QUARKONIUM PRODUCTION AND DECAY IN PERTURBATIVE QCD

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1. Scales in quarkonium
2. NRQCD
3. Onium decay
4. Onium production
5. Testing NRQCD Factorization
   - decay rates from 1st principles
   - global analysis of onium production
   - spin alignment of ψ, ψ' at the Tevatron

Scales in Quarkonium Physics

	$c\bar{c}$	$b\bar{b}$
M_Q	1.5 GeV	4.5 GeV
$M_Q v$	0.9 GeV	1.5 GeV

well-separated if $v^2 \ll 1$

kinematic scales: p_T, $\sqrt{s}$, ...

Short Distance: $\ell \lesssim \frac{1}{M_Q}$

- perturbation expansion in $\alpha_s(M_Q)$

Long Distance: $\ell \gtrsim \frac{1}{M_Q v}$

- lattice gauge theory
- phenomenology

How to separate SD from LD?

Answer: NRQCD

Non Relativistic QCD

an effective field theory
equivalent to full QCD
at momenta $p \ll M_Q$

$$\mathcal{L}_{QCD} = \mathcal{L}_{light} + \overline{\Psi}^{+}(i\not{D} - M_Q)\Psi$$

Ψ = Dirac field for $Q, \bar{Q}$

$$\mathcal{L}_{NRQCD} = \mathcal{L}_{light} + \psi^{+}(iD_0 + \frac{\vec{D}^2}{2M_Q})\psi + \chi^{+}(iD_0 - \frac{\vec{D}^2}{2M_Q})\chi + \text{corrections}$$

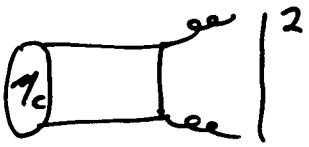
ψ = Pauli field for Q
χ = " $\bar{Q}$

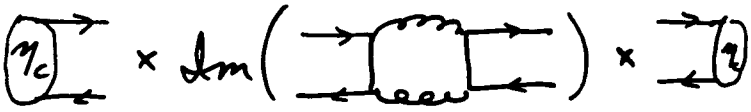
$\Rightarrow$ conservation of Q, $\bar{Q}$
heavy quark spin symmetry

Quarkonium Decay

in Full QCD

η_c decays at order α_s^2 via $c\bar{c} \to gg$

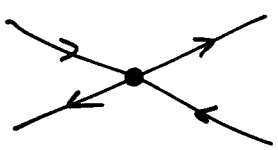
$$2M\Gamma(\eta_c) = \int \Big|$$  $$\Big|^2$$

$$=$$ 

in NRQCD

$c\bar{c} \to gg$ is forbidden,
but $c\bar{c} \to c\bar{c}$ can have imaginary part!

can reproduce annihilation part of $c\bar{c} \to c\bar{c}$
by pointlike 4-fermion interaction



$$\Delta\mathcal{L} = f_1(^1S_0)\, \psi^\dagger\chi\chi^\dagger\psi + \cdots$$

matching

$$\mathrm{Im}\left(\;\right)_{QCD} = \mathrm{Im}\left(f_1(^1S_0)\;\times + \cdots\right)_{NRQCD}$$

$$\Longrightarrow \mathrm{Im}\, f_1(^1S_0) = \frac{4\alpha_s}{3m_c}$$

decay rate

$$2M\Gamma(\eta_c) = \eta_c \times \mathrm{Im}\left(f_1(^1S_0)\;\times\right) \times \eta_c$$

$$= \mathrm{Im}\, f_1(^1S_0) \times \eta_c \;\eta_c$$

$$\langle \eta_c | \psi^\dagger \chi \chi^\dagger \psi | \eta_c \rangle \approx 6\,\frac{|R(0)|^2}{4\pi}$$

$$2M\Gamma(\eta_c) = \frac{4\alpha_s}{3m_c}\;\langle \eta_c | \psi^\dagger \chi \chi^\dagger \psi | \eta_c \rangle$$

NRQCD Factorization Formula

a systematic expansion in α_s and in v^2

$$2M\,\Gamma(H) = \sum_n C_n(\alpha_s, m_c)\,\langle H|\mathcal{O}_n|H\rangle$$

short-distance coefficients C_n

- involves scales $l \approx \frac{1}{M_Q}$
- calculable using PQCD
- expansion in powers of $\alpha_s(M_Q)$

NRQCD matrix element $\langle H|\mathcal{O}_n|H\rangle$

- involves scales $l \gtrsim \frac{1}{M_Q v}$
- scales like v^{3+2N}, where N depends on $\mathcal{O}_n$ and H
- calculable using lattice simulations of NRQCD

