty$$

Q_T-spectrum (as $Q_T \to 0$) is completely changed!

- We just resummed (exponentiated) an infinite series of soft gluon emissions – double logarithms

$$e^{-\alpha_s L^2} \approx 1 - \alpha_s L^2 + \frac{(\alpha_s L^2)^2}{2!} - \frac{(\alpha_s L^2)^3}{3!} + \ \ldots$$

$L \propto \ln\left(Q^2/Q_T^2\right)$



Soft gluon emission treated as uncorrelated

Still a wrong Q_T-distribution

❑ **Experimental fact:** $\frac{d\sigma}{dy\,dQ_T^2} \Rightarrow$ finite [neither ∞ nor 0!] as $Q_T \to 0$

- Double Leading Logarithms Approximation (DLLA)

  radiated gluons are both soft and collinear with strong ordering in their transverse momenta

- Strong ordering in transverse momenta in DLLA

  – overly constrains the phase space ofthe emitted gluons

  – ignores the overall transverse momentum conservation

  $\Rightarrow$ DLLA over suppresses small Q_T region

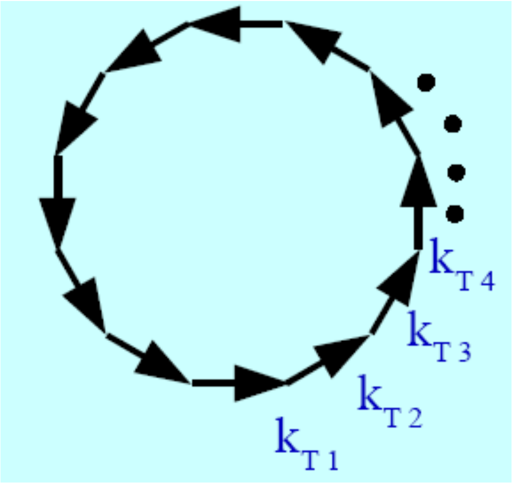
Resummation of uncorrelated soft gluon emission leads to a too strong suppression at $Q_T = 0$!

Still a wrong Q_T-distribution

- ❑ Why?

  Particle can receive many finite k_T kicks via soft gluon radiation yet still have $Q_T = 0$

  – Need a vector sum!



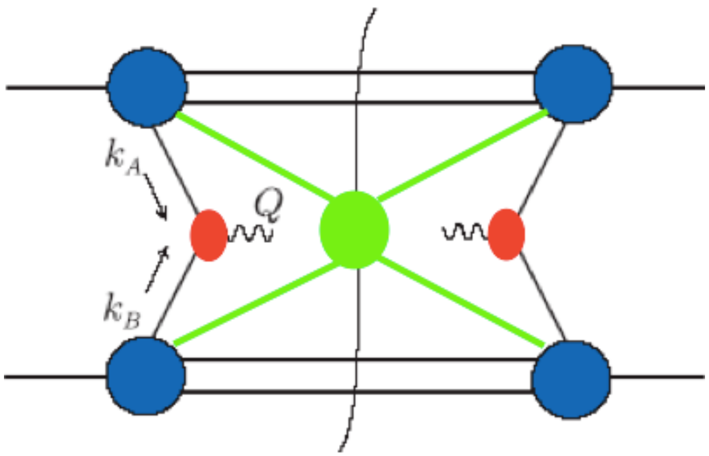
- ❑ Subleading logarithms are equally important at $Q_T = 0$
- ❑ Solution:

  To impose the 4-momentum conservation at each step of soft gluon resummation → TMD factorization

CSS b-space resummation formalism

Collins, Soper, Sterman, 1985

❑ **TMD-factorized cross section:**



$$\frac{d\sigma_{AB}}{dQ^2 dQ_T^2} = \sum_f \int d\xi_a d\xi_b \int \frac{d^2k_{A_T} d^2k_{B_T} d^2k_{s,T}}{(2\pi)^6}$$

$$\times P_{f/A}(\xi_a, k_{A_T}) P_{\bar{f}/B}(\xi_b, k_{B_T}) H_{f\bar{f}}(Q^2) S(k_{s,T})$$

$$\times \delta^2(\vec{Q}_T - \vec{k}_{A_T} - \vec{k}_{B_T} - \vec{k}_{s,T})$$

$$\boxed{\delta^2(\vec{Q}_T - \prod_i \vec{k}_{i,T}) = \frac{1}{(2\pi)^2}\int d^2b\, e^{i\vec{b}\cdot\vec{Q}_T} \prod_i e^{-i\vec{b}\cdot\vec{k}_{i,T}}}$$

❑ **Factorized cross section in "impact parameter b-space":**

$$\frac{d\sigma_{AB}(Q,b)}{dQ^2} = \sum_f \int d\xi_a d\xi_b \overline{P}_{f/A}(\xi_a, b, n) \overline{P}_{\bar{f}/B}(\xi_b, b, n) H_{f\bar{f}}(Q^2) U(b,n)$$

❑ **Resummation:** **Two equations, two resummation of log's**

$$\mu_{\text{ren}} \frac{d\sigma}{d\mu_{\text{ren}}} = 0 \qquad\qquad n^\nu \frac{d\sigma}{dn^\nu} = 0$$

CSS b-space resummation formalism

- Solve those two equations and transform back to Q_T:

$$\frac{d\sigma_{AB}}{dQ^2 dQ_T^2} \equiv \frac{1}{(2\pi)^2}\int d^2b\, e^{i\vec{b}\cdot\vec{Q}_T}\tilde{W}_{AB}(b,Q) + Y_{AB}(Q_T^2,Q^2)$$

