HOW WELL SHOULD PQCD JETS MATCH DATA

- QCD VS PERTURBATIVE QCD

  (IS NOT QCD ≡ NLO QCD ??)

- VALIDITY OF PQCD JET PREDICTIONS

  (WHY IS NLO QCD NOT AGREEING WITH DATA AT $E_T^{JET/PHOTON/Z\ldots} \gtrsim 10$ GEV AND WHY IS THIS NOT A PROBLEM)

- REDEFINING PQCD WHEN FACING DEVIATIONS

  (SHOWER MC., RESUMMATION & MORE)

W. GIELE
CTEQ WORKSHOP
NOV 1996.

QCD vs PERTURBATIVE QCD

- GIVEN QCD LAGRANGIAN → $P\bar{P} \to H_1(P_1) H_2(P_2) \cdots H_N(P_N)$

  $\to J_1(P_1) \cdots J_M(P_M)$

  → ONLY PARAMETER $\alpha_s(M_Z)$

  → NO PARTON DENSITY FUNCTIONS,
  FRAGMENTATION FUNCTIONS,
  LO/NLO/SHOWERING/RESUMMATIONS/HIGHER TWIST

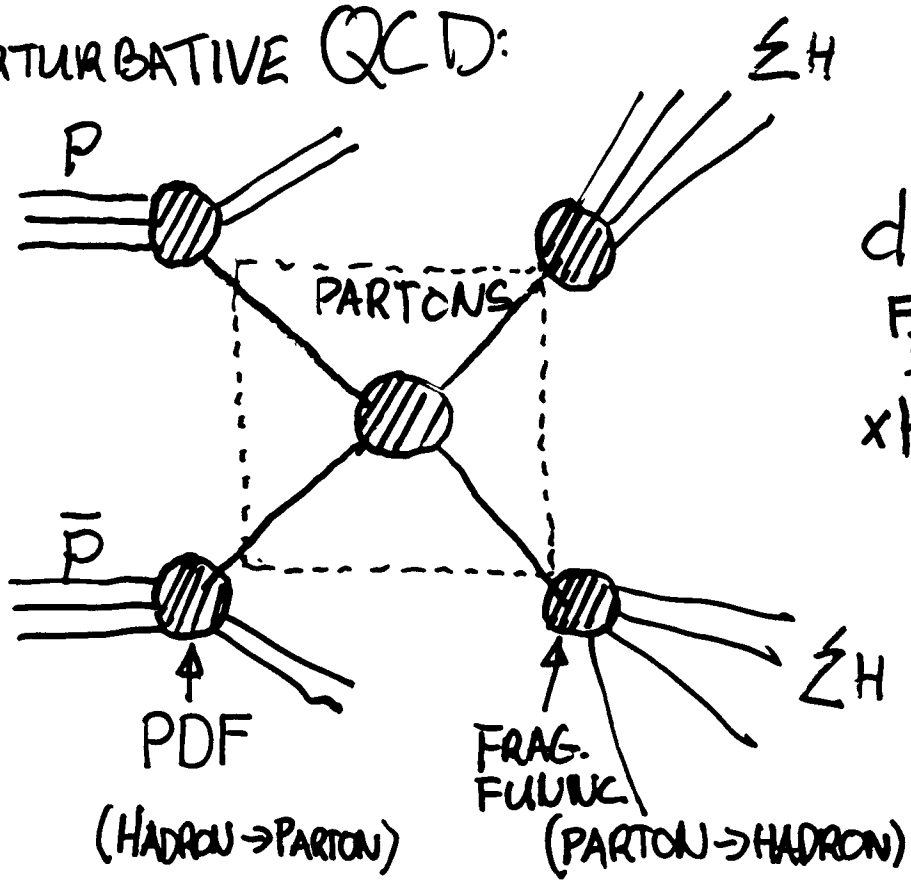
- SO GIVEN α_s QCD PREDICTS EVERYTHING FROM HADRON SPECTRUM TO HIGH MOMENTUM JET PRODUCTION.

- UNFORTUNATELY THEORISTS CANNOT SOLVE QCD EXACTLY → WE NEED APROXIMATIONS (VALID/USEFUL IN SPECIFIC SITUATIONS)

  → LATTICE QCD, ... → HADRON SPECTRA
  → DECAY RATES

  → PERTURBATIVE QCD → HIGH MOMENTUM TRANSFER QCD

PERTURBATIVE QCD:



$$d\sigma(P\bar{P} \to J_1 J_2) = F_{P \to P_1} F_{\bar{P} \to P_2} d\hat{\sigma}(P_1 P_2 \to P_3 P_4) \times H_{P_3 \to \Sigma H} H_{P_4 \to \Sigma H}$$

APPROXIMATION #1: FACTORIZATION (EVEN IF WE COULD CALCULATE THE PARTONIC CROSS SECTION UP TO ALL ORDERS, IT IS STILL AN APPROXIMATION TO QCD)

