Vector Boson and Direct Photon Production

Lecture 2

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Take-home messages from lecture one

- X-section with identified hadron(s), such as DIS, Drell-Yan, is NOT perturbatively calculable

- QCD factorization is necessary, but, is an approximation!
  - ✧ Collinear factorization for x-section with ONE large scale
  - ✧ TMD factorization for x-section with TWO different scales

- Drell-Yan x-section of inclusive massive vector boson production is factorizable for leading power contribution

- Theory and experiment are consistent for inclusive massive vector boson production, including

$$\sigma^{\text{total}}(Q = M_V),\ \frac{d\sigma}{dy},\ \frac{d\sigma}{dy\,dq_T^2},\ \frac{d\sigma}{dQ^2},\ \frac{d\sigma}{dy\,dQ^2},\ \frac{d\sigma}{dy\,dQ^2\,dq_T^2}$$

Resummation needed, and works

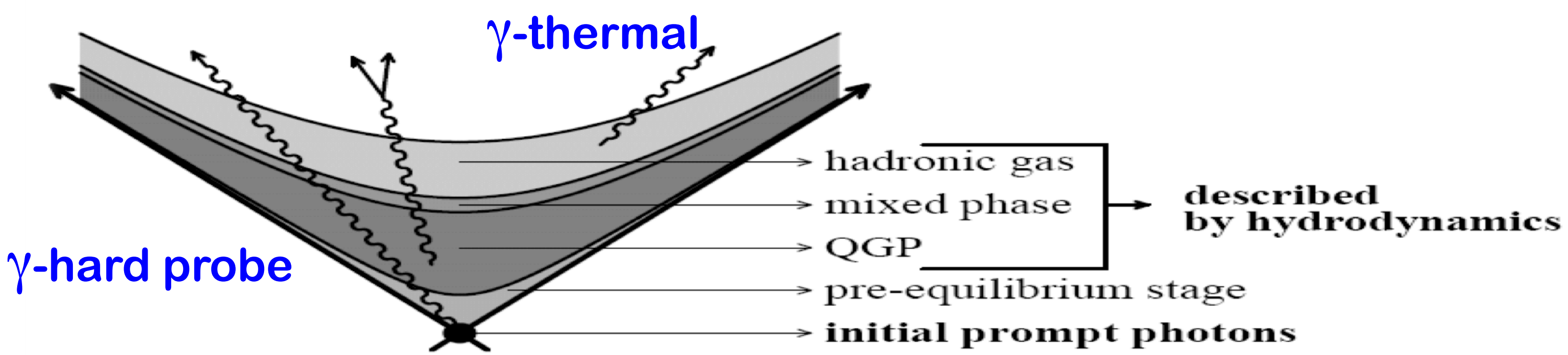
- Excellent probe for PDFs, hadron structure, …

Outline of the two lectures

- Lecture one:
  - ✧ Basics of vector bosons
  - ✧ Drell-Yan like production process
  - ✧ Cross section with a single hard scale – precision
  - ✧ Cross section with two different scales – resummation
- Lecture two:
  - ✧ Photon production at high pT – direct vs fragmentation
  - ✧ Isolation cut – the need and its complication
  - ✧ Photons from fixed target to collider energies
  - ✧ Multi-boson associated production at collider energies

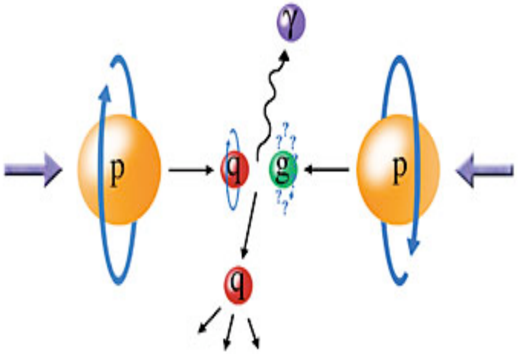
Why photons?

- Photon is a EM probe:
  - It can be produced at any stage of the collision
  - It does not interact strongly once produced
- Good probe of short-distance strong interaction:
  - Isolated or “direct” photon is produced at a distance $1/p_T \ll$ fm
  - “snap shot” of what happened at the distance scale $1/p_T$
  - Key signal, as well as background of Higgs production: $H^0 \to \gamma + \gamma$
- Photon can tell the full history of heavy ion collision:



Theory behind the high p_T photon

❑ Production mechanism – leading power factorization:



$$\frac{d\sigma_{AB}}{dydp_T^2} = \int dx f_{a/A}(x,\mu) \int dx' f_{b/B}(x',\mu) \frac{d\hat{\sigma}_{ab}(\alpha_s(\mu))}{dydp_T^2} + \text{frag contribution} + \mathcal{O}\left(\frac{1}{p_T^n}\right)$$

Hard part: $\hat{\sigma}_{ab}(\alpha_s(\mu)) = \hat{\sigma}^0_{ab}\alpha_s^m(\mu) + \hat{\sigma}^1_{ab}(\log(\mu))\alpha_s^{m+1}(\mu) + ...$

❑ Predictive power:

✧ Short-distance part is Infrared-Safe, and calculable

✧ Long-distance part at the leading power is Universal – PDFs, FFs

❑ Factorization and renormalization scale dependence:

✧ NLO is necessary

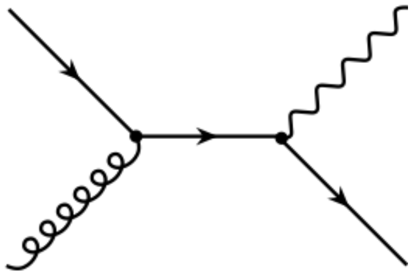
❑ Power correction could be important at low p_T

Direct photon is sensitive to gluon

❑ Sensitive to gluon at the leading order – hadronic collision:

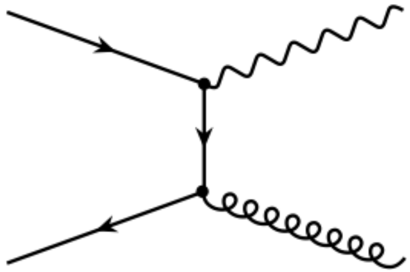
✧ Lowest order direct $\mathcal{O}(\alpha_{em}\alpha_s)$:

Compton: $q(\bar{q}) + g \rightarrow \gamma + q(\bar{q})$



+ …

Annihilation: $q + \bar{q} \rightarrow \gamma + g$



+ …

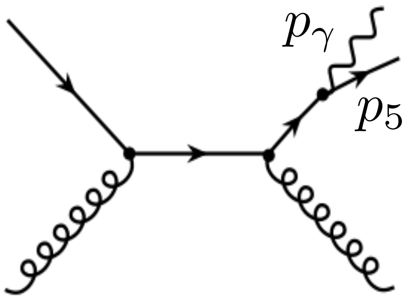
✧ Compton dominates in pp collision:

$$f_{g/p}(x,\mu^2) \gg f_{\bar{q}/p}(x,\mu^2)$$ for all x

Direct photon production could be a good probe of gluon distribution

Complication from high orders

❑ Final-state collinear singularity:



$$\overline{\sum}|M(qg\to\gamma qg)|^2 \approx \frac{\alpha_{em}}{2\pi}\mathcal{P}^{(0)}_{q\to\gamma}(z)\,\frac{1}{s_{\gamma q}}\,\overline{\sum}|M(qg\to qg)|^2$$

$$\mathcal{P}^{(0)}_{q\to\gamma}(z) = \frac{1+(1-z)^2}{z}$$

$$s_{\gamma q} = (p_\gamma + p_5)^2 \to 0 \qquad \textbf{when} \quad p_\gamma \parallel p_5$$

An internal quark line goes on-shell signaling long-distance physics

❑ Fragmentation contribution:

$$\frac{d\sigma^{\rm Frag}_{AB\to\gamma}}{dy\,dp_T^2} = \sum_{abc}\int\frac{dz}{z^2}D_{c\to\gamma}(z,\mu)\int dx\,f_{a/A}(x,\mu)\int dx'\,f_{b/B}(x',\mu)\,\frac{d\hat{\sigma}^{\rm Frag}_{ab\to c}}{dy\,dp_T^2}$$

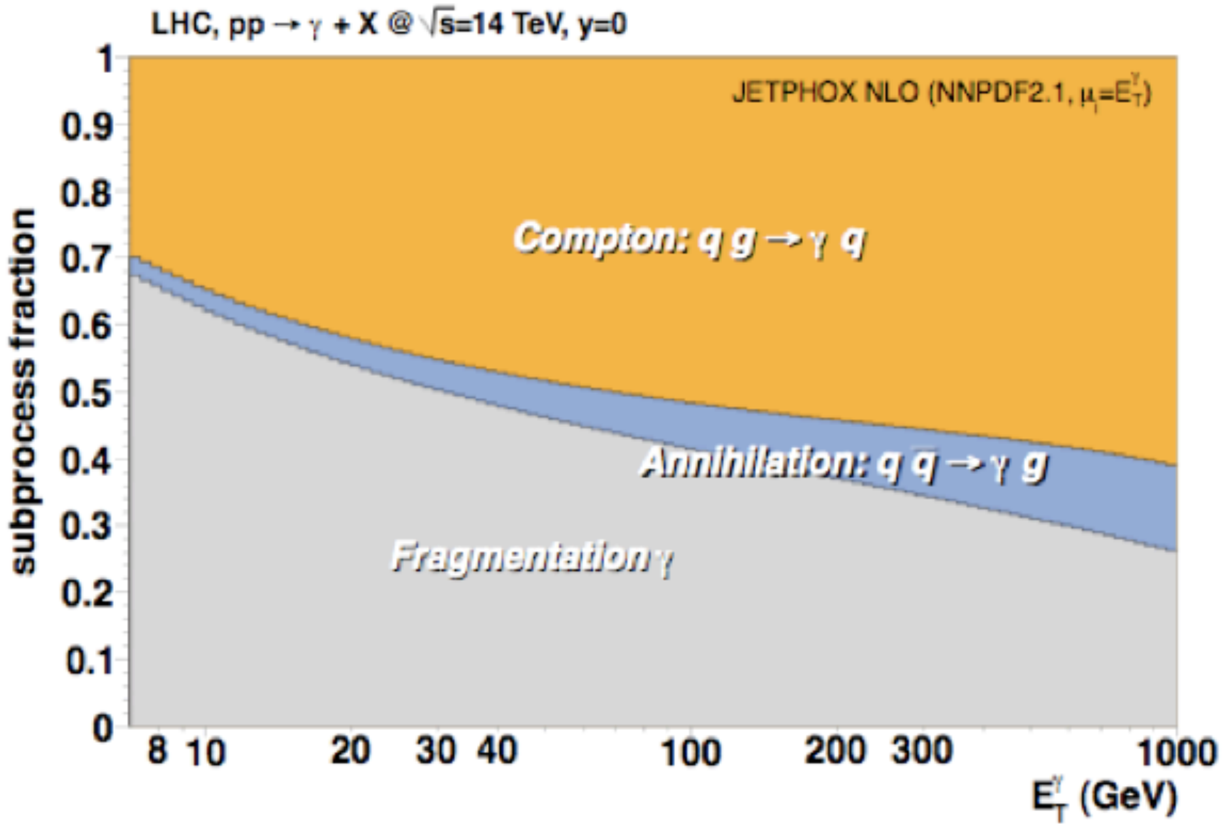
❑ Photon fragmentation functions – inhomogeneous evolution:

$$\frac{\partial D_{c\to\gamma}(z,\mu)}{\partial\log(\mu)} = \frac{\alpha_{em}}{2\pi}\mathcal{P}_{c\to\gamma}(z) + \sum_{a=q\bar{q}g}\frac{\alpha_s}{2\pi}P_{ac}(z)\otimes D_{a\to\gamma}(z,\mu)$$

Size of fragmentation

Campbell, CTEQ SS2013

❑ Inclusive direct photon:



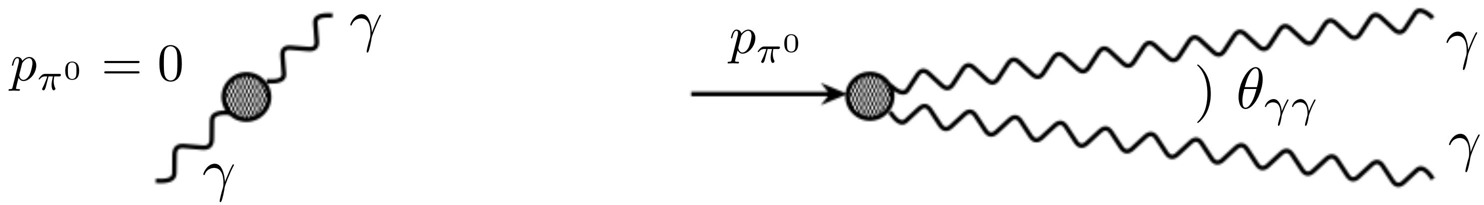
✧ Production at NLO – available, e.g., in MCFM and JETPHOX (shown here)

✧ Fragmentation contribution is huge for inclusive production:

$\sigma^{\text{Frag}} / \sigma^{\text{Total}}$ > 50% at pT=20 GeV @ LHC (role of FF!)

Complication from the measurement

- Separation the signal photon from $\pi^0 \to \gamma\gamma$:



  - ✧ When $p_{\pi 0}$ increases, the opening angle $\theta_{\gamma\gamma}$ decreases
  - ✧ Two photons could be misidentified as one photon at high p_T

- Isolation cut – algorithms (like jet):
  - ✧ Cone algorithm – reduction of fragmentation contribution

    Require that there is less then 1 GeV hadronic transverse energy in a cone of radius (CDF): $R = \sqrt{(\Delta\eta)^2 + (\Delta\phi)^2} \sim 0.7$

Complication from the measurement

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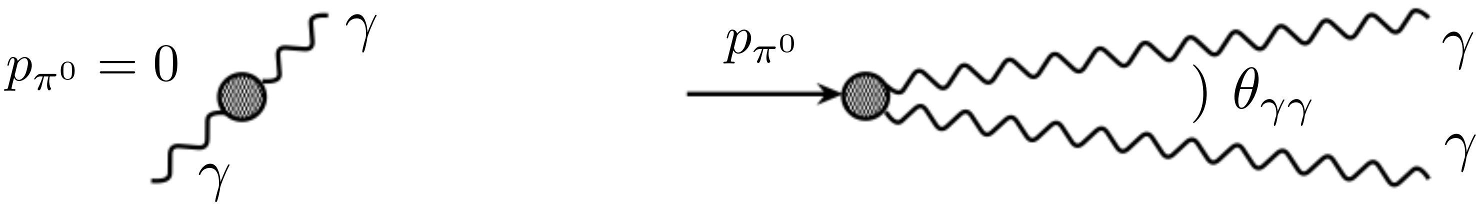


✧ When $p_{\pi 0}$ increases, the opening angle $\theta_{\gamma\gamma}$ decreases

✧ Two photons could be misidentified as one photon at high p_T

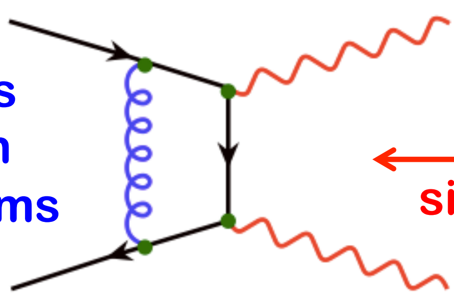
❑ Isolation cut – algorithms:

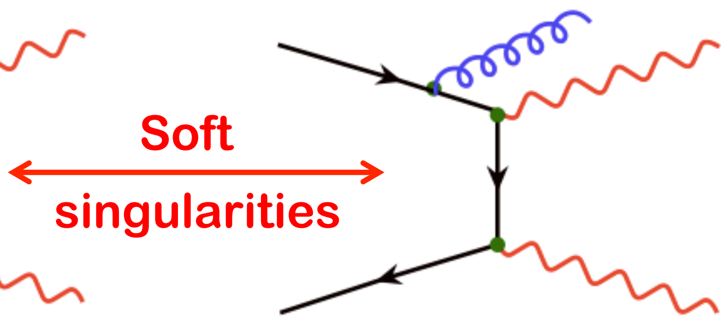
Needed for IR safety

✧ Cone algorithm – reduction of fragmentation contribution

Require that there is less then 1 GeV hadronic transverse energy in a cone of radius (CDF): $R = \sqrt{(\Delta\eta)^2 + (\Delta\phi)^2} \sim 0.7$

Isolation has No effect on virtual diagrams

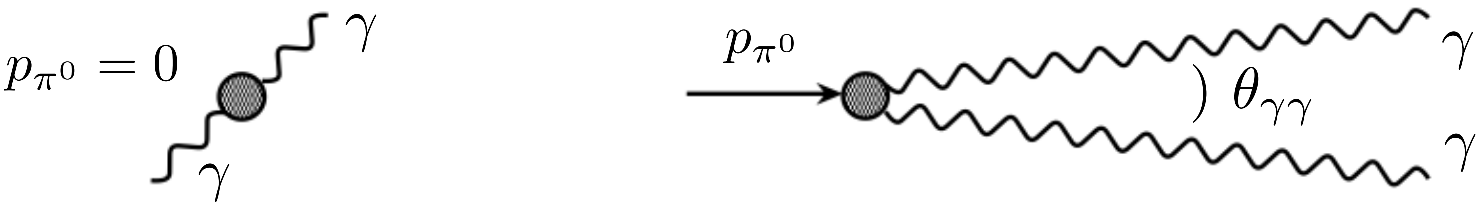




Isolation limits phase space of real diagrams

Complication from the measurement

- Separation the signal photon from $\pi^0 \to \gamma\gamma$:



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  - Cone algorithm – reduction of fragmentation contribution

    Require that there is less then 1 GeV hadronic transverse energy in a cone of radius (CDF): $R = \sqrt{(\Delta\eta)^2 + (\Delta\phi)^2} \sim 0.7$

  - Modified cone algorithm – NO fragmentation contribution

$$\sum_{R_{j\gamma} \in R_0} E_T(\mathrm{had}) < \epsilon_h p_T^\gamma \left(\frac{1 - \cos R_{j\gamma}}{1 - \cos R_0} \right)$$

