Can perturbative QCD and Heavy Quark Effective Theory describe inclusive heavy hadron decays?

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CTEQ Symposium

Answer: Yes, but what are the dark secrets you need to worry about?

Duality
Renormalons and pole masses
Semileptonic vs. nonleptonic decays

Semileptonic $B \to X_c \ell \nu$ inclusive

Total rate, computed at parton level:

$$\Gamma = \frac{G_F^2 |V_{cb}|^2}{192\pi^3} m_B^5 \times 0.369 \quad \leftarrow \text{phase space}$$

$$\times \Big[1 - 1.54 \frac{\alpha_s}{\pi} + \cdots \qquad \leftarrow O(1)$$

$$- 1.65 \frac{\bar{\Lambda}}{m_B} + 1.44 \frac{\bar{\Lambda}}{m_B} \frac{\alpha_s}{\pi} + \cdots \leftarrow O\left(\tfrac{1}{m_B}\right)$$

$$- 0.95 \frac{\bar{\Lambda}^2}{m_B^2} - 3.18 \frac{\lambda_1}{m_B^2} - 0.02 \frac{\lambda_2}{m_B^2} + \cdots$$

$$\nwarrow O\left(\tfrac{1}{m_B^2}\right)$$

$$+ \cdots \Big]$$

Simultaneous expansion in $\frac{1}{m_B}$, $\frac{\alpha_s}{\pi}$

Questions:

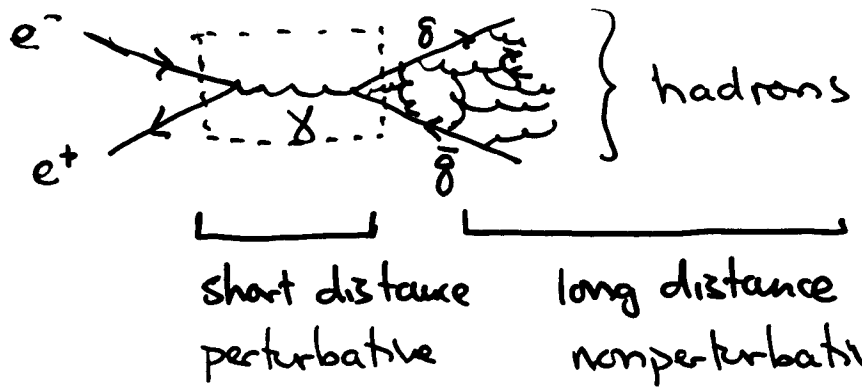
1) Does this have anything to do with the real world?

2) How well do we know inputs α_s, $\bar{\Lambda}$, λ_1, λ_2, ...

3) How accurate is it to truncate at this order?

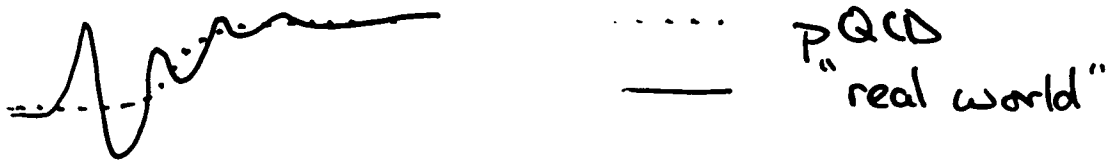
1) Why is a parton-level calculation sensible?

Recall $e^+e^- \to$ hadrons at c.o.m. energy $\sqrt{s}$



When does a perturbative calculation work?