→ THE HADRONIC CROSS SECTION FACTORIZES INTO A HADRONIC PART (WHICH DESCRIBES THE HADRON/PARTON TRANSISION AND HAS TO BE MEASURED) AND A PARTONIC CROSS SECTION ($gg \to gg$, $gq \to gq$, ……)

→ IN OTHER WORDS WE HAVE PDF'S (AND FRAG. FUNC.) WHICH ARE INDEPENDENT OF THE PARTONIC HARD SCATTERING PROCESS

→ NOTES:

- FACTORIZATION IS ONLY CORRECT UP TO POWER SUPPRESSED TERMS (I.E. TERMS OF ORDER $\mathcal{O}(\frac{1}{Q}) \sim \mathcal{O}(\frac{1}{E_T^{JET}})$). IN OTHER WORDS FACTORIZATION IS AN APPROXIMATION IN ITSELF.
- THE ABOVE STATEMENTS HAVE BEEN PROVE FOR VERY SPECIFIC PROCESSES (E.G. DY PRODU NOT FOR E.G. 2 JET PRODUCTION (BUT GENERALLY BELIEVED TO BE CORRECT).
- PDF'S AND FRAG. FUNCTIONS ONLY EXIST WITHIN THE FRAMEWORK OF FACTORIZATION AND ARE IN THIS SENSE THEORETICAL CONSTRUCTS

APPROXIMATION #2: PERTURBATIVE EXPANSION

(EVALUATING THE PARTONIC CROSS SECTION USING FEYNMAN RULES)

→ WE EXPAND THE QCD PARTONIC SCATTERING IN TERMS OF α_s

$$d\hat{A}_{PARTONIC} = \alpha_s^N (a_0 + \alpha_s a_1 + \alpha_s^2 a_2 + \dots)$$

ASSUMING $a_0 \gg \alpha_s a_1 \gg \alpha_s^2 a_2 \gg \dots$

$\left(d\hat{A}^{LO}_{PARTONIC} = \alpha_s^N a_0 \; ; \; d\hat{A}^{NLO}_{PARTONIC} = \alpha_s^N (a_0 + \alpha_s a_1) \; ; \dots \right)$

→ THIS ENABLES US TO USE FEYNMAN-RULES/GRAPHS AND WE CAN CALCULATE THE PARTONIC CROSS SECTION ORDER BY ORDER.

→ ALSO BY EXPRESSING THE PARTONIC CROSS SECTION IN TERMS OF FEYNMAN-GRAPHS WE CAN APPLY RESUMMATION TECHNIQUES ETC... (i.e. EXPAND IN $\alpha_s LOG(x)$; $x \ll 1$)

→ NOTES:

- α_s IS USUALLY DEFINED THROUGH Λ_{QCD} AT HIGH MOMENTUM TRANSFER (IE HIGH E_T^{SET}) THIS GIVES A LARGE SENSITIVITY TO THE RUNNING (i.e. 1-LOOP, 2-LOOP, ...)

  FOR HIGH E_T-PHYSICS IT IS FAR MORE CONVENIENT TO DEFINE α_s AT SOME FIXED SCALE

AT THE MOMENT THIS WOULD BE $\alpha_s(M_Z)$.

THEN $$\alpha_s(Q) = \frac{\alpha_s(M_Z)}{1+\alpha_s(M_Z) L^{(n)}(Q/M_Z)}$$

WITH $L^{(n)}$ AN EXPANTION IN $\alpha_s(M_Z)$

1-LOOP RUNNIG: $L^{(1)}(\lambda) = b_0 LOG(\lambda)$

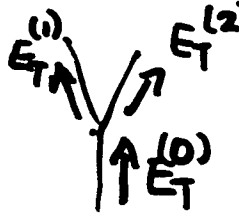
2-LOOP. $L^{(2)} = [b_0 + b_1 \alpha_s(M_Z)] LOG(\lambda)$

3-LOOP $L^{(3)} = (b_0 + b_1 \alpha_s(M_Z) + b_2^{\overline{MS}} \alpha_s(M_Z)) LOG(\lambda)$
$- \frac{b_0 b_1}{2} \alpha_s^2(M_Z) LOG^2(\lambda)$