  - Parton is softer as it closer to photon
  - No contribution at CO singularity

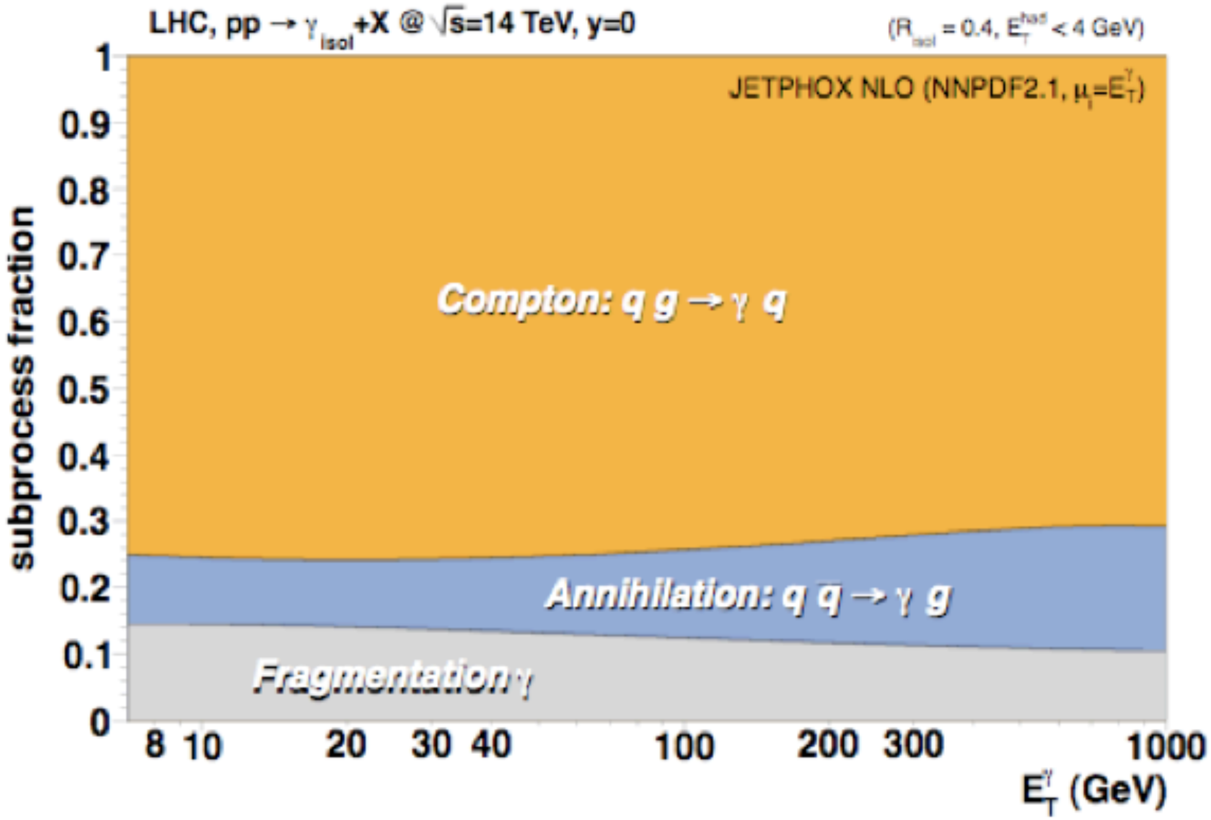
Hard to implement experimentally (detector resolution)

S. Frixione, 1998

Size of fragmentation

Campbell, CTEQ SS2013

- Isolated direct photon:



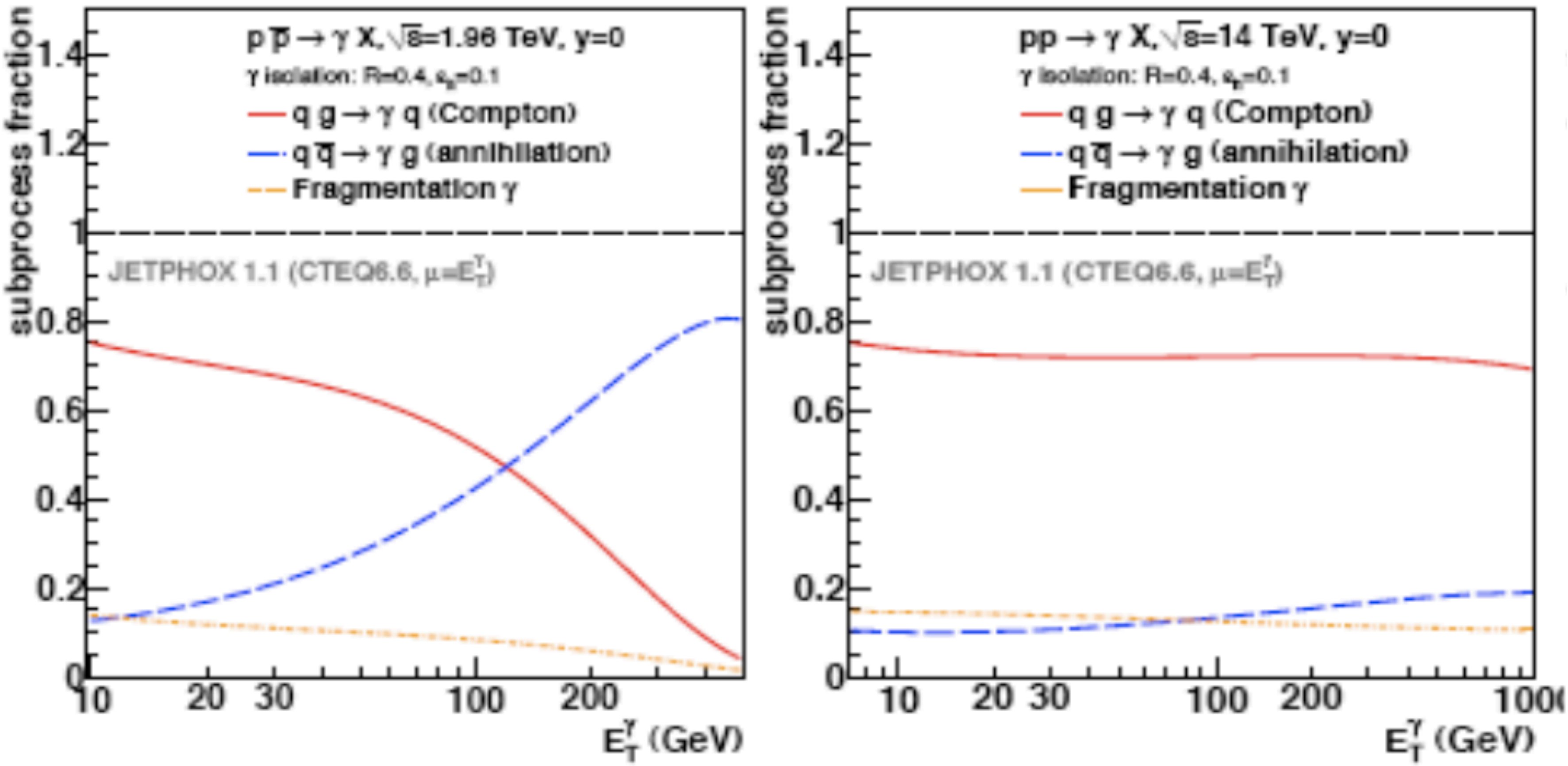
✧ Isolation removes the most of fragmentation contribution! (down to 10%)

✧ About 75% of production rate is from gluon initiated subprocesses

Potentially, a useful probe of gluon PDF

Role of gluon in pp collision

❑ pp vs p$\bar{p}$:

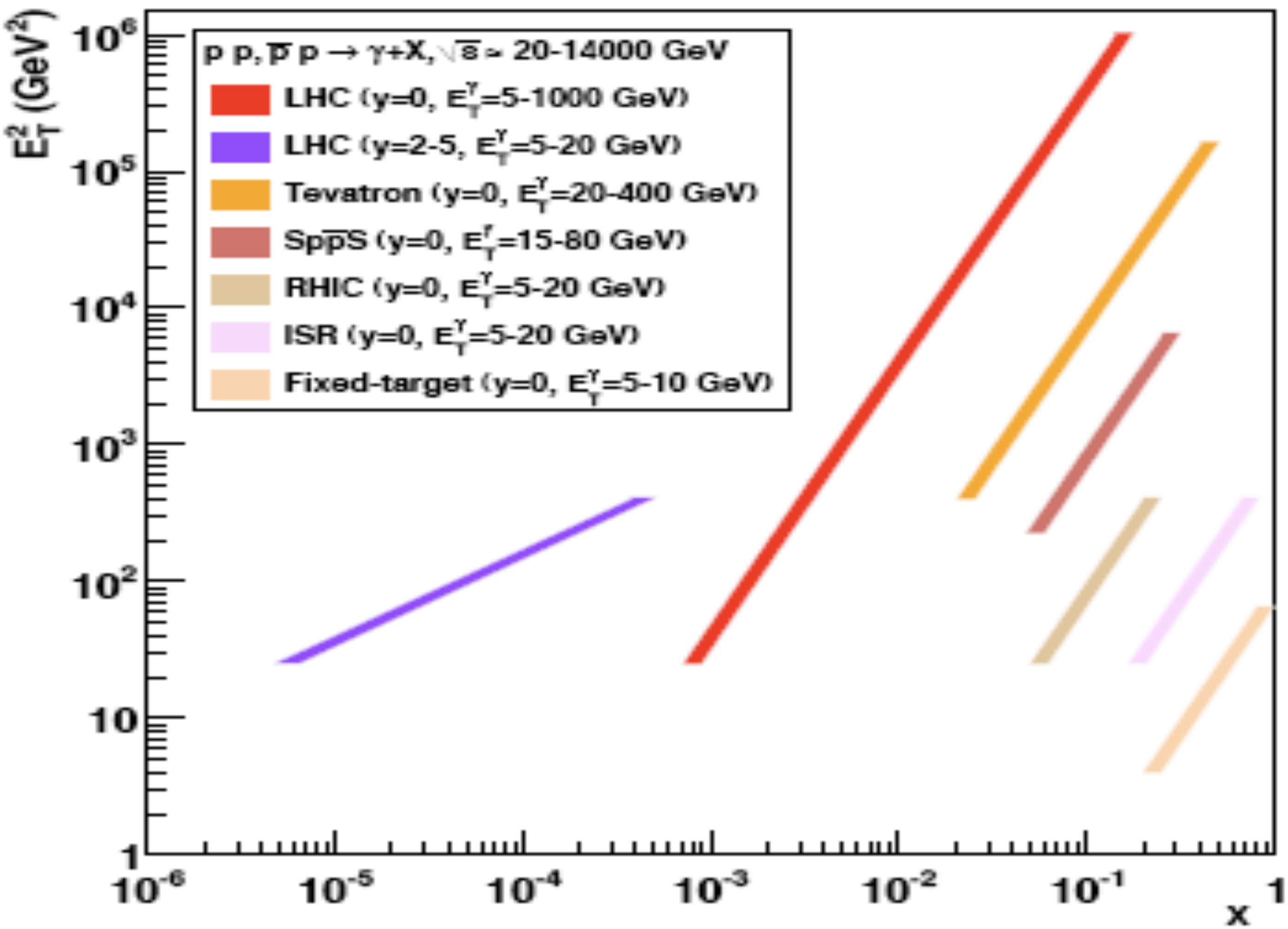


✧ Dominant role of the gluon in pp collision!

✧ Even more dominance in the forward region!

Direct photon covers a wide range of x and Q^2

- Photon energy vs gluon momentum fraction x:



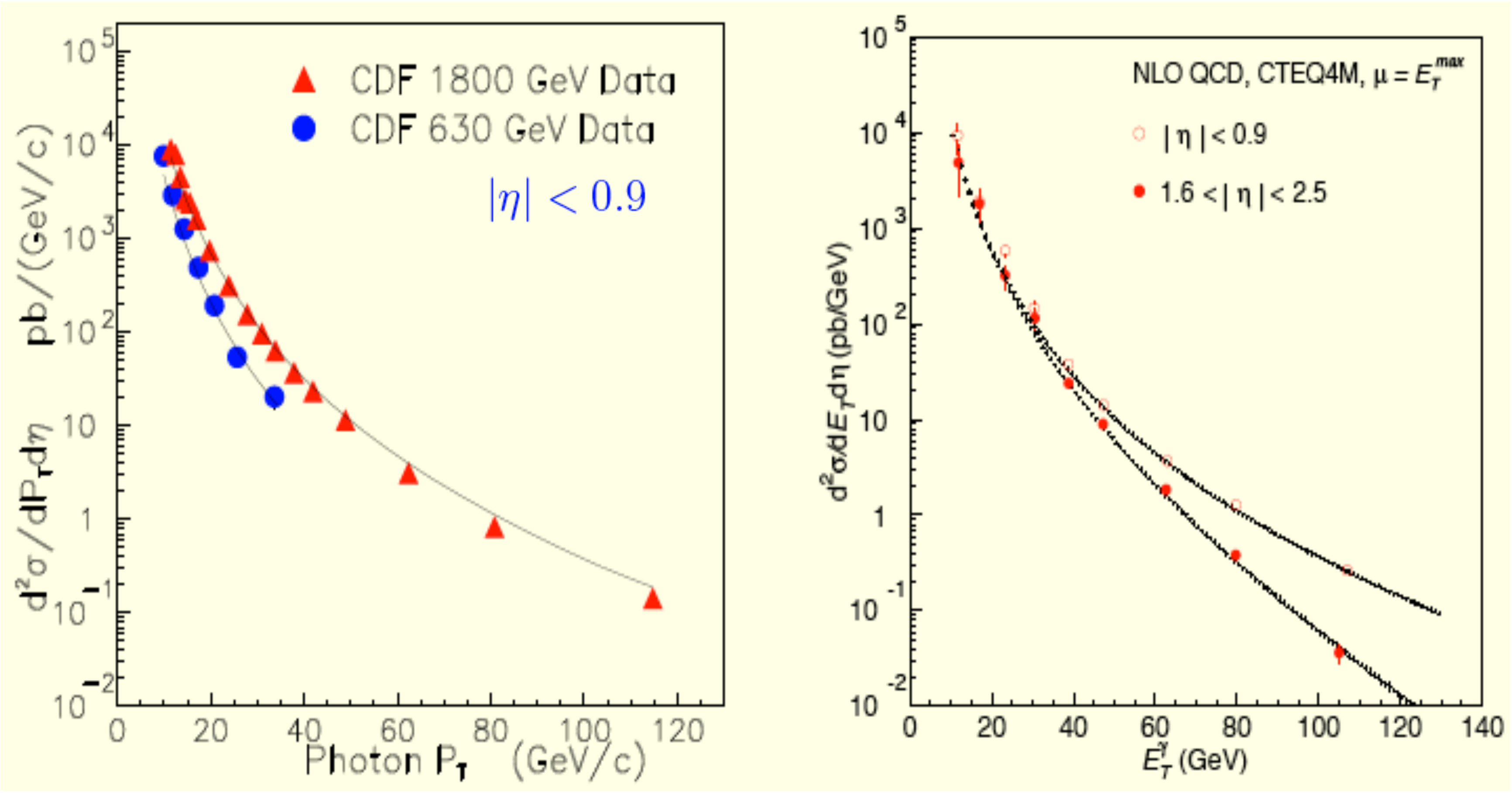
Ichou and D'Enterria, arXiv:1005.4529

Direct photon data

- Fixed target energies $\sqrt{s} = 20 - 40$ GeV:
  - With p_T = 3-10 GeV, data have high $x_T = \frac{2p_T}{\sqrt{s}}$
  - Challenge for NLO theory to fit data – wrong shape!
- Collider energies:
  - pp at ISR with $\sqrt{s} = 44 - 62$ GeV
  - pp at CERN and Fermilab with $\sqrt{s} = 540 - 1960$ GeV
  - $p\bar{p}$ at RHIC with $\sqrt{s} = 200 - 500$ GeV, dA and AA as well
  - pp at LHC with $\sqrt{s} = 7 - 14$ TeV, and PbPb as well
- Data sources:
  - Data review by W. Vogelsang and M.R. Whalley, J. Phys. G23, Suppl. 7A, A1 (1997)
  - Online database at http://durpdg.dur.ac.uk/HEPDATA

Theory vs experimental data

❑ Tevatron data:

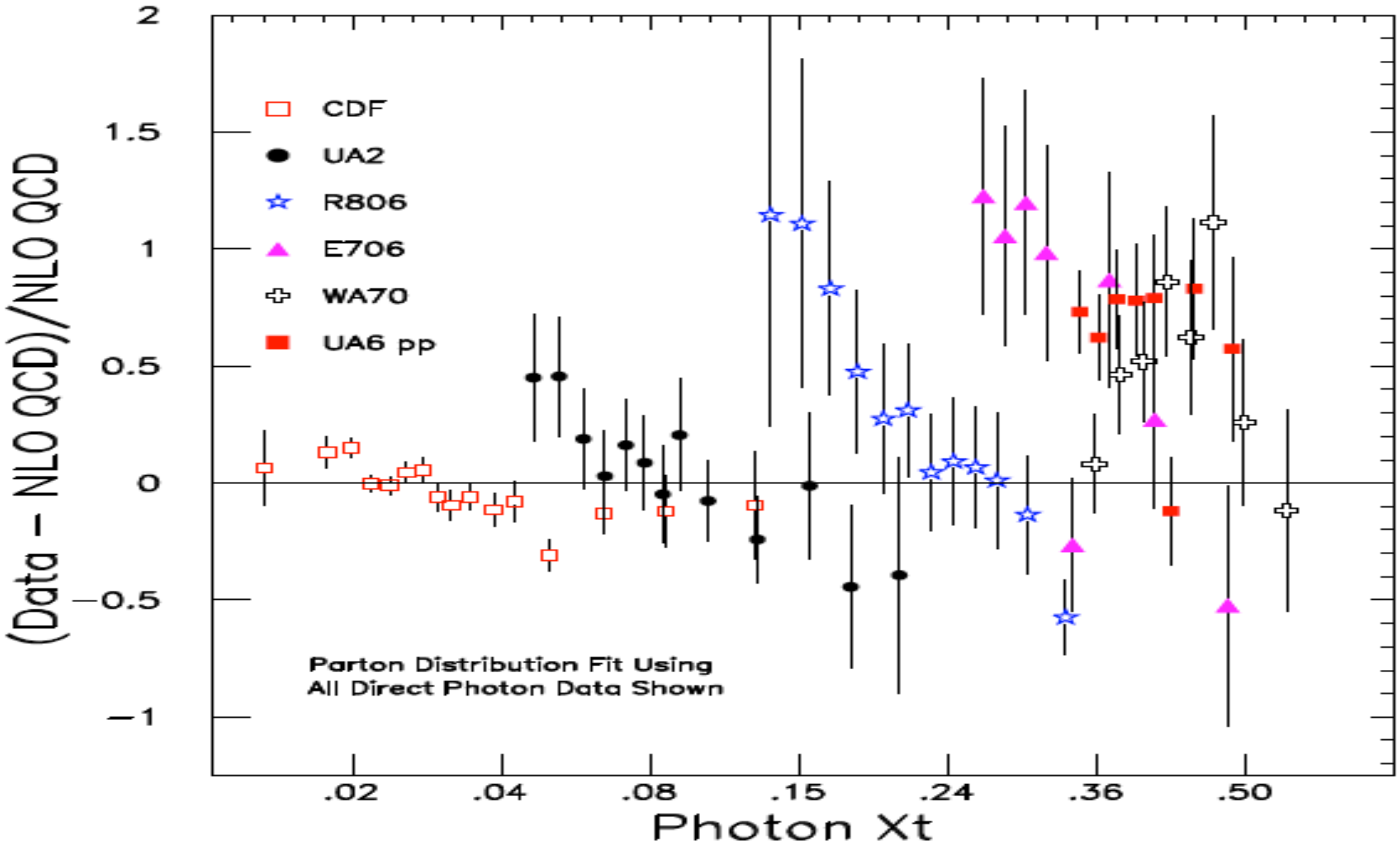


- ✧ Agreement looks good when plotted on a logarithmic scale
- ✧ QCD description of direct photon production works

Compare with data from different expt's

❑ CTEQ global analysis:

CTEQ Huston et al.