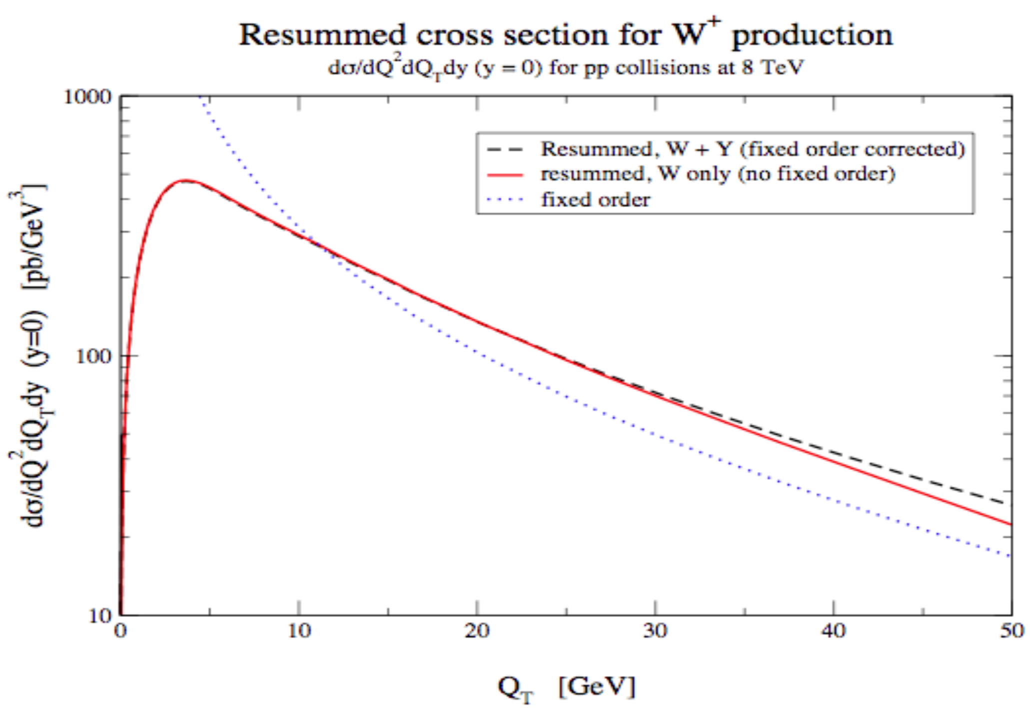
resummed

No large log's

$$= \frac{1}{(2\pi)}\int_0^\infty db\, J_0(bQ_T)\, b\tilde{W}_{AB}(b,Q) + \left[\frac{d\sigma_{AB}^{(\mathrm{Pert})}}{dQ^2 dQ_T^2} - \frac{d\sigma_{AB}^{(\mathrm{Asym})}}{dQ^2 dQ_T^2}\right]$$

- Role of each term:

implemented in RESBOS code



CSS b-space resummation formalism

❑ **b-space distribution:**

$$\tilde{W}_{AB}(b,Q) \equiv \sum_{i,j} \tilde{W}_{ij}(b,Q)\hat{\sigma}_{ij}(Q)$$

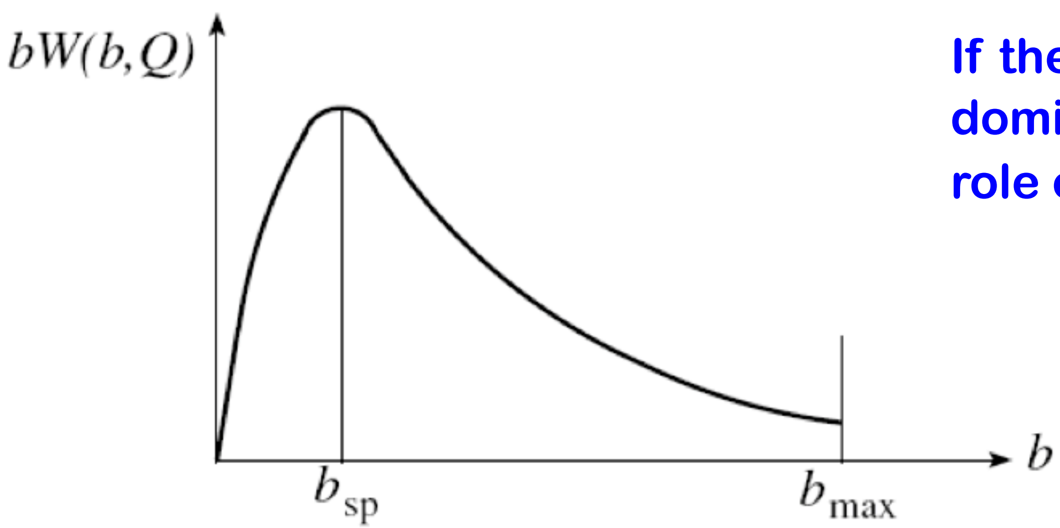
❑ **Perturbative contribution ($b \ll 1/\Lambda_{\rm QCD}$):**

Sudakov Form factor

$$W_{AB}^{\rm Pert}(b,Q) = \sum_{a,b,i,j} \sigma_{ij\to V}^{\rm LO} \left[\phi_{a/A} \otimes C_{a\to i}\right] \otimes \left[\phi_{b/B} \otimes C_{b\to j}\right] e^{-S(b,Q)}$$

Collinear PDFs

❑ **Nonperturbative contribution from large b-region:**



If the area under the curve is dominated by samll-b region, the role of large-b region is minimal

✧ Large Q, and/or

✧ Large $\sqrt{S}$

Qiu, Zhang, 2001

Role of the nonperturbative input

❑ For the region where b > 1/GeV:

1) Work in Q_T-space directly to some approximation
   The originals: Dokshitzer, Diakanov & Troyan
   Revived by Ellis & Veseli Kulesza & Stirling
   who re-derived it from b-space.