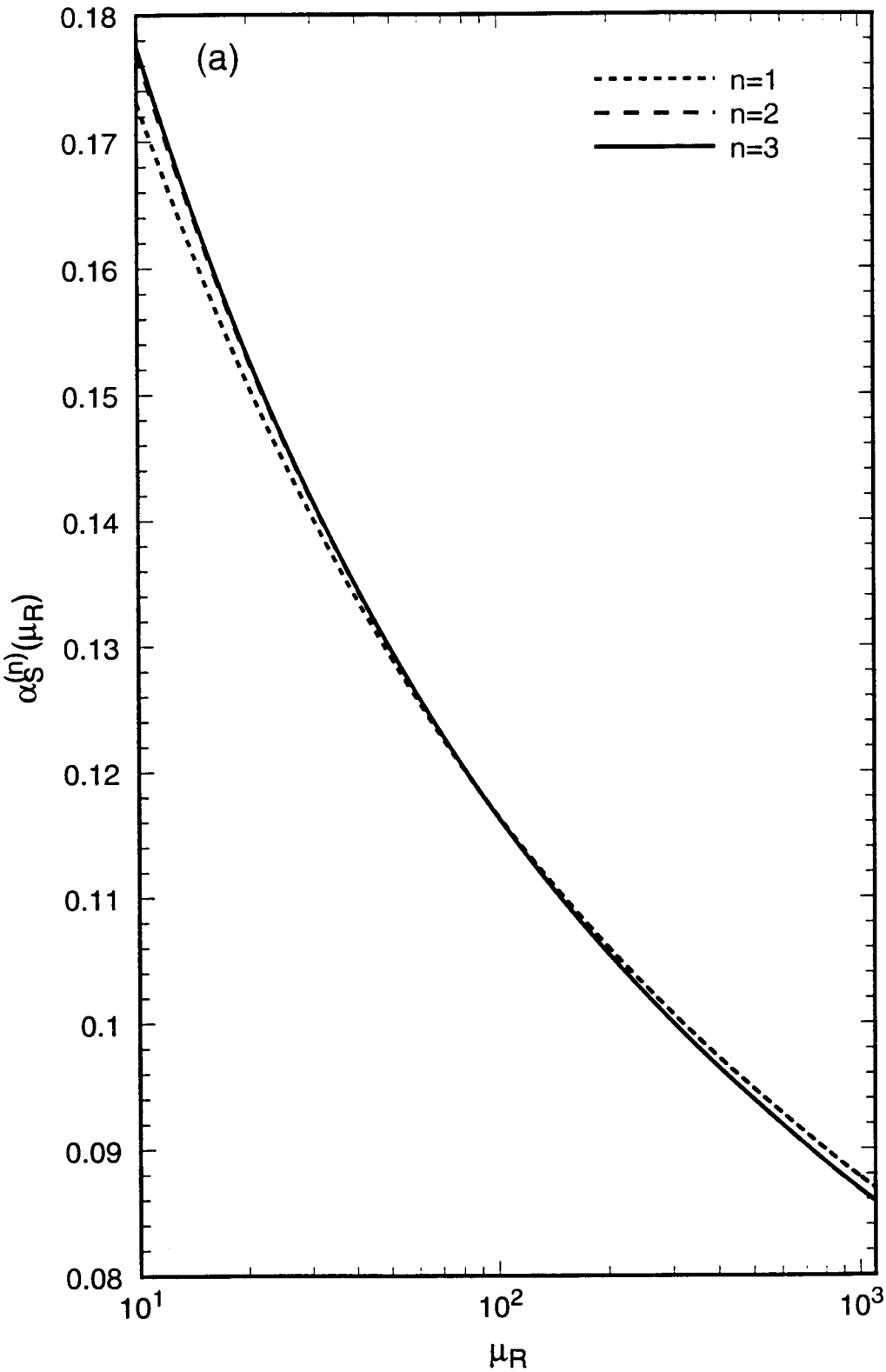
(VERY) SMALL DEPENDENCE ON RUNNING ORDER (AS LONG AS $E_T \sim \mathcal{O}(M_Z)$) (would also be nice for PDF's)

