FRAGMENTATION FUNCTIONS

GRECO

Progress in Phenomenology and Theory

(i) Introduction on FF's to NLO.

(ii) Light mesons.

(iii) Heavy Quarks - Perturbative FF's.

(iv) D, D^* production.

(v) Fragmentation for heavy quarkonia.

Fermilab - 7 Nov. '95

2. One-particle inclusive production at next-to-leading order

Let us consider the inclusive production of a hadron H via the generic reaction $A+B\to H$ where A and B stand for hadrons and/or leptons. The cross section can be written as a convolution of the fragmentation functions $D_l^H(z, M_f^2)$ with the partonic cross section

$$E_H\frac{d\sigma_{A+B\to H}}{d^2P_H}=\sum_l\int_{z_h}^1\frac{dz}{z^2}D_l^H(z, M_f^2)E_l\frac{d\sigma_{A+B\to l}}{d^2P_l}\left(\frac{z_H}{z}, \theta, \alpha_s(\mu^2), M_f^2,\dots\right), \tag{1}$$

hadronic — partonic

where z_H is the reduced energy of the hadron H: $z_H=2E_H/\sqrt{S}$ and θ is the scattered angle of the parton l. The inclusive production of the parton l via the reaction $A+B\to l$ has the following perturbative development:

$$E_l\frac{d\sigma_{A+B\to l}}{d^3P_l}\left(\frac{z_H}{z}, \theta, \alpha_s(\mu^2), M_f^2,\dots\right) \qquad \text{NLO}$$

$$=\sigma^0_{A+B\to l}\left(\frac{z_h}{z}, \theta\right)+\frac{\alpha_s(\mu^2)}{2\pi}\sigma^1_{A+B\to l}\left(\frac{z_h}{z}, \theta, M_f^2\right)+\dots. \tag{2}$$

Finally $D_l^H(z, M_f^2)$ represents the number of hadrons H inside the parton l carrying the fraction of momentum z from H, evolved at the scale M_f^2. These fragmentation functions satisfy Altarelli–Parisi type evolution equations:

NLO evolut. eqs.

$$\frac{\partial D_q^H(z, M_f^2)}{\partial \ln M_f^2}=\frac{\alpha_s(M_f^2)}{2\pi}\int_z^1\frac{dy}{y}\left[P_{qq}^T(y, \alpha_s(M_f^2))D_q^H\left(\frac{z}{y}, M_f^2\right)\right.$$

$$\left.+P_{gq}^T(y, \alpha_s(M_f^2))D_g^H\left(\frac{z}{y}, M_f^2\right)\right], \tag{3}$$

$$\frac{\partial D_g^H(z, M_f^2)}{\partial \ln M_f^2}=\frac{\alpha_s(M_f^2)}{2\pi}\int_z^1\frac{dy}{y}\left[P_{qg}^T(y, \alpha_s(M_f^2))D_q^H\left(\frac{z}{y}, M_f^2\right)\right.$$

$$\left.+P_{gg}^T(y, \alpha_s(M_f^2))D_g^H\left(\frac{z}{y}, M_f^2\right)\right]. \tag{4}$$

The evolution kernels have the perturbative development:

$$P_{ij}^T(x, \alpha_s(M_f^2))=P_{ij}^0(x)+\frac{\alpha_s(M_f^2)}{2\pi}P_{ij}^{T1}(x)+\dots.$$

$e^+e^- \to l + X$

The partonic cross sections from e^+e^- collisions read at NLO

$$\longrightarrow \quad E_{q_i}\frac{d\sigma_{e^++e^-\to q_i}}{d^2P_{q_i}}(y,\theta,\alpha_s(\mu^2),M_f^2)$$

$$=\frac{6\sigma_0}{\pi Q^2 y}e_i^2\left\{\frac{3}{8}(1+\cos^2\theta)\left[\delta(1-y)+\frac{\alpha_s(\mu^2)}{2\pi}\left(P_{qq}^0(y)\ln\frac{Q^2}{M_f^2}+K_q^T(y)\right)\right]\right.$$

$$\left.+\frac{3}{4}(1-\cos^2\theta)\frac{\alpha_s(\mu^2)}{2\pi}K_q^L(y)\right\}, \qquad (8)$$

$$\longrightarrow \quad E_g\frac{d\sigma_{e^++e^-\to g}}{d^2P_g}(y,\theta,\alpha_s(\mu^2),M_f^2)$$

$$=\frac{12\sigma_0}{\pi Q^2 y}\sum_{i=u,d,s,c,\ldots}e_i^2\left\{\frac{3}{8}(1+\cos^2\theta)\left[\frac{\alpha_s(\mu^2)}{2\pi}\left(P_{gq}^0(y)\ln\frac{Q^2}{M_f^2}+K_g^T(y)\right)\right]\right.$$

$$\left.+\frac{3}{4}(1-\cos^2\theta)\frac{\alpha_s(\mu^2)}{2\pi}K_g^L(y)\right\}, \qquad (9)$$

Altarelli, Ellis, Martinelli, Pi

where σ_0 is the usual pointlike cross section

$$\sigma_0=\frac{4\pi\alpha^2}{3Q^2},$$

α is the QED coupling constant and Q^2 is the invariant mass of the e^+e^- pair. the functions K_q^T, K_q^L, K_g^T and K_g^L have been extracted from ref. [12] (see also ref.