2) Insert a "soft landing" on the k_T integral by replacing

$$1/b \rightarrow \sqrt{1/b^2 + 1/b_*^2}$$

   for some fixed b_*. (CS, CSS "b_*" prescription, ResBos)

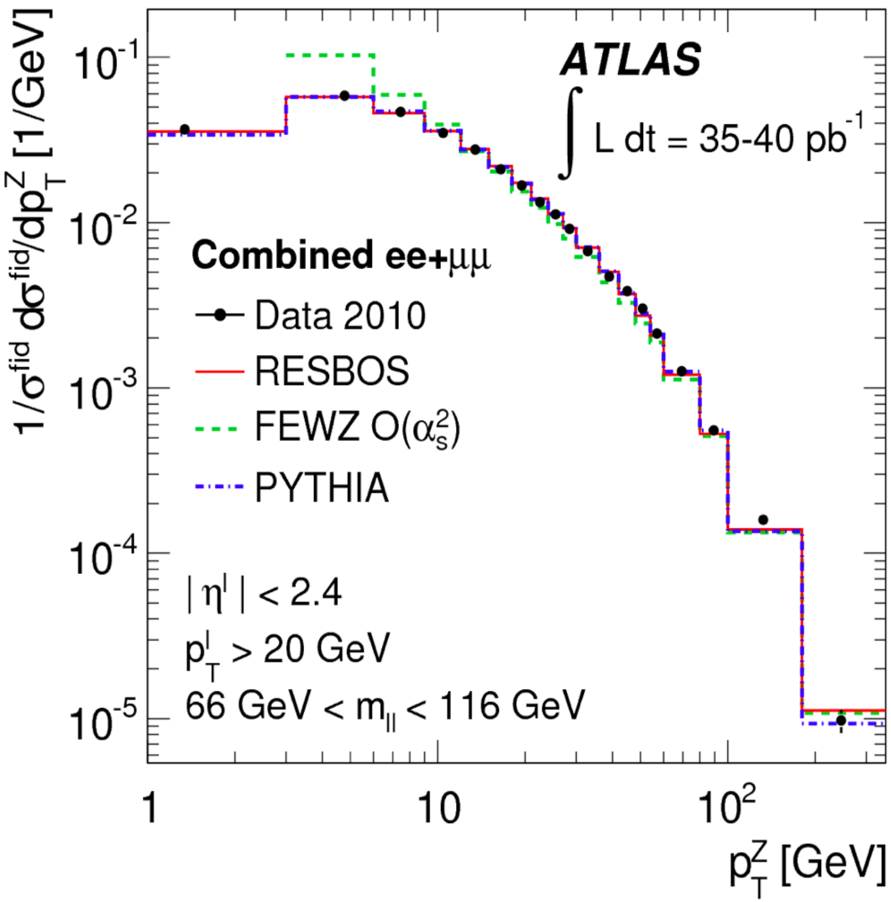
3) Extrapolation of E^{PT} into NP region (Qiu, Zhang)

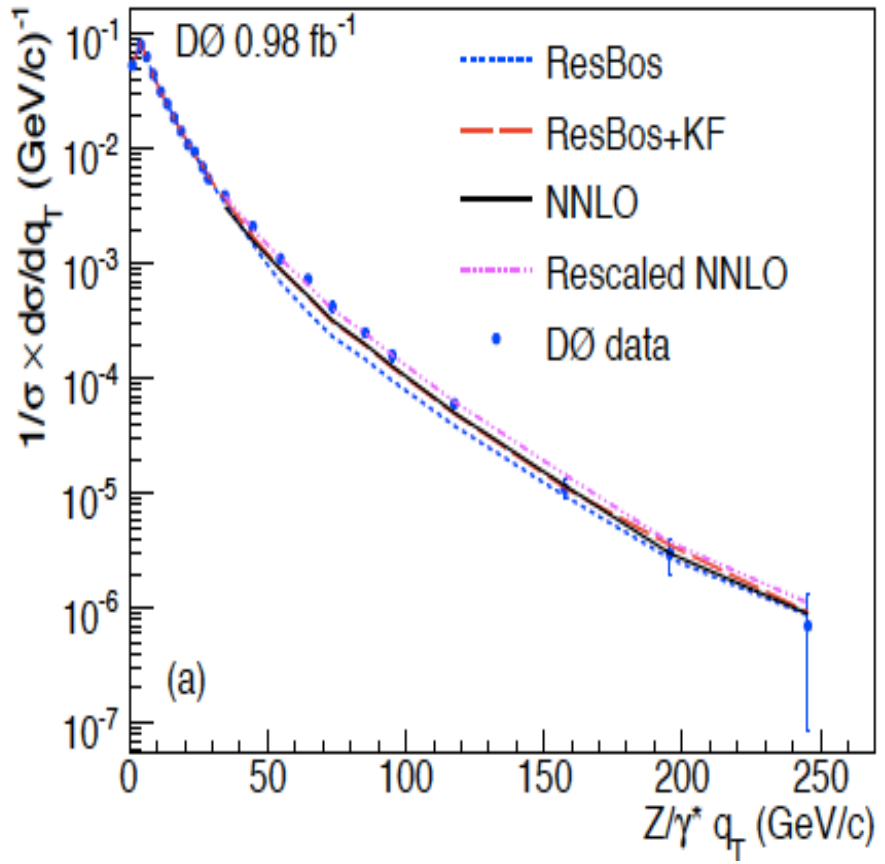
4) Minimal: avoid the singularity at $1/b = \Lambda_{\mathrm{QCD}}$
   by monkeying with the b-space contour integral
   (This technique introduced in threshold resummation;
   then adapted by Laenen, GS and Vogelsang,
   and Bozzi, Catani, de Florian and Grazzini.)