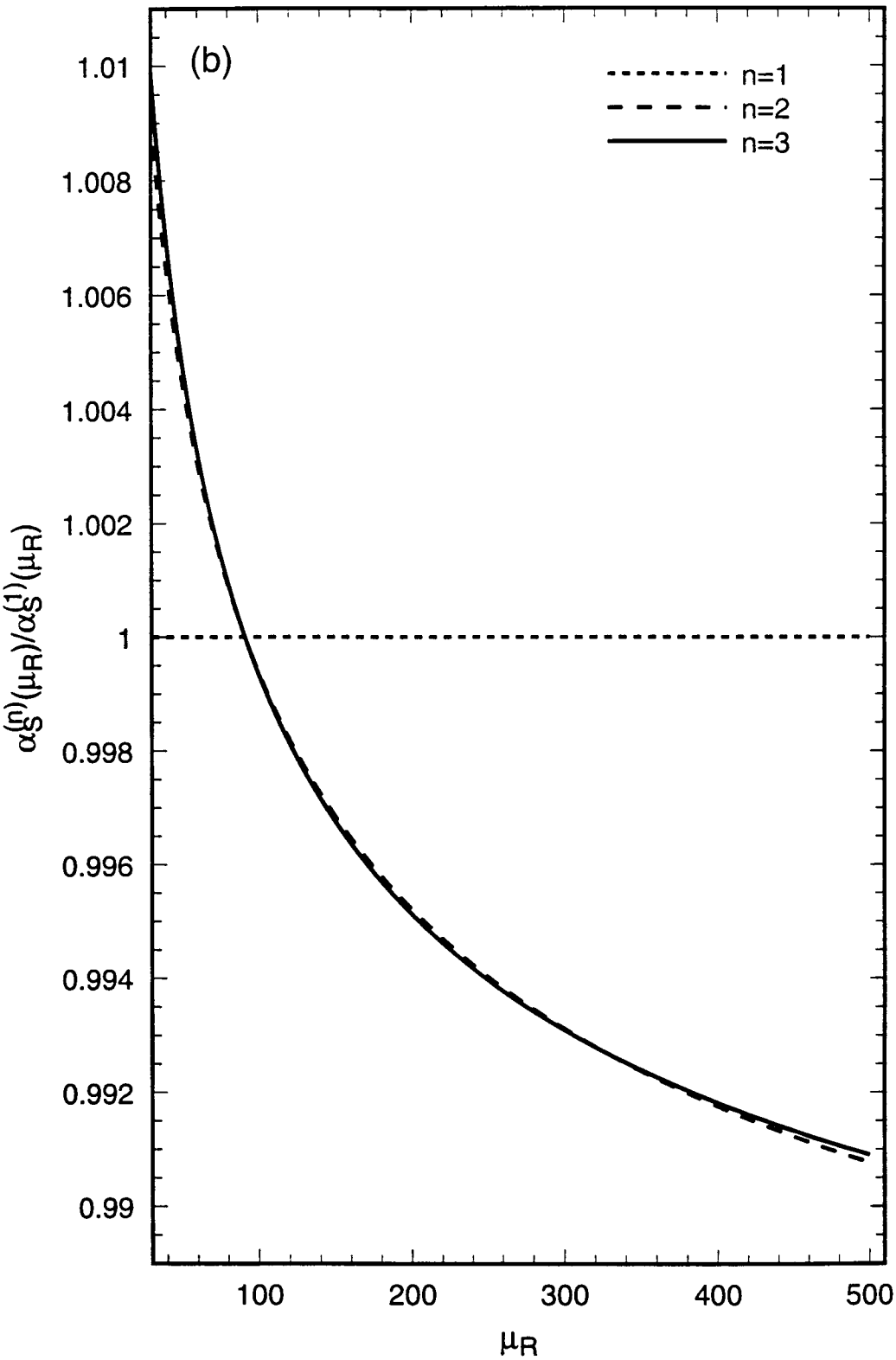
GRAPH 1^a / 1^b

- A FURTHER SIMPIFICATION OFTEN MADE (BY THEORISTS) IS REMOVING THE FRAGMENTATION FUNCTION (ESPECIALLY AT NLO & BEYOND), MAKING THE IDENTIFICATION PARTONIC JET-MOMENTUM = HADRONIC JET
(ASSUMES: • JET IS "OPAQUE", NO QUESTIONS ABOUT WHAT HAPPENS INSIDE A JET.
• NO HADRONIC-DEPENDENT DETECTOR EFFECTS
• "ENERGY/MOMENTUM" CONSERVATION IN JET ALG



$$E_T^{(0)} = E_T^{(1)} + E_T^{(2)} = \sum_{i=1}^{N} E_T^{(i)}$$


(a)
n=1
n=2
n=3
0.18
0.17
0.16
0.15
0.14
0.13
0.12
0.11
0.1
0.09
0.08
$\alpha_S^{(n)}(\mu_R)$
10^1
10^2
10^3
μ_R


(b)
n=1
n=2
n=3
1.01
1.008
1.006
1.004
1.002
1
0.998
0.996
0.994
0.992
0.99
100
200
300
400
500
μ_R
$\alpha_S^{(n)}(\mu_R)/\alpha_S^{(1)}(\mu_R)$

SUMMARIZING:

PERTURBATIVE QCD

- → POWER EXPANTION IN $1/E_T^{JET}$
  - → PDF'S & FRAG. FUNCT.
  - → PARTONIC CROSS SECTION (THROUGH FACTORIZATION)

  (UNFORTUNATELY NO SYSTEMATIC MANNER TO CALCULATE $1/Q^n$ TERMS)
- → α_s - EXPANTION (ORDERLY MANNER OF CALCULATION AND UNCERTAINTY ESTIMATES)
  - → CALCULATING PARTONIC CROSS SECTIONS
  - → PDF-EVOLUTION

VALIDITY OF P.Q.C.D JET CROSS SECTI

WE WILL TAKE THE ONE JET INCLUSIVE TRANSVERS ENERGY DISTRIBUTION AS AN EXAMPLE:

→ WE WILL SEPARATE THE "COMPARISONS"/UNCERTAINTIES INTO TWO PARTS:

- NORMALIZATION ($\alpha_s, \mu_R, \ldots$)
- SHAPE (LARGE LOGS)

PHENOMELOGICALLY WE KNOW FROM MANY MULTIJET COMPARISONS (WITH OR WITHOUT VECTOR BOSONS) THAT THE SHAPES AT LO USUALLY WORK, WHILE THE NORMALIZATION OF THE CROSS SECTION IS AT BEST PROBLEMATIC.

