CTEQ Symposium

Nov 7-9, 1996

S. Brodsky

Towards a Precise QCD Calculus: Scale-Setting as Resummation

"BLM" Method : SJB, G.P. Lepage, P. Mackenzie

Phys. Rev. D<u>28</u>, 228 (1983)

Commensurate Scale Relations:

SJB + H.J. Lu

Phys. Rev. D<u>51</u>, 3652 (1995)

Precision tests of a theory:

relate one observable to another

- all theoretical conventions irrelevant

e.g.
- calculational method
- renormalization procedure
- choice of cutoffs
- ⋮

$$\mathcal{L}_{QCD} \underset{\substack{\text{UV}\\ \text{regularization}}}{\Rightarrow} \mathcal{L}^{(\Lambda)}\left[m_0(\Lambda), \alpha_0(\Lambda)\right]$$

Many possible renormalization schemes

Choice of Λ irrelevant

Convention: dim regularization

$$\mu^{2\epsilon}\int d^{4-2\epsilon}k$$

$$\alpha_{\overline{MS}}(\mu), \quad m_{\overline{MS}}(\mu)$$

Physical quantities indep of
UV scheme, renormal. scale

$$\frac{d}{d\mu}\mathcal{O} = 0, \quad \mathcal{O}_A \Rightarrow \mathcal{O}_B$$

Parameters in $\mathcal{L}_{eff} = \mathcal{L}^{(\Lambda)}$

are theoretical constructs

- dependent on cutoff conventions

* Relations between observables must be

independent of theoretical conventions

* Use observables to fix parameters

Grunberg

Use physical observables to define "effective charges" $\alpha_A(Q^2)$

$$\bar{R}_{e^+e^-}(s) \equiv R_0(s)\left[1 + \frac{\alpha_R(s)}{\pi}\right]$$

smeared $\quad\quad \sum e_q^2$

$$\int_0^1 dx \left[g_1^p(x,Q^2) - g_1^n(x,Q^2)\right] \equiv \frac{g_A}{3}\left[1 - \frac{\alpha_{g_1}}{\pi}\right]$$

Bj sum rule

$$V(Q^2) \equiv \frac{-4\pi C_F \alpha_V(Q^2)}{Q^2}$$

Heavy Quark Potential

"Running couplings"

QCD: $\boxed{\alpha_A(Q_A^2) \Rightarrow \alpha_B(Q_B^2)}$

indep of μ, renorm. scheme!

Effective Charges / Running Couplings obey

$$\frac{d}{d\ln Q^2}\alpha_A(Q^2) = \beta_A(\alpha_A(Q^2))$$

* First two terms β_0, β_1 universal!

<u>QCD Program</u>: choose

any effective charge

as "standard candle"

e.g. $\alpha_V(Q_0^2)$

→ predict all other observables in terms of $\alpha_V(Q^2)$

* 1) No μ, scheme dependence

* 2) No scale ambiguity

$\alpha_V(Q^2)$: $Q^2 = -t = -q^2$

Quarkonium Physics:

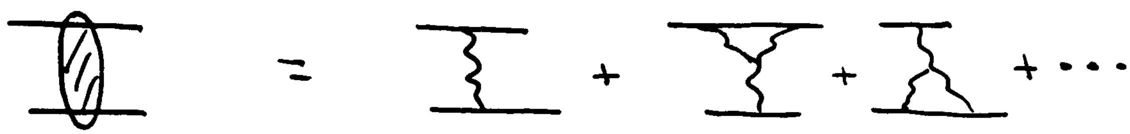
Scales and Schemes

Natural renormalization scheme $\alpha_V(Q)$

$$V(Q^2) \equiv -4\pi \frac{C_F \, \alpha_V(Q)}{Q^2}$$

Potential for infinitely heavy test charges

$t = -Q^2$



Obeys standard RGE:

$$* \qquad \frac{\partial}{\partial \ln Q} \alpha_V(Q) = \beta\left[\alpha_V(Q)\right]$$

β_0, β_1 universal

quark mass dependence as in QED

* $\alpha_V(Q)$: sums all v.p. contributions

no Scale ambig's!

Input from LGTh:

$$\alpha_V^{(n_f=3)}[Q = 8.2\,\mathrm{GeV}]$$

$$= \begin{cases} 0.1945\,(30) & \Upsilon \\ 0.1940\,(67) & \Psi. \end{cases}$$

Davies, et al

$$\alpha_{\text{Lattice}} \Rightarrow \alpha_P \overset{\sim}{\Rightarrow} \alpha_V$$

Use BLM

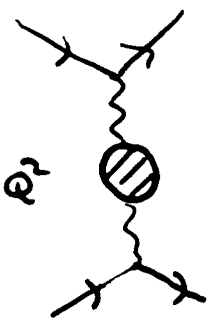
$$\alpha_{\overline{MS}}(M_Z) = \begin{cases} 0.117\,(2) & \text{NNLO L+W.} \\ 0.115\,(2) & \text{O} \end{cases}$$

Lüsher + Weisz NNLO $(n_f = 0)$

$\alpha_{\overline{MS}} \to \alpha_P$

Application of BLM to QED

$t = -Q^2$



Coulomb scatt of heavy lepton

$$V(Q^2) \equiv -\frac{4\pi\alpha_V(Q^2)}{Q^2}$$

$$Q^2 = 0: \quad \alpha_V^{-1} = 137.0359895(6).$$

$$\alpha_V(Q^2) = \frac{\alpha_V(Q_0^2)}{1 - \Pi(Q^2, Q_0^2)}$$

↖ sums all VP from

Compute observables in terms of α_V

Lautrup
de Rafael
BLM

$$a_\ell = \frac{\alpha(Q^*)}{2\pi} + C_{20}\frac{\alpha^2(Q^{**})}{\pi^2} + \cdots$$

universal form!

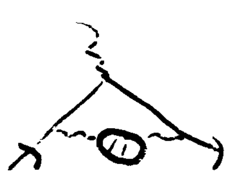
J. H. Field

$$Q^* = e^{-5/4}\, m_\ell$$

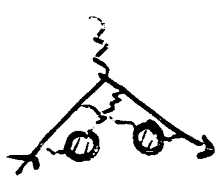
(also MVTh)

C_{20} indep of n_F^{VP}, conformal coef

sums
all VP



Q^*



Q^{**}

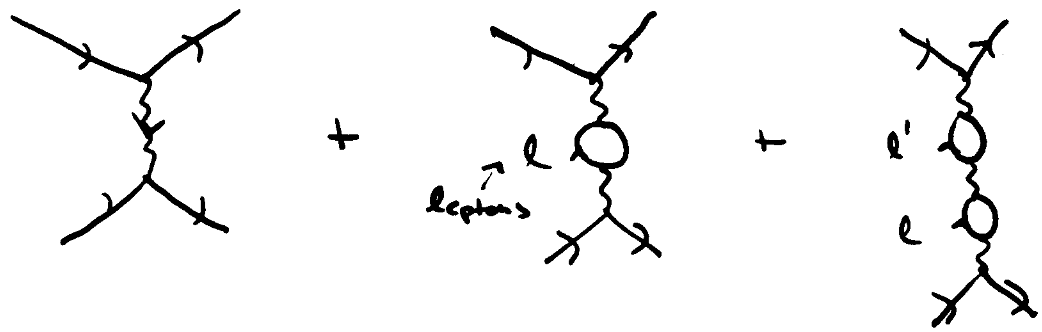
* In QED: no scale ambiguity!

Measure in Coulomb Scattering of Heavy Charges

$$\alpha(q^2) = \frac{\alpha(q_0^2)}{1 - \pi(q^2, q_0^2)}$$

"running" coupling sums all Vac. Pol.

$t = q^2$



$+ \cdots$

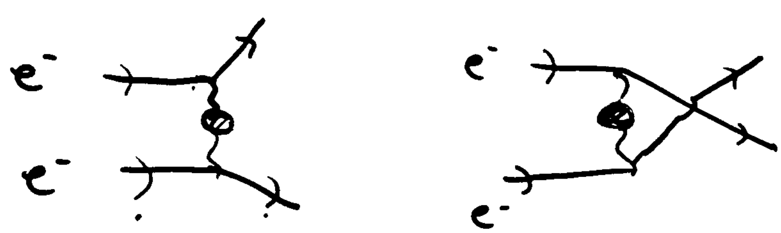
$$\pi(q^2, q_0^2) = \sum_\ell \frac{\alpha(q_0^2)}{3\pi} \log q^2/q_0^2 + \cdots$$

$(|q^2|, |q_0^2| \gg 4m_\ell^2)$

Example: $e^- e^- \to e^- e^-$ scattering

$$\mathcal{M}^{\uparrow\uparrow \to \uparrow\uparrow} = 8\pi \frac{s}{t}\alpha(t) + 8\pi \frac{s}{u}\alpha(u)$$

two scales in $d\sigma/dt$



"Dressed Skeleton" Expansion

$\alpha(t)$ $\alpha(u)$

unambiguous

In general QCD processes involve many invariants.

many scales

$$\mathcal{M}^{\uparrow\uparrow\to\uparrow\uparrow}(s,t) = 8\pi \frac{s}{t}\alpha(t) + 8\pi \frac{s}{u}\alpha(u)$$

must use t, u — otherwise $\alpha \ne$ V.P.

cannot use single scale p_T^2

$\alpha(Q^2)$ sums all V.P to all orders.