Cartoon of a threshold region: $R = \dfrac{e^+e^- \to \text{hadrons}}{e^+e^- \to \mu^+\mu^-}$

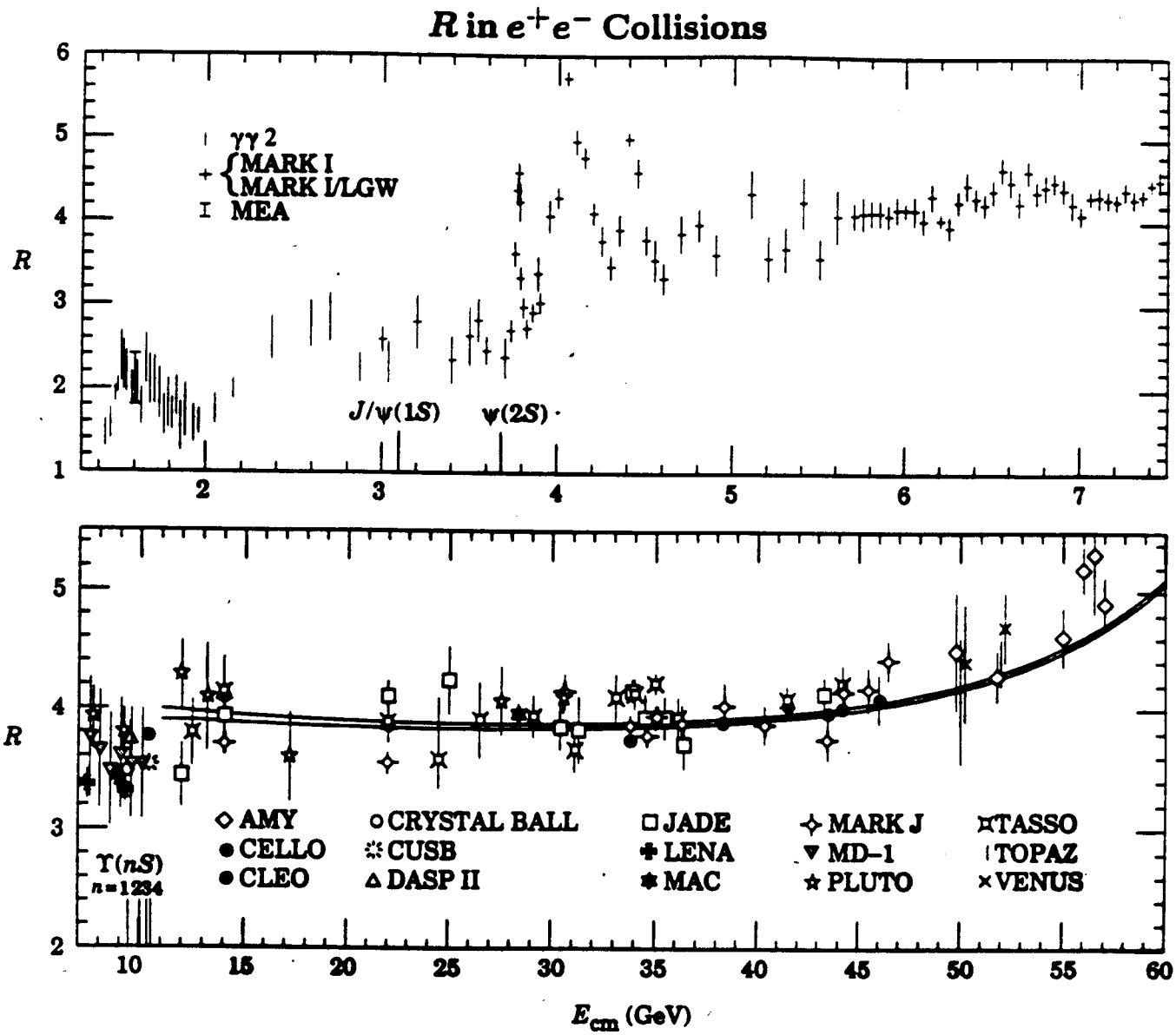


It works a) away from resonances, point-by-point

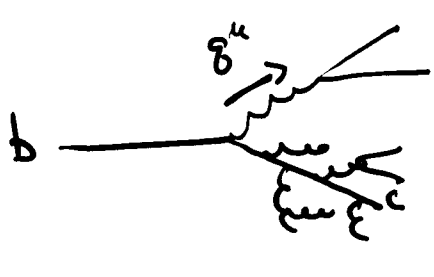
b) averaged in a resonance regime

"Local duality": point by point, holds sufficiently far above resonance regime

"Global duality": averaged over $\sqrt{s}$, can hold when local duality fa


R in e+e− Collisions
γγ 2
MARK I
MARK I/LGW
MEA
J/ψ(1S)
ψ(2S)
R
AMY
CELLO
CLEO
CRYSTAL BALL
CUSB
DASP II
JADE
LENA
MAC
MARK J
MD-1
PLUTO
TASSO
TOPAZ
VENUS
Υ(nS)
n=1234
Ecm (GeV)