→ WE WILL LOOK AT LO VS NLO AND WHAT WE CAN DEDUCE FROM THAT RELATED TO THE UNCERTAINTY RESULTING FROM THE EXPANTION IN α_s.

NORMALIZATION UNCERTAINTY:

AT LO WE HAVE: $\frac{d\sigma}{dE_T} = \alpha_s^2(\mu_R) A^{(0)}(x_T) \quad x_T = 2E_T/\sqrt{s}$

→ IN THE USUAL DEFINITION OF α_s USING Λ_{QCD} WE HAVE: $\alpha_s = \alpha_s(\mu_R/\Lambda_{QCD})$

- CHANGING $\mu_R \to \lambda\mu_R$ IS IDENTICAL TO CHANGING $\Lambda_{QCD} \to \lambda^{-1}\Lambda_{QCD}$ (OR REDEFINING $\alpha_s(M_Z) \to \alpha_s'(M_Z)$)
- THIS ACTUALLY MEANS THAT μ_R & Λ_{QCD} AT LO ARE NOT WELL DEFINED QUANTITIES.

  CHANGING $\mu_R \leftrightarrow$ CHANGING $\Lambda_{QCD} \leftrightarrow$ CHANGING $\alpha_s(M_Z)$
- SO FOR THE NORMALIZATION UNCERTAINTY WE CAN JUST AS WELL TAKE A RANGE OF $\alpha_s^{(M_Z)}$-VALUES, SET μ_R TO THE CANONICAL SCALE

  $\left(\mu_R = E_T \;;\; \alpha_s(M_Z) = 0.13 - 0.15 \;;\; \alpha_s(E_T) = \frac{\alpha_s(M_Z)}{1+\alpha_s(M_Z) L^{(1)}\left(\frac{E_T}{M_Z}\right)}\right)$

→ HOW TO CHOOSE α_s IN LO CALCULATIONS

(CLAIM: IN VIEW OF ABOVE IT DOES NOT REALLY MATTER.)

A GOOD EXAMPLE IS THE W+n JET / Z+n JET CROSS SECTION. (NORMALIZATION OF LO WELL "STUDIED" BECAUSE OF TOP QUARK SEARCH)

.DF: • USE 2-LOOP α_s & Λ_{QCD} FROM PDF'S
(THIS TYPICALLY WOULD GIVE $\alpha_s(M_Z) \sim 0.114$)
• TO FIT NORMALIZATION TO TOP-DEPLETED DATA ONE HAS TO CHOOSE A SMALL RENORMALIZATION SCALE ($\mu_R \approx 20$ GeV)
• THE ACTUAL VALUE OF α_s USED IN THE EVENT EVALUATION $\alpha_s(20\,\text{GeV}) \sim 0.14$

DØ: • USE 1-LOOP α_s & Λ_{QCD} FROM PDF'S
(THIS TYPICALLY WOULD GIVE $\alpha_s(M_Z) \sim 0.14$)
• FIT TO DATA NOW GIVES THE "CANONICAL" SCALE ($\mu_R \approx 80 - 90$ GeV)
• THE ACTUAL VALUE OF α_s USED IN THE EVENT EVALUATION IS THE SAME AS CDF !

MORE SYSTEMATIC:
• DETERMINE THE EFFECTIVE LO. α_s FROM THE DATA:

$$\rightarrow \frac{W+n\text{-JET}}{W+(n-1)\text{JET}} = \alpha_s^{TRUE} \times K \times A^{LO} = \alpha_s^{LO} \times A^{LO}$$

(OR Z)

$$\Rightarrow \alpha_s^{LO} = K \times \alpha_s^{TRUE}$$

• THIS WILL GIVE YOU $\alpha_s^{LO} \sim 0.14$ (INDEPENDENT OF JET MULTIPLICITY)

• NO μ_R, "FORM" OF α_s, Λ_{QCD}, ...

• WE PARAMETRIZE PART OF THE HIGHER ORDER CORRECTIONS IN α_s^{LO}

→ THIS EXAMPLE SHOWS THAT AT LO THERE IS A LARGE AMBIGUITY WHAT TO DO WITH α_s. ALL THESE METHODS ARE EQUIVALENT. (ACTUALLY IN THIS SENSE μ_R IS IRRECEVANT AT LO AS IT SHOULD)

→ AT NLO THIS ALL DISAPPEARS:

$$\frac{d\sigma}{dE_T} = \alpha_s^2\left(\frac{\mu_R}{\Lambda}\right)\left(A^{(0)}(x_T) + \alpha_s(\mu_R)\, A^{(1)}\left(x_T, \mu_R/E_T\right)\right)$$

$\Lambda_{QCD} \Leftrightarrow \alpha_s(M_Z)$

μ_R IS NOW DECOUPLED FROM VALUE OF α_s

→ WHAT ABOUT PDF'S?