Relate Observable to Observable

No dependence on renormalization scheme or scale at any order of pert. theory

* $$\alpha_A(Q_A) = \alpha_B(Q_B)\left[1 + c_1 \frac{\alpha_B}{\pi} + \cdots\right.$$

coefficients $\{c_n\}$

* identical to theory with $\beta = 0$

"conformal coefficients"

* all $\beta \neq 0$ terms contained in α_B

Commensurate Scale Relation:

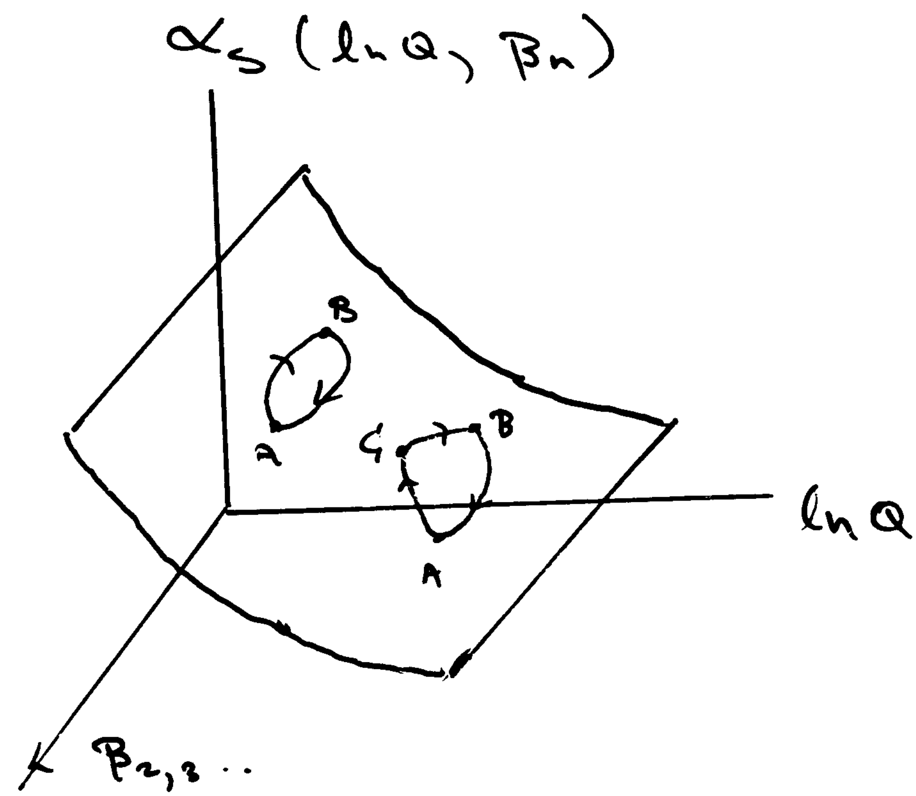
$$* \quad \alpha_B(Q_B) = \alpha_A(Q_A)\left[1 + r_1^{B/A}\,\frac{\alpha_A}{\pi} + \cdots\right]$$

conformal coeff.

$$Q_B/Q_A = \lambda_{B/A}$$

Peterman Stückelberg Renormalization "Group"

$$\left\{\begin{array}{lll} \lambda_{B/A} = \lambda_{B/C}/\lambda_{A/C} & & \text{transitive} \\ \lambda_{B/A} = \lambda_{A/B}^{-1} & & \text{symmetry} \\ \lambda_{A/A} = I & & \text{identity} \end{array}\right.$$



SJB
Lepage
Mackenzie

SJB + HJLu

Kataev + Gorishny

J. Rathsman

BLM :

* All $\beta \neq 0$ contributions resummed
into scale of running couplings

* coefficients of perturbative series
same as conformally-invariant
theory.

Commensurate Scale Relations:

* Relations between observables

* no scheme, scale ambiguity

* generalized Crewther relation

$\beta_0 = 11 - 2/3\, n_f$

BLM scale fixing
(any effective charge / scheme)

$$\alpha_s(Q) = \frac{\alpha_s(Q_0)}{1 + \frac{\beta_0}{2\pi}\alpha_s(Q_0)\ln\frac{Q}{Q_0} + \cdots}$$

Given: $\rho(Q) = k\,\alpha_s(\mu)\left[1 + \frac{\alpha_s(\mu)}{\pi}(An_f + B) + \cdots\right.$

write $An_f + B = -3/2\,\beta_0 A + \left(\frac{33}{2}A + B\right)$

$*$ shift $\mu \to \mu\, e^{3A(\mu)} = Q^*$
$\leftarrow \mu$ indep

Then

$*$ $\rho(Q) = k\,\alpha_s(Q^*)\left[1 + \frac{\alpha_s(Q^*)}{\pi} C + \cdots\right.$

$*$ $C = \frac{33}{2}A + B$ conformal coeff.

Note: $Q^* = \mu\, e^{3A}$

determined from n_f coeff

Proof of BLM Scale-Setting

In α_V-scheme, all VP summed in $\alpha_V(Q$

$\therefore$ coefficients n_f-independent!

any eff. charge

$$\alpha_A(Q_A) = \alpha_V(\mu)\left[1 + (A_{VP}\, n_f + B)\frac{\alpha_V}{\pi} + \cdots\right]$$

must shift argument of α_V:

$$Q_V = e^{3A_{VP}(\mu)}\,\mu$$

$$\alpha_A(Q_A) = \alpha_V(Q_V)\left[1 + r_1^{A/V}\frac{\alpha_V}{\pi} + \cdots\right]$$

* then $r_1^{A/V} = B + \frac{33}{2} A_{VP}$

  is independent of n_f.

* $\lambda_{A/V} = \dfrac{Q_A}{Q_V}$ fixed, unique

* All n_f-VP summed into α_V resummation

Repeat for another effective charge

$$\alpha_B(Q_B) = \alpha_V(Q_V)\left[1 + r_1^{B/V}\frac{\alpha_V}{\pi} + \cdots\right]$$

$$\alpha_A(Q_A) = \alpha_V(Q_V)\left[1 + r_1^{A/V}\frac{\alpha_V}{\pi} + \cdots\right]$$

$\therefore$

$$\boxed{\alpha_B(Q_B) = \alpha_A(Q_A)\left[1 + r_1^{B/A}\frac{\alpha_A}{\pi} + \cdots\right]}$$

$$r_1^{B/A} = r_1^{B/V} - r_1^{A/V}$$ indep of n_f, β_0

$$Q_B/Q_A = \lambda_{B/A} = \frac{\lambda_{B/V}}{\lambda_{A/V}}$$ transitive reflective symm

* Alternatively, compute Q_B/Q_A by requiring $r_1^{B/A}$ to be n_f-indep. $\Rightarrow$ BLM.

* Result is independent of intermediate scheme P+S

* Can use $\overline{MS}$:

$$\alpha_V(Q) = \alpha_{\overline{MS}}(e^{-5/6}Q)\left[1 - 2\frac{\alpha_{\overline{MS}}}{\pi}\cdots\right]$$

S. Brodsky + H. J. Lu

<u>Example of Commensurate Scale Relation:</u>

"effective charge"

$$R_{e^+e^-}(Q^2) \equiv R^0_{e^+e^-}(Q^2)\left[1 + \frac{\alpha_R(Q)}{\pi}\right]$$

$$\Gamma^{p-n}(Q^2) = \int_0^1 dx \left[g_1^p(x,Q^2) - g_1^n(x,Q^2)\right]$$

$$\equiv \frac{g_A}{3}\left[1 - \frac{\alpha_{g_1}(Q)}{\pi}\right]$$

$$\boxed{\alpha_{g_1}(Q) = \alpha_R(0.52\,Q)\left[1 - \frac{\alpha_R}{\pi} + \cdots\right]}$$

Relate physical observable to physical observable

Relative scale fixed

No scale or scheme ambiguity

Expansion in finite quantity, not

$$\alpha_{\overline{MS}}(Q) = \frac{4\pi}{\beta_0 \ln Q^2/\Lambda^2} = \infty \text{ at } Q$$

Grunberg

Define "Effective Charge" $\alpha_R(s)$

$$R_{e^+e^-}(s) \equiv R_0(s)\left[1 + \frac{\alpha_R(s)}{\pi}\right]$$

$R_0(s) = 3\sum_q e_q^2 \quad (s \gg m_q^2)$

$$\boxed{\frac{\alpha_R(s)}{\pi} \equiv \frac{R_{e^+e^-}(s) - R_0(s)}{R_0(s)}}$$

* $\alpha_R(s)$ determined from expt.

* $\alpha_R(s)$ obeys usual RGE in PQCD (important test of theory)

* $\alpha_R(s)$ defines a renormalization scheme if we use $\alpha_R(s)$ as expansion.