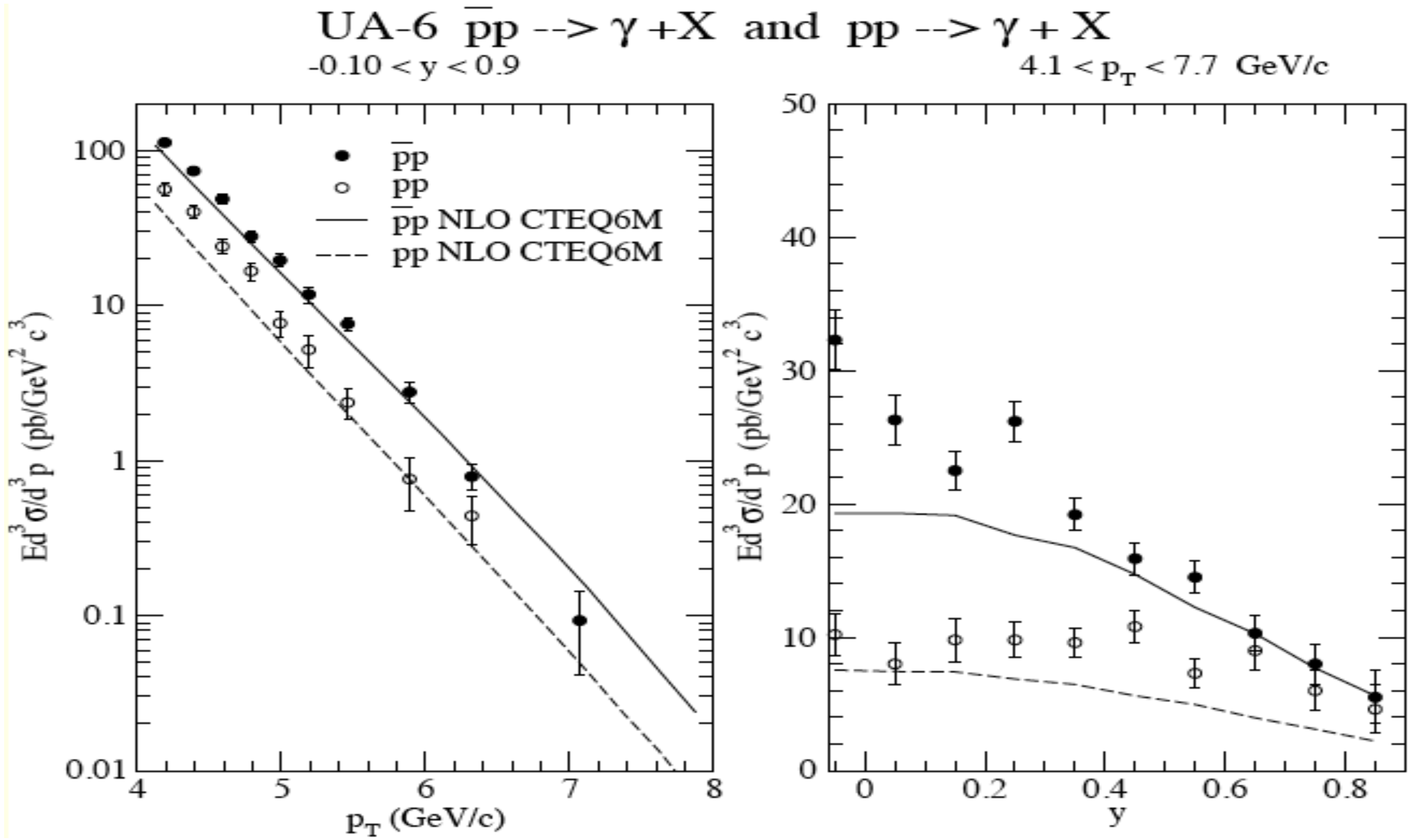
$$x_T = \frac{2p_T}{\sqrt{s}}$$

✧ Neither PDFs nor photon FFs can significantly improve the shape

✧ Direct photon data were excluded from most global fits

Experiments with both pp and $p\bar{p}$

❑ **UA6:** both pp and $\bar{p}p$ at $\sqrt{s} = 24.3$ GeV

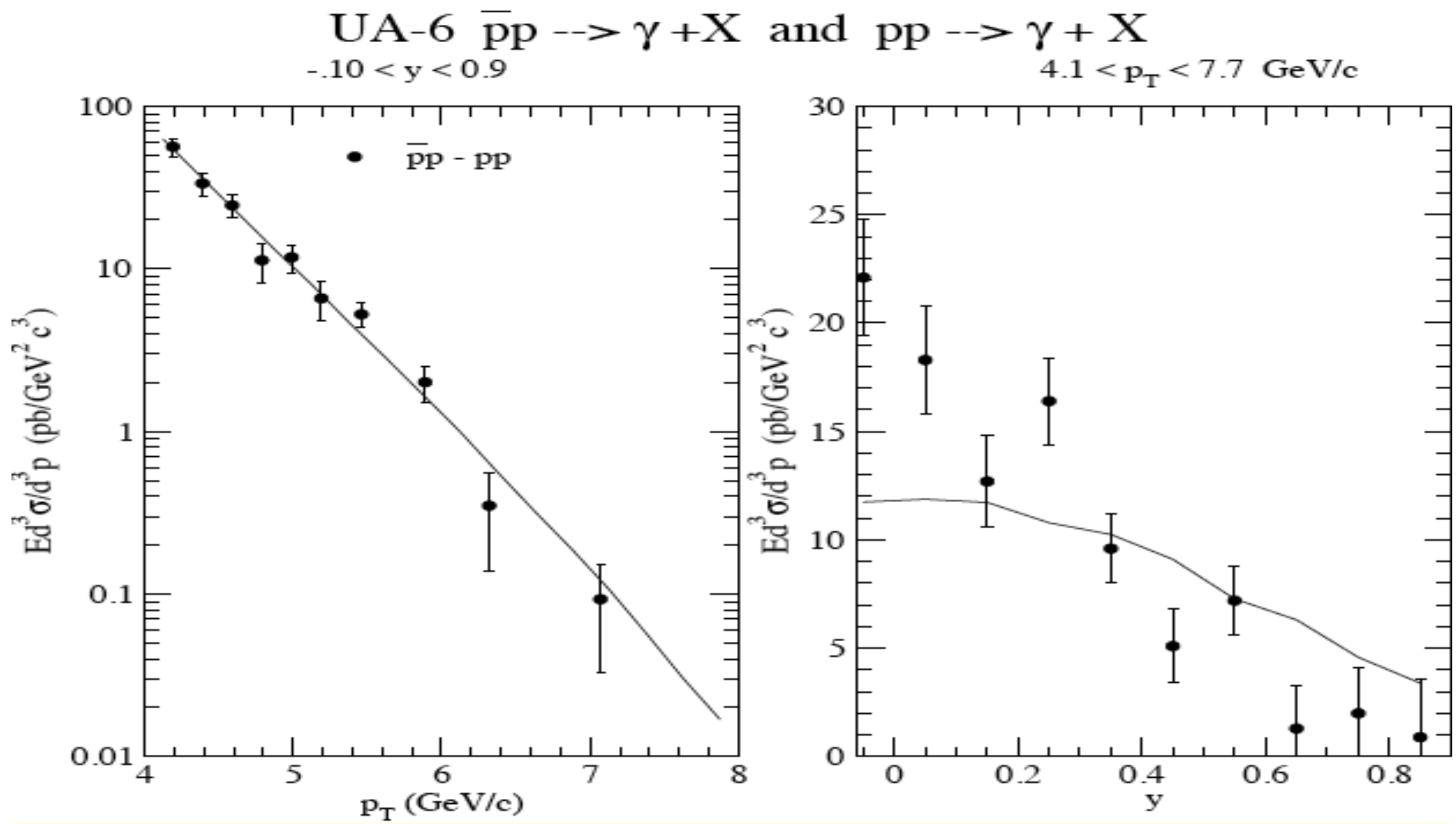


✧ **Theory curves are below the data**

✧ **Rapidity curves are flatter**

Role of gluon distribution?

❑ **UA6: $\bar{p}p$ - pp** both pp and $\bar{p}p$ at $\sqrt{s} = 24.3$ GeV

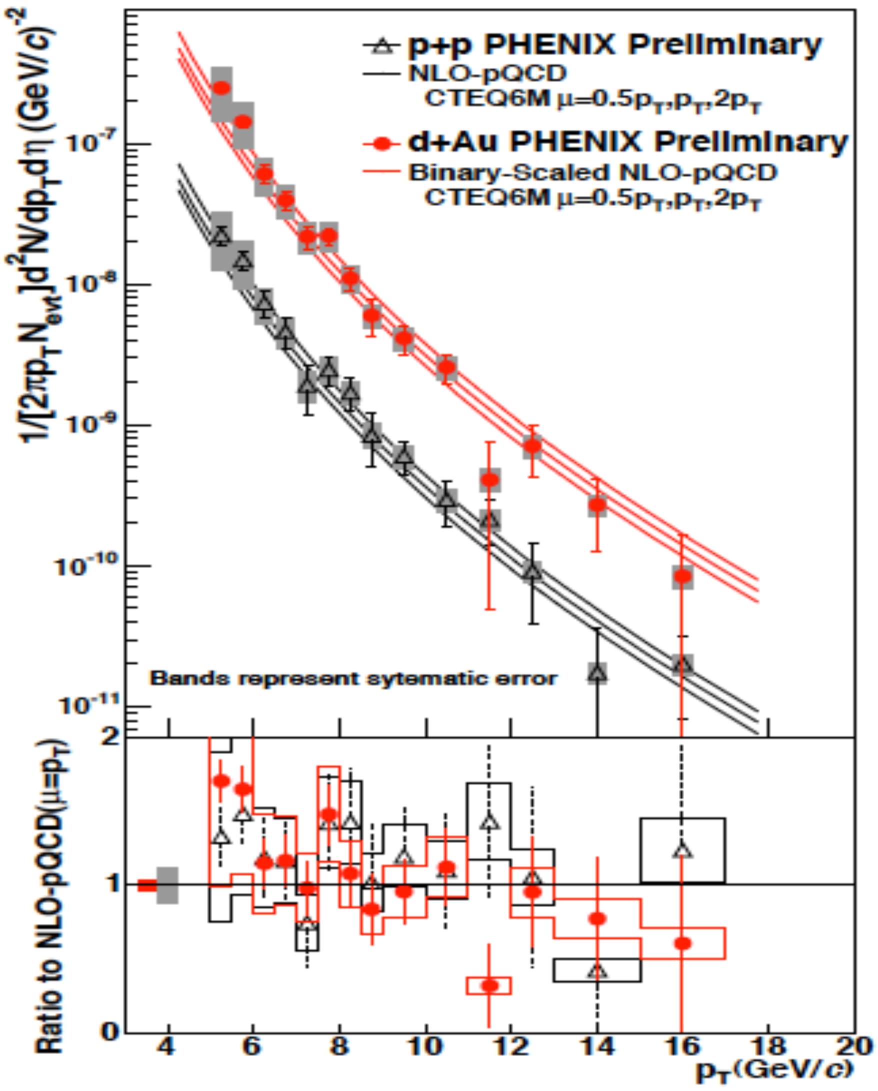


✧ **NO gluon contribution to the difference!**

✧ **Theory matches the data better – role of gluon?**

Theory works well at RHIC energy

PHENIX



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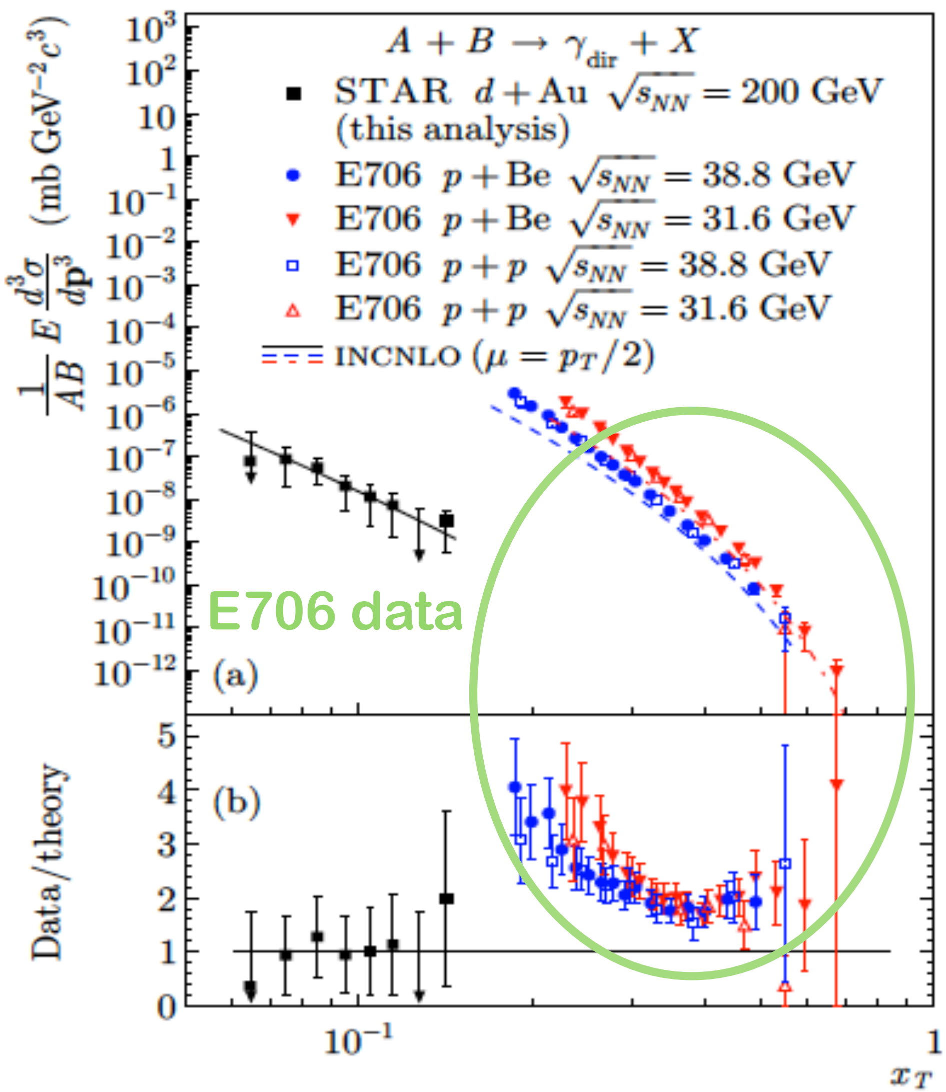
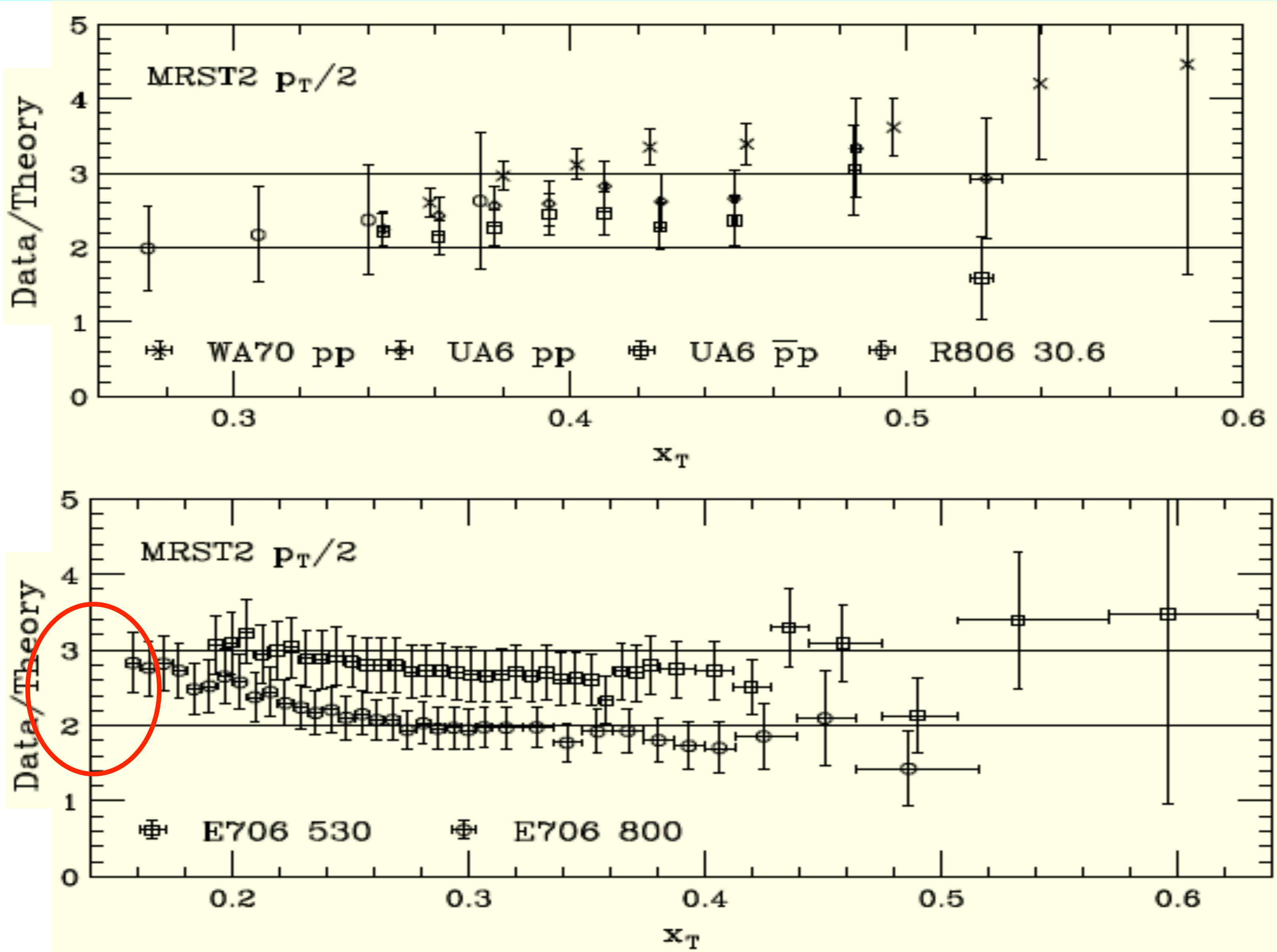
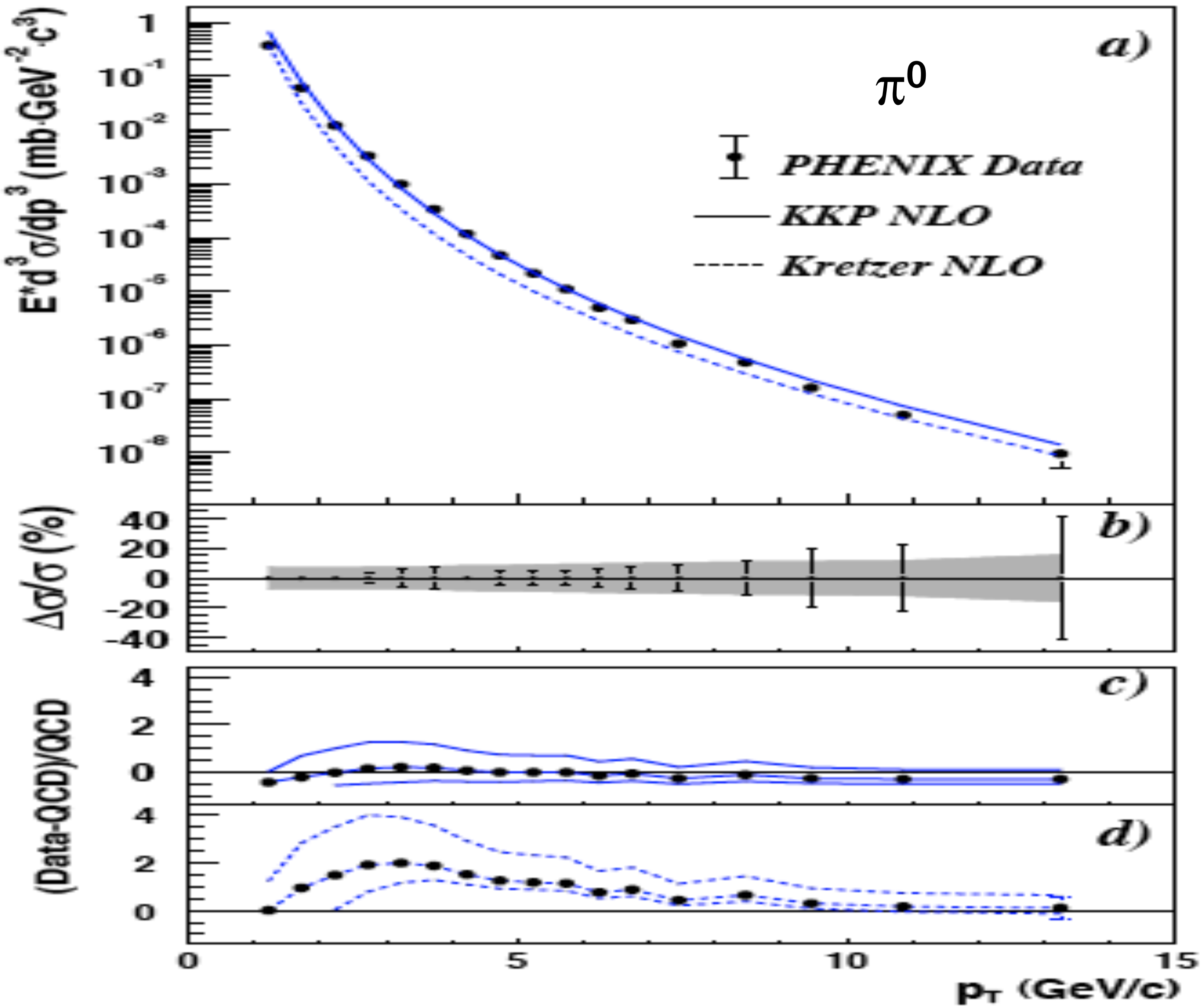