Semileptonic b decay



$p_H = p_b - q$

$\Rightarrow s_H = m_b^2 - 2p_b \cdot q + q^2$

Integral over q (lepton phase space)

⇓

Average over s_H

⇓

Need global duality only

(This can be made more formal ...)

2) Can we pin down the inputs?

a) $\alpha_s(m_b) \approx 0.21$ ✓

Is m_b the correct scale at which to evaluate α_s?

Hint from two loops: $\alpha_s(\frac{m_b}{2})$?

More later...

b) $\lambda_2 = \frac{1}{4}(m_{B^*}^2 - m_B^2) \simeq 0.12\ \text{GeV}^2$ ✓

c) $\lambda_1 = -\frac{1}{2M_B} \langle B| \bar{b}(iD)^2 b |B\rangle$

$\simeq -$ (kinetic energy of b quark in B rest frame)

Models/prejudice: $\lambda_1 \simeq -(0.1-0.4)\ \text{GeV}^2$

Corrections from $\lambda_1, \lambda_2 \approx$ few percent

$\Rightarrow$ don't need too much accuracy

d) $\bar{\Lambda} = \left[m_B - m_b^{pole} \right]_{m_b \to \infty}$

Either $\bar{\Lambda}$ or m_b^{pole} appears in $\Gamma_{s.l.}$.

Phase space $\sim m_b^2 (m_b - m_c)^3$

Problem: there is no physical pole in the b propagator! (confinement)

So m_b^{pole} is not defined...

Solution? Define

$$m_b^{pole} = m_b^{pole} \Big|_{\text{fixed order in } \alpha_s}$$

To order α_s^n, there $\underline{\underline{\text{is}}}$ a pole in the b propagator

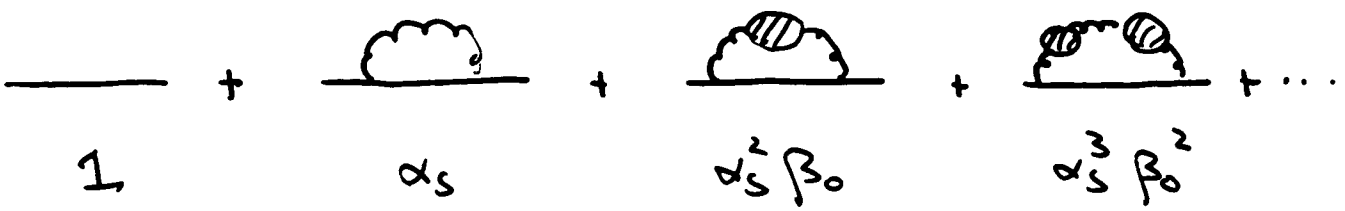
Truncate series $\Rightarrow$ ambiguity of order α_s^{n+1}

Remove ambiguity by taking limit $n \to \infty$

Problem: the limit does not exist!

QCD is an <u>asymptotic</u> series

Interesting subset of graphs:



$\propto \alpha_s \beta_0$, gluon self-energy, one loop

$\beta_0 \simeq 9$

$$\Rightarrow \quad m_b^{pole} = m_b(m_b)\left[1 + \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \beta_0^{n-1}\, n!\, c_n\right]$$

Best accuracy => truncate at smallest term

Lesson: perturbation theory does not contain all the physics!