- PDF'S ARE EXTRACTED AT A REFERENCE SCALE $Q^2 \simeq 4\,GeV^2$ AND TYPICALLY USED AT $Q^2 \simeq 10^4\,GeV^2$
- LONG EVOLUTION "ARM", SENSITIVE TO ORDER OF RUNNING.
- I COULD (IN PRINCIPLE) EXTRACT PDF'S DIRECTLY AT $Q^2 \simeq 10^4\,GeV^2$.
- IF WE WOULD USE LO PDF'S WITH 1-LOOP RUNNING THESE TWO SETS WILL SIGNIFICANTLY DIFFERENT
- BETTER: USE "BEST" PDF WITH "BEST" RUNNING TO EVOLVE TO CHARACTERISTIC SCALE AND TAKE THAT AS THE "INPUT"-PDF'S (FORGET HOW THEY WERE OBTAINED) (ONE CAN THEN STILL DETERMINE HOW TO EVOLVE AROUND THE CHARACTERISTIC SCALE)

SHAPE UNCERTAINTY:

→ SOFAR WE HAVE LOOKED AT UNCERTAINTIES IN A RATHER QUALATATIVEL WAY.
NLO WILL ENABLE US TO HAVE MORE QUANTITATIVE ESTIMATES OF THE UNCERTAINTIE DUE TO TRUNCATION IN α_s-EXPANTION

→ THIS IS VERY IMPORTANT, BECAUSE WE GIVE APREDICTION BASED ON APPROXIMATIONS ONE CAN ARGUE THAT THE MOST IMPORTANT ASPECT OF THE PREDICTION IS THE ESTIMATION OF THE UNCERTAINTY.

→ HAVING JUST THE FIRST TERM IN AN EXPANTION MAKES IT VERY DIFFICULT TO ESTIMATE THE UNCERTAINTY

$$F(x) = a_0(x) + \alpha a_1(x) + \alpha^2 a_2(x) + \dots$$

HAVING THE SECOND TERM IS IMPORTANT AND WILL ENABLE US TO GIVE AN ESTIMATE OF THE UNCERTAINTY DUE TO THE TRUNCATION AND PHASE SPACE REGIONS OF CONVERGENCE.

→ WE WILL LOOK AT THE ONE-JET INCLUSIVE:

STEP 1: IDENTIFYING REGIONS OF PHASE SPACE WHERE FIXED ORD. PQCD BREAKS DOWN:

- THIS IS USUALLY EASY AND ASSOCIATED WITH RESTRICTING THE SOFT RADIATION IN SOME MANNER → EDGES OF PHASE SPACE.
- $\left.\frac{d\sigma}{dE_T}\right|_{LO} = \alpha_s^2 A^{(0)}(x_T)$ { $\lim_{x_T \to 0} A^{(0)}(x_T) = \infty$ ← typical for indications of problems. $\lim_{x_T \to 1} A^{(0)}(x_T) = 0$ ←

STEP 2: QUANTIFYING THE "LARGE LOG"-REGIONS AND ESTIMATING THE UNCERTAINTIES.

- $\left.\frac{d\sigma}{dE_T}\right|_{NLO} = \alpha_s^2 \left(A^{(0)}(x_T) + \alpha_s A^{(1)}(x_T)\right)$