* Other perturbatively-calculable expt. quantities can be used to define effective charg

$$R(Q) \equiv 3 \sum_f Q_f^2 \left[1 + \frac{\alpha_R(Q)}{\pi} \right]$$

$\mu \to Q$

$$\frac{\alpha_R(Q)}{\pi} = \frac{\alpha_{\overline{\rm MS}}(Q)}{\pi} + \left(\frac{\alpha_{\overline{\rm MS}}(Q)}{\pi}\right)^2 \left[\left(\frac{41}{8} - \frac{11}{3}\zeta_3\right) C_A - \frac{1}{8} C_F + \left(-\frac{11}{12} + \frac{2}{3}\zeta_3\right) f \right]$$

$$+ \left(\frac{\alpha_{\overline{\rm MS}}(Q)}{\pi}\right)^3 \left\{ \left(\frac{90445}{2592} - \frac{2737}{108}\zeta_3 - \frac{55}{18}\zeta_5 - \frac{121}{432}\pi^2\right) C_A^2 \right.$$

$$+ \left(-\frac{127}{48} - \frac{143}{12}\zeta_3 + \frac{55}{3}\zeta_5\right) C_A C_F - \frac{23}{32} C_F^2$$

$$+ \left[\left(-\frac{970}{81} + \frac{224}{27}\zeta_3 + \frac{5}{9}\zeta_5 + \frac{11}{108}\pi^2\right) C_A \right.$$

$$\left. + \left(-\frac{29}{96} + \frac{19}{6}\zeta_3 - \frac{10}{3}\zeta_5\right) C_F \right] f$$

$$+ \left(\frac{151}{162} - \frac{19}{27}\zeta_3 - \frac{1}{108}\pi^2\right) f^2$$

$$\left. + \left(\frac{11}{144} - \frac{1}{6}\zeta_3\right) \frac{d^{abc} d^{abc}}{C_F d(R)} \frac{\left(\sum_f Q_f\right)^2}{\sum_f Q_f^2} \right\}$$

assumes
$\mu = Q$

light-by-light

$d(R) = N$

$C_A = N$

Gorishny
Kataev
Larin

$d_{abc} d^{abc} = \frac{40}{3}$

$C_F = \frac{N^2-1}{2N}$

$\sum Q_f = 2/3 - 1/3 - 1/3 = 0$

Surguladze
Samuel

$T = 1/2$

$$\int_0^1 dx\,\left[g_1^{ep}(x,Q^2) - g_1^{en}(x,Q^2)\right] \equiv \frac{1}{3}\left|\frac{g_A}{g_V}\right|\left[1 - \frac{\alpha_{g_1}(Q)}{\pi}\right]$$

$$\frac{\alpha_{g_1}(Q)}{\pi} = \frac{\alpha_{\overline{\rm MS}}(Q)}{\pi} + \left(\frac{\alpha_{\overline{\rm MS}}(Q)}{\pi}\right)^2 \left[\frac{23}{12}C_A - \frac{7}{8}C_F - \frac{1}{3}f\right]$$

$$+ \left(\frac{\alpha_{\overline{\rm MS}}(Q)}{\pi}\right)^3 \left\{ \left(\frac{5437}{648} - \frac{55}{18}\zeta_5\right) C_A^2 + \left(-\frac{1241}{432} + \frac{11}{9}\zeta_3\right) C_A C_F + \frac{1}{32}C_F^2\right.$$

$$+ \left[\left(-\frac{3535}{1296} - \frac{1}{2}\zeta_3 + \frac{5}{9}\zeta_5\right) C_A + \left(\frac{133}{864} + \frac{5}{18}\zeta_3\right) C_F\right] f$$

$$\left. + \frac{115}{648}f^2\right\}.$$

assumes:

$\mu = Q$

S. A. Larin

J. A. M. Vermaseren

NLO BLM formulas in terms of beta functions

Any effective charge in perturbative QCD can be written in the following form

$$\frac{\alpha_1(Q)}{\pi} = \frac{\alpha_{\overline{\rm MS}}(Q)}{\pi} + \left(A_1 + B_1\beta_0\right)\left(\frac{\alpha_{\overline{\rm MS}}(Q)}{\pi}\right)^2 + \left(C_1 + D_1\beta_0 + E_1\beta_0^2 + \tfrac{1}{4}B_1\beta_1\right)\left(\frac{\alpha_{\overline{\rm MS}}(Q)}{\pi}\right)^3 + \cdots. \tag{1}$$

where the light-by-light contributions should be included in the C_1 coefficient. Notice the appearance of the $\frac{1}{4}B_1\beta_1$ term in the NNLO coefficient.

Similarly, given a second effective charge $\alpha_2(Q)$, we can put it in the form

$$\frac{\alpha_2(Q)}{\pi} = \frac{\alpha_{\overline{\rm MS}}(Q)}{\pi} + \left(A_2 + B_2\beta_0\right)\left(\frac{\alpha_{\overline{\rm MS}}(Q)}{\pi}\right)^2 + \left(C_2 + D_2\beta_0 + E_2\beta_0^2 + + \tfrac{1}{4}B_2\beta_1\right)\left(\frac{\alpha_{\overline{\rm MS}}(Q)}{\pi}\right)^3 + \cdots. \tag{2}$$

The two effective charges $\alpha_1(Q)$ and $\alpha_2(Q)$ are related by the following series,

$$\frac{\alpha_1(Q)}{\pi} = \frac{\alpha_2(Q)}{\pi} + \left(A_{12} + B_{12}\beta_0\right)\left(\frac{\alpha_2(Q)}{\pi}\right)^2 + \left(C_{12} + D_{12}\beta_0 + E_{12}\beta_0^2 + + \tfrac{1}{4}B_{12}\beta_1\right)\left(\frac{\alpha_2(Q)}{\pi}\right)^3 + \cdots, \tag{3}$$

where the coefficients $A_{12}, B_{12}, C_{12}, D_{12}$ and E_{12} are given by:

$$\begin{aligned}
A_{12} &= A_1 - A_2 \\
B_{12} &= B_1 - B_2 \\
C_{12} &= C_1 - C_2 - 2(A_1 - A_2)A_2 \\
D_{12} &= D_1 - D_2 - 2(A_1B_2 + A_2B_1) + 4A_2B_2 \\
E_{12} &= E_1 - E_2 - 2(B_1 - B_2)B_2.
\end{aligned} \tag{4}$$

Applying the BLM procedure we obtain

$$\frac{\alpha_1(Q)}{\pi} = \frac{\alpha_2(Q^*)}{\pi} + A_{12}\left(\frac{\alpha_2(Q^{**})}{\pi}\right)^2 + C_{12}\left(\frac{\alpha_2(Q^{***})}{\pi}\right)^3 + \cdots, \tag{5}$$

conformal coefficients

where the renormalization scales are

$$\begin{aligned} \log\left(Q^{*2}/Q^2\right) &= -4\ B_{12} + 4\ \beta_0\ \left(B_{12}^2 - E_{12}\right)\left(\frac{\alpha_2(Q)}{\pi}\right), \\ \log\left(Q^{**2}/Q^2\right) &= -4\ \frac{D_{12}}{A_{12}}. \end{aligned} \tag{6}$$

$$\frac{\alpha_{g_1}(Q)}{\pi} = \frac{\alpha_R(Q^*)}{\pi} - \frac{3}{4}C_F\left(\frac{\alpha_R(Q^{**})}{\pi}\right)^2$$

0 for 3 flavors

$$+\left[\frac{9}{16}C_F^2 - \left(\frac{11}{144} - \frac{1}{6}\zeta_3\right)\frac{d^{abc}d^{abc}}{C_F N}\frac{\left(\sum_f Q_f\right)^2}{\sum_f Q_f^2}\right]\left(\frac{\alpha_R(Q^{***})}{\pi}\right)^3$$

$$Q^* = Q\exp\left[\frac{7}{4} - 2\zeta_3 + \left(\frac{11}{96} + \frac{7}{3}\zeta_3 - 2\zeta_3^2 - \frac{\pi^2}{24}\right)\left(\frac{11}{3}C_A - \frac{2}{3}f\right)\frac{\alpha_R(Q)}{\pi}\right]$$

$$Q^{**} = Q\exp\left[\frac{523}{216} + \frac{28}{9}\zeta_3 - \frac{20}{3}\zeta_5 + \left(-\frac{13}{54} + \frac{2}{9}\zeta_3\right)\frac{C_A}{C_F}\right]$$

Note: Abelian light-by-light
not summed.

$$\widehat{\alpha}_{g_1}(Q) = \widehat{\alpha}_R(Q^*) - \widehat{\alpha}_R^2(Q^{**}) + \widehat{\alpha}_R^3(Q^{***})$$

where

$$\widehat{\alpha}_{g_1}(Q) = \frac{3C_F}{4\pi}\alpha_{g_1}(Q)$$

$$\widehat{\alpha}_R(Q) = \frac{3C_F}{4\pi}\alpha_R(Q)$$

Independent of Nc!

See also:

Single Scale

Kataev, et al

Rathsman

* why is the relation between

α_R and α_{g_1}

so simple ?.

{ Kataev H.J. Lu 813

Consider conformal limit $\beta_0 \Rightarrow 0$, $\beta_1 \neq 0$

$$CSR \Rightarrow (1 + \hat{\alpha}_R)(1 - \hat{\alpha}_{g_1}) = 1$$

* Follows from Crewther relation !