$\phi\ p \longrightarrow l + X$

The partonic cross sections for hadronic collisions are given by [2]

$$E_l \frac{d\sigma_{p+p\to l}}{d^3P_l}(y, \theta, \alpha_s(\mu^2), M_f^2)$$

$$= \frac{1}{\pi S}\sum_{i,j}\int_{VW}^{V}\frac{dv}{1-v}\int_{VW/v}^{1}\frac{dw}{w}$$

$$\times\left\{F_i^p(x_1, M^2)F_j^p(x_2, M^2)\left[\frac{1}{v}\left(\frac{d\sigma^0}{dv}\right)_{ij\to l}(s, v)\delta(1-w)\right.\right.$$

$$\left.\left.+\frac{\alpha_s(\mu^2)}{2\pi}K_{ij\to l}(s, v, w; \mu^2; M^2, M_f^2)\right] + (x_1 \leftrightarrow x_2)\right\}. \qquad (16)$$

The variables V, W are defined by

$$V = 1 - \frac{y}{2}(1-\cos\theta), \quad W = \frac{y(1+\cos\theta)}{2-y(1-\cos\theta)},$$

and we also have

$$x_1 = \frac{VW}{vw}, \quad x_2 = \frac{1-V}{1-v}$$

Aversa, Chiappetta, M.G., Guillet

LIGHT MESONS ($\pi^0, \pi^\pm, K^\pm, \eta, K_s^0$)

(π^0) NLO analysis - Predictions for LHC.

Chiappetta, M.G., Guillet, Rolli, Werlen

⇒ Input of F.F. at reference scale M_{f0}.

(i) e^+e^- data (CELLO, TASSO, JADE, TPC, ...)

I. Simple ansatz for valence, sea, gluon (SU(2) sym.)

$M_{f_0}^2 = 2\ GeV^2$ $\quad D_i(z, M_{f_0}^2) \sim N_i\, z^{\alpha_i}(1-z)^{\beta_i}$ (3F.F.)

"Natural scales": $M_f = \mu = \sqrt{s}$ (e^+e^-)

$M = M_f = \mu = c\, p_T$ (pp)

$0.5 \lesssim c \lesssim 5$ too large

Fixed target, ISR, WA 2

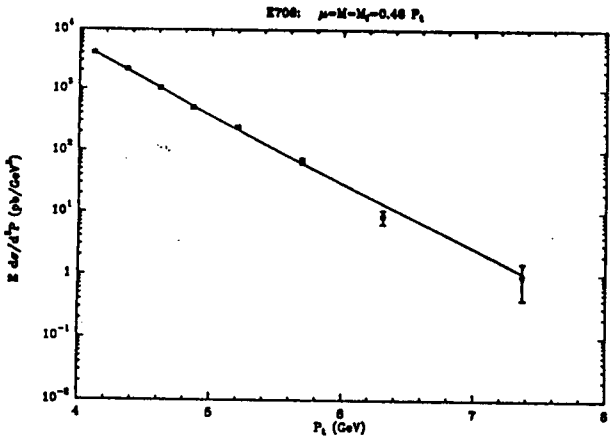
II. "Optimized scales" Politzer-Stevenson

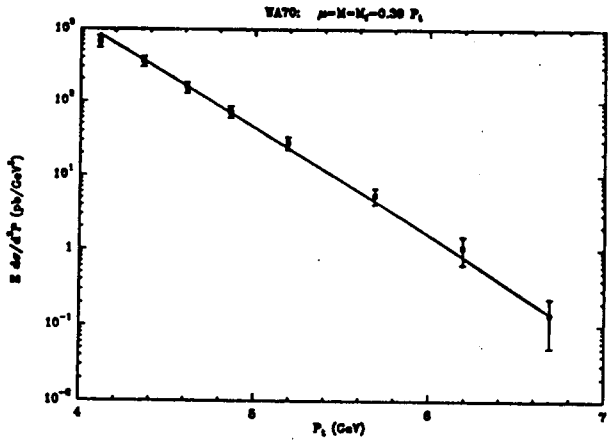
(4F.F. wscg)

$$\frac{\partial}{\partial \ln \mu^2}\left(E\frac{d\sigma}{d^3p}\right)^{pp\to\pi^0+X} = \frac{\partial}{\partial \ln M^2}\left(E\frac{d\sigma}{d^3p}\right) = \frac{\partial}{\partial \ln M_f^2}\left(E\frac{d\sigma}{d^3p}\right) = 0$$