$A^{(1)}(x_T) = c_1 LOG^2(x_T) + c_2 LOG^2(1-x_T) + B(x_T)$

(FORCED) SOFT RADIATION

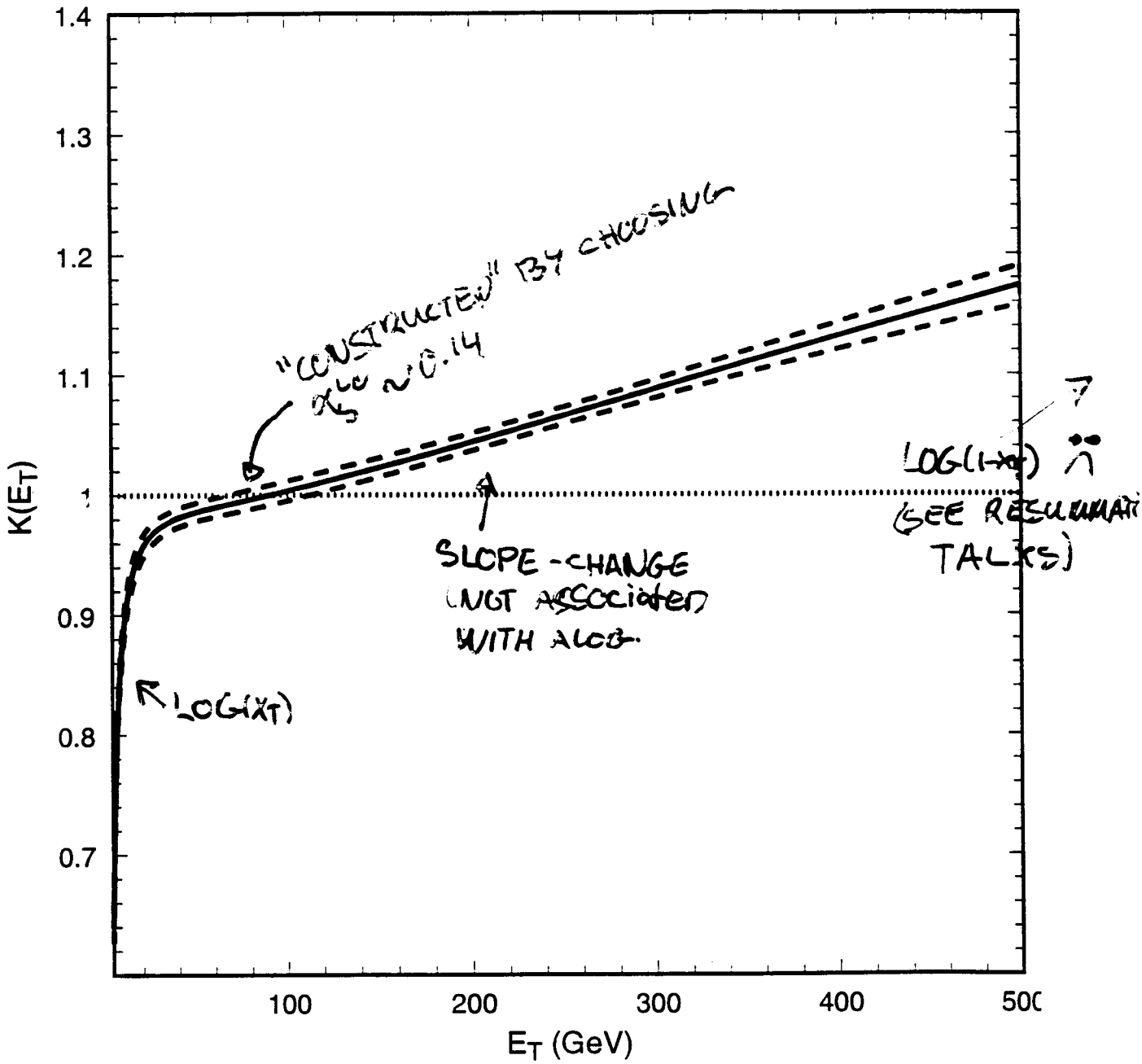
- HOW LARGE ARE THESE TERMS RELATIVE TO EACH OTHER AND HOW LARGE IS $A^{(0)}(x_T)$ RELATIVE TO $\alpha_s A^{(1)}(x_T)$?

FOR WHICH x_T IS $\alpha_s A^{(1)}(x_T) \ll A^{(0)}(x_T)$

(AND WHAT ABOUT THE NORMALIZATION)