Quarkonium Production

1. must produce $Q, \bar{Q}$ with small relative momentum
   ... involves Short Distances $\ell \lesssim \frac{1}{M_Q}$

2. $Q, \bar{Q}$ must bind to form quarkonium
   ... involves Long Distances $\ell \gtrsim \frac{1}{M_Q v}$

For sufficiently inclusive cross sections,*

SD and LD effects

can be factored using standard

factorization methods of PQCD

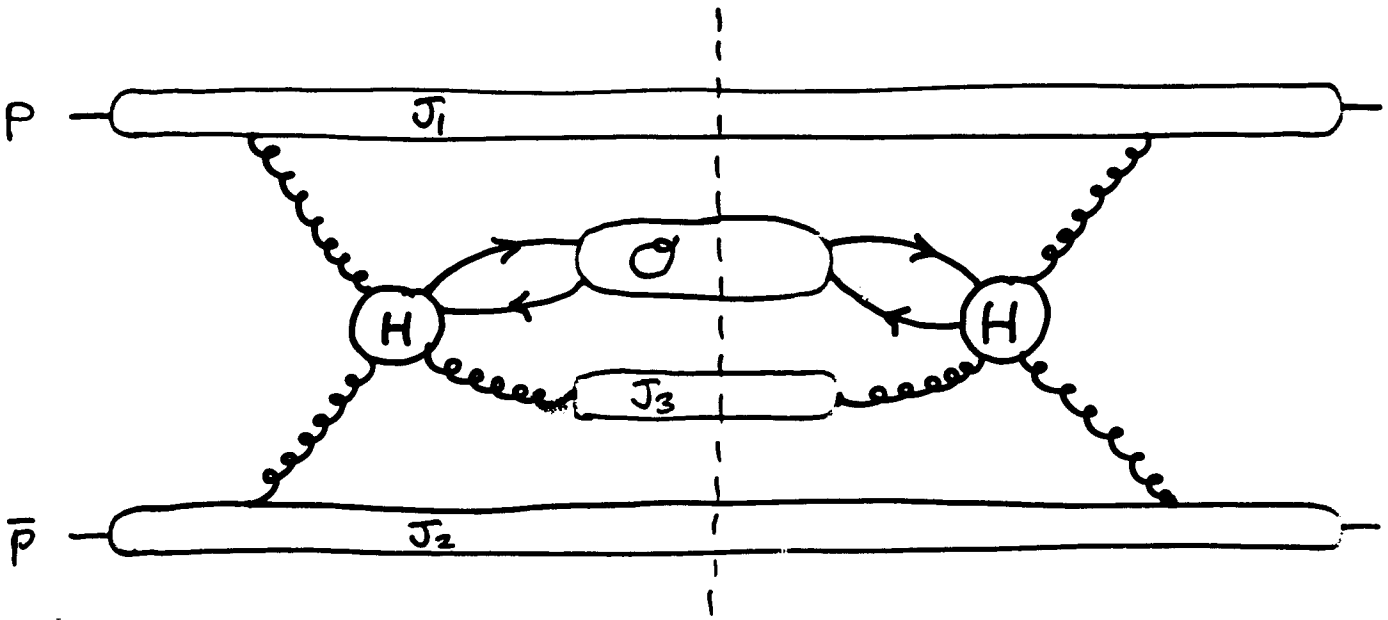
* plus kinematical constraints
  if there are hadrons in the initial state

Dominant diagrams factor into

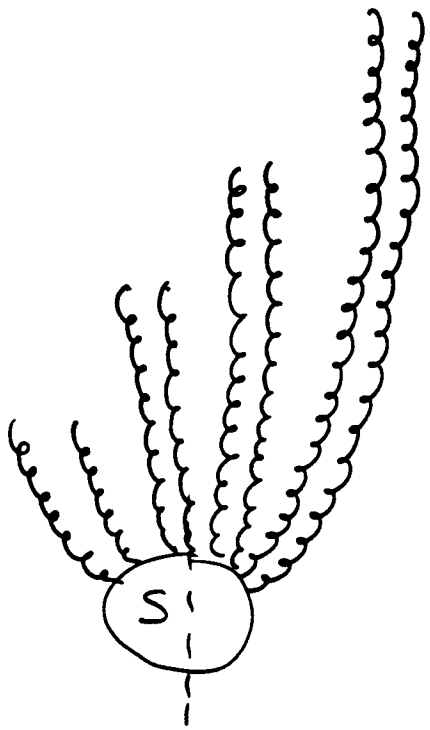
hard-scattering amplitudes H

jet subdiagrams J_i

onium subdiagram $\mathcal{O}$



connected by soft gluons S



but soft gluons cancel

after summing over all connections

(Ward identities, KLN theorem)