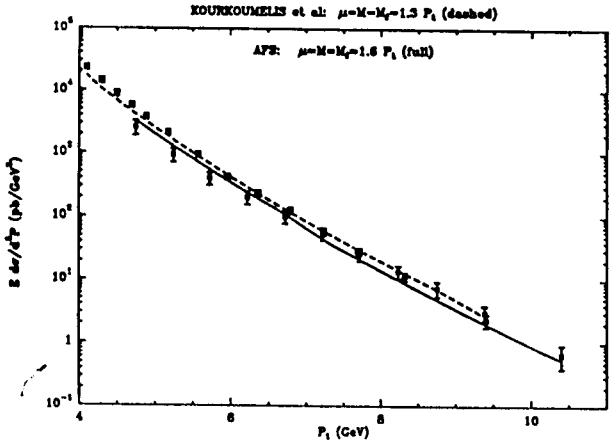
(ii) Parton Shower M.C. (HERWIG) $\begin{cases} e^+e^- \to q\bar{q} \to \cdots \\ gg \to gg \to \cdots \end{cases}$

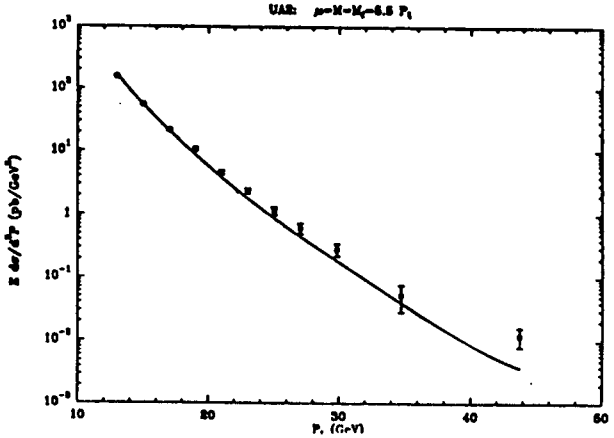
$D_i(z, M_{f_0}^2) = N_i\, z^{\alpha_i}(1-z)^{\beta_i} \qquad i = q_s', g$





$pp \to \pi^0$ Set I Chiappetta et al.





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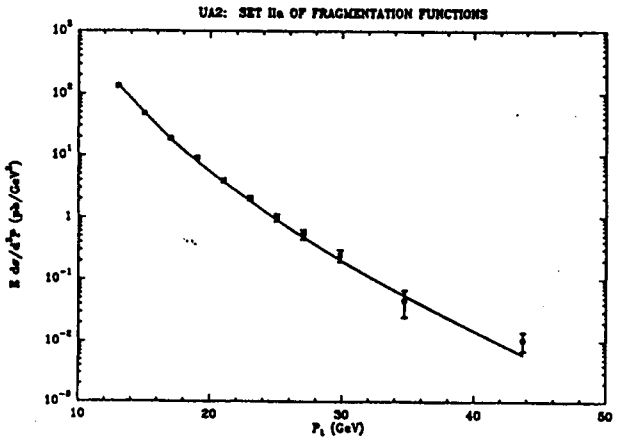
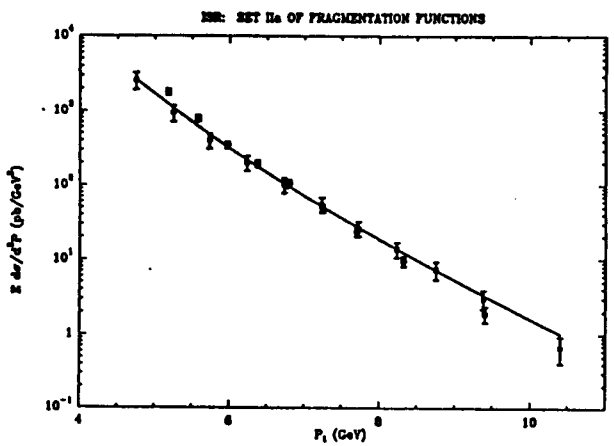
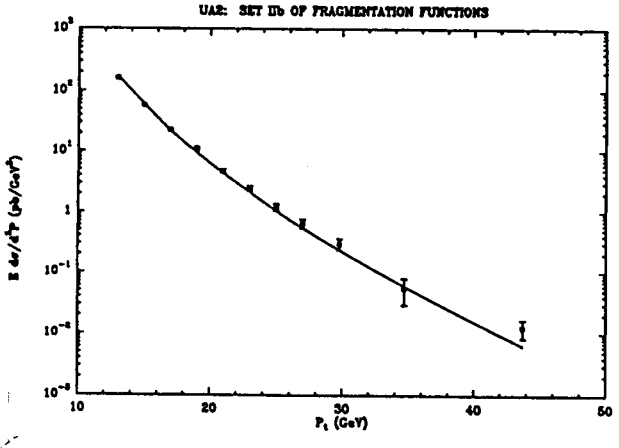
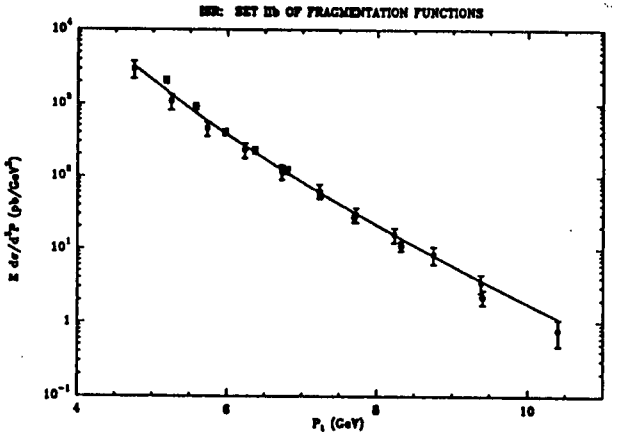
Fermilab - 7 Nov. '96