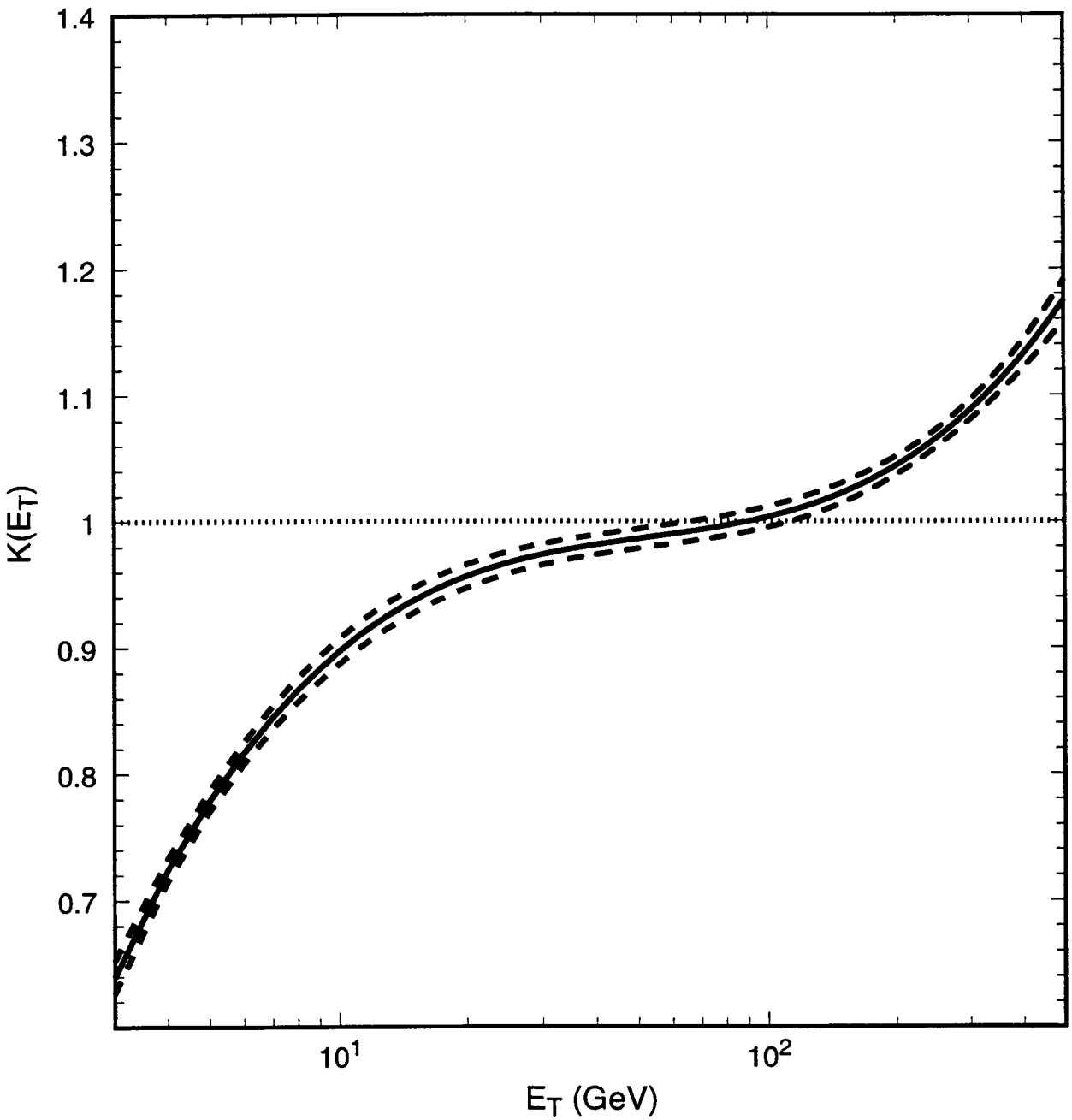
- FIG 2a & 2c

- BECAUSE OF THE LO NORMALIZATION PROBLEMS THE RELEVANT BEHAVIOR IN THE K-FACTOR ARE INDICATIONS OF LOGARITHIC DIVERGENCIES (i.e. RAPID CHANGES IN THE K-FACTOR.)
- QUANTIFYING THE UNCERTAINTIES:
  - → IN THE REGION $45\text{GeV} \lesssim E_T \lesssim 300\text{GeV}$ (RELEVANT FOR THE HIGH STATISTICS DATA) THE SHAPE CHANGES NIC %.
    - ⇒ THE CONSERVATIVE ESTIMATE WOULD BE THAT CORRECTIONS BEYOND NLO ARE SMALLER THAN 10%
  - → BELOW $E_T \lesssim 30\text{GeV}$ NLO (OR ANY FIXED ORDER FOR THAT MATTER) WILL/MUST DEVIATE FROM DATA. (I.E. THE EXPANSION IN α_S SIMPLY BREAKS DOWN)
    (IS THERE A PROBLEM WITH QCD IF CDF/DØ DATA DEVIATES FROM FIXED ORDER PQCD FOR $P_T^{JET}, P_T^{\gamma}, P_T^{W}, P_T^{Z}, \ldots \lesssim 30\text{GeV}$?
    NO, THIS IS ACTUALLY WHAT IS SUPPOSED TO HAPPEN ! (THE PROBLEM IS HOW DO WE PREDICT THAT REGION)



1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
K(E_T)
100
200
300
400
500
E_T (GeV)
"CONSTRUCTED" BY CHOOSING
α_s^LO ~ 0.14
SLOPE-CHANGE
(NOT ASSOCIATED
WITH A LOG.
LOG(1-x_T)
(SEE RESUMMATI
TALKS)
LOG(x_T)

FIG 2


1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
K(E_T)
10^1
10^2
E_T (GeV)

→ ABOVE $E_T \gtrsim 500$ GEV WE WILL AGAIN RUN INTO LARGE LOGARITHMS (SEE TALKS ON SATERDAY)

To Summarize:

→ PQCD is based on two approximations

* Factorization
* α_s expantion

→ The most important aspect about NLO is the ability to quantify the uncertainties in the α_s-expantion

(Both in normalization (defines α_s/PDF's/Frag. Func.) and shape (large logs))

→ How to proceed:

* More NLO (uncertainty estimates) — with experimental data becoming more and more accurate this is crucial for any predict
* Shower MC & fixed order to a large extend complimentary.

We need to develop a systematic (and correct) way to merge approaches beyond LO partonic cross sections

→ BEYOND NLO?

↓

WE NEED TO BETTER UNDERSTAND UNCERTAINTIES RESULTING FROM PDF'S