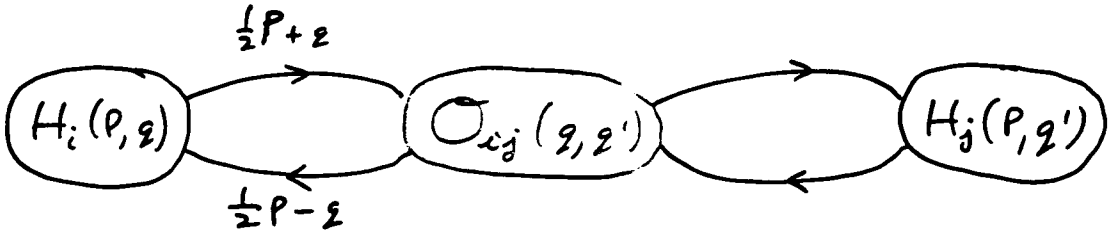
Involves integral over

relative momentum q of $Q, \bar{Q}$

Simplify by expanding

hard-scattering amplitude

in powers of q/M_Q



$=$

where m,n include color indices

spin "

orbital-angular-momentum "

Use NRQCD to define $\langle \mathcal{O}_{nm} \rangle$

$\Longrightarrow$ gauge-invariant separation

of SD and LD

NRQCD Factorization Formula

systematic expansion in $\alpha_s(M_Q)$ and v^2

$$d\sigma(H(P)+X) = \frac{1}{4E_1E_2v_{12}} \sum_{nm} C_{nm} \langle \mathcal{O}^H_{nm} \rangle$$

Short-distance coefficients C_{nm}

- functions of $P, P_1, P_2, \ldots$
- expansion in powers of $\alpha_s(M_Q)$

NRQCD matrix elements $\langle \mathcal{O}^H_{nm} \rangle$

- phenomenological parameters
- scale like v^{3+2N}, where N depends on $\mathcal{O}^H_{nm}$
- can be simplified using NRQCD
  - rotational symmetry
  - heavy-quark spin symmetry
  - vacuum-saturation approx.
  - velocity-scaling rules

NRQCD factorization formula provides
a general <u>model-independent</u> framework
consistent with perturbative QCD
at short distances

<u>Color-singlet Model</u>: leading color-singlet $\langle \mathcal{O} \rangle$ dominates

$\langle \mathcal{O}_1(^1S_0) \rangle$ for η_c
$\langle \mathcal{O}_1(^3S_1) \rangle$ ψ
$\langle \mathcal{O}_1(^3P_J) \rangle$ χ_{cJ}

<u>Color-evaporation Model</u>: S-wave $\langle \mathcal{O} \rangle$'s dominate

$\langle \mathcal{O}_1(^1S_0) \rangle, \langle \mathcal{O}_1(^3S_1) \rangle, \langle \mathcal{O}_8(^1S_0) \rangle, \langle \mathcal{O}_8[^3S$

$\langle \mathcal{O} \rangle \sim v^{3+D}$, D = # of derivatives

<u>NRQCD</u>: $\langle \mathcal{O} \rangle \sim v^{3+(D+E+2M)}$,

D = # of derivatives
E = # of chromoelectric transition
M = # of chromomagnetic transition

Testing NRQCD Factorization

1. Decay Rates from 1st Principles

$$2M\Gamma(H) = \sum_n C_n(\alpha_s, m_c) \langle H|\mathcal{O}_n|H\rangle$$

- compute C_n using PQCD

  Bodwin, Braaten, & Lepage (1994)
  Petrelli (1996)
  Huang + Chao (1996)

- compute $\langle H|\mathcal{O}_n|H\rangle$ using Lattice NRQCD

  Bodwin, Sinclair, + Kim (1996)

only phenomenological inputs

are α_s and m_Q !

Precise measurements of decay rates:

E760/E835, BES

2. Global Analysis of Onium Production

NRQCD matrix elements $\langle \mathcal{O}_{mn}^{H} \rangle$ are universal!

The same matrix elements must describe production in...