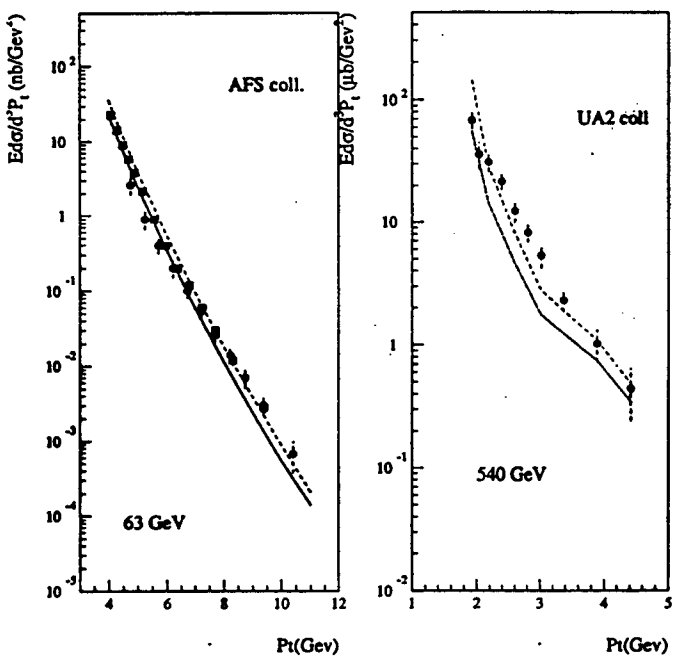


$pp \to \pi^0$ Set II Chiappetta et al.





π^0 production at $p\bar{p}(p)$ colliders



FF from HERWIG

Predictions at LHC from various sets of FF agree within a factor 2. other uncertainties much smaller.

MORE DATA ARE NEEDED!

Similar analysis has been performed for η, $\pi^{\pm}$, $k^{\pm}$, k^{0}_{S}. S. Rolli, M.G.
S. Rolli, M.G., A. Vicini

- η/π^0 ratio (agree with E706)
- $\pi^0/\langle jet \rangle$ (" CDF)
- k^s_o (" CDF)