Same excess seen in π^0 production



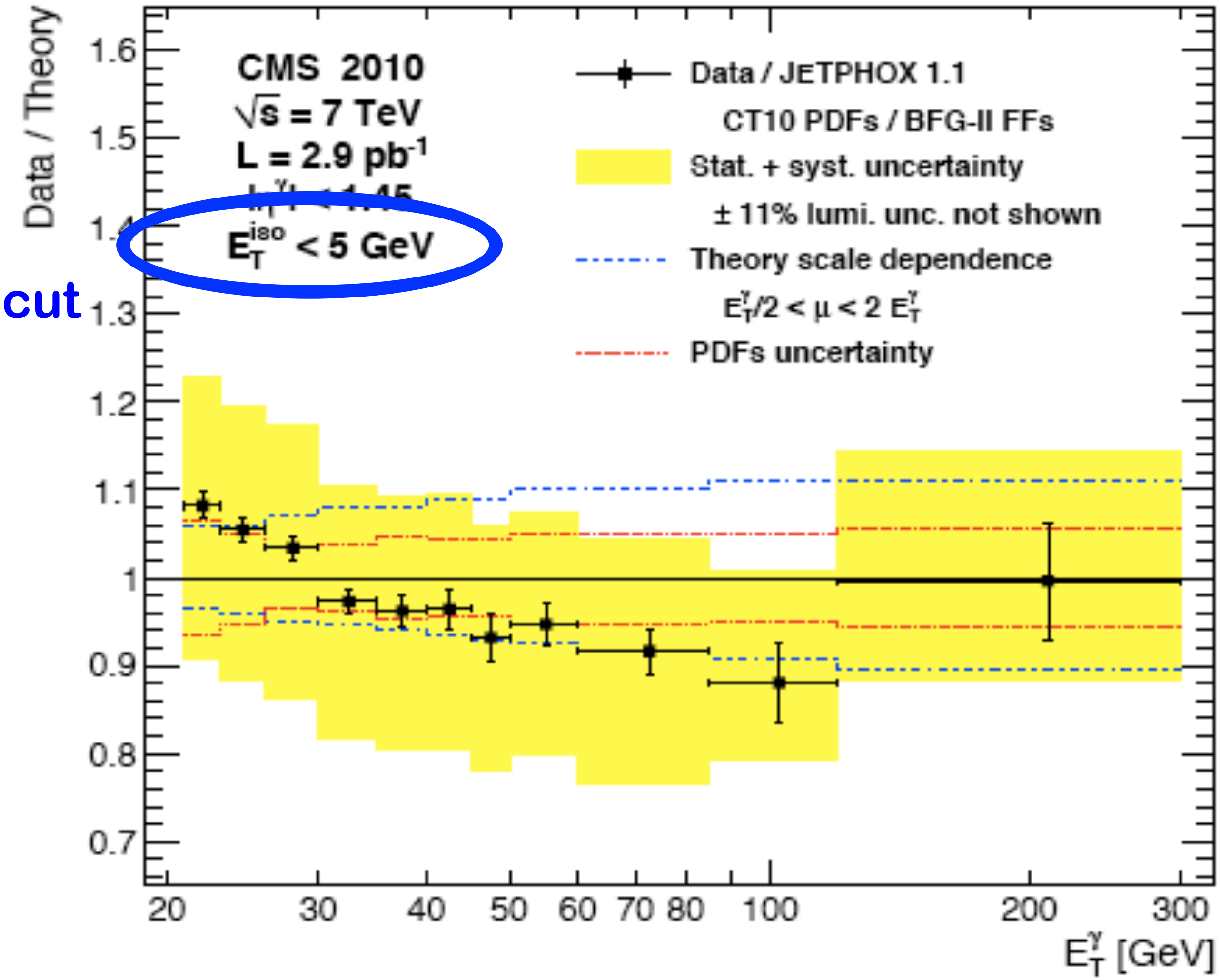
But, works at RHIC energy



How about at the LHC?

- CMS:

Isolation cut

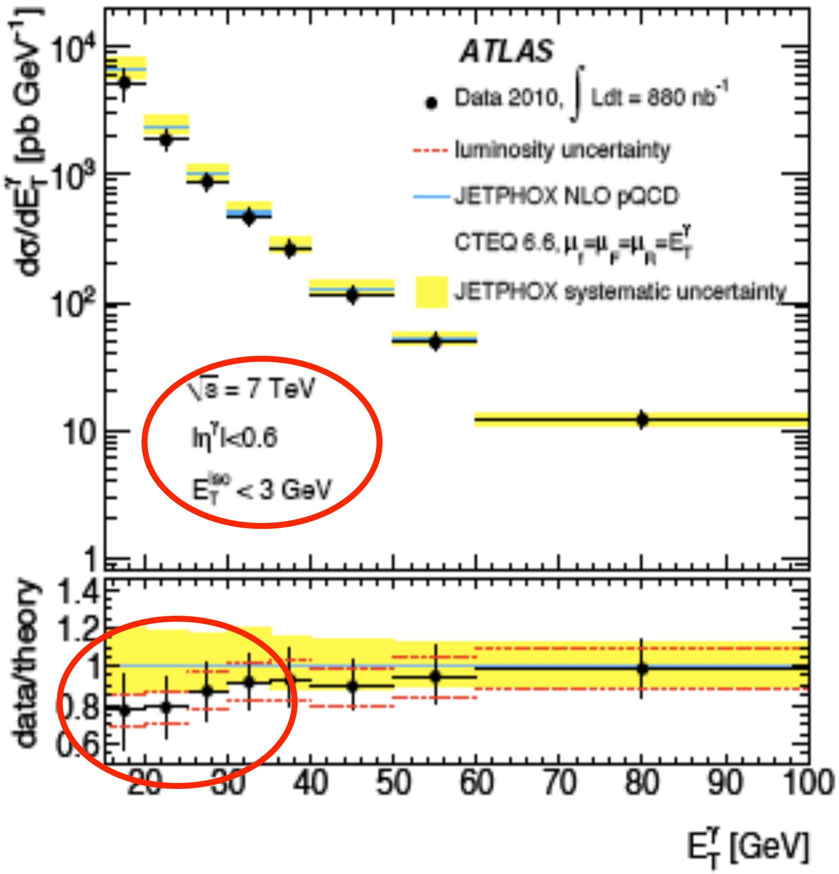


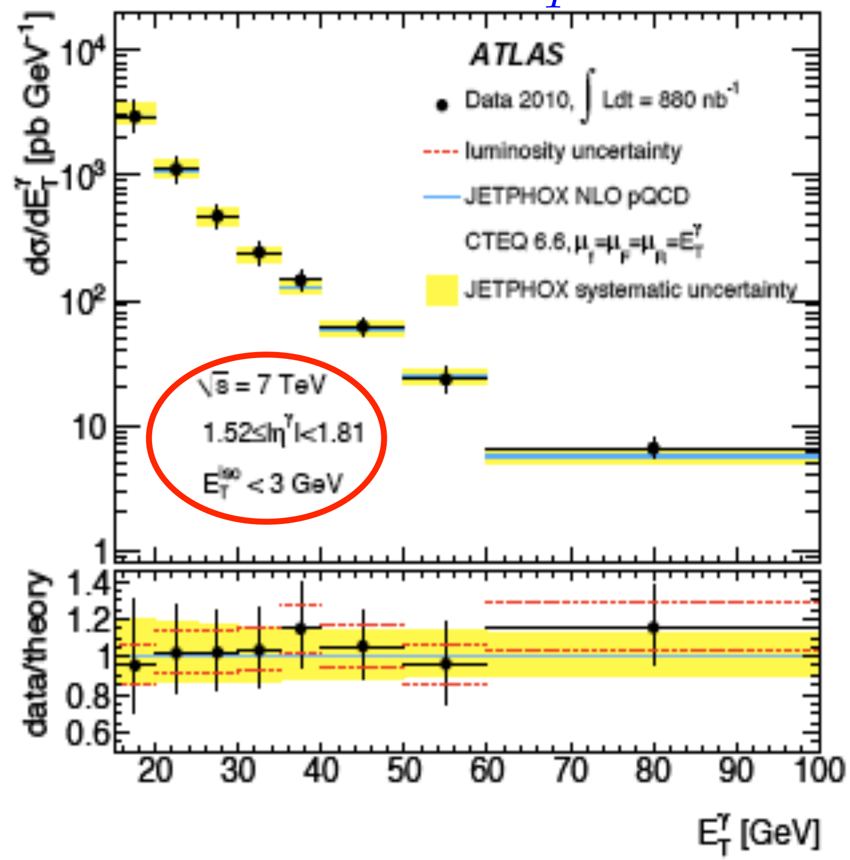
✧ Shape in x_T – within the PDF uncertainty?

Rapidity dependence at the LHC

❑ **ATLAS:**

Note: CMS has $E_T^{\rm iso} < 5$ GeV



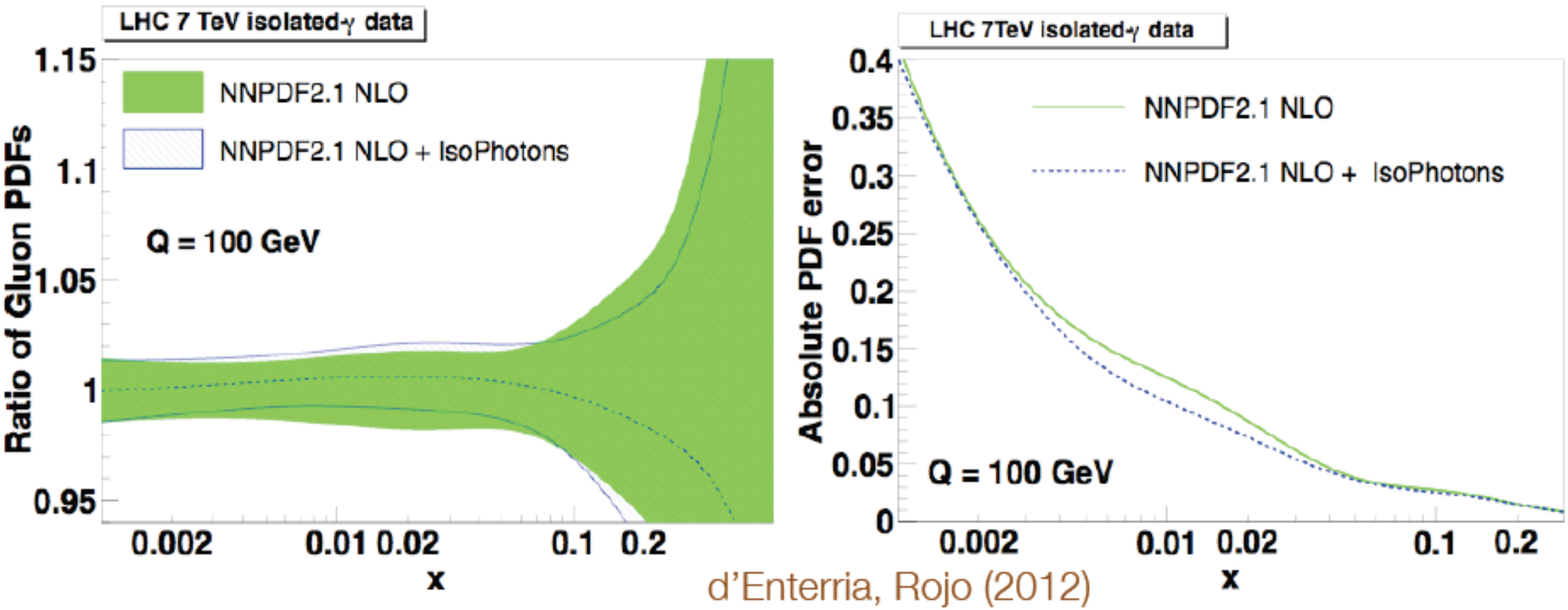


✧ **Data seems to be lower than theory at central** η^γ **and small** E_T^γ

Overall consistency is better at collider energies!

Role of direct photon in PDF fits

❑ Impact to NNPDF:



d'Enterria, Rojo (2012)

- ✧ Show slight improvement in gluon uncertainty
- ✧ Potential for improvement with more data from the LHC (gluon dominance)
- ✧ Some caveats:

Only at NLO – NNLO becoming the standard, nonperturbative FFs, …

Where do we stand?

- ❑ Agreement between theory and data improves with increasing energy and is excellent at $\sqrt{s}$ = 200 GeV

- ❑ Situation with fixed target direct photon data is confusing:
  - ✧ Disagreement between experiments
  - ✧ A reassessment of systematic errors on the existing fixed target photon experiments might help resolve the discrepancies

- ❑ We need an improved method of calculating single particle inclusive cross sections in the fixed target energy
  – Threshold resummation helps

- ❑ All experiments see an excess of data over theory at fixed target energies, but, less than theory at low pT at the LHC

More data from the LHC should help (the gluon dominance)!

Di-photon production

- **Principle background to Higgs production channel $H^0 \to \gamma\gamma$:**

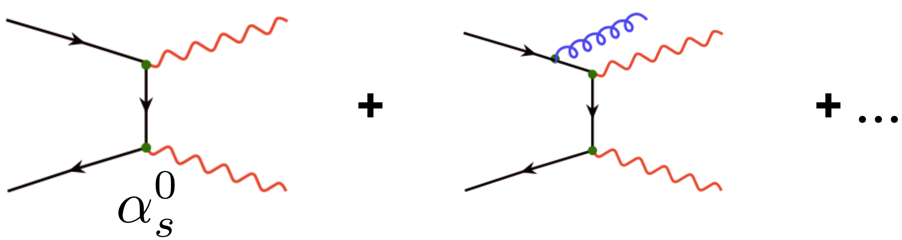
  Although the background is subtracted with a fitting procedure, it is also important to have some control of this process ab initio

- **Experimentally,**

  Significant contamination from the production of jets, or photon +jet, where jets are mis-identified as photons

  Jet production rate is so much higher photon, care is needed even with mis-identification rate as small as 10^{-4}!

- **Theoretically,**



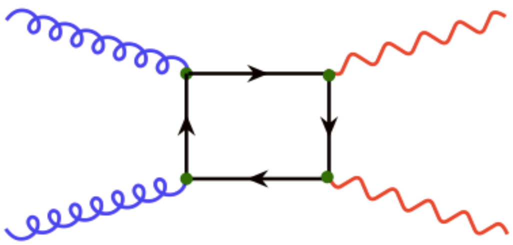
  Implementation of isolation cut with two photons

  Back-to-back kinematics – angular distribution – TMD factorization?

Di-photon production

❑ High order corrections:

- ✧ NLO corrections included in DIPHOX and MCFM
- ✧ A particular class of NNLO contributions is separately gauge-invariant, and, numerically important at the LHC – more gluons



Contribute at $\mathcal{O}(\alpha_s^2)$ to the x-section

NO tree-level $gg \to \gamma\gamma$

N³LO correction with NLO technology

- ✧ Contributes approximately 15-25% of the NLO total, depending on exact choice of photon cuts, scale choice, etc.
- ✧ TMD factorization vs collinear factorization?