... fixed target: pN, πN, KN, γN, eN

... colliders: e^+e^-, $p\bar{p}$, ep, pp

... decays: Z^0, Υ, B

For ψ, ψ', $\langle \mathcal{O}_1(^3S_1) \rangle$

$\langle \mathcal{O}_8(^3S_1) \rangle$, $\langle \mathcal{O}_8(^1S_0) \rangle$, $\langle \mathcal{O}_8(^3P_J) \rangle$

$\vdots$

For χ_{cJ}, $\langle \mathcal{O}_1(^3P_0) \rangle$, $\langle \mathcal{O}_8(^3S_1) \rangle$

$\vdots$

See proceedings of UIC Workshop on Quarkonium Physics (June 1996)

Spin Allignment of Prompt ψ, ψ' at the Tevatron

$$p\bar{p} \rightarrow \psi(\lambda) + X$$

1. At $p_T >> M_\psi$, σ is dominated by gluon fragmentation

Braaten + Yuan 1993

$$d\sigma(p\bar{p} \rightarrow \psi(P) + X) = \int_0^1 dz\, d\sigma(p\bar{p} \rightarrow g\left(\frac{P}{z}\right) + X)\, D_{g \rightarrow \psi}(z)$$

2. Gluon fragmentation may be dominated by color-octet matrix element

Braaten + Fleming 1995

$$D_{g \rightarrow \psi}(z) = \frac{\pi \alpha_s}{24 m_c^3} \delta(1-z) \langle O_8(^3S_1) \rangle$$

3. At leading order in α_s,

gluon fragmentation produces ψ's that are 100% transversely polarized

Cho + Wise 1995

4. Beyond leading order in α_s

Beneke + Rothstein 1995

polorization from gluon fragmentation

$> 90\%$

5. Beyond fragmentation

Beneke + Kramer 1996
Leibovich 1996

dominant corrections to polarization come from $1/p_T^2$ corrections

Beneke + Kramer hep-ph/9611218

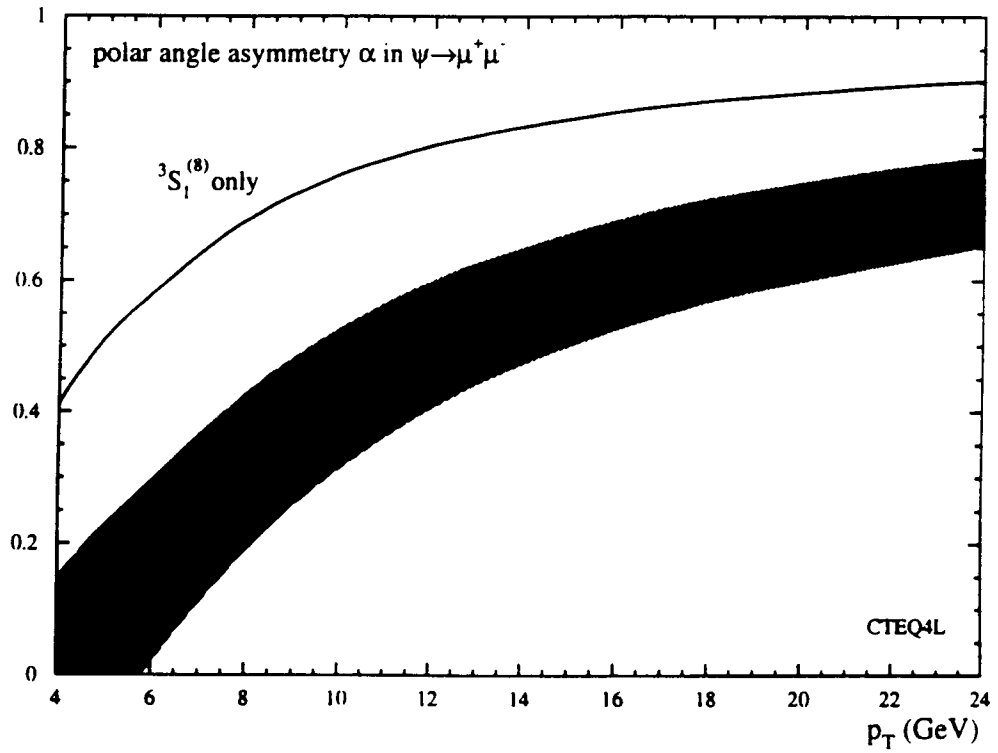


Figure 3: α as a function of p_t. Parameter specifications as for Fig. 1. The band is obtained from Eq. (8) and the $\mathcal{O}(\alpha_s^4)$ fragmentation contributions from [6] as described in the text. The solid curve is obtained with $M_{3.5}^w(^1S_0^{(8)}, ^3P_0^{(8)}) = 0$ and shows the importance of this parameter for polarization. The band does not include theoretical uncertainties in the short-distance coefficients, Tab. 2.

Conclusions

Quarkonium physics *is* under control in perturbative QCD

Perturbative factorization + NRQCD

$\Longrightarrow$ separation of SD and LD

systematic expansion in $\alpha_s(M_Q)$ and v

Can calculate...

... annihilation decay rates

... sufficiently inclusive cross sections

Tests of NRQCD Factorization

- decay rates from 1st principles
- global analysis of production
- spin alignment of ψ, ψ' at the Tevatron