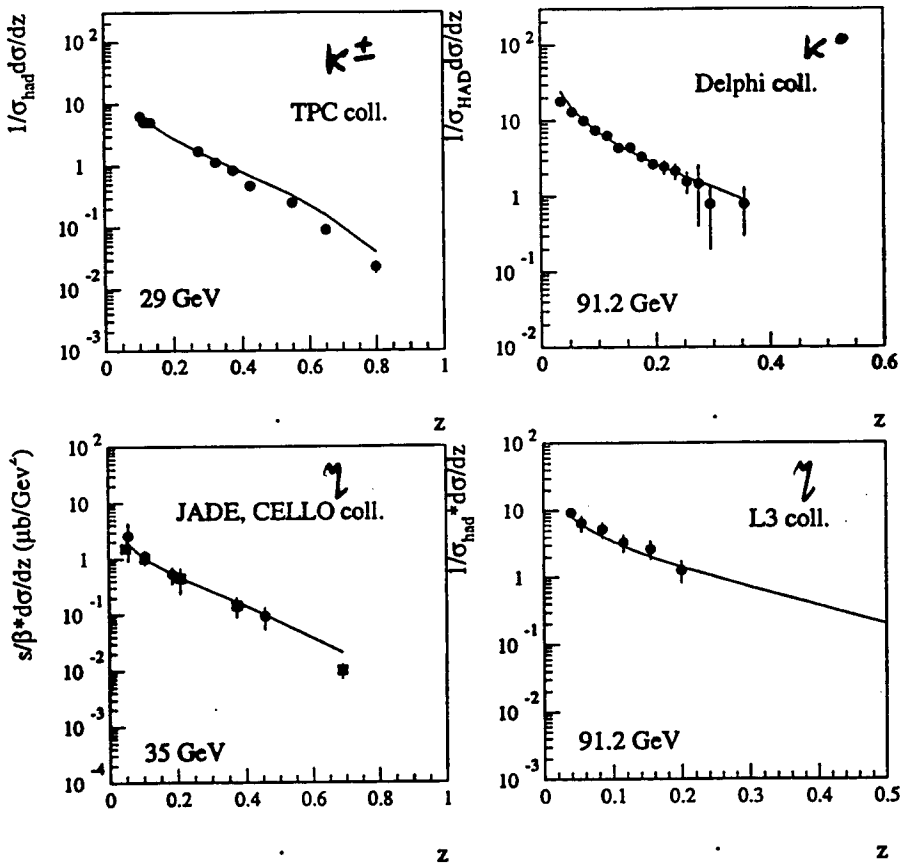
Similar approach followed by:

Borgumati, Knieh!, Kramer (LO)
Knieh!, Kramer (LO)
Binnewies, Knieh!, Ramon (BKK) (NLO)

Work in progress:

A global fit à la CTEQ (T. Morfin, S. Rolli) to e^+e^-, $p\bar{p}^{(-)}$, ep and fixed target data. Determination of realistic errors and extrapolation to LHC energies.