Qiu et al. PRL 2011

$$\frac{d\sigma}{d^4 q_{\gamma\gamma} d\Omega_{\gamma\gamma}}$$

When $q_{T\gamma\gamma} \ll \sqrt{q^2_{\gamma\gamma}}$, or imposing photon pT cut

Linear polarized gluon impacts $\Omega_{\gamma\gamma}$ distribution

NNLO results

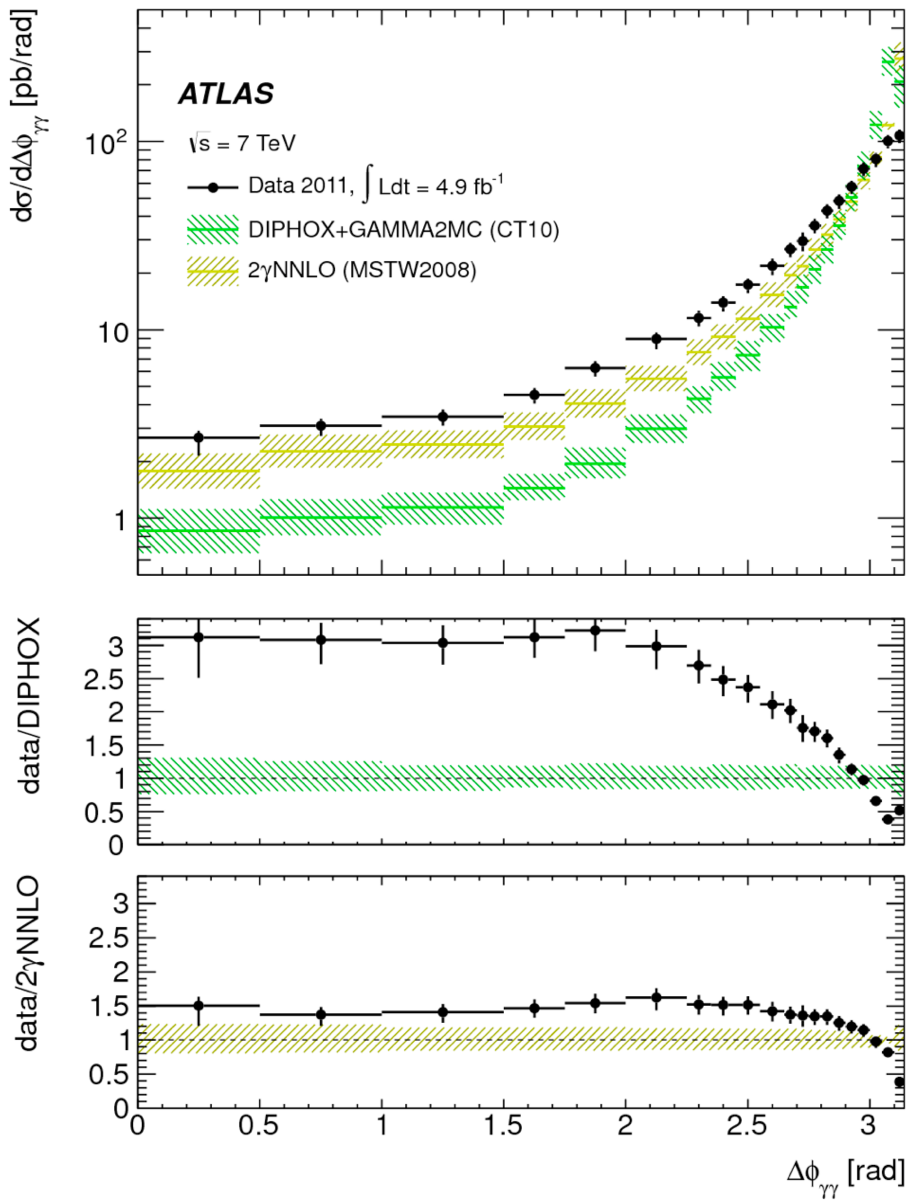
- Full NNLO calculation performed in the "Frixione" scheme, i.e. no need for fragmentation contributions

  Catani et al (2012)

- Better description of kinematic regions that are poorly described or inaccessible at NLO, e.g., azimuthal angle between photons

- Even better description would require either higher orders or inclusion in parton shower

  → not yet feasible.



Photon + jet angular distribution

- **QCD Compton and annihilation subprocess:**

$$\frac{d\sigma}{d\hat{t}} \sim (1-\cos(\theta^*))^{-1} \quad \text{as} \quad \cos(\theta^*) \to 1$$

- **Other QCD subprocess,** $qq \to qq, qg \to qg, gg \to gg,$ etc. **more relevant to jet+jet angular distribution:**

$$\frac{d\sigma}{d\hat{t}} \sim (1-\cos(\theta^*))^{-2}$$

$$\text{as} \;\; \cos(\theta^*) \to 1$$

- **Prediction:**

Photon-jet angular distribution should be flatter than that observed in jet-jet final states

$$\cos(\theta^*) = \tanh\left(\frac{\eta_\gamma - \eta_{jet}}{2}\right)$$

Photon + jet angular distribution

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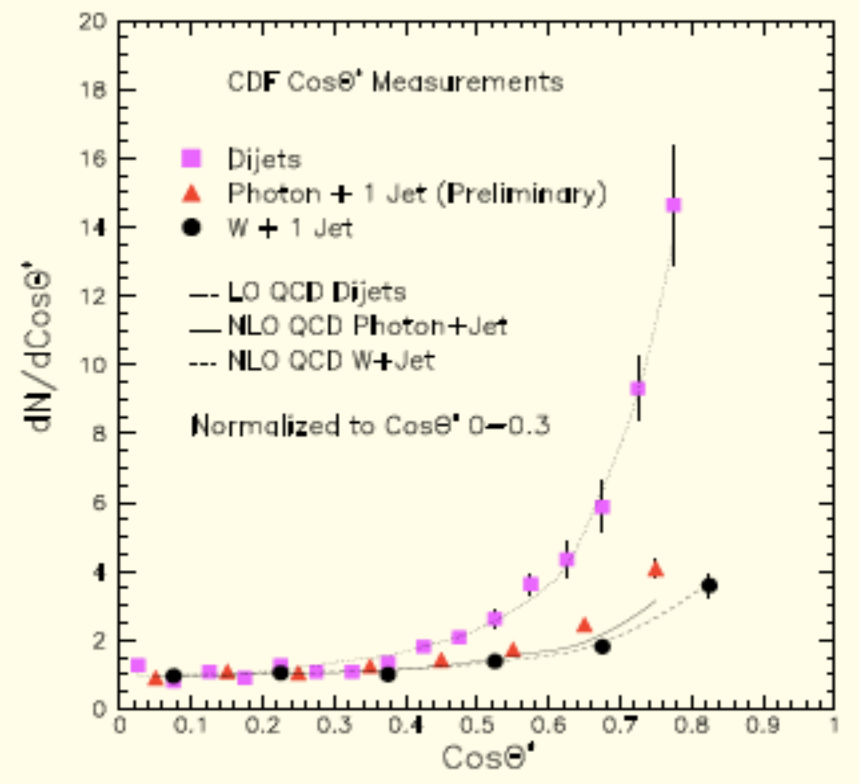
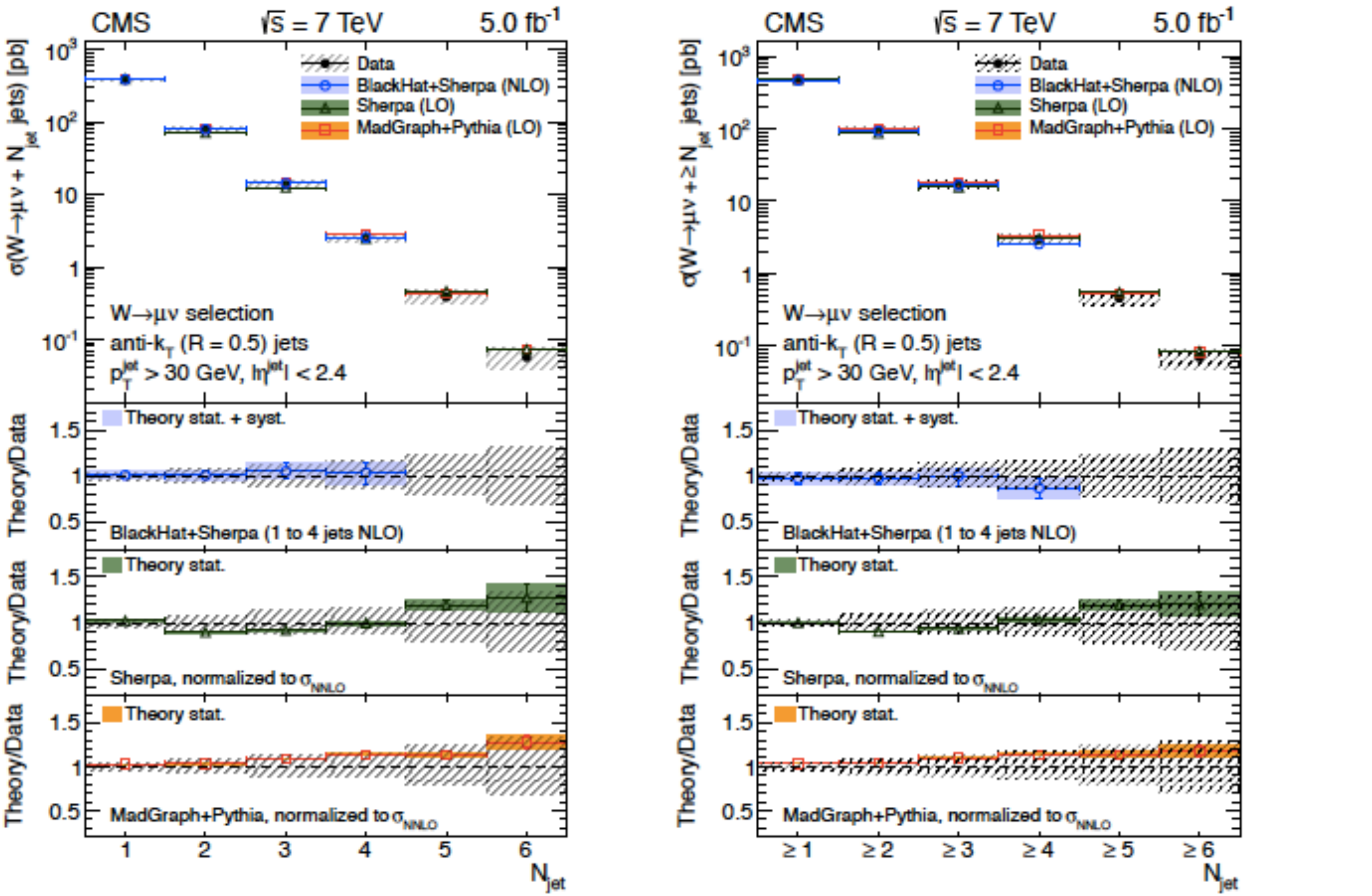


W-boson + jets

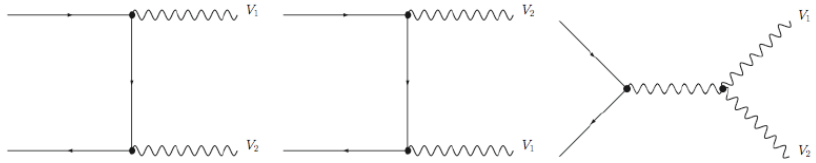
CMS – 1406.7533

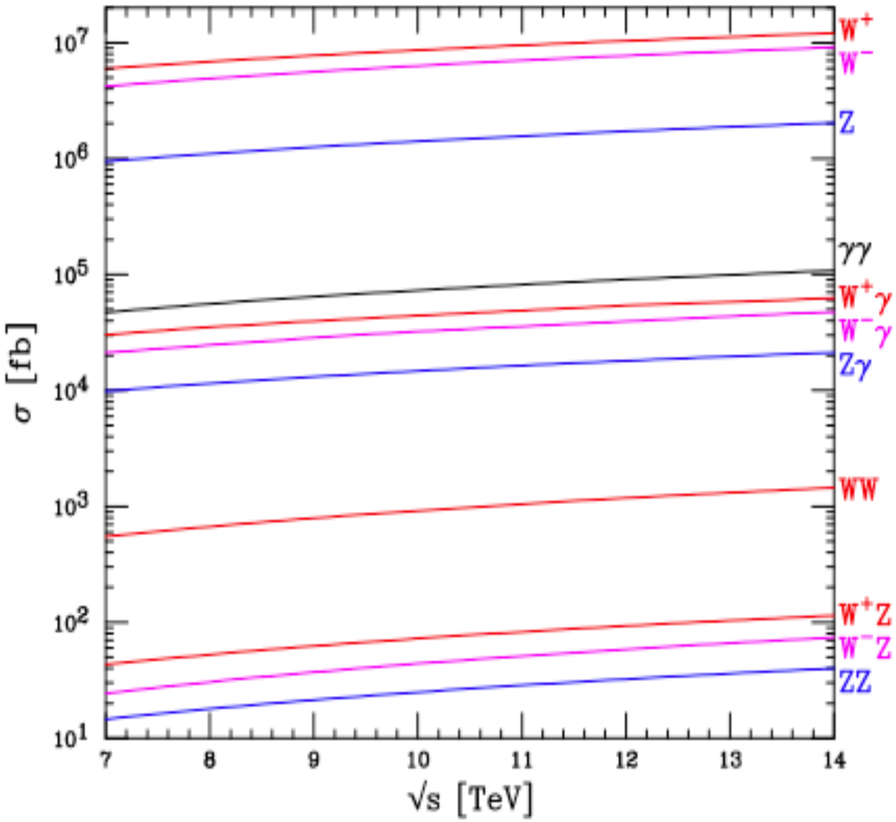


Di-boson hadronic production

Campbell, CTEQ SS2013

❑ Triple gauge boson interaction:



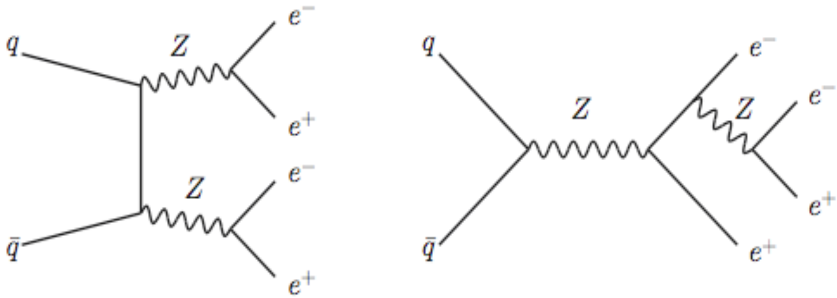


- ✧ Triple gauge coupling present for all processes except Z γ
- ✧ Processes involving photons dependent on photon pT (and rapidity) cut, strongly
- ✧ NLO corrections known analytically, included in MCFM, VBFNLO (also POWHEG NLO MC)

Two bosons with single-resonant

Campbell, CTEQ SS2013

- **Two Z's:** $q\bar{q} \to ZZ \to e^+e^-e^+e^-$

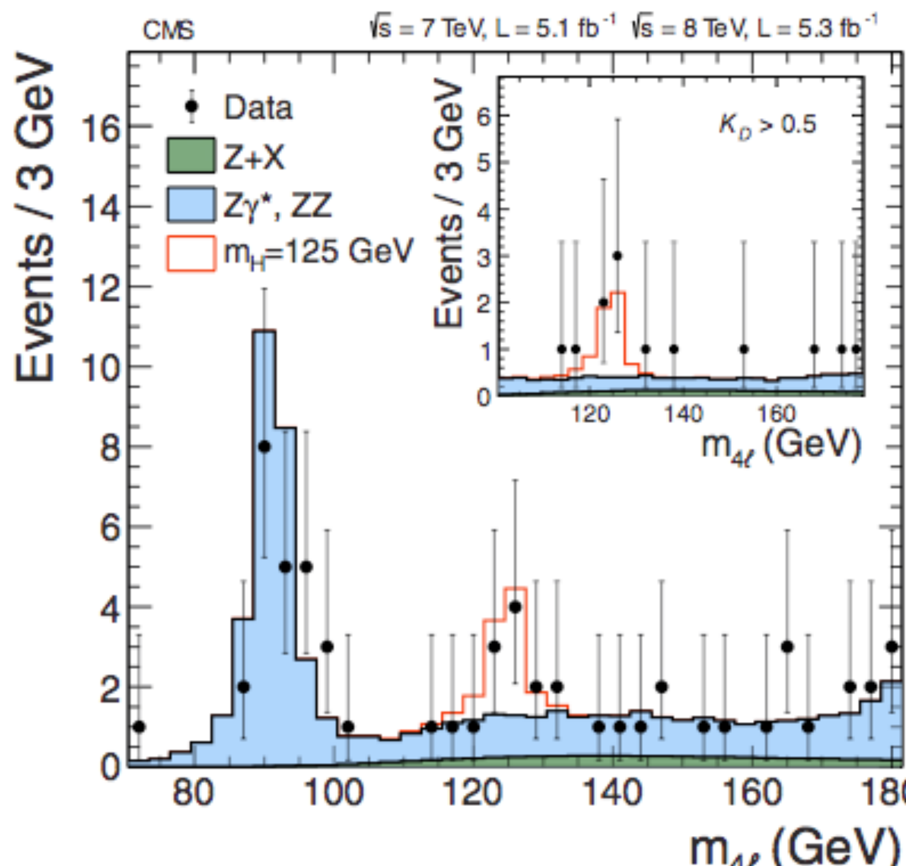


"double"-resonant

"single"-resonant

Plus diagrams with Z replaced by photon

- ✧ Inclusive cross section is dominated by the double-resonant contribution
- ✧ Notably: invariant mass of 4 leptons
- ✧ One of the cross-checks in Higgs search



W+photon – Radiation Zero

Campbell, CTEQ SS2013

- W+photon amplitude:

$$\mathcal{M}_{\bar{u}(p_1)+d(p_2)\to W^+ +\gamma(p_3)} \propto \left(Q_u + Q_d \frac{p_1 \cdot p_3}{p_2 \cdot p_3}\right)$$

In c.m. frame: $p_2 \cdot p_3 \propto (1+\cos\theta^*)$ $\quad p_1 \cdot p_3 \propto (1-\cos\theta^*)$

$$\mathcal{M}_{\bar{u}(p_1)+d(p_2)\to W^+ +\gamma(p_3)} \propto Q_u(1+\cos\theta^*) + Q_d(1-\cos\theta^*)$$

Amplitude vanishes if $\cos\theta^* = \frac{Q_u + Q_d}{Q_d - Q_u} = -\frac{1}{3}$

(Independent of photon energy)