$\beta = 0$ chiral

$$3S = K R' = K\left(\tfrac{4}{3} R\right)$$

$\uparrow \pi^0 \to \gamma\gamma$ $\quad \uparrow \mathcal{B}_J$ GLS $\quad \uparrow \hat{R}_{e^+e^-}$ $\quad \uparrow \check{R}_{e^+e^-}$

Deviations from Crewther Relation

proportional to β

Kataev Broadhurst

Test of Commensurate Scale Relation

* Generalized Crewther Relation

$$\frac{R_{e^+e^-}(s)}{3\sum e_q^2} \cdot \int_0^1 dx \, \frac{F_3^{\nu p}(x,Q^2) + F_3^{\bar\nu p}(x,Q^2)}{6}$$

$$= 1 + \underbrace{\Delta \beta_0 \frac{\alpha_R^3}{\pi^3}}_{\sim 7\times 10^{-4}}$$

where $\sqrt{s} = Q^* \cong 0.38\, Q$ commensurate scales

$O[\alpha_s^3]$ correction comes from $Q^{**} \neq Q^*$

* Check at $Q^2 = 3\ \mathrm{GeV}^2$:

$$\begin{array}{ccccc} 1+\hat\alpha_R(s) & & 1-\hat\alpha_{F_3}(Q) & & 1+\Delta\beta_0\hat\alpha^3 \\ 1.20 & \cdot & \dfrac{2.50 \pm 0.13}{3} & = & 1.00 \pm .04 \end{array}$$

from R smoothed Mattingly + Stevenson $\sqrt{s} = .38\, Q = .66\ \mathrm{GeV}$

from CCFR measurement of GLLS $Q = \sqrt{3}\ \mathrm{GeV}$

SOB
Kataev
Lu
Gabadze

$$\frac{1}{3\sum_f Q_f^2} R_{e^+e^-}(\sqrt{s} = 5.0 \text{ GeV}) = 1.08 \pm 0.0$$

$$\therefore \quad \frac{\alpha_R^{expt}}{\pi}(\sqrt{s} = 5.0 \text{ GeV}) = 0.08 \pm 0.0$$

Predict:

$$\frac{\alpha_{g_1}}{\pi}(Q = 12.33 \pm 1.20 \text{ GeV})$$

$$= \frac{\alpha_{GLS}}{\pi}(Q = 12.33 \pm 1.20 \text{ GeV})$$

$$= 0.074 \pm 0.026$$

Expt:

$$\frac{\alpha_{GLS}}{\pi}(Q = 12.25 \text{ GeV}) = 0.093 \pm 0.0$$

Using BLM to fix scale:

we can relate PQCD observables

$$\boxed{\alpha_{g_1}(Q) = \alpha_R(0.520\, Q)\left(1 - \frac{\alpha_R}{\pi} + \cdots\right)}$$

* test QCD by tracking Q^2-dep, normalization

$\alpha_{g_1}(Q)$, $\alpha_R(Q)$ "effective charges"

scheme choices

obey RGE for $\alpha_s(Q^2)$ $\quad \underbrace{\beta_0, \beta_1}_{\text{scheme-indep}}, \beta_2 \cdots$ $\quad \beta_0 = 11 - 2/3\, n_f$

* No scale ambiguity = physical scale

* thresholds at heavy quarks correct

* Eliminate $\overline{MS}$: unphysical scheme, scale

* Some Higher-Twist also related

Conformal Coefficients: $(\beta = 0)$

$$\frac{\alpha_A}{\pi} = \frac{\alpha_B}{\pi} + C_1 \frac{\alpha_B^2}{\pi^2} + \cdots$$

Contain no IR renormalon

$$n! \, \beta_0^n \, \alpha_s^n$$

May be well-behaved

e.g Generalized Crewther Relation

$$\frac{\alpha_g}{\pi} = \frac{\alpha_R}{\pi} - \frac{\alpha_R^2}{\pi^2} + \frac{\alpha_R^3}{\pi^3} +$$

Geometric Series

Relation between $R(s)$ and $\Gamma_{\tau \to \text{hadrons} + \nu_\tau}$

$$R_\tau = 3\left[1 + \frac{\alpha_\tau}{\pi}(m_\tau)\right]$$

$$R(Q) = 3\sum_f Q_f^2\left[1 + \frac{\alpha_R(Q)}{\pi}\right]$$

Gorishny
Kataev
Larin

$$\alpha_\tau(Q) = \frac{\alpha_{\overline{MS}}(Q)}{\pi}$$

$$+ \left(\frac{\alpha_{\overline{MS}}(Q)}{\pi}\right)^2 \left[\left(\frac{947}{144} - \frac{11}{3}\zeta_3\right)C_A - \frac{C_F}{8} + \left(-\frac{85}{72} + 2\zeta_3\right.\right.$$

$$+ \left(\frac{\alpha_{\overline{MS}}(Q)}{\pi}\right)^3 \left\{\left(\frac{559715}{10368} - \frac{2591}{72}\zeta_3 - \frac{55}{18}\zeta_5 - \frac{121}{432}\pi^2\right)C\right.$$

$$+ \left(-\frac{1733}{576} - \frac{143}{12}\zeta_3 + \frac{55}{3}\zeta_5\right)C_A C_F - \frac{23}{32}C_F^2$$

$$+ \left[\left(-\frac{24359}{1296} + \frac{73}{6}\zeta_3 + \frac{5}{9}\zeta_5 + \frac{11\pi^2}{108}\right)C_A\right.$$

$$\left. + \left(-\frac{125}{288} + \frac{19}{6}\zeta_3 - \frac{10}{3}\zeta_5\right)C_F\right]f$$

$$\left. + \left(\frac{3935}{2592} - \frac{19}{18}\zeta_3 - \frac{\pi^2}{108}\right)f^2\right\} + \cdots$$

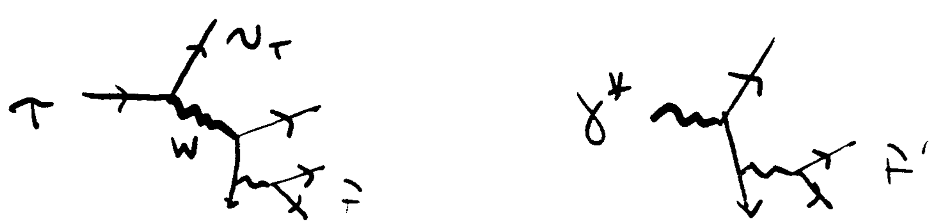
Apply NLO BLM: [f=3]

$$\boxed{\frac{\alpha_\tau(m_\tau)}{\pi} = \frac{\alpha_R(Q^*)}{\pi}}$$

$$Q^* = m_\tau \exp\left[-\frac{19}{24} - \frac{169}{128}\frac{\alpha_R(m_\tau)}{\pi}\right]$$

Note that all ζ_3, ζ_5, π^2 disappear!

Up to NNLO: α_τ, α_R related by scale shift



* $\Gamma_{\tau\to\nu_\tau \text{ hadrons}}$ ~ integral over $R_{e^+e^-}(Q)$

* MVT: $\frac{Q^*}{m_\tau} = e^{-19/24} + \cdots$ adjusts for $f \neq f'$ threshold

* BLM: discover hidden conformal relation

$$\alpha_\tau(m_\tau) = \alpha_R(c\, m_\tau)$$

$$c\, m_\tau = Q^*_{BLM}.$$

Suppose $\quad c \Rightarrow c' = c + \delta c$

$$\alpha_\tau(m_\tau) = \alpha_R(c'\, m_\tau)$$

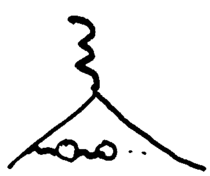
$$+ \sum_1 n!\, \beta_0^n \alpha_s^n (\delta c)$$

IR renormalon induced by choosing incorrect scale.

Infrared Renormalons

$$\rho = \sum_n c_n \, \alpha_s^n(\mu)$$

$$c_n \sim n! \, \beta_0^n \quad !$$



Divergent series generated by β_0 [V.P.] insertions

Eliminated iff $\mu = Q^*_{BLM}$

Remaining series $\Rightarrow$ conformal theory

Q^*_{BLM} expansion may be divergent

QCD: Padé, Borel sum

BLM + Resummation

BLM:

$$\boxed{\mathcal{O} = \sum_{n=0}^{\infty} c_n \, \alpha_V^n(Q_n^*)}$$

Observable

c_n : identical to coefficients in conformal theory $\beta = 0$

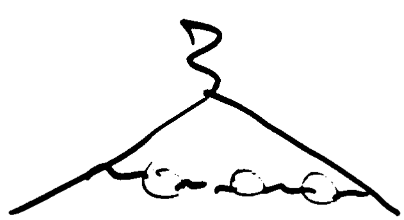
Any other scale: $Q_n \neq Q_n^*$

$n! \, (\beta_0 \alpha_s)^n$

Q_n^* resums $\beta_0, n_f, \ldots$

Renormalon growth from

$$\int dk^2 \, \alpha(k^2) \, f(k^2)$$



$\frac{1}{1-\Pi}$

$Q_n^* \Leftrightarrow$ MVT

resums all β terms

BLM equivalent to $\underline{MVT}$

$$\int d^4 l \, F(l) \, \alpha_V(l^2)$$

$$= \int d^4 l \, F(l) \, \alpha_V(Q^{*2})$$

$$\alpha_{\overline{MS}}(Q^2) = \alpha_V(e^{5/3} Q^2)\left[1 + 2\frac{\alpha_V}{\pi}(Q^2)\right]$$

See also: Braun + Beneke

Neubert

BLM

Lepage + Mackenzie LGT.

BLM

Lepage + Mackenzie
PRD 48, 2250 (

Mean value theorem approach

Want $\int d^4q \, f(q) \, \alpha_V(q) = \alpha_V(q^*) \int d^4q \, f(q)$

Better: $\alpha_V(q) = \alpha_V(\mu)\left[1 + \beta_0 \ln \frac{\mu^2}{q^2} \frac{\alpha_V(\mu)}{4\pi} + \cdots \right]$

$\therefore \int d^4q \, f(q) \ln \frac{\mu^2}{q^2} = \ln \frac{\mu^2}{q^{*2}} \int d^4q \, f(q)$

$* \qquad \ln q^{*2} = \dfrac{\int d^4q \, f(q) \ln q^2}{\int d^4q \, f(q)}$

sets physical scale in α_V scheme.