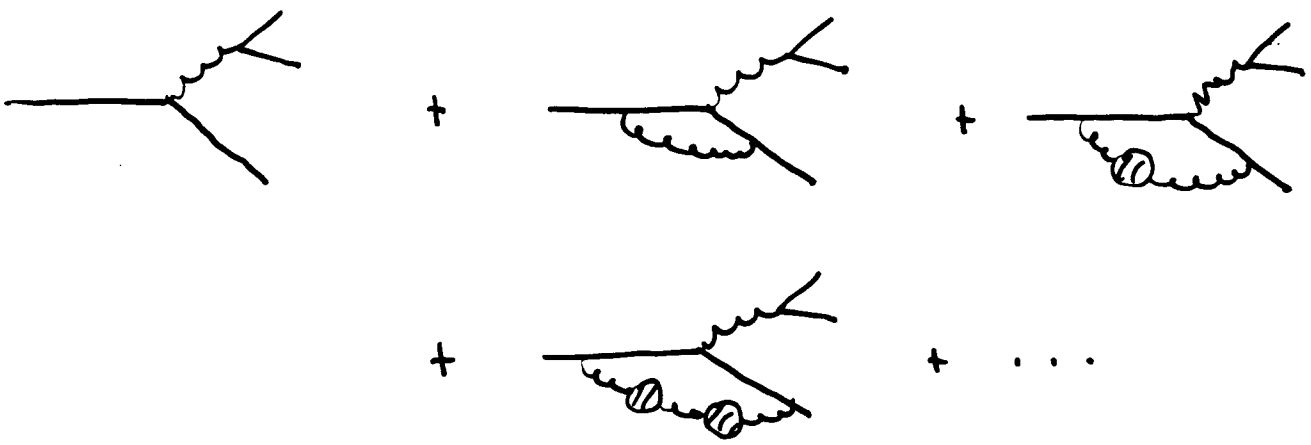
Ambiguity in perturbation theory: infrared renormalon

$\bar{\Lambda} = m_B - m_b^{pole}$ has a renormalon ambiguity, too

Renormalon in $\bar{\Lambda} \gtrsim 100$ MeV

$\Rightarrow$ ambiguity in $\Gamma_{s.l.}$ of order $\frac{\Lambda_{QCD}}{m_B}$

Another problem: renormalon in $\Gamma_{s.l.}$ itself!



$$\Gamma_{s.l.} = \Gamma_0 \left[1 + \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \beta_0^{n-1}\, n!\, a_n \right]$$

$\Rightarrow$ another ambiguity in $\Gamma_{s.l.}$ of order $\frac{\Lambda_{QCD}}{m_B}$

Wonderful Fact

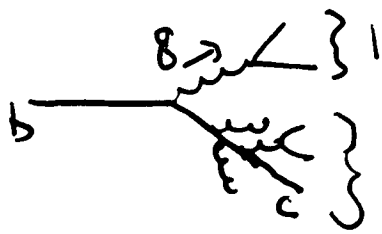
These renormalon ambiguities are related,

and when treated appropriately, cancel each other!

Both renormalons associated with appearance of m_b^{pole} (or $\bar{\Lambda}$).

Eliminate this dependence!

For example, measure $\langle s_H \rangle$ in $B \to X_c l \nu$



$$\langle s_H - \bar{m}_D^2 \rangle = m_B^2 \left[0.051 \frac{\alpha_s}{\pi} + \cdots \right.$$
$$+ 0.23 \frac{\bar{\Lambda}}{m_B} + 0.092 \frac{\bar{\Lambda}}{m_B} \frac{\alpha_s}{\pi} + \cdots$$
$$+ 0.26 \frac{\bar{\Lambda}^2}{m_B^2} + 1.01 \frac{\lambda_1}{m_B^2} - 0.31 \frac{\lambda_2}{m_B^2} + \cdots$$
$$\left. + \cdots \right]$$

Same ambiguities as $\Gamma_{s.l.}$, of order $\frac{\Lambda_{QCD}}{m_B}$

But eliminate $\bar{\Lambda}$:

$$\Gamma_{s.l.} = \Gamma_0 \left[1 - 7.17 \frac{\langle s_H - \bar{m}_D^2 \rangle}{m_B^2} - 1.17 \frac{\alpha_s}{\pi} - \cdots \right]$$

no ambiguity at this order!