$$e^+e^- \to h + X$$



S. Rolli, M.G.

FF from HERWIG

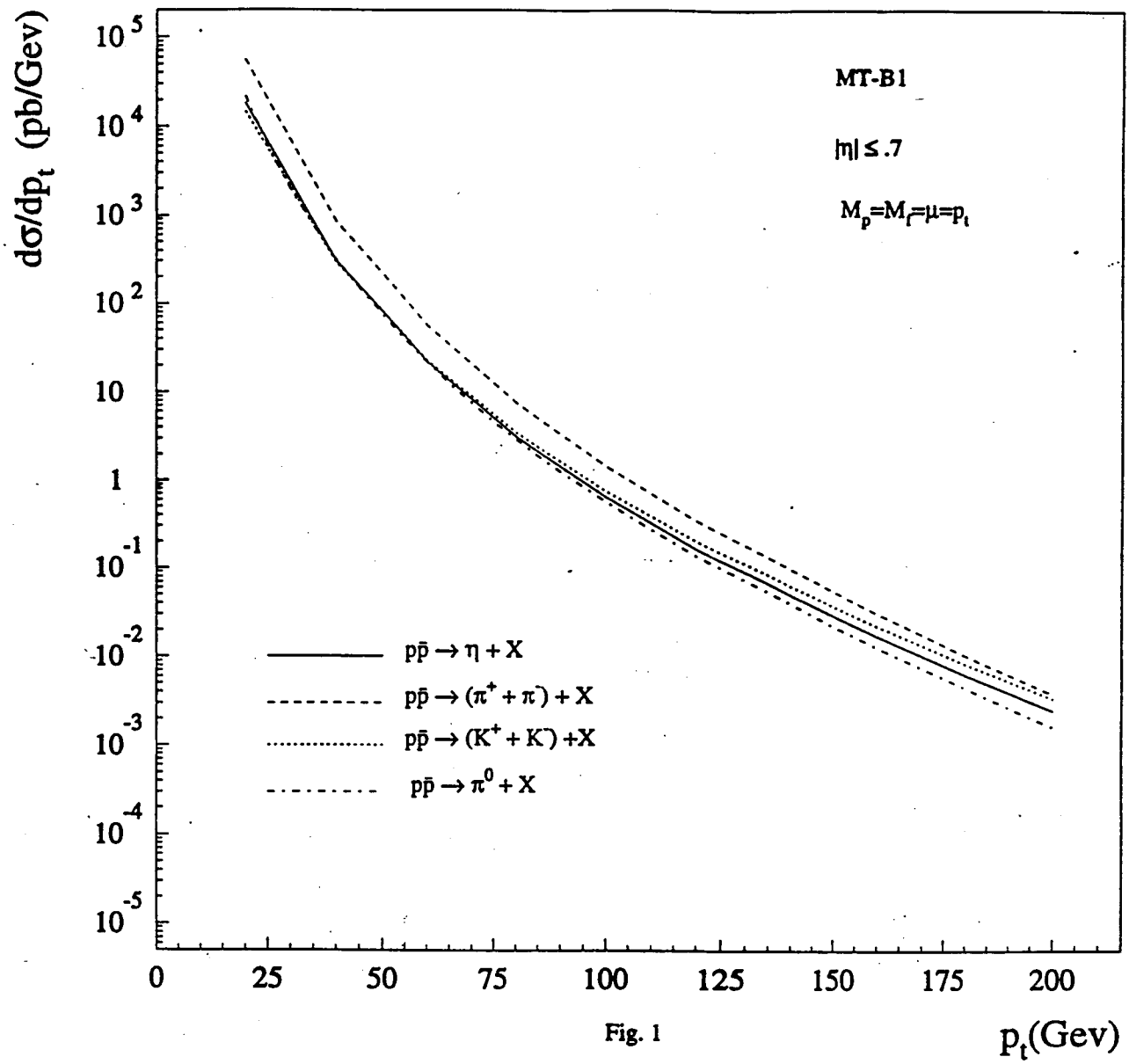


Fig. 1

S. Rolli, M.G.

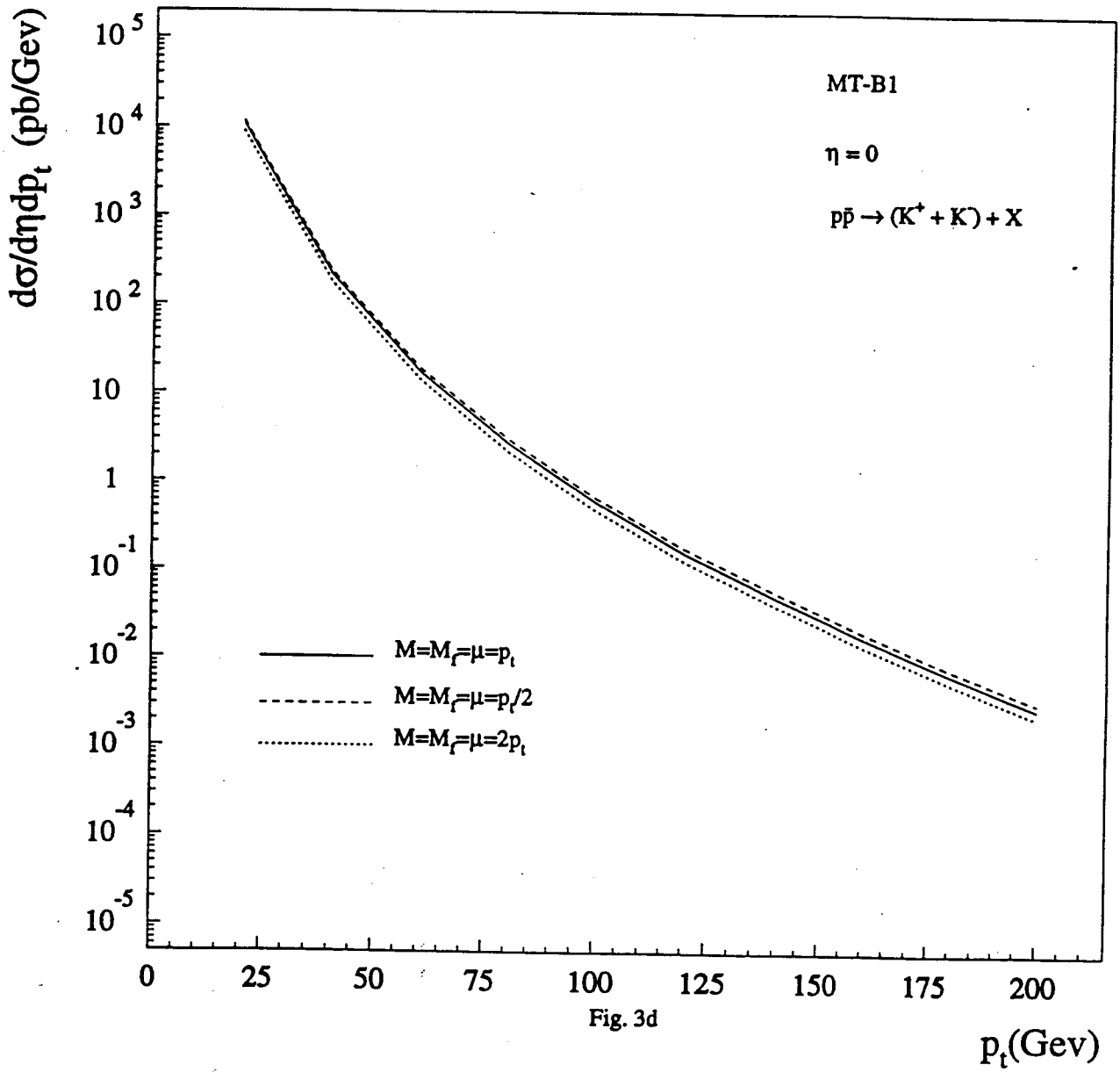

$d\sigma/d\eta dp_t$ (pb/Gev)
10^5
10^4
10^3
10^2
10
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
MT-B1
$\eta = 0$