$\Rightarrow$ all n_f summed in $\alpha_V(q^*)$
not in coeffs

BLM

HW

Commensurate Scaling Relations

$$
\begin{tikzcd}[column sep=small]
 & \alpha_V(Q) \arrow[dl, leftrightarrow] \arrow[dr, leftrightarrow] & \\
\alpha_{\overline{MS}}(0.435Q) \arrow[d, leftrightarrow] & & \alpha_R(0.614Q) \arrow[d, leftrightarrow] \\
\alpha_{g_1}(1.18Q) \arrow[d, leftrightarrow] & & \alpha_\tau(1.36Q) \arrow[d, leftrightarrow] \\
\alpha_{M_2}(0.904Q) \arrow[rr, leftrightarrow] & & \alpha_{\Upsilon \to \gamma X}(2.77Q)
\end{tikzcd}
$$

e.g.

$$\alpha_{g_1}(Q) = \alpha_R(0.520Q)\left(1 - \frac{\alpha_R}{\pi} + \cdots\right)$$

$$\left.\begin{array}{l}\text{transitive}\\ \text{reflexive}\\ \text{symmetric}\\ \text{identity}\end{array}\right\} \text{scheme transformation :}$$

Renormalization Group

Peterman / Stueckelberg

We can define a "running coupling" for $\overline{MS}$:

S&B
Gill
Mirabelli

$$\tilde{\alpha}_{\overline{MS}}(Q^2) \equiv \alpha_V(e^{5/3} Q^2)$$
$$\left[1 + 2\frac{\alpha_V}{\pi} + C_2 \frac{\alpha_V^2}{\pi^2}\right]$$

computed by M. Peter

analytic extension $n_f(Q)$ contin

Q^2 scale determined : no ambiguity

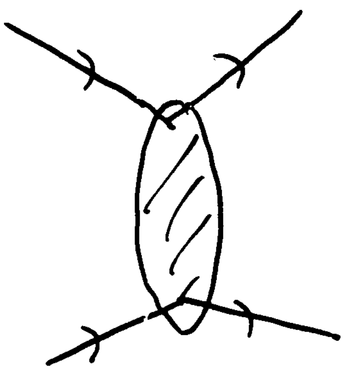
flavor thresholds unambiguous

$e^{5/3}$ due to $\overline{MS}$ convention

$$2\frac{\alpha_V}{\pi} = 2\frac{C_A}{3}\frac{\alpha_V}{\pi} \quad \text{non-Abelian}$$

* Use BaBar to determine

fundamental QCD coupling $\alpha_V(Q^2)$

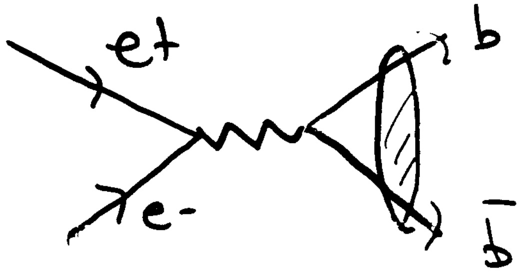


$$V(Q^2) \equiv -\frac{4\pi \overset{4/3}{C_F}\, \alpha_V(Q^2)}{Q^2}$$

potential for scattering of heavy quark test charg

$$\alpha_V(Q^2) \Rightarrow \alpha_{QED}(Q^2)$$

(Abelian theory unification)



sensitive to

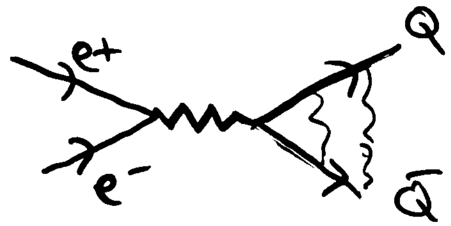
$$\alpha_V(\beta^2 s)$$

$$\beta^2 = 1 - \frac{4m_Q^2}{s}$$

Hoang. Teubner
Kühn SJB

PQCD corrections for slow heavy quarks

SJB
A. H. Hoang
J. H. Kühn
T. Teubner
M. Voloshin



$$\beta^2 = \frac{P_{cm}^2}{s} << 1$$

radiation suppressed

Strong corrections near threshold – relate to $\alpha_V(Q^2)$

$$|F_1(s) + F_2(s)|^2 = \frac{X}{1-e^{-X}}\left[1 - 4C_F\frac{\alpha_V(m^2e^{3/4})}{\pi}\right]$$

Use BLM scale-fixing

$$X = \frac{\pi C_F \alpha_V(4m^2\beta^2)}{\beta}$$

$$|F_1(s)|^2 = \frac{X'}{1-e^{-X'}}\left[1 - 3C_F\frac{\alpha_V(m^2e^{7/6})}{\pi}\right]$$

$$X' = \pi C_F \frac{\alpha_V(4m^2\beta^2/e)}{\beta}$$

$$\frac{dN}{d\Omega} \propto (1 + A\cos^2\theta)$$: Strong corrections to Anisotrop. $A(\beta)$

$$\boxed{\frac{dN}{d\cos^2\theta} = 1 + A(\beta^2)\cos^2\theta}, \qquad A = \frac{\tilde{A}}{1-\tilde{A}}$$

$\underline{e^+e^- \to Q\bar{Q}}$ $\qquad (\beta^2 << 1)$

double resummation

$\Rightarrow (\beta_0 \alpha_s)^n$

Coulomb rescattering

$$* \quad \tilde{A}_{QCD}(\beta^2) = \frac{\beta^2}{2} \; \frac{1 - 4\frac{\alpha_V(m^2 e^{7/6})}{\pi}}{1 - \frac{16}{3}\frac{\alpha_V(m^2 e^{3/4})}{\pi}} \; \frac{\frac{x'}{1-e^{-x'}}}{\frac{x}{1-e^{-x}}}$$

$$x = \frac{4\pi}{3}\frac{\alpha_V(4m^2\beta^2)}{\beta}, \qquad x' = \frac{4\pi}{3}\frac{\alpha_V(4m^2\beta^2/e)}{\beta}$$

* Relates observable to observable

$$\underline{A(\beta^2) \Longleftrightarrow V(Q^2)}$$

measured in HQLGTh

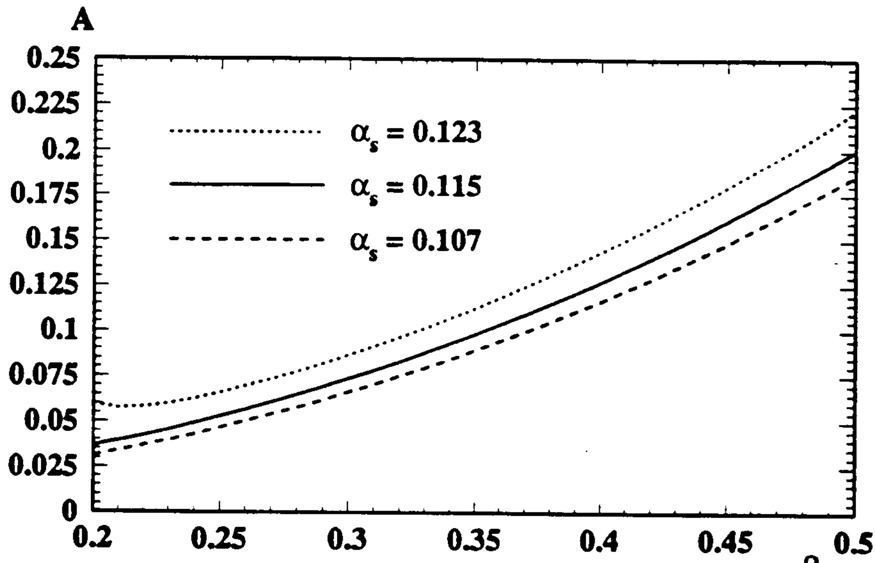
Davies et al

* No scale or scheme ambiguity! – four scales!

* Measures α_V at very low scales!

$$4m^2\beta^2 \to 0$$

$$\boxed{\frac{dN}{d\cos\theta}\left(e^+e^- \to b\bar{b}\right) \propto 1 + A(\beta^2)\cos^2\theta}$$



(a)

$$A = \frac{\tilde{A}}{1-\tilde{A}} \qquad \alpha_s = \alpha^{(5)}_{\overline{MS}}(M_Z)$$

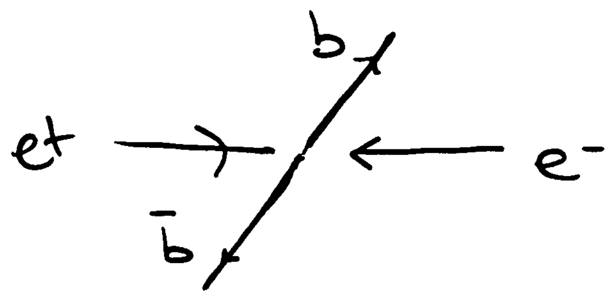
$$\tilde{A} = \frac{\frac{\beta^2}{2}\left(1 - 4\frac{\alpha_V(m^2e^{7/6})}{\pi}\right)}{\left(1 - \frac{16}{3}\frac{\alpha_V(m^2e^{3/4})}{\pi}\right)} \; \frac{1-e^{-x}}{1-e^{-x'}} \; \frac{\alpha_V(4m^2\beta^2/e)}{\alpha_V(4m^2\beta^2)}$$