Canceling the renormalon

⇓

Improved behaviour of pQCD at high orders

⇓

Truncation induces ambiguity of order $\frac{\Lambda_{QCD}^2}{m_B^2}$

(series is still asymptotic)

In principle, can continue improvement program to eliminate subleading renormalons, too.
(more physical observables)

The improvement has been checked explicitly, to five loops, in $B \to X_u l \nu$

Consider the simpler decay, $B \to X_u \ell \nu$

Compute terms $\alpha_s^n \beta_0^{n-1}$ to five loops...

$$\Gamma = \frac{G_F^2 V_{ub}^2}{192\pi^3} m_B^5 \Big[1 - 2.4 \frac{\alpha_s}{\pi} - 3.0 \left(\frac{\alpha_s}{\pi}\right)^2 \beta_0 - 4.4 \left(\frac{\alpha_s}{\pi}\right)^3 \beta_0^2 - 7.7 \left(\frac{\alpha_s}{\pi}\right)^4 \beta_0^3 - 15.7 \left(\frac{\alpha_s}{\pi}\right)^5 \beta_0^4 - \cdots - 5 \frac{\bar{\Lambda}}{m_B} + \cdots \Big]$$

$\alpha_s(m_b) = 0.21$, $\beta_0 = 9$ $\Rightarrow$

$$\Gamma = \frac{G_F^2 V_{ub}^2}{192\pi^3} m_B^5 \Big[1 - 0.06 - 0.12 - 0.11 - 0.11 - 0.14 - \cdots - 5 \frac{\bar{\Lambda}}{m_B} + \cdots \Big]$$

This is real trouble!

Beyond perturbation theory: renormalon can be absorbed into $\bar{\Lambda}$

Consider series for $\langle s_H \rangle$ in $B \to X_u \ell \nu$:

$$\langle s_H \rangle = m_B^2 \Big[0.20 \tfrac{\alpha_s}{\pi} + 0.35 \left(\tfrac{\alpha_s}{\pi}\right)^2 \beta_0 + 0.64 \left(\tfrac{\alpha_s}{\pi}\right)^3 \beta_0^2 + 1.29 \left(\tfrac{\alpha_s}{\pi}\right)^4 \beta_0^3 + 2.95 \left(\tfrac{\alpha_s}{\pi}\right)^5 \beta_0^4 + \cdots + \tfrac{7}{10} \tfrac{\bar{\Lambda}}{m_B} + \cdots \Big]$$

$$= m_B^2 \big[0.0135 + 0.0141 + 0.0156 + 0.0189 + 0.0261 + \cdots + \tfrac{7}{10} \tfrac{\bar{\Lambda}}{m_B} \big]$$

Just as bad... but eliminating $\bar{\Lambda}$ will eliminate renormalon, too.

$$\Gamma = \frac{G_F^2 V_{ub}^2}{192\pi^3} m_B^5 \Big[1 - 7.14 \tfrac{\langle s_H \rangle}{m_B^2} - 0.064 - 0.020 - \underline{0.002} - 0.022 - 0.047 - \cdots \Big]$$

Truncate at smallest term

Remaining renormalon is $\frac{1}{m_b^2}$, not so important...

Bound on V_{cb} from semileptonic inclusive decay

Take $\langle s_H - \bar{m}_D^2 \rangle \geq 0$

Will improve in future

Data on $\Gamma(B \to X_c \ell \nu)$

$\Rightarrow$

$$\boxed{V_{cb} \geq 0.037 \cdot \left(\frac{\tau_B}{1.5 \text{ ps}}\right)^{-1/2}}$$

- two loop corrections under control
- leading renormalon eliminated
- $1/m_b^2$ corrections a few percent, positive

Alternative

Eliminate renormalon by writing $\Gamma_{s.l.}$

in terms of a short distance mass,

such as $\overline{m}_b(m_b)$.

1. Very clean, theoretically
2. How do you determine such an unphysical parameter with the necessary accuracy?

Careful of lattice, QCD sum rules!

accurate m_b (lattice)

$\Downarrow$ ← asymptotic perturbation series

accurate $\overline{m}_b(m_b)$?