$p\bar{p} \rightarrow (K^+ + K^-) + X$
$M=M_f=\mu=p_t$
$M=M_f=\mu=p_t/2$
$M=M_f=\mu=2p_t$
0
25
50
75
100
125
150
175
200
p_t(Gev)

Fig. 3d

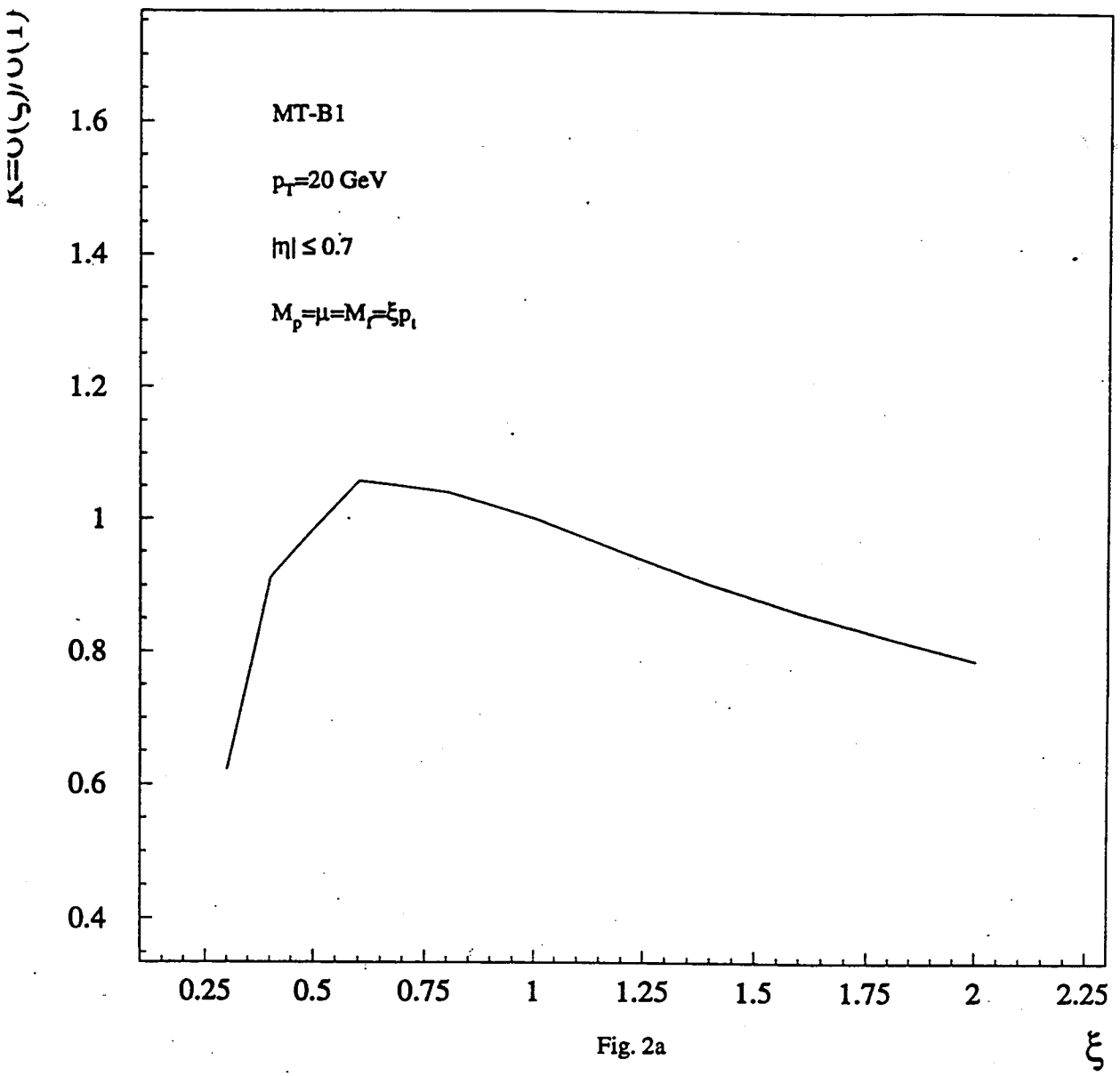

K=σ(ξ)/σ(1)
MT-B1
p_T=20 GeV
|η| ≤ 0.7
$M_p=\mu=M_f=\xi p_t$
1.6
1.4
1.2
1
0.8
0.6
0.4
0.25
0.5
0.75
1
1.25
1.5
1.75
2
2.25
ξ

Fig. 2a

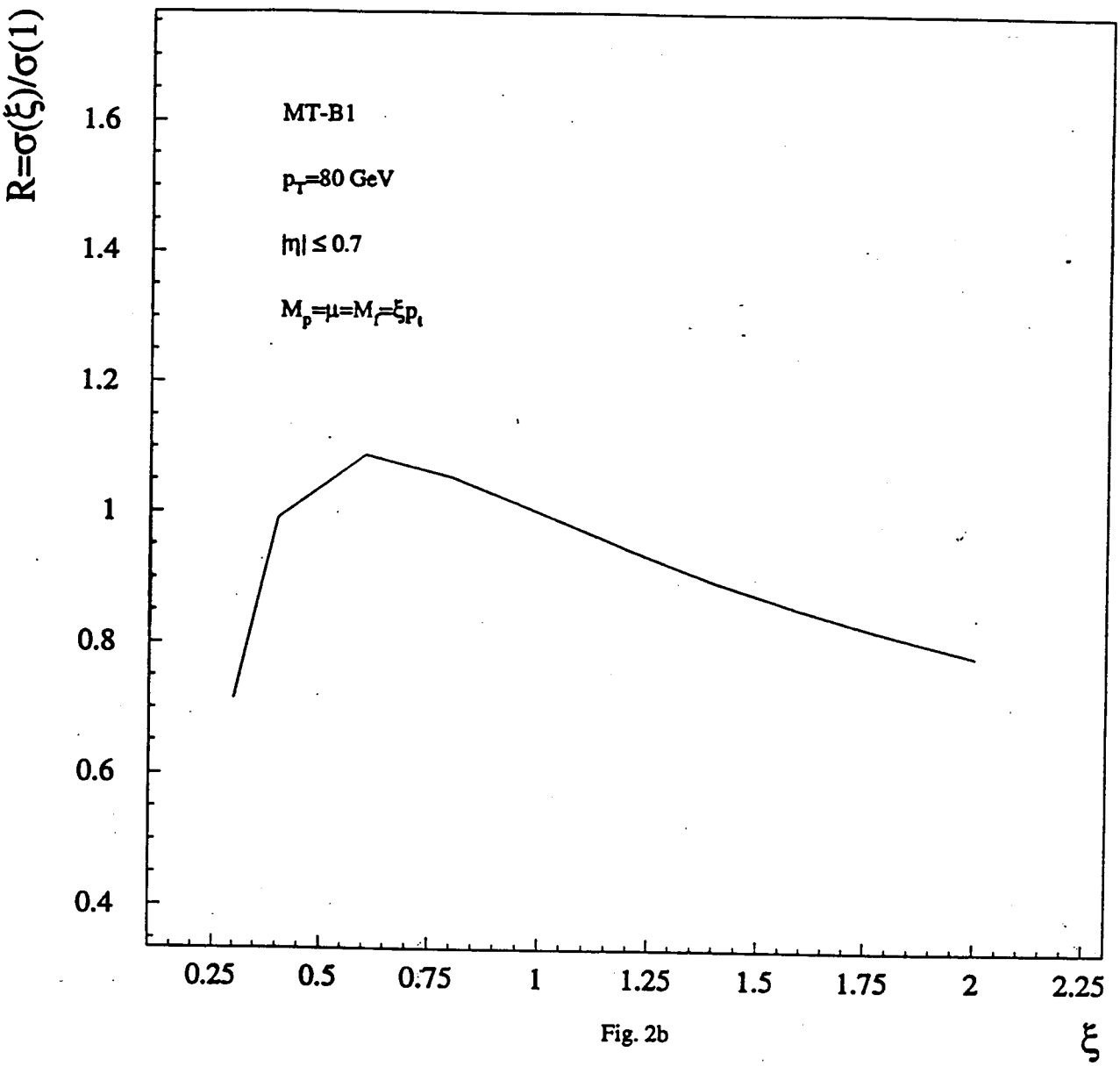

R=σ(ξ)/σ(1)
MT-B1
p_T=80 GeV
|η| ≤ 0.7
$M_p=\mu=M_f=\xi p_t$
1.6
1.4
1.2
1
0.8
0.6
0.4
0.25
0.5
0.75
1
1.25
1.5
1.75
2
2.25
ξ

Fig. 2b

Jet Fragmentation