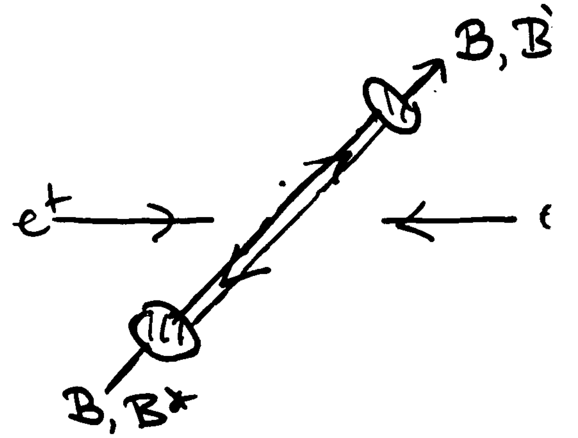
$$x = \frac{4\pi}{3}\frac{\alpha_V(4m^2\beta^2)}{\beta}$$

$$x' = \frac{4\pi}{3}\alpha_V(4m^2\beta^2/e)$$

* Ansatz : HQET

heavy hadrons follow direction of <u>heavy</u> quark





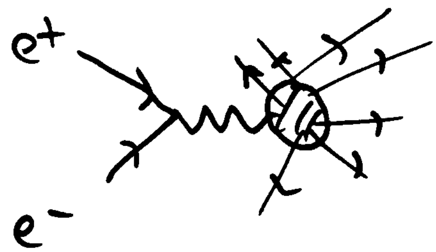
$e^+e^- \to b\bar{b}$

$e^+e^- \to B\bar{B},\ B\bar{B}^*,\ B$

Use <u>sum</u> over channels to determine

$$\frac{dN}{d\cos\theta} \propto 1 + A(\beta^2)\cos^2\theta$$

$$A(\beta^2) \Rightarrow \alpha_V(\beta^2 s)$$



$$(p_i + p_j)^2 < ys$$

$$f_n = \frac{\sigma_{n\text{-jet}}(y, s)}{\sigma_{TOT}(s)}$$

$$f_2(s, y) = 1 - c_{21}(y)\,\alpha_s(\mu) + c_{22}\left(y, \ln\frac{s}{\mu^2}\right)$$

$$f_3(s, y) = c_{31}(y)\,\alpha_s(\mu) + c_{32}\left(y, \ln\frac{s}{\mu^2}\right)\alpha_s^2$$

$$f_4(s, y) = c_{42}(y)\,\alpha_s^2(\mu) + \cdots$$

G. Kramer + B. Lampe Z. Phys A. 339, 189(1

$\alpha_s = \alpha_{\overline{MS}}$ ren. scheme

μ "arbitrary" renormalization scale

* BLM principle :

running coupling sums all V.P.

$\therefore$ coefficients must be independent of n_f, β_0

This fixes $\mu = Q^*(s, y)$

$$n_f \left[-\tfrac{2}{3} \ln \frac{s}{\mu^2} C_{21}(y) + \frac{C_F}{2\pi} Z_T(y) \right] = 0$$

$$Q^* = \sqrt{s}\, \exp\left(\frac{\frac{3}{4} \frac{C_F}{2\pi} Z_T(y)}{C_{21}(y)} \right)$$

special $\overline{MS}$

$\cong \frac{1}{2} [\ln y - \frac{11}{12} +$ small

$$\therefore \quad Q^*(y) \cong \sqrt{ys}\, e^{-\frac{11}{12}}$$

for small y

See also Ingelman Matsuura

Physically correct ! only sensitive to produced flavor

Kramer + Lampe

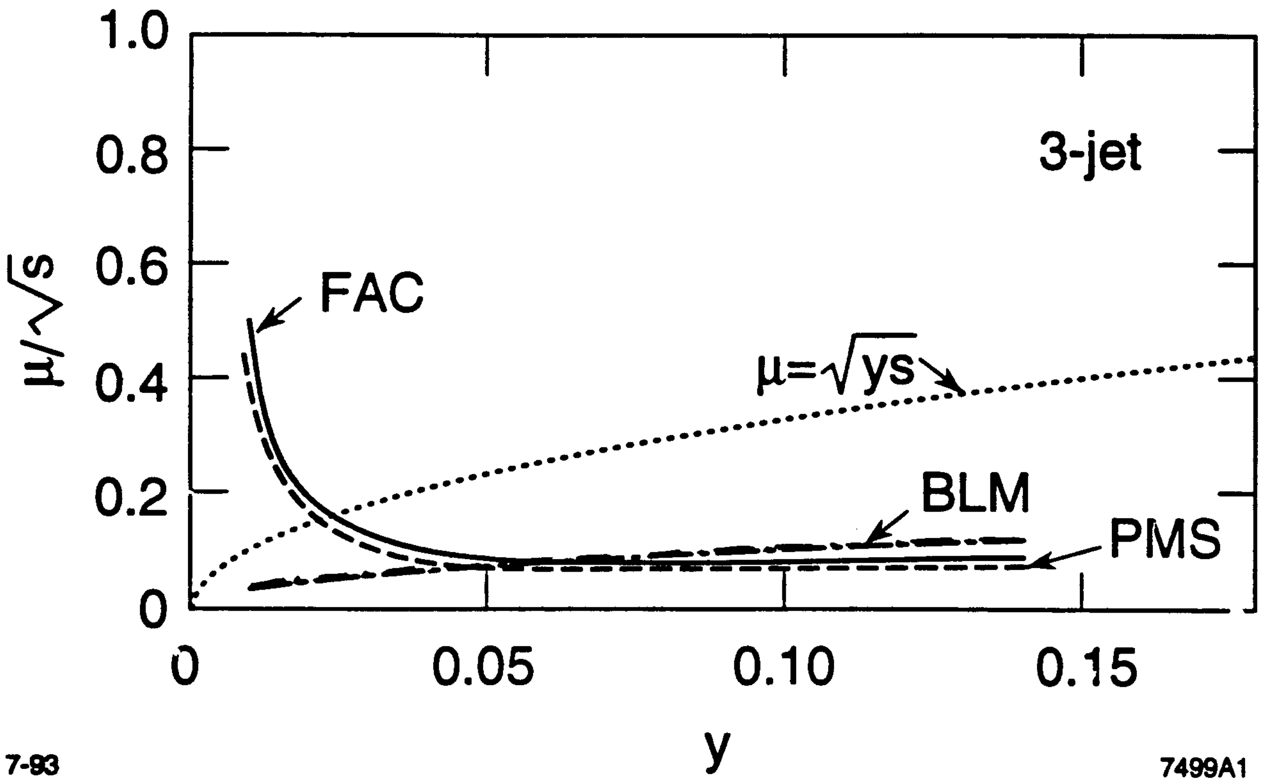
ms Shen

$\} \text{jet}$

- $\dfrac{\sigma_{3\text{-jet}}(y)}{\sigma_{TOT}} = C\,\alpha_s(\mu)\left[1 + (A n_f + B)\dfrac{\alpha_s}{\pi} + \cdots\right]$

$(p_i + p_j)^2 < y s$

$C(y),\ A(y),\ B(y)$

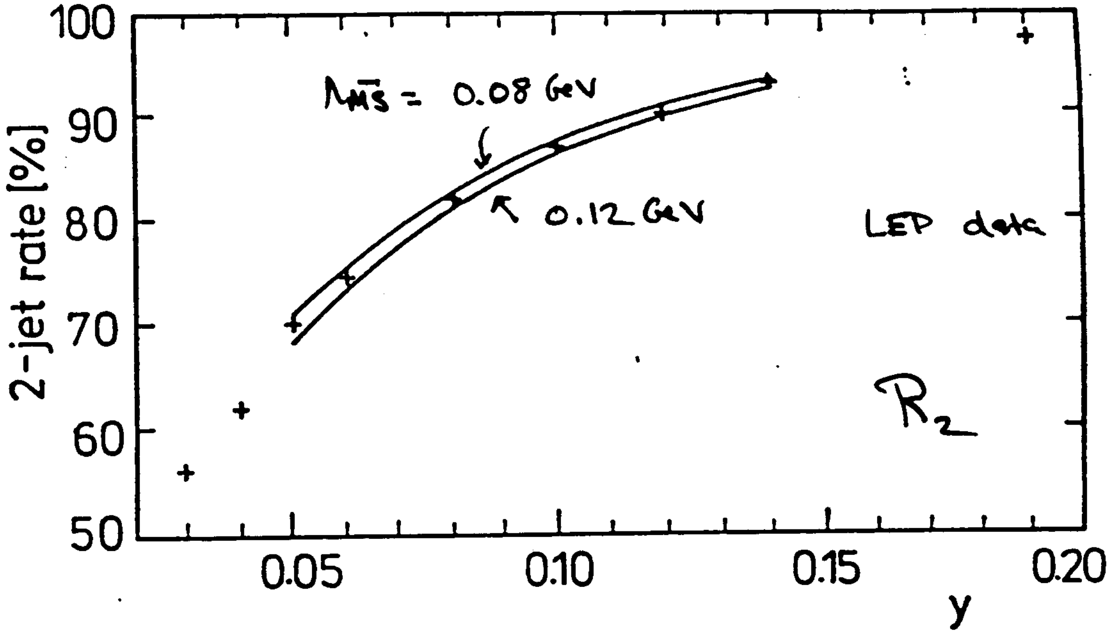


$\mu_{BLM}(y) = \mu\, e^{3A(y)} \propto \sqrt{ys}$

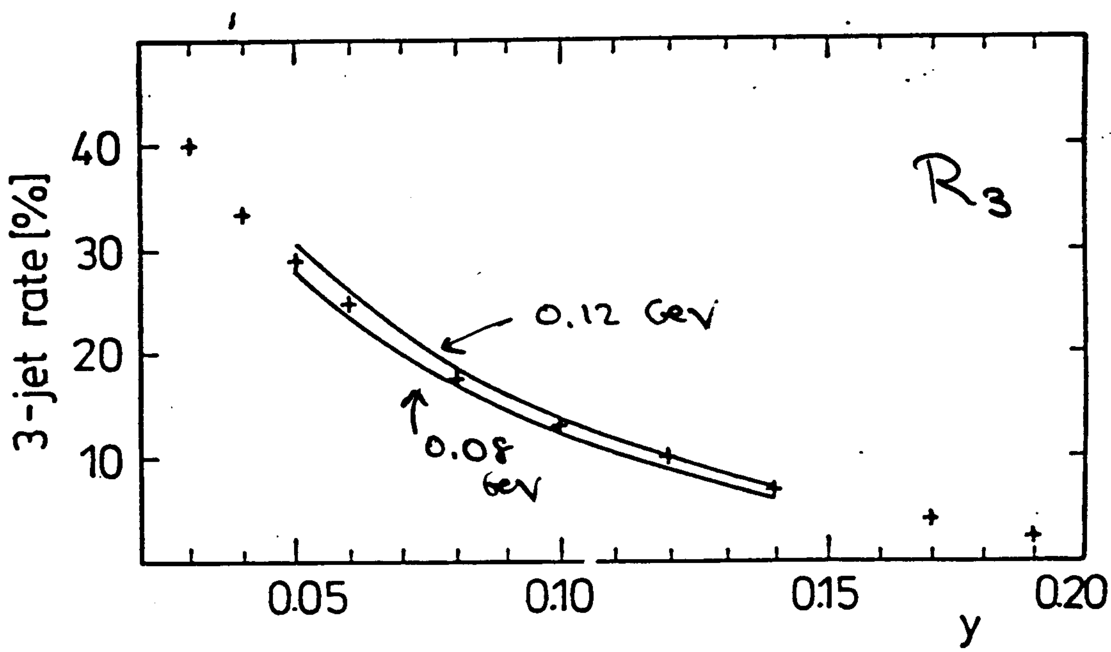
$\Rightarrow$ all n_f dep in α_s

Kramer + Lampe

Jet fractions: $(p_i - p_j)^2 < yS \Rightarrow$ same jet



$\ast$ $\Lambda_{\overline{MS}} = 0.10 \pm 0.0$ GeV



$\ast$ $\alpha_{\overline{MS}}(M_Z) = 0.107 \pm .00$

n_f indep.

BLM:
$$\frac{\sigma_{3\text{-jet}}(y)}{\sigma_{TOT}} = K_3 \frac{\alpha_{\overline{MS}}(Q^*)}{\pi}\left[1 + C_3 \frac{\alpha_{\overline{MS}}}{\pi} + \cdots\right]$$

$\mu_{BLM} = Q^* \propto \sqrt{yS}$ for $y \to 0$

Two-Jet Fraction at $y \to 0$

$$f_2(s,y) = 1 - c_{21}(y)\,\alpha_s(\mu) + c_{22}\,\alpha_s^2 + \cdots$$

$$\underset{y\to 0}{=} 1 - \frac{C_F}{2\pi}\, 2\ln^2\frac{1}{y}\,\alpha_s(\mu)$$

$$+ \frac{C_F^2}{(2\pi)^2}\, 2\ln^4\frac{1}{y}\,\alpha_s^2 + \cdots$$

$$= e^{-\frac{C_F}{\pi}\ln^2\frac{1}{y}\,\alpha_s(\mu)}$$

* Sudakov form factor:

probability of two jets $\Rightarrow 0$

for $(p_i - p_j)^2 < ys \quad y \to 0$

* Correct physics even if $\beta = 0$!

* BLM: $\alpha_s(\mu) \Rightarrow \alpha_s(Q^*)$

$$Q^* = \sqrt{ys}\; e^{-11/12}$$

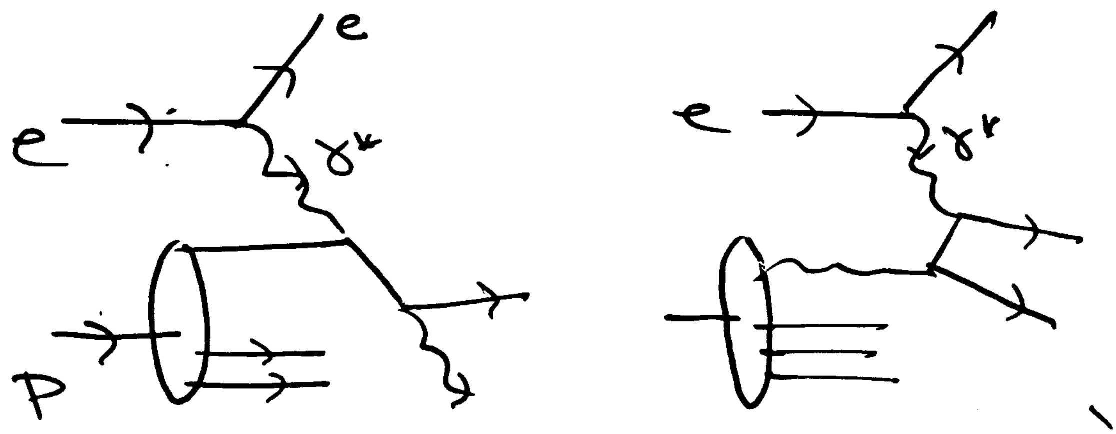
$\overline{MS}$

Lothman & Ingelman:

Elimination of Renormalization

Scale uncertainty

in DIS Jet Cross Sections



Compton Fusion

ratio of 2+1 jets to 1+1 jets

$$r = G(Q, x, P_T, y, \ldots)\, \alpha_s(\mu) + \cdots$$

choose scheme to define α_s e.g. $\overline{MS}$

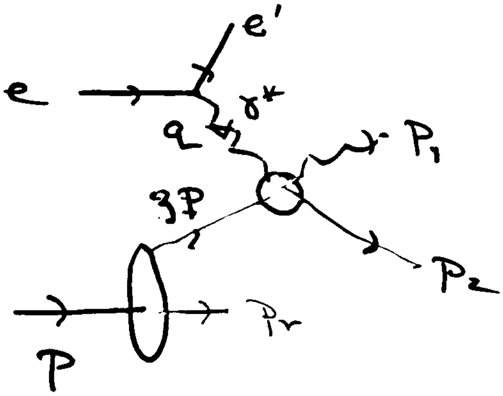
problem: what is μ

BLM: Separate scales for Compton
Fusion

Jets in Deep Inelastic Scattering

– A multi-scale problem

QCD Compton:



$$\xi = x\left[1+\frac{(P_1+P_2)^2}{Q^2}\right] = \frac{x}{x_p}$$

$$z = \frac{P_1 \cdot P}{q \cdot P}$$

$$(P_i+P_j)^2 > y_{cut}\, W^2$$

QCD: Compton $\gamma^* q \to g q$ QCD: Fusion $\gamma^* g \to q\bar{q}$

$\overline{MS}$

$$\frac{d\sigma_{compton}}{dx\,dQ^2 \ldots} = A\,\frac{\alpha_s(Q^2)}{\pi}\left[1+(B N_f + C)\frac{\alpha_s}{\pi}+\ldots\right]$$

Graudenz
Brodkorb, Körner, Mirkes

$$= A\,\frac{\alpha_s(Q^*)}{\pi}\left[1+D\,\frac{\alpha_s}{\pi}+\ldots\right]$$

comb. coeff

{Ingelman, Rathsman} * $Q^{*2} = \min\begin{cases} e^{-5/3}\, y_{cut}\, Q^2\, \frac{1-x}{x} \\ e^{-5/3}\,(1-z)\, Q^2\, \frac{1-x_p}{x} \end{cases}$ Same for QG Comp

BLM

$$A = A(x, Q^2, y_{cut}), \quad D = D(x, Q^2, y_{cut})$$

Ingelman + Rothsman

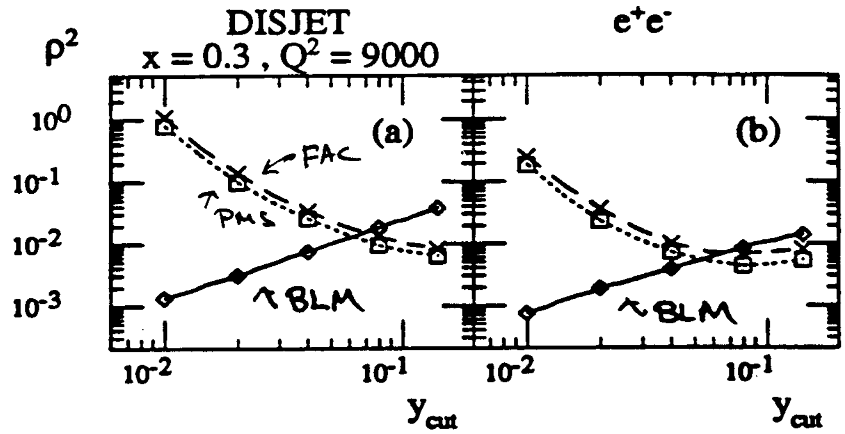


Figure 7: *Comparison of the scales obtained from the BLM (solid), FAC (dashed) and PMS (dotted) methods for (a) 2+1 jet production in DIS at* $x = 0.3$, $Q^2 = 9000\ GeV^2$ *and (b) 3-jet production in* e^+e^- *as given in ref. [22].*

$x = .001$

$Q^2 = 90\ GeV^2$

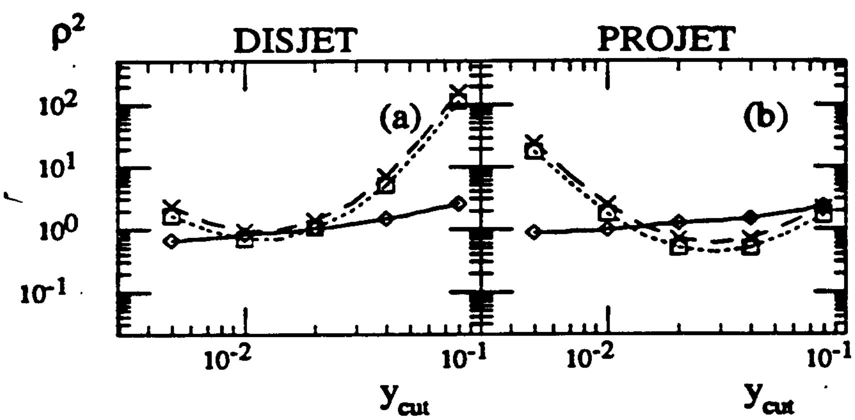


Figure 8: *Comparison of the scales obtained in* $x = 0.001$, $Q^2 = 90GeV^2$ *according to the BLM (solid), FAC (dashed) and PMS (dotted) methods with (a) the complete* α_s^2 *cross-section (DISJET) and (b) the partial result (PROJET).*

$$\rho^2 = \frac{Q^{*2}}{Q^2}$$

from $\alpha_V \Rightarrow \alpha_{\overline{MS}}$

DIS: Find $\rho^2_{BLM} \simeq \frac{\overline{W}^2}{Q^2} y_{cut}\, e^{-5/2}$

$$W^2 = Q^2 \frac{1-x}{x} = (q+p)^2$$

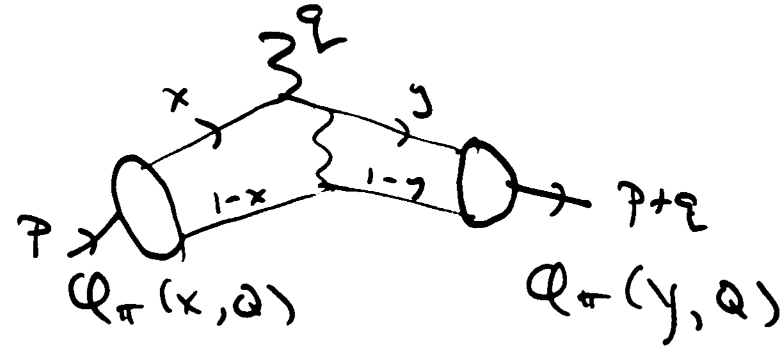
Application to Exclusive Processes

Form Factors

Elastic Scattering

Heavy Hadron Decays

e.g. $F_\pi(Q^2)$



$$\int_0^1 dx \int_0^1 dy \; \phi_\pi(x,Q)\, \phi_\pi(y,Q)\, T_H$$

Ji
Pang
Lu
820

$$T_H = \frac{\alpha_V\big((1-x)(1-y)Q^2\big)}{(1-x)(1-y)Q^2}$$

$$\alpha_V(Q^{*2}) \qquad Q^{*2} \cong e^{-3} Q^2$$

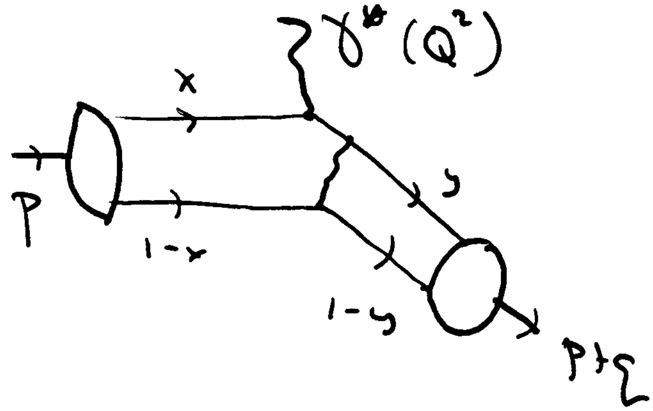
small scale even at large Q^2

$\alpha_V \sim$ const ? $\Rightarrow$ dim. counting

SJB
HLu
CRJi
A. Pang

Exclusive Amplitudes in QCD

$F_\pi(Q^2)$:



large Q^2 :

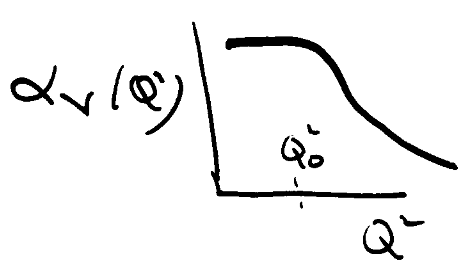
$$F_\pi(Q^2) = \int_0^1 dx \int_0^1 dy \; \phi_\pi(x,Q)\, \phi_\pi(y,Q) \; \frac{4\pi C_F \, \alpha_V\left[(1-x)(1-y)Q^2\right]}{(1-x)(1-y)Q^2}$$

For $\phi_\pi(x) \propto x(1-x)$

BLM scale

$$F_\pi(Q^2) = c_1 \frac{\alpha_V(e^{-3} Q^2)}{Q^2} + \text{h.o.}$$

very low scale!
$\sim \frac{1}{20} Q^2$



If $\alpha_V \sim$ const

$Q^2 F_\pi \sim c_1$

in $Q^2 < 20\, Q_0^2$.

$\alpha_V(Q^2)$ poorly determined at low Q^2

$$\alpha_V \sim \frac{4\pi}{\beta_0 \ln Q^2/\Lambda_V^2}$$ only valid at $Q^2 \gg \Lambda_V^2$

Program:

$$\bar{\alpha}_V^{(n)}(Q_0^2) = \int_0^{Q_0^2} dQ^2 \, \frac{(Q^2)^n}{(Q_0^2)^{n+1}} \, \alpha_V(Q^2)$$

Dokshitzer
Webber

physical parameters
constrain from expt.

BLM, CSR

Resum β terms into scale Q^*

Equivalent to n_f resummation

Large n_f resummation $\Rightarrow$ β_0 resummation

(Only n_f assoc. with renormalization)

Commensurate Scale Relations

* Relate Observable to Observable
* Convention - Independent Tests of QCD
* No scale ambiguity at any finite order
* Identifies underlying conformal relations
* Transitive : any intermediate scheme / scale
* Consistent with BLM, large n_f, naive non-abelianization
* Consistent with Abelian ($N_c \to 0$)
* Expansion in finite effective charges
* I.R. Renormalon summed into scale
* Q^* expansion can have renormalon
* Extends $\overline{MS}$ to analytic scale-fixed scheme
* α_V : special advantages

QCD Program

* Adopt standard effective charge

e.g: $\alpha_V(Q^2)$ many advantages

* Use observables to set parameters, e.g.

$$\frac{1}{Q_0^2}\int_0^{Q_0^2} dQ^2\, \alpha_V(Q^2) = \bar{\alpha}_V(Q_0^2)$$

* calculate from LGTh LCQ

* Commensurate Scale Relations:

Precise, convention-independent tests of QCD

Summary: PQCD / S.M.

* Connect Observable to Observable

$\Rightarrow$ no scale or scheme ambiguities

at any finite order

Commensurate Scale Relations

* "True" coefficients = conformal coefficients

$\mu = Q^*_{BLM}$ resums $\beta \neq 0$

$Q^{*2}_{BLM} = \langle k_g^2 \rangle_V$ (MVT)

* $\mu \neq Q^*_{BLM}$: $\sum_n (\beta_0 \alpha_s)^n \, n!$

$y \to 0$: $Q^*_{FAC,PMS} \to \infty$ unphysical behavior

sums conformal physics

* Error estimate from BLM / FAC

* $\alpha_V(Q)$: physical defin $\Leftrightarrow$ LGTh.

BLM Scale Fixing

* Physical Scales : test fermions probe gluon virtuality

* Multiple scales okay

* Commensurate scales:
  consistent treatment of quark thresholds

* Coefficients given by conformal-inv. theory

* Coefficients give meaningful criteria for convergence ; reflect physics

* Applications to all areas of Standard Model
  Uniform treatment of QED, EW, QCD

* BLM: resum β_0, β_1 into α_s

  $\rightarrow$ no criterion to minimize high order terms

Recent Developments   CSR-BLM   Scale Setting

*	Ingelman Rathsman	HERA 2+1 Jets
*	Smith + Voloshin Sirlin	Γ_{top}
*	Zerwas, Spira, Graudenz	Γ_{Higgs}
*	Luke, Savage, Wise	Γ_b
*	Braun + Beneke Mueller Neubert	UV Renormalons Q^*, Higher Twist
*	SJB C-R Ji	Exclusive Processes
*	SJB Kai Wong	$\frac{\partial}{\partial \ln Q^2} M_n(Q^2)$
*	SJB G. Mirabelli	Finite mass evolution $\alpha_s(Q)$
*	SJB H.J. Lu A. Kataev	Crewther Relation