Nonleptonic Decays

$b \to c \bar{c} s$, $b \to c \bar{u} d$

Parton-level calculation is similar to semileptonic decay

- perturbative corrections more complicated
- important differences in "spectator" effects, arise at order Λ_{QCD}^3 / m_b^3

Interesting problems:

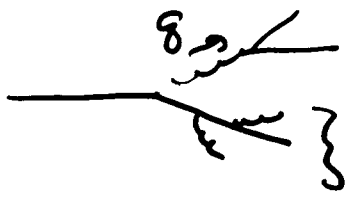
- B decays: nonleptonic branching ratio (high)
  charm multiplicity (low)
- Λ_b decays: lifetime $\tau(\Lambda_b)$ (short)

Almost certainly involve QCD, not "new physics"!

Tests of pQCD and HQET in nonleptonic decays.

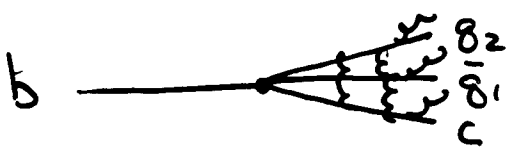
Duality in nonleptonic decays

Semileptonic decay:



Integral over q → average over s_H

Nonleptonic decay:



Need local duality, since no external momentum to average over

- $m_b^2 \gg \Lambda_{QCD}^2$, local duality holds as $m_b \to \infty$
- Is $m_b \simeq 4.5$ GeV large enough to be away from resonances in final state? (c.f. global duality → average over resonances)

Local duality is "reasonable", but if your calculation doesn't give the right answer ... ?

Perturbative corrections

One loop: $\frac{\alpha_s}{\pi}$ take $\alpha_s(m_b) \approx 0.21$

- $b \to c l \nu$ $\sim 10\%$
- $b \to c \bar{u} d$ $\sim 10\%$ quite dependant on m_c
- $b \to c \bar{c} s$ $\sim 30\%$ very dependant on m_c

Two loops: $\left(\frac{\alpha_s}{\pi}\right)^2 \beta_0$

dominant part of two loop term?

- $b \to c l \nu$ $\sim 3\%$ after $\bar{\Lambda}$ eliminated
- $b \to c \bar{u} d$ $\sim 5\%$
- $b \to c \bar{c} s$ $\sim 30\%$

Trouble for $b \to c \bar{c} s$?

(Also, correction $\sim \pi^2 \left(\frac{\alpha_s}{\pi}\right)^2$ as $m_c \to m_b/2$)

What about charm decays?

$\tau(D^+) \neq \tau(D^0)$

$\Rightarrow$ large spectator effects in nonleptonic D decays

- failure of local duality?
- large corrections of order $\frac{\Lambda_{QCD}^3}{m_D^3}$

Semileptonic D decays

- large perturbative corrections, $\frac{\alpha_s(m_c)}{\pi}$, $\left[\frac{\alpha_s(m_c)}{\pi}\right]^2 \beta_0$ make precision tests difficult
- one test in which large corrections cancel,

$$\langle s_H \rangle_{D_s} - \langle s_H \rangle_D = \begin{cases} 0.18 \pm 0.02\ \text{GeV}^2 & \text{(theory} \\ 0.19 \pm 0.03\ \text{GeV}^2 & \text{(expt.)} \end{cases}$$

works well, but by itself this proves little

$\Rightarrow$ The jury is still out!

So... do pQCD and HQET describe B decays?

Semileptonic: pretty well

- global duality
- eliminate renormalon by eliminating $\bar{\Lambda}$
- full two loop calculation needed

⇒ eventual accuracy at ~ 5% level for $B \to X_c \ell \nu$

Nonleptonic: maybe, but not as solid

- local duality
- larger two loop corrections, esp. for $b \to c\bar{c}s$
- outstanding problems ($Br(SL)$ of B, $\tau(\Lambda_b)$) test the formalism

⇒ failure at 25% level or higher would not be surprising for B decays