5) SCET, …

Phenomenology

- ❑ **Compare with the LHC data:**

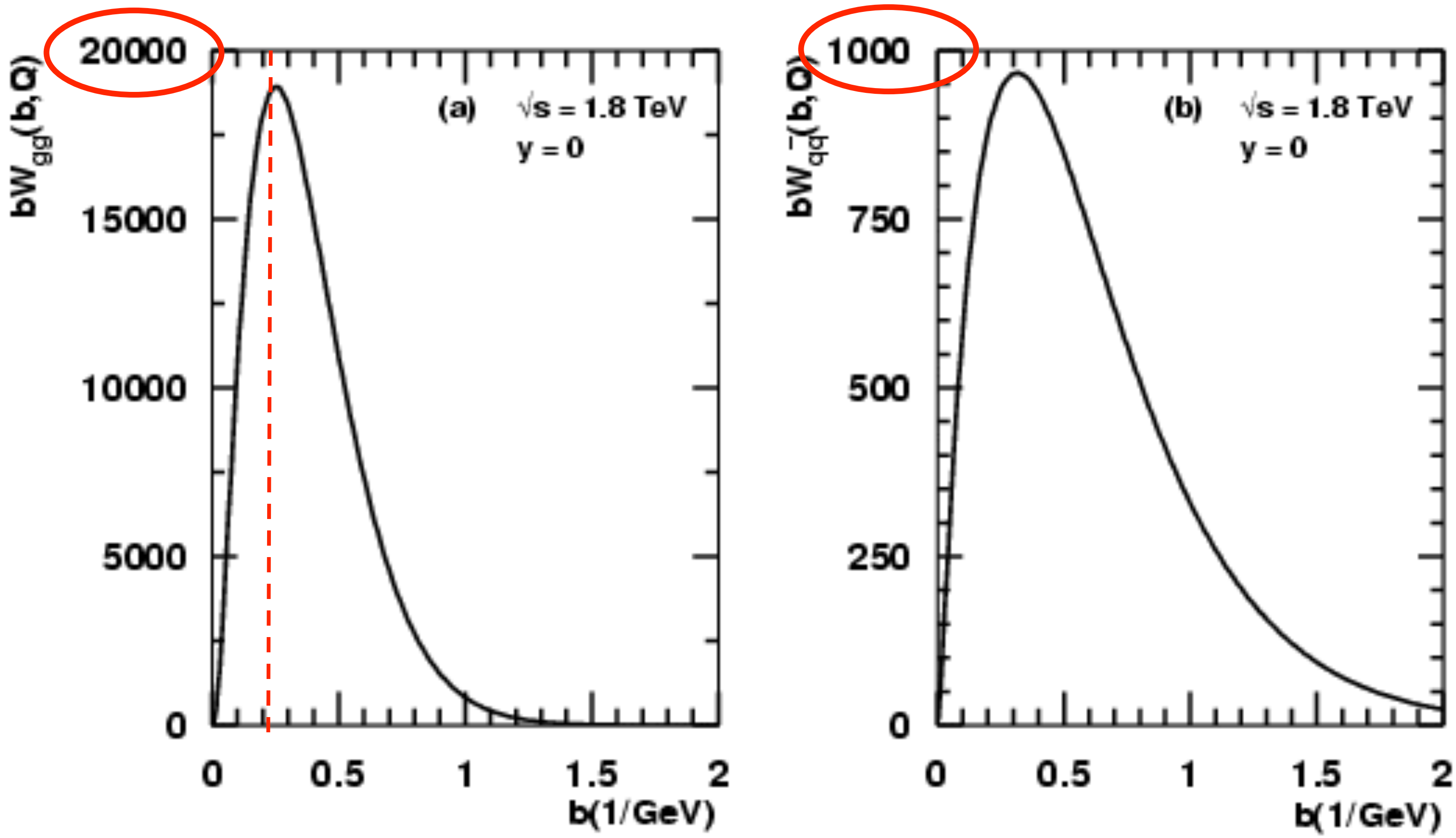




Phenomenology

Berger, Qiu, Wang, 2005

- ❑ Upsilon production (low Q, large phase space):