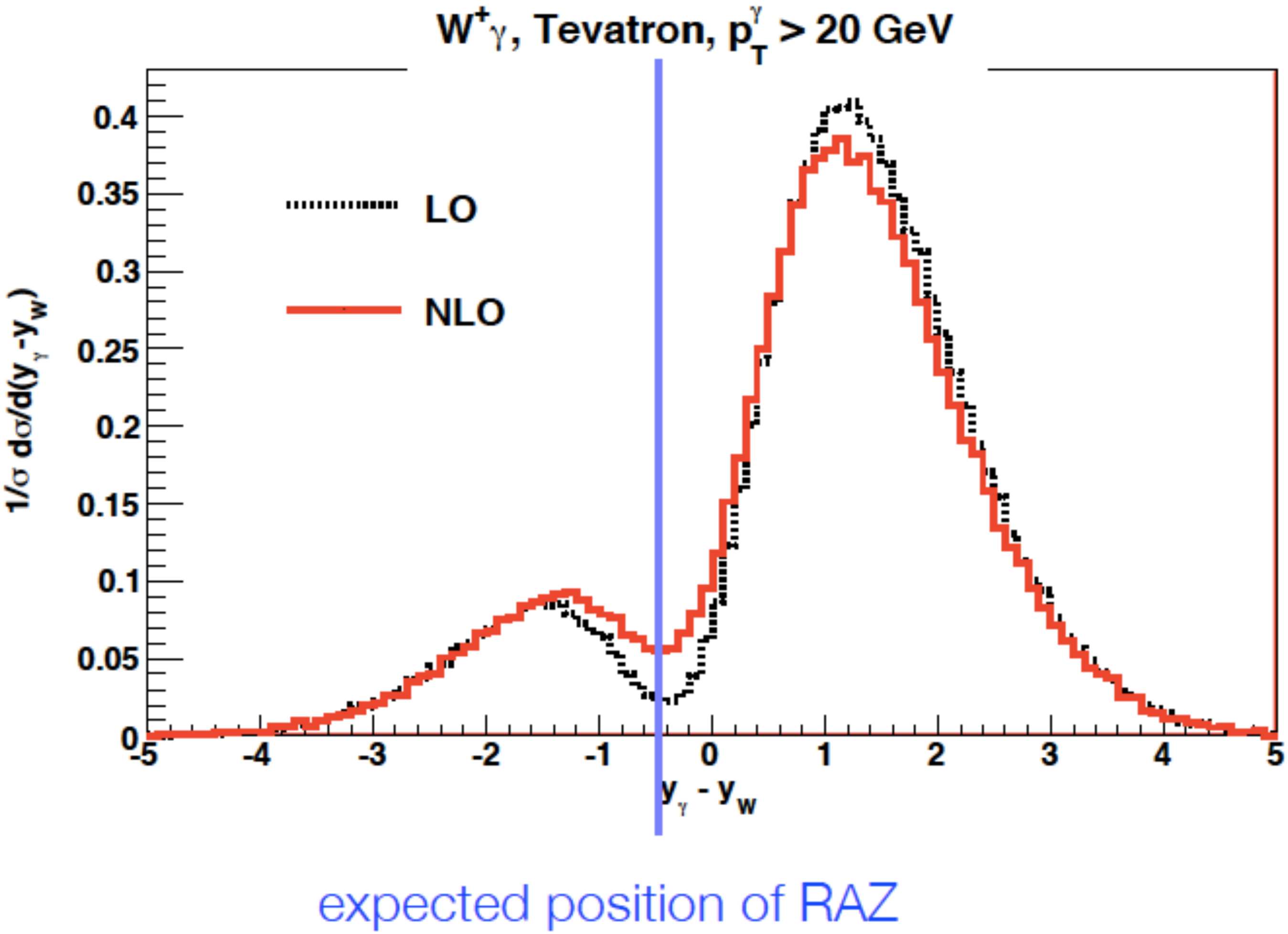
- "Radiation amplitude zero" (RAZ):

General feature of photon in multi-boson processes

  - Result of interference between diagrams
  - Corresponding photon rapidity: $y^*_\gamma = \frac{1}{2}\ln\left(\frac{1+\cos\theta^*}{1-\cos\theta^*}\right) \approx -0.35$
  - Boost invariant rapidity difference: $\Delta y^* = y^*_\gamma - y^*_W$

In c.m. frame: $y^\star_W \approx \frac{1}{2}\log\left(\frac{m_W - p_T^\gamma \cos\theta^\star}{m_W + p_T^\gamma \cos\theta^\star}\right)$ when photon pT << m_W

Effect of PDFs for RAZ

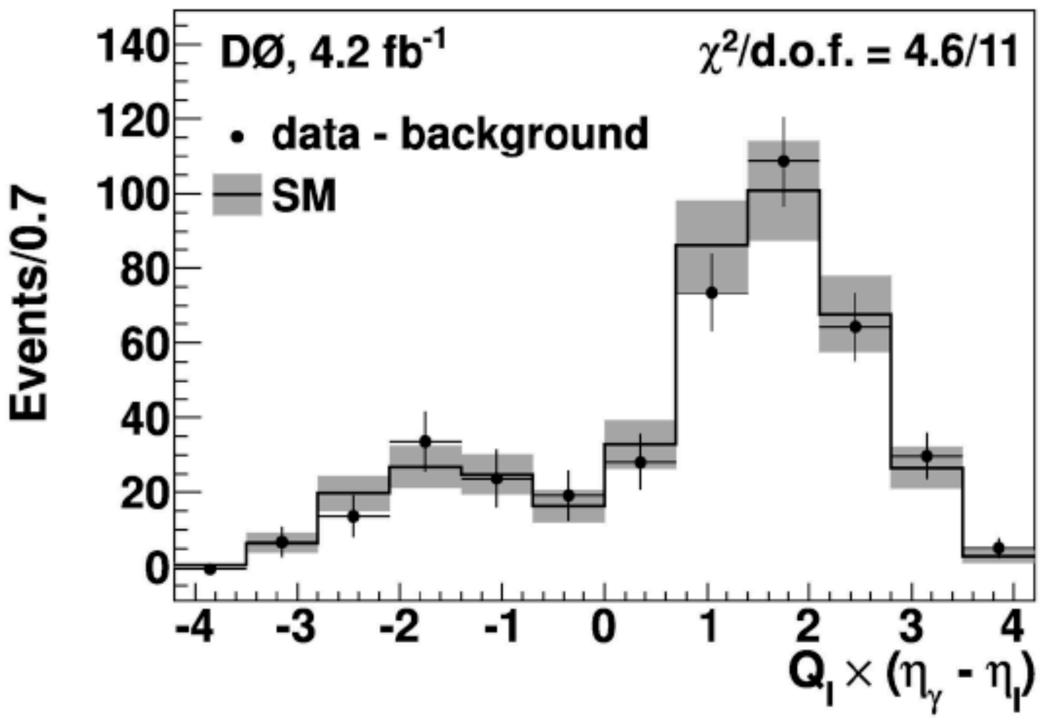


Amplitude zero a feature of the LO amplitude only
→ partially washed out at higher orders

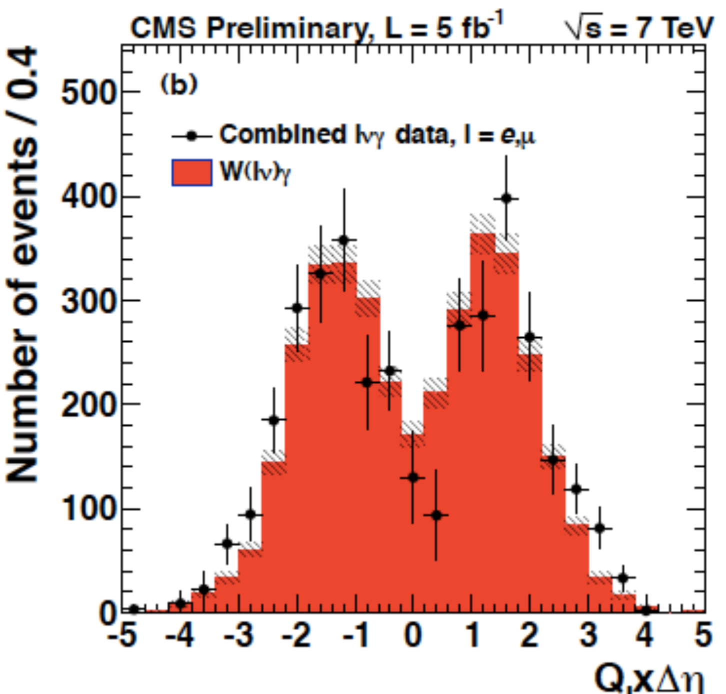
Experimental evidence for RAZ

- Experimental issues that wash out dip:
  - ✧ use of lepton rapidity rather than reconstructing W (retains most information)
  - ✧ contamination from photon radiation in W decay

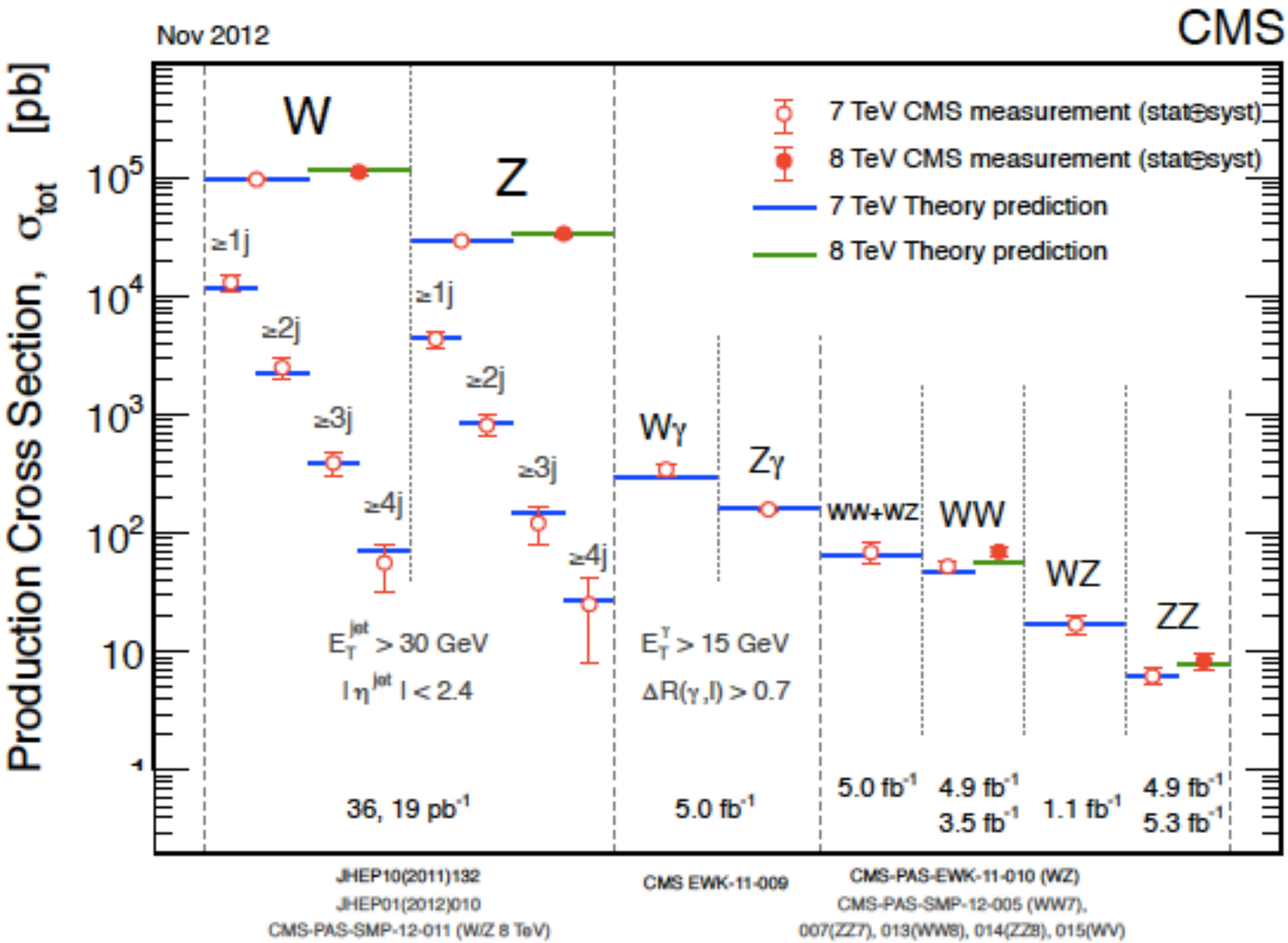
D0, arXiv 1109.4432



CMS, PAS-EWK-11-009

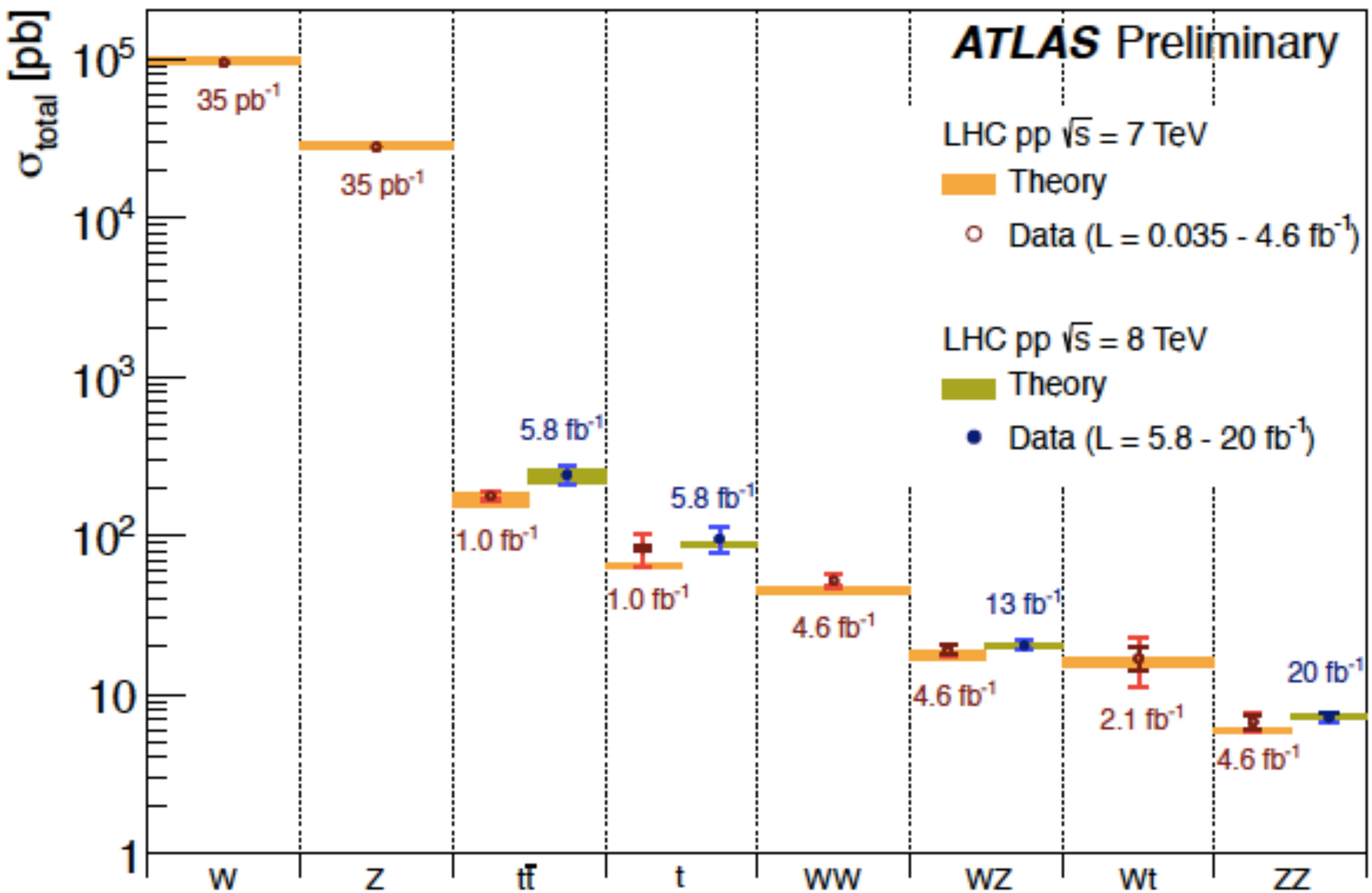


Vector bosons: experimental summary



Good consistency with theory expectations of NNLO (W/Z), and NLO (di-bosons) for all processes in both experiments

Vector bosons: experimental summary



Good consistency with theory expectations of NNLO (W/Z), and NLO (di-bosons) for all processes in both experiments

Thank you
for
your attention!

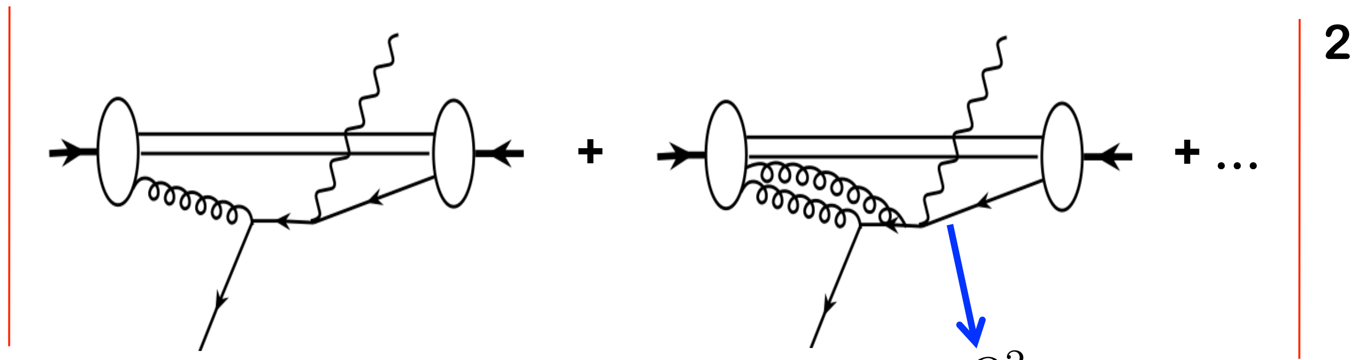
Please feel free to ask me questions
jqiu@bnl/gov

Backup slides

Factorization is an approximation

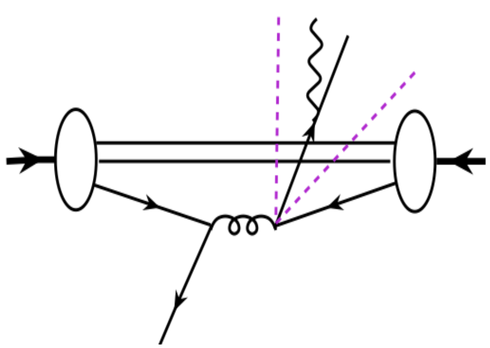
❑ Multiple scattering and power correction:

$\sigma(P_T) \sim$



$$\propto \hat{\sigma}(P_T, x_1, x_2, \mu) \otimes \phi(x_1, \mu) \otimes \phi(x_2, \mu) + \mathcal{O}\left(\frac{Q_s^2}{p_T^2}\right)$$

❑ Fragmentation function and isolation cut:



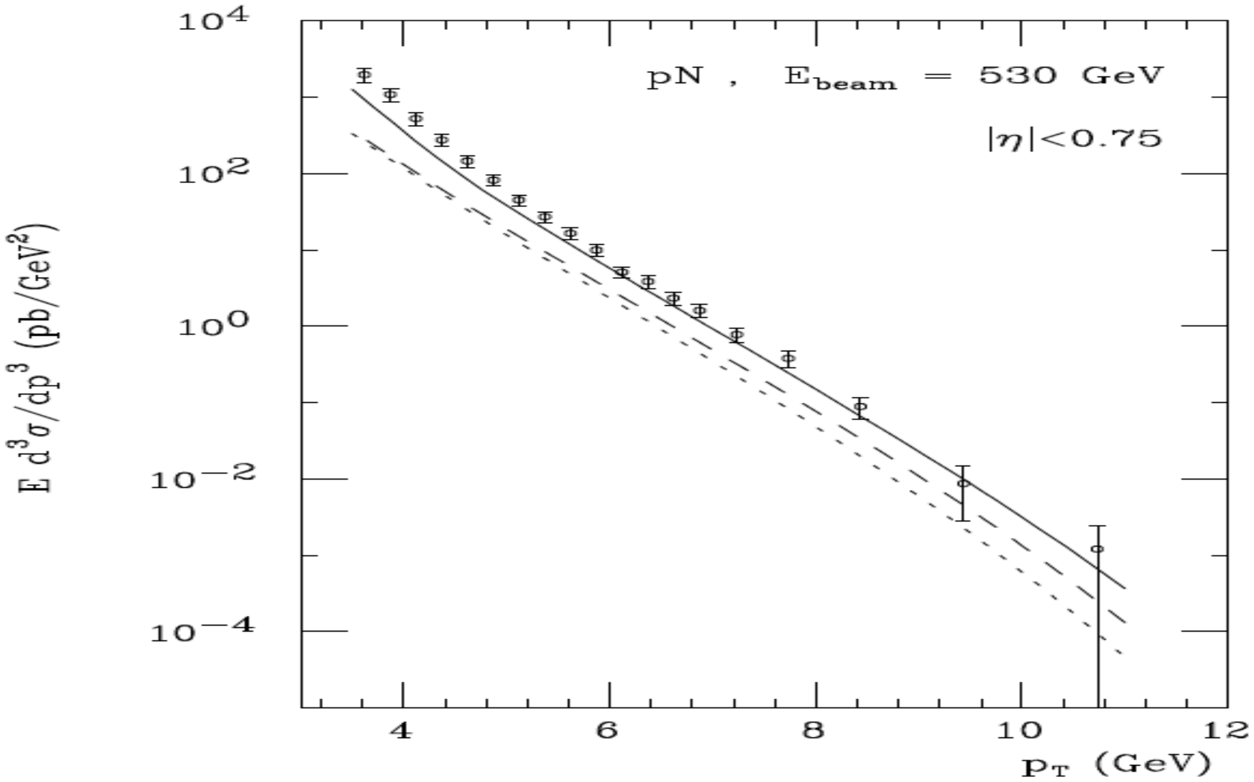
$$\sigma(P_T) \quad \propto \quad \hat{\sigma}(P_T, x_1, x_2, \mu) \otimes \phi(x_1, \mu) \otimes \phi(x_2, \mu) \otimes D(z)$$
$$+ \quad \mathcal{O}\left(\frac{Q_s^2}{p_T^2}\right)$$

Note: $\ln(R)$ **Cone size cannot be too small**

$\ln(E_h/E_\gamma)$ → E_h/E_γ **Not too small**

Threshold resummation could help

- Threshold resummation – rate at fixed target energy:

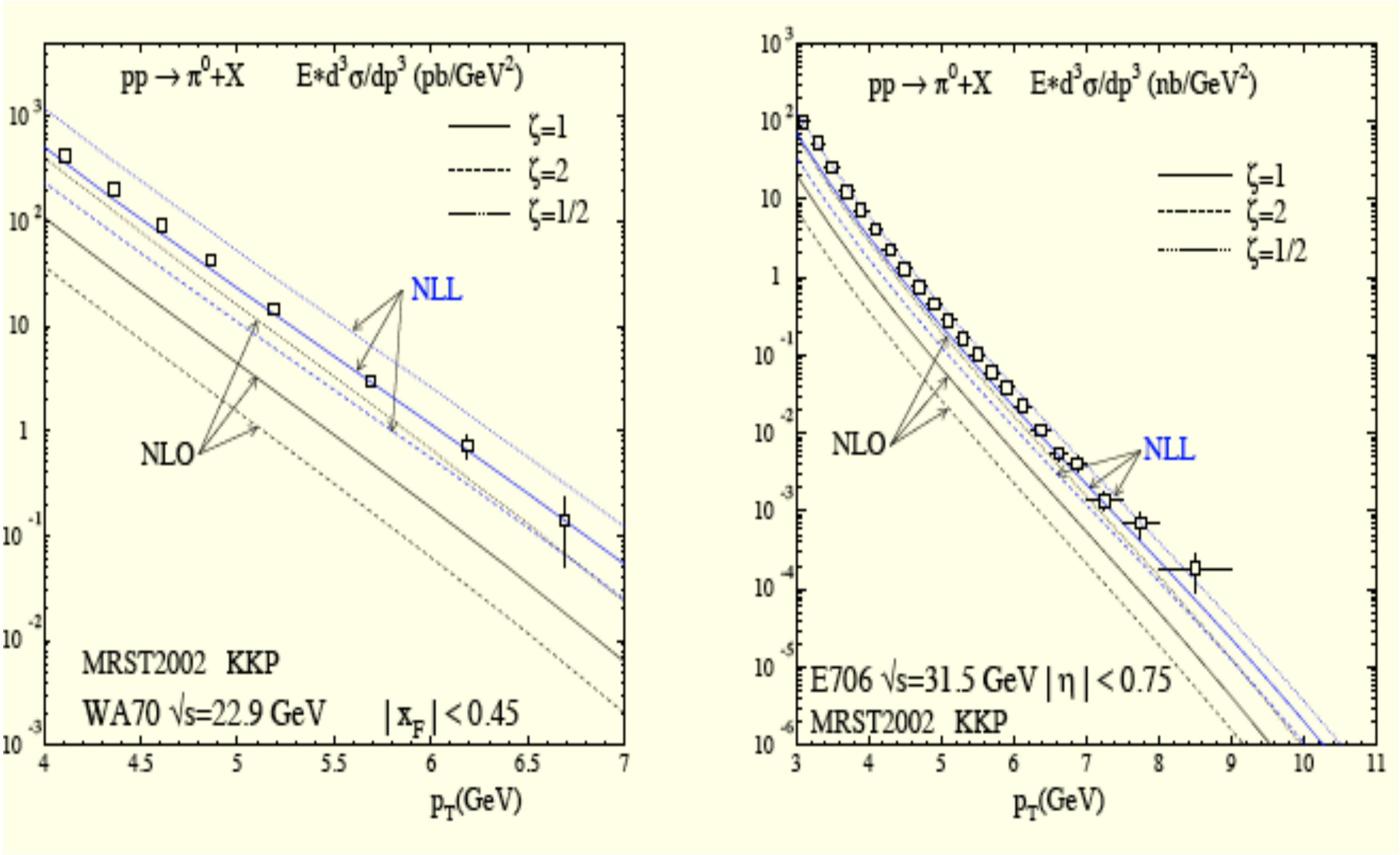


Laenen, Sterman, Vogelsang, 2008

CTEQ Huston et al.

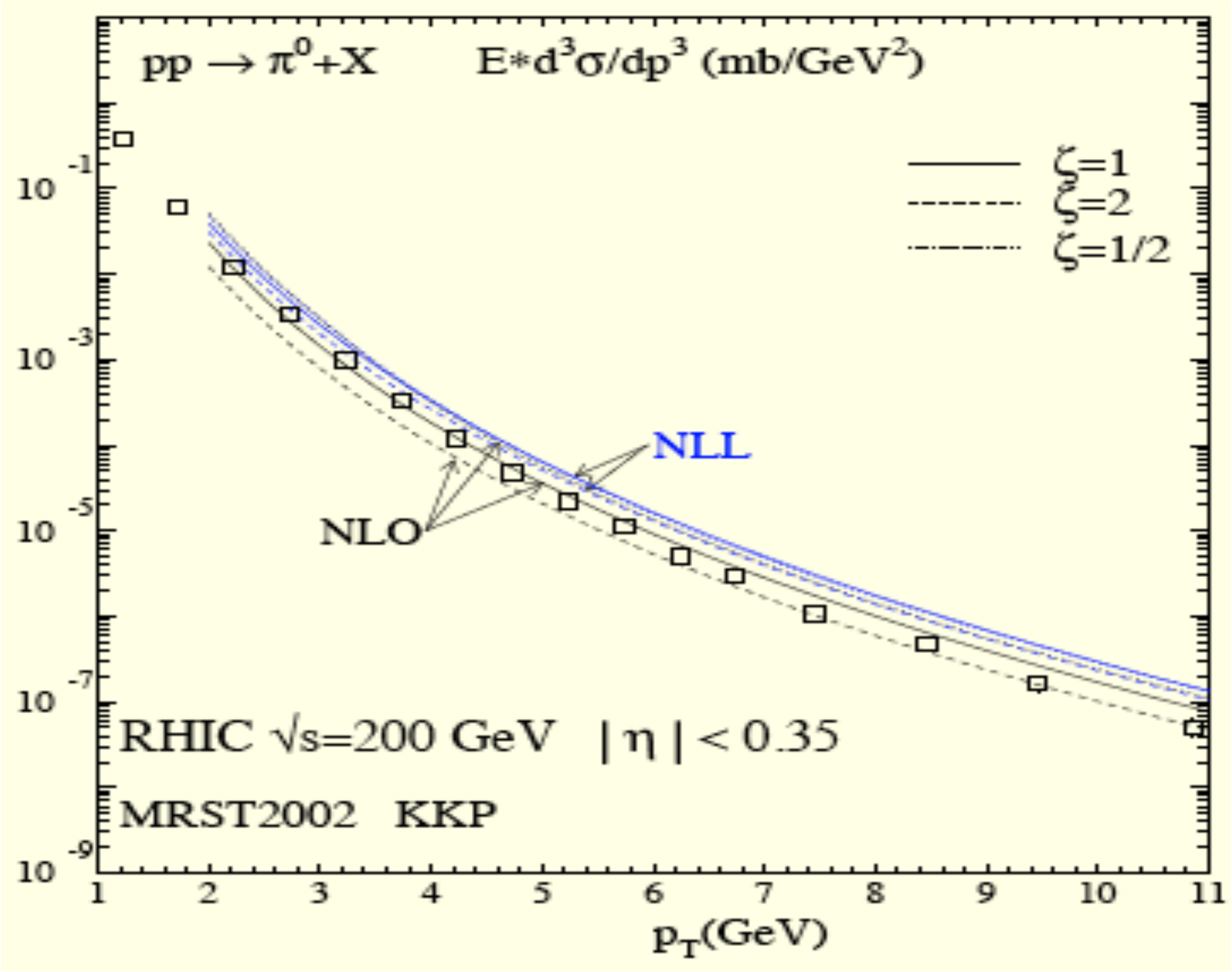
- Intrinsic k_T – x_T dependence at fixed target energy:
  - ✧ Mimic the resummation of initial-state gluon shower
  - ✧ Large effect on a steep falling P_T distribution

Resummation helps π^0 cross section too



de Florian and Vogelsang, hep-ph/0501258

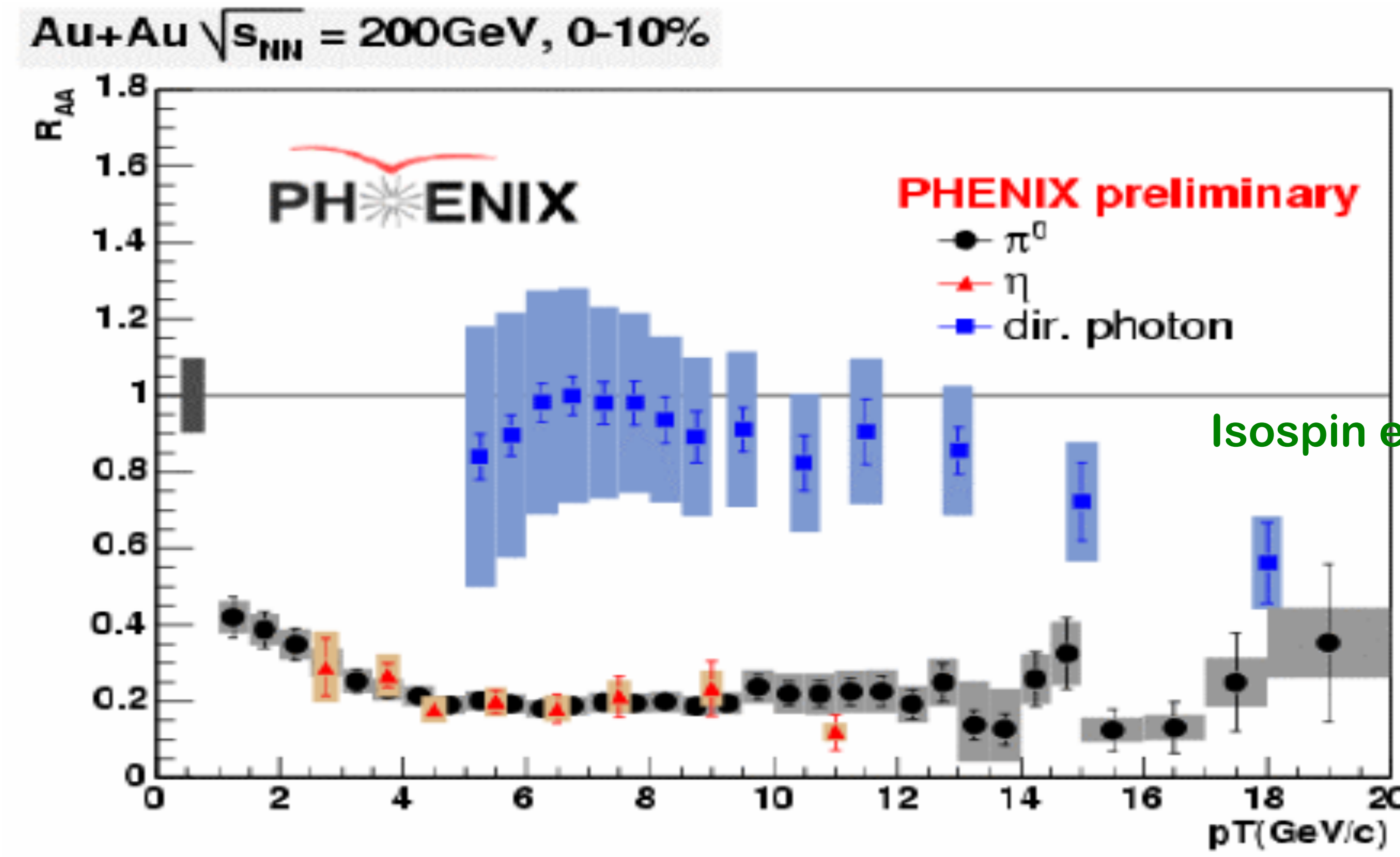
What happens at RHIC energy?



Reduced enhancement at RHIC energies than fixed target energies

Photon can penetrate the medium

- Photon tells the history:



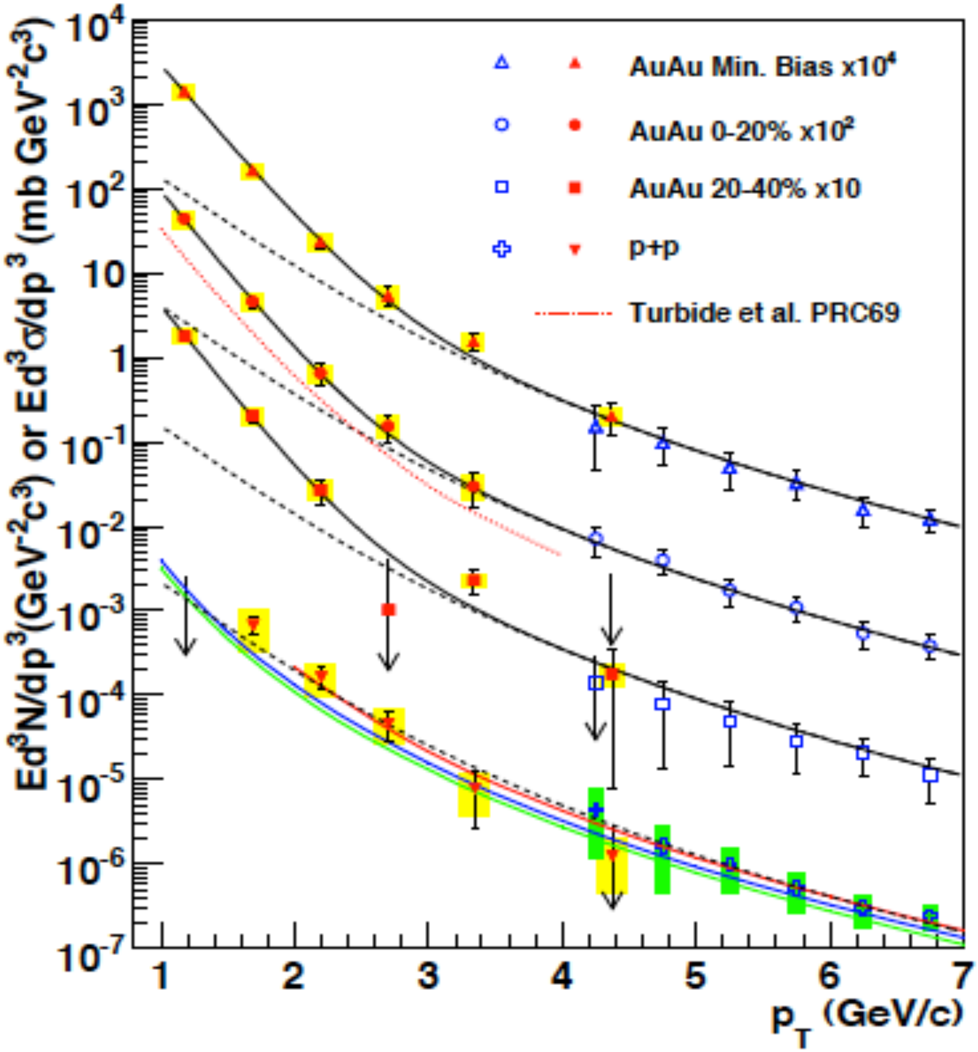
Isospin effect

High P_T photon penetrates the medium without suppression

“Photon” at low p_T in Au-Au collisions

❑ Low mass e^+e^- pairs ⟶ direct photon production:

arXiv:0804.4168 (PRL in press)



$$\frac{d^2 n_{ee}}{dm_{ee}} = \frac{2\alpha}{3\pi}\frac{1}{m_{ee}}\sqrt{1-\frac{4m_e^2}{m_{ee}^2}}\left(1+\frac{2m_e^2}{m_{ee}^2}\right) S dn_\gamma$$