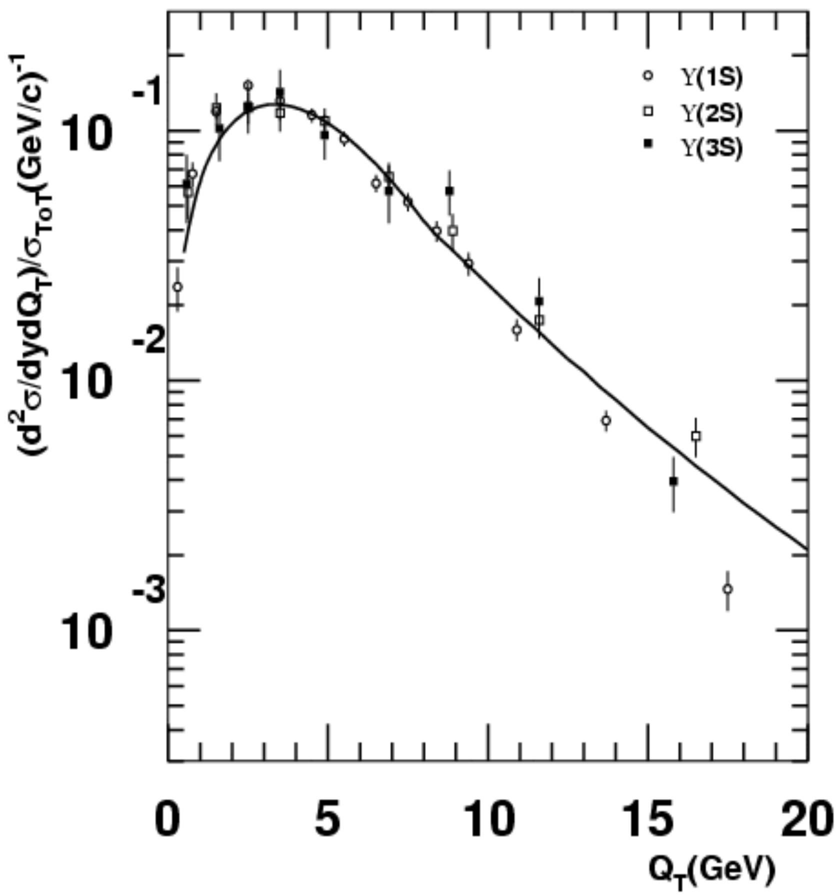
Gluon-gluon dominate the production

Dominated by perturbative contribution even M_{Υ}~10 GeV

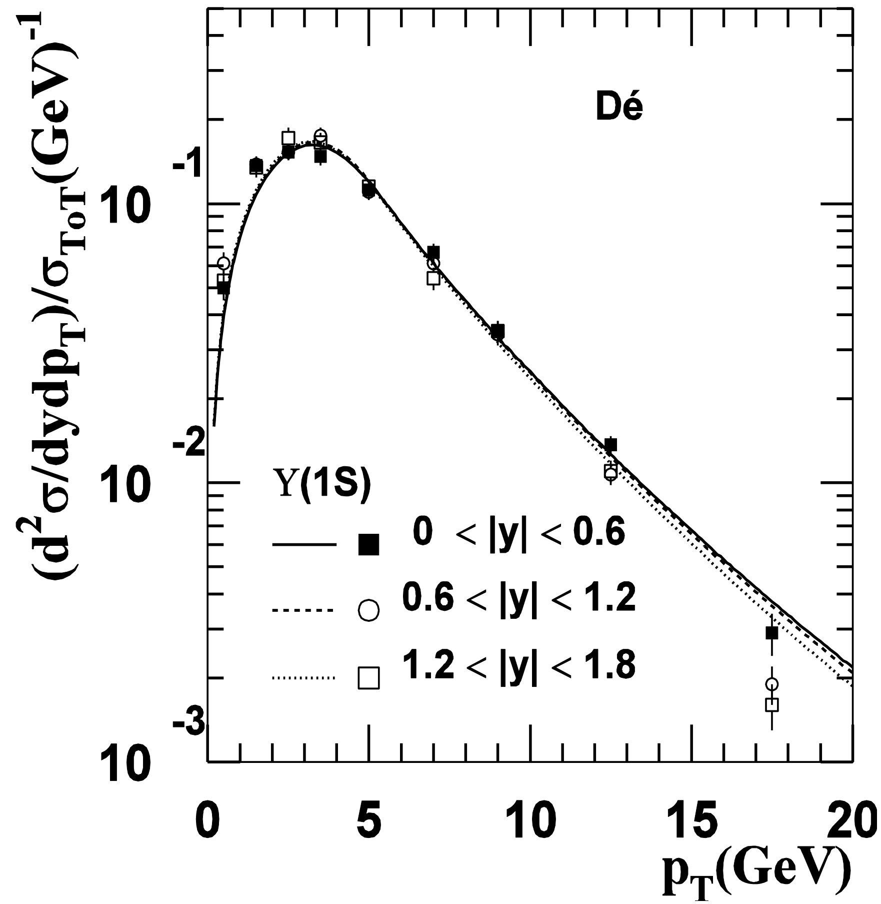
Phenomenology

Berger, Qiu, Wang, 2005

- Prediction vs Tevatron data:



CDF Run-I data

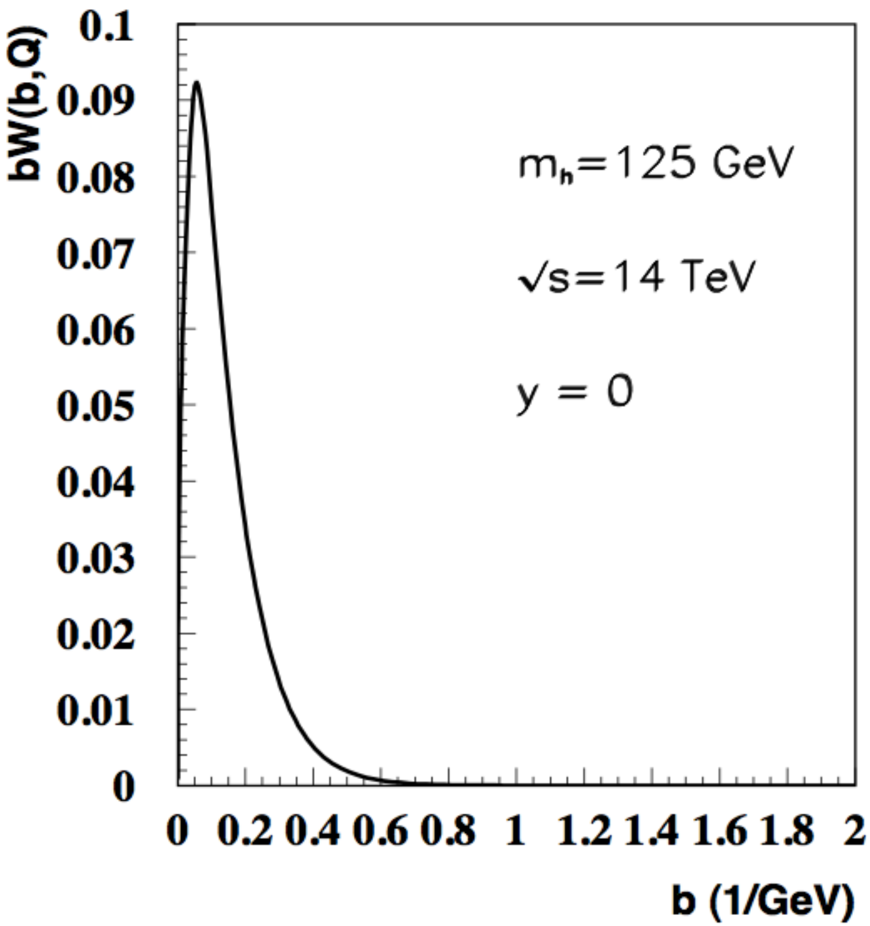


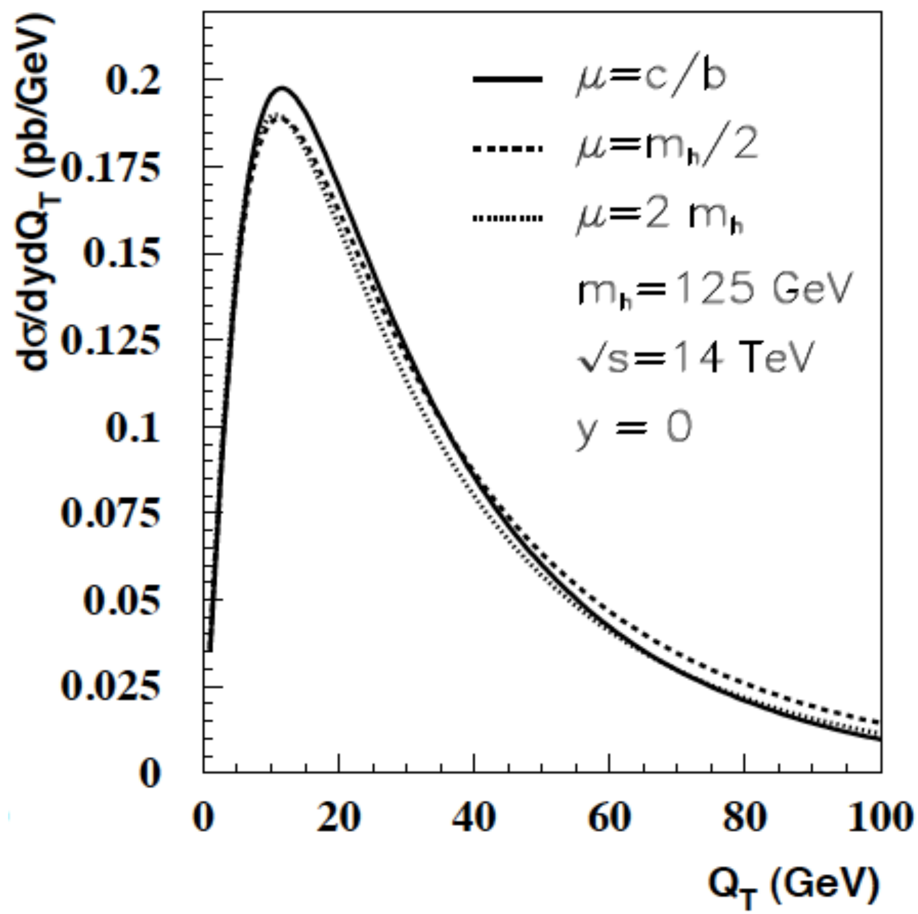
DO Run-II data

Phenomenology

Berger, Qiu, 2003

- Higgs at the LHC:





Effectively no non-perturbative uncertainty!

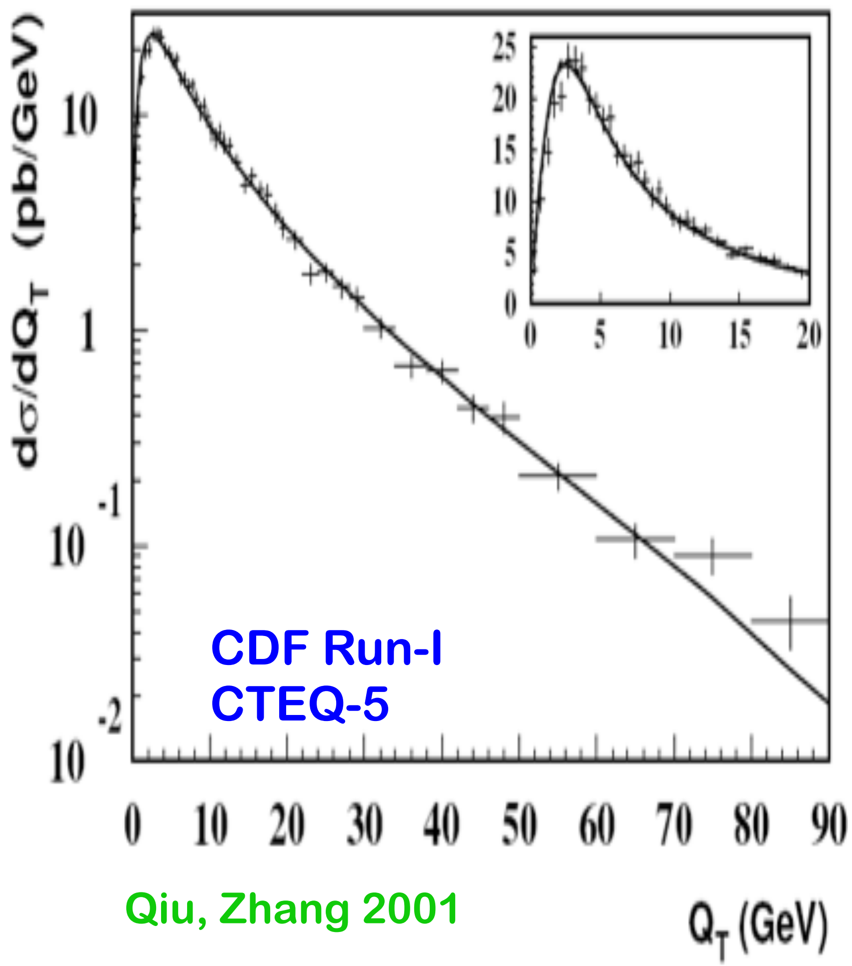
Thank you!

See you at the recitation tonight

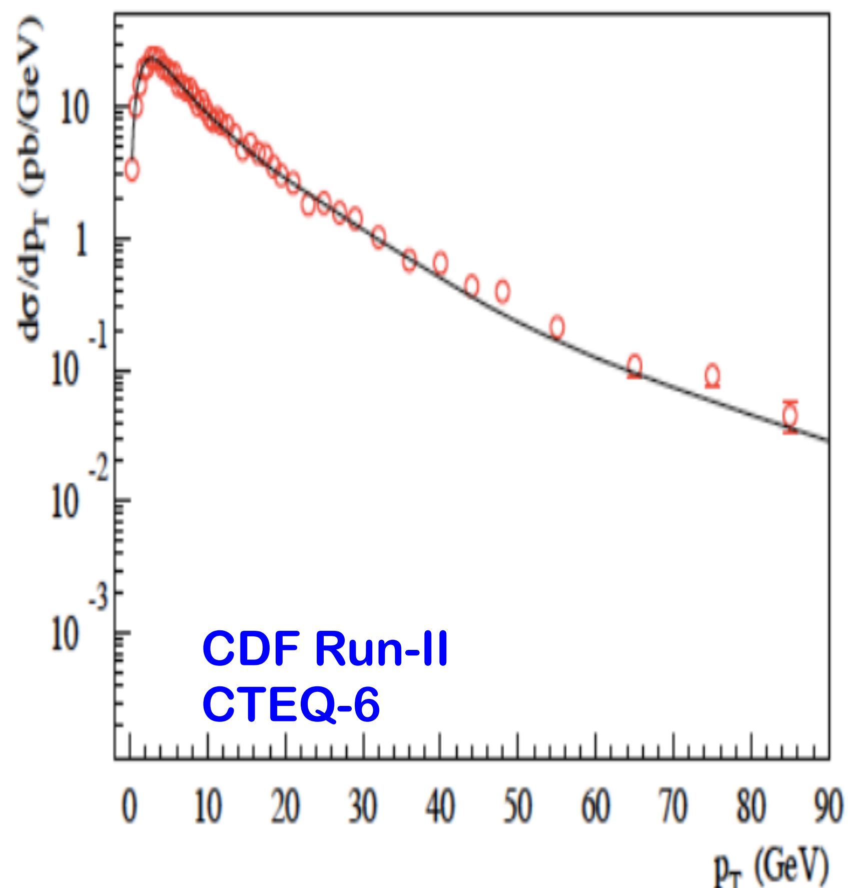
Backup slices

Phenomenology

- Compare with the Tevatron data:



Qiu, Zhang 2001



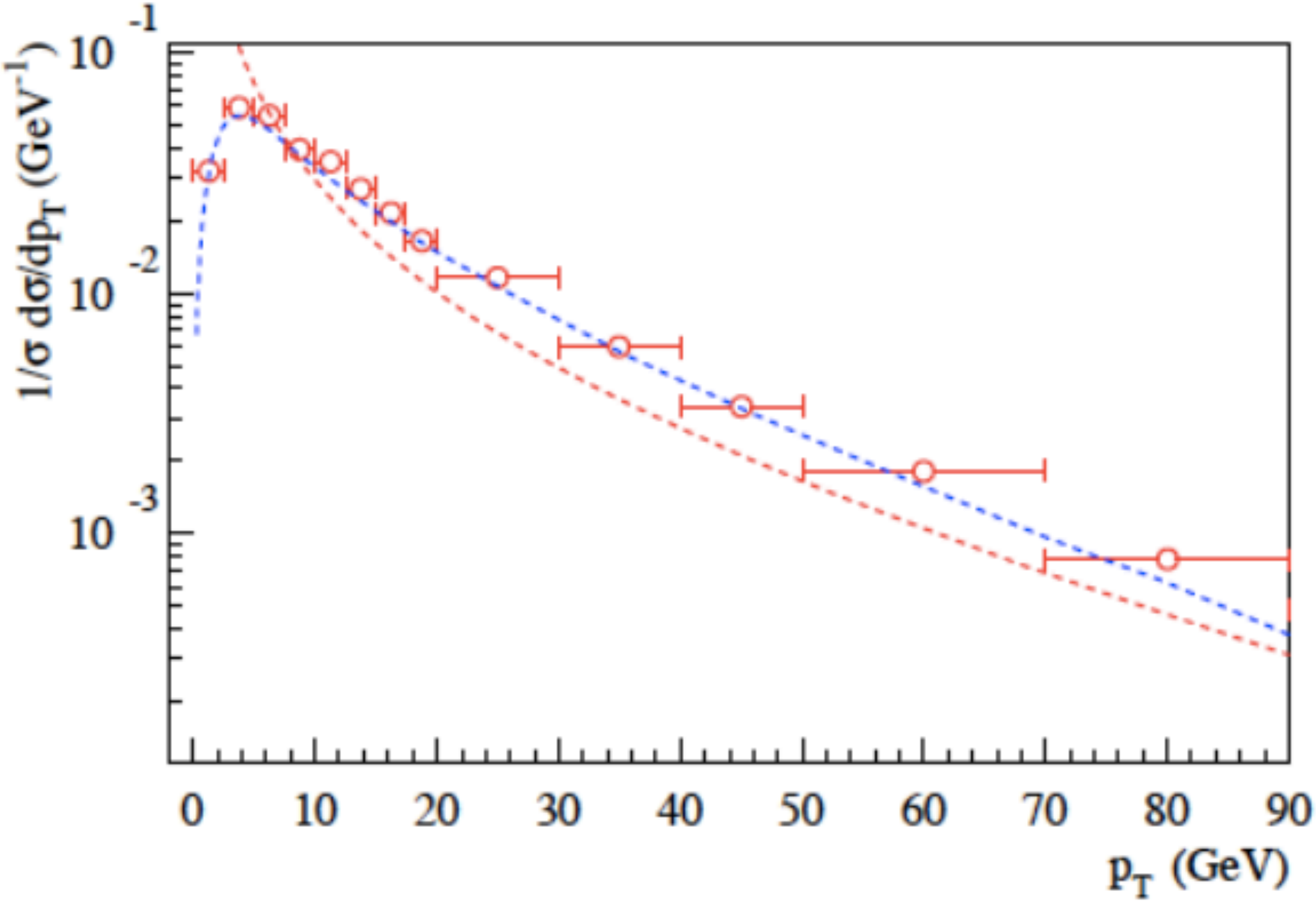
Kang, Qiu 2012

No free fitting parameter!

Phenomenology

Kang, Qiu, 2012

- Compare with the LHC data:



Effectively no non-perturbative uncertainty!

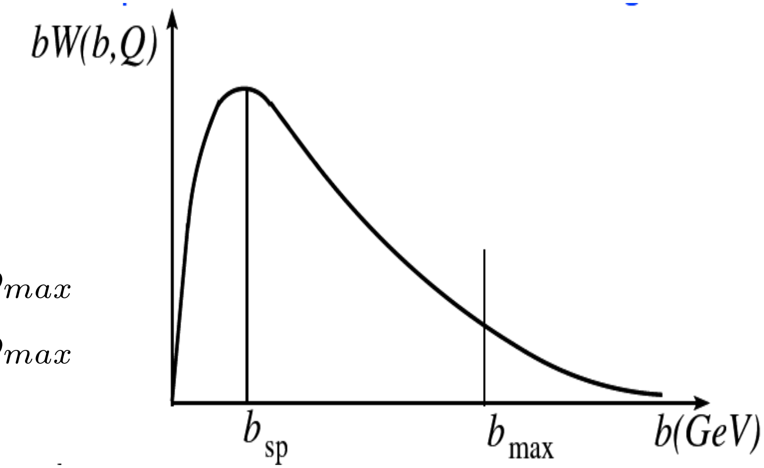
Phenomenology

Qiu, Zhang, 2001

❑ Resummed cross section:

$$\frac{d\sigma^{\text{resum}}_{AB\to Z}}{dq_T^2} \propto \int_0^\infty db\, J_0(q_T b)\, b\, W(b,Q)$$

$$W(b,Q) = \begin{cases} W^{\text{pert}}(b,Q) & b \le b_{max} \\ W^{\text{pert}}(b_{max},Q)\, F^{NP}_{QZ}(b,Q,b_{max}) & b > b_{max} \end{cases}$$



❑ Resummed cross section:

$$F^{NP}_{QZ}(b,Q;b_{max}) = \exp\left\{ -\ln\left(\frac{Q^2 b^2_{max}}{c^2}\right)\left[g_1\left((b^2)^\alpha - (b^2_{max})^\alpha\right) + g_2\left(b^2 - b^2_{max}\right)\right] - \bar{g}_2\left(b^2 - b^2_{max}\right)\right\}$$

Leading twist

Intrinsic power corrections

Dynamical power corrections

❑ Predictive power:

- ✧ Larger Q
- ✧ Larger S

Smaller b_{sp} → Better prediction