S : process dependent factor

$\sqrt{s} = 200$ GeV

$m_{ee} < 0.3$ GeV/c

$1 < p_T < 5$ GeV/c

Difference pp vs AA – thermal photon

➡ Temperature

$T = 221 \pm 19^{\mathrm{stat}} \pm 19^{\mathrm{syst}}$ MeV

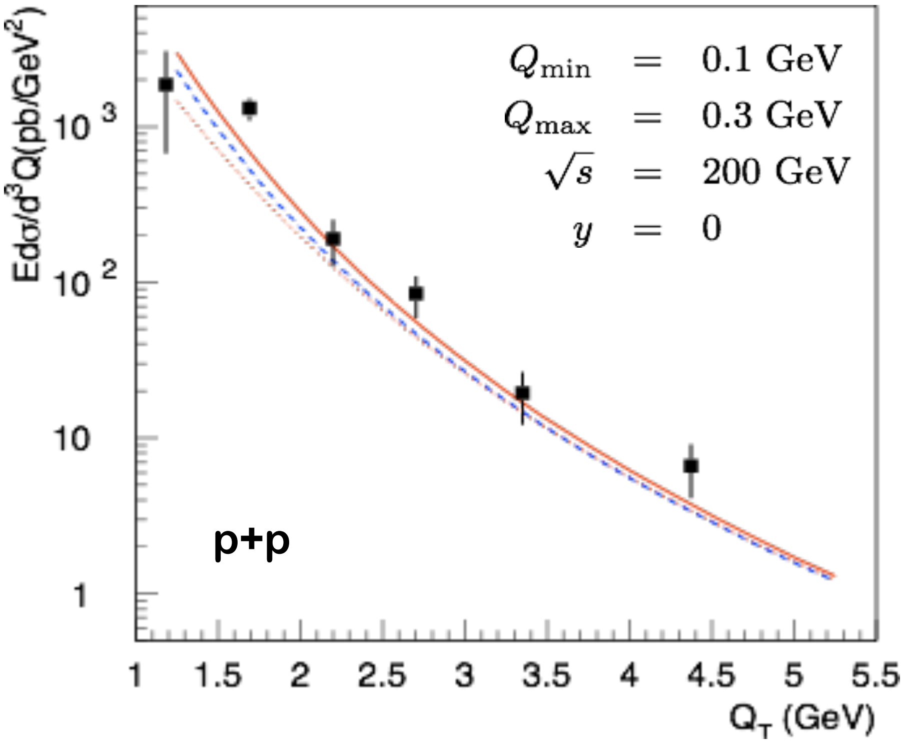
Invariant cross section in pp collision

Kang, Qiu, Vogelsang, PRD 2009

❑ **Definition:**

$$E\frac{d\sigma_{AB\to\ell^+\ell^-(Q)X}}{d^3Q} \equiv \int_{Q^2_{\rm min}}^{Q^2_{\rm max}} dQ^2\,\frac{1}{\pi}\,\frac{d\sigma_{AB\to\ell^+\ell^-(Q)X}}{dQ^2\,dQ_T^2\,dy}$$

❑ **Role of non-perturbative fragmentation function:**



Data from PHENIX: arXiv:0804.4168

✧ **Input FF:**

$$D(z,\mu_0) = D^{\rm QED}(z) + \kappa\, D^{\rm NP}(z)$$

✧ **QED alone (dotted):**

$\kappa = 0$ at $\mu_0 = 1$ GeV

✧ **QED + hadronic input (solid):**

$\kappa = 1$ at $\mu_0 = 1$ GeV

Hadronic component of fragmentation is very important at low Q_T

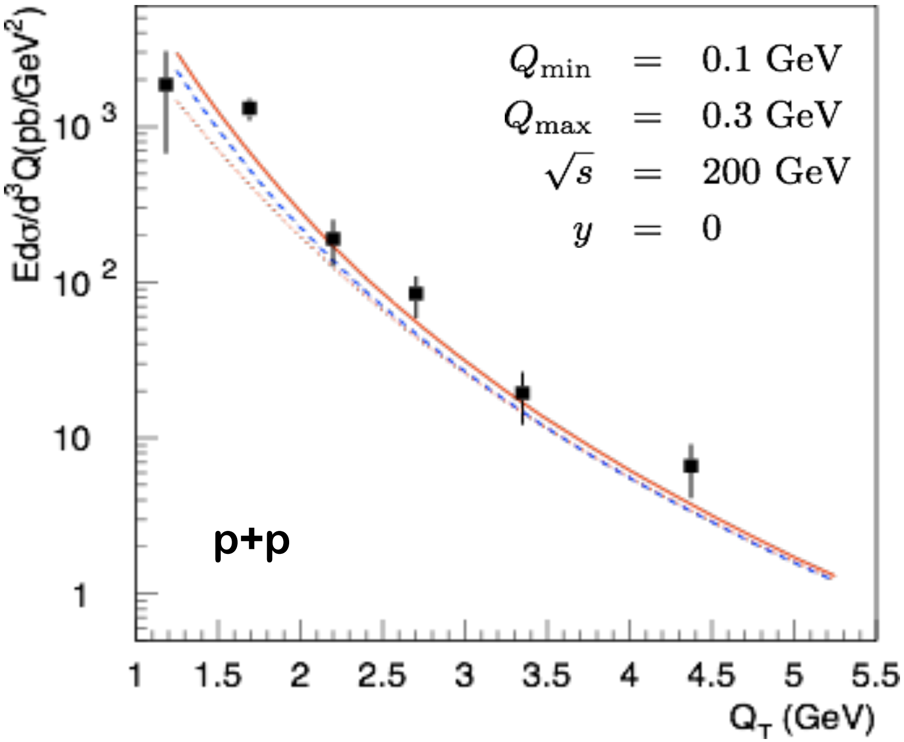
"Direct photon" approximation

❑ Dilepton production vs direct photon production:

$$E\frac{d\sigma_{AB\to\ell^+\ell^-(Q)X}}{d^3Q} \approx \frac{d\sigma_{AB\to\gamma(\hat{Q})X}}{dQ_T^2dy}\int_{Q^2_{min}}^{Q^2_{max}} dQ^2\left(\frac{\alpha_{em}}{3\pi^2Q^2}\right)\sqrt{1-\frac{4m_\ell^2}{Q^2}}\left(1+\frac{2m_\ell^2}{Q^2}\right)$$

$$\approx \frac{\alpha_{em}}{3\pi}\ln\left(\frac{Q^2_{max}}{Q^2_{min}}\right)E_\gamma\frac{d\sigma_{AB\to\gamma(\hat{Q})X}}{d^3Q}$$

← Direct photon cross section



Data from PHENIX: arXiv:0804.4168

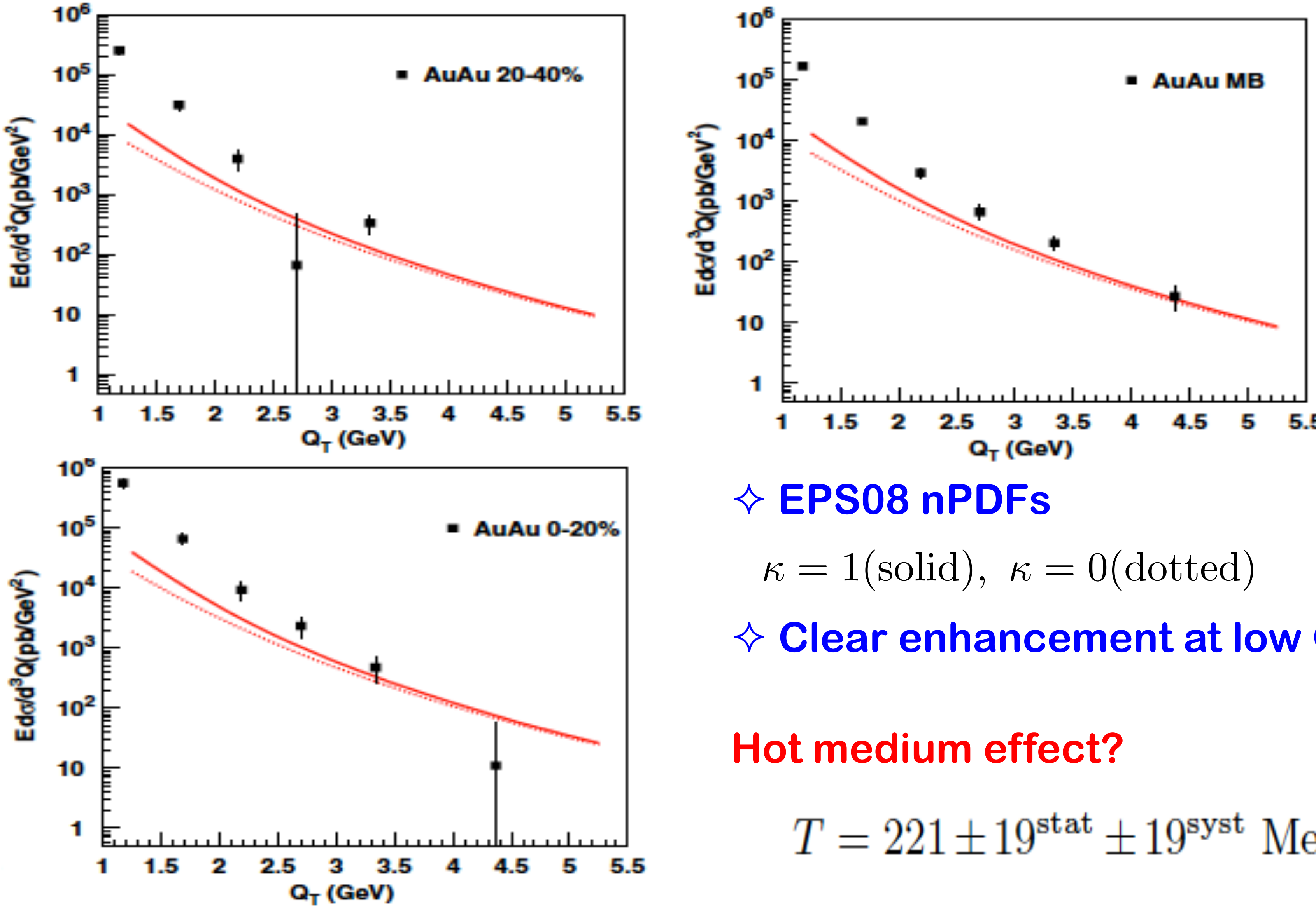
✧ **Inclusive NLO direct photon (blue-dashed)**

Gordon, Vogelsang, 1993

✧ **Direct photon code has similar non-perturbative fragmentation functions**

✧ **Low mass dilepton ~ inclusive photon production**

Au-Au data: beyond shadowing + isospin



Data from PHENIX: arXiv:0804.4168

✧ **EPS08 nPDFs**

$$\kappa = 1(\text{solid}),\ \ \kappa = 0(\text{dotted})$$

✧ **Clear enhancement at low Q_T**

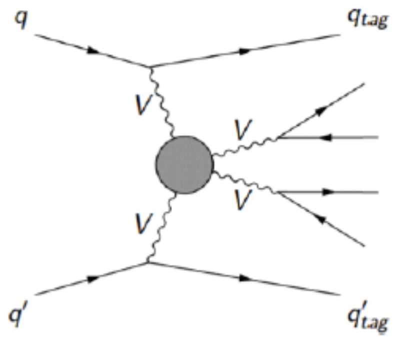
Hot medium effect?

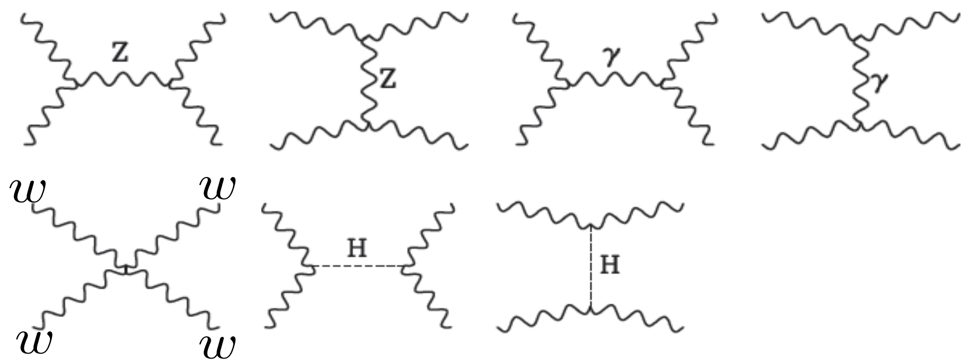
$$T = 221 \pm 19^{\text{stat}} \pm 19^{\text{syst}}\ \text{MeV}$$

Kang, Qiu, Vogelsang, PRD 2009

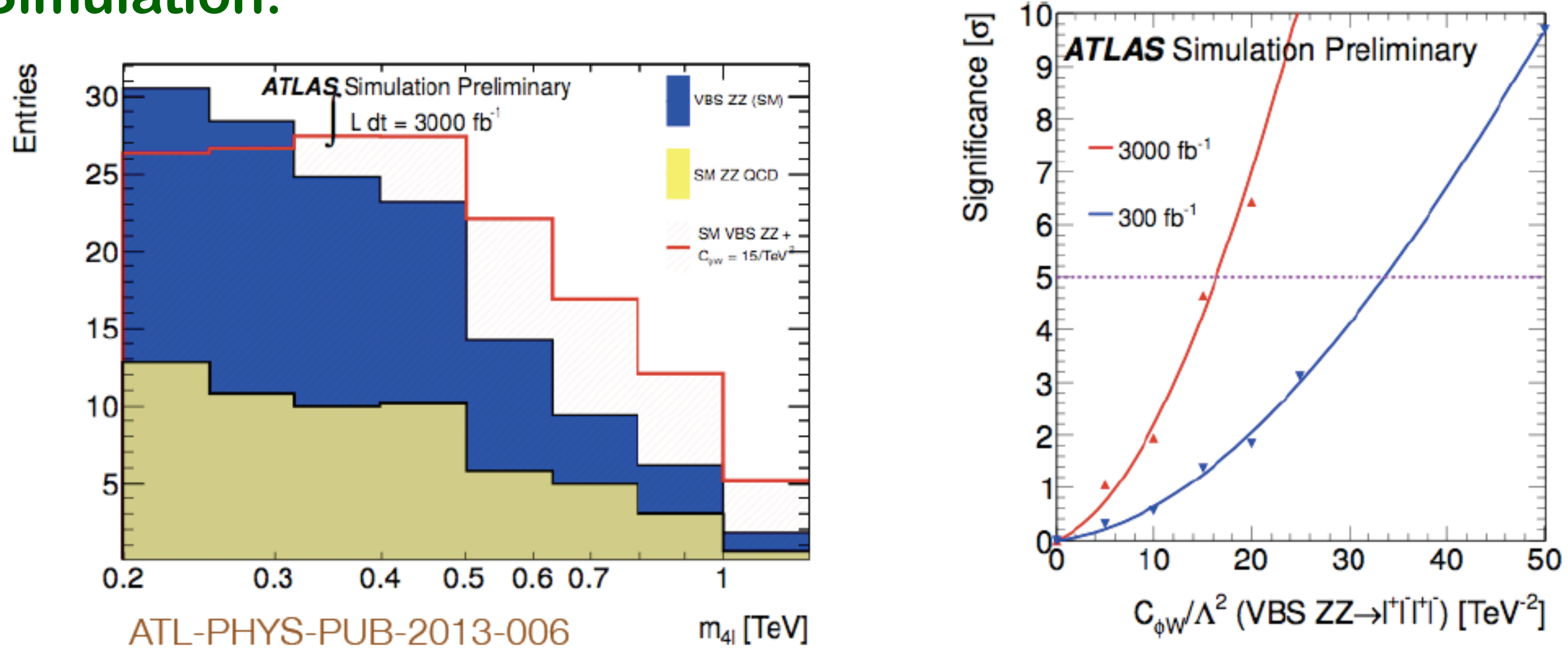
Vector boson scattering

- ❑ Another way to probe EW sector:





- ❑ Simulation:

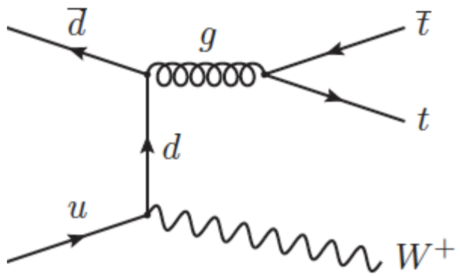


ATL-PHYS-PUB-2013-006

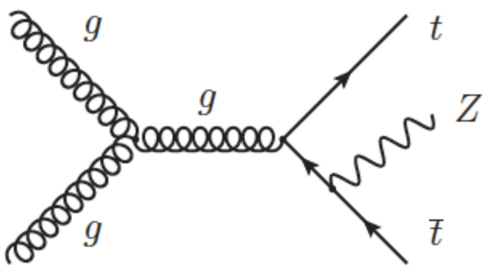
Top pair in association with W or Z

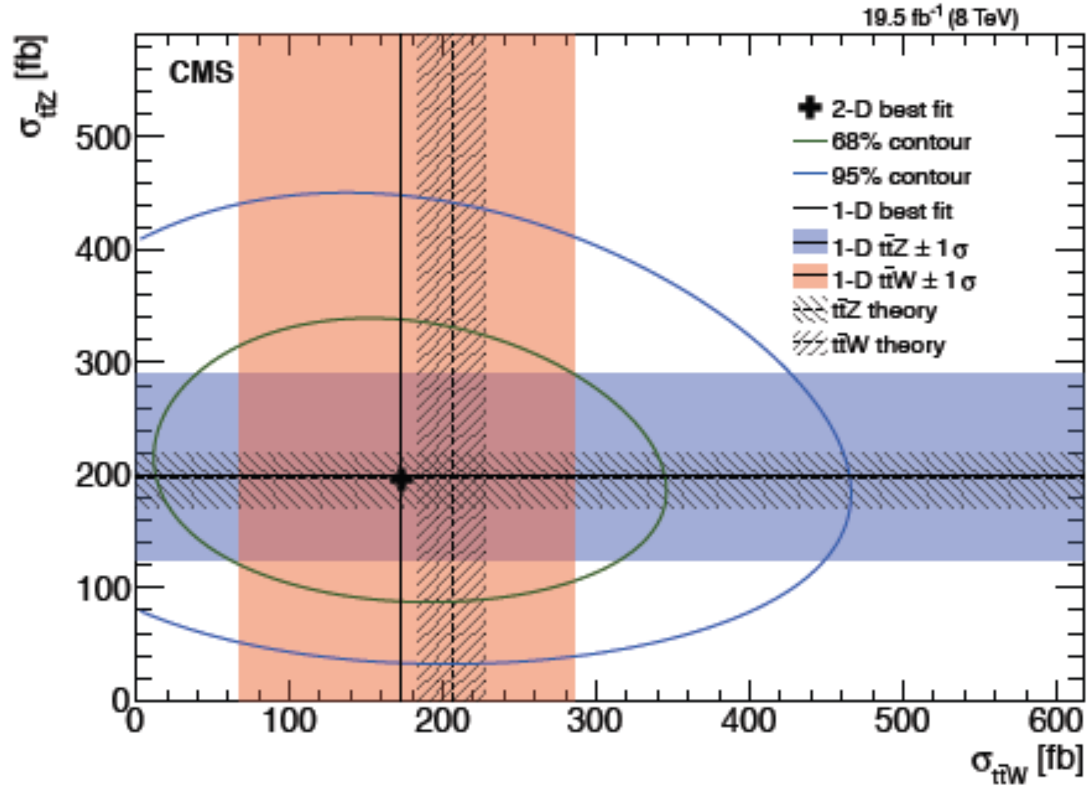
CMS-1406.7830

❑ CMS - results:



Channels used	Process	Cross section
2ℓ	$t\bar{t}W$	$170^{+90}_{-80}\,(\mathrm{stat}) \pm 70\,(\mathrm{syst})$ fb
$3\ell+4\ell$	$t\bar{t}Z$	$200^{+80}_{-70}\,(\mathrm{stat})^{+40}_{-30}\,(\mathrm{syst})$ fb
$2\ell+3\ell+4\ell$	$t\bar{t}W + t\bar{t}Z$	$380^{+100}_{-90}\,(\mathrm{stat})^{+80}_{-70}\,(\mathrm{syst})$ fb





Theory - NLO

Slightly off for $\sigma_{t\bar{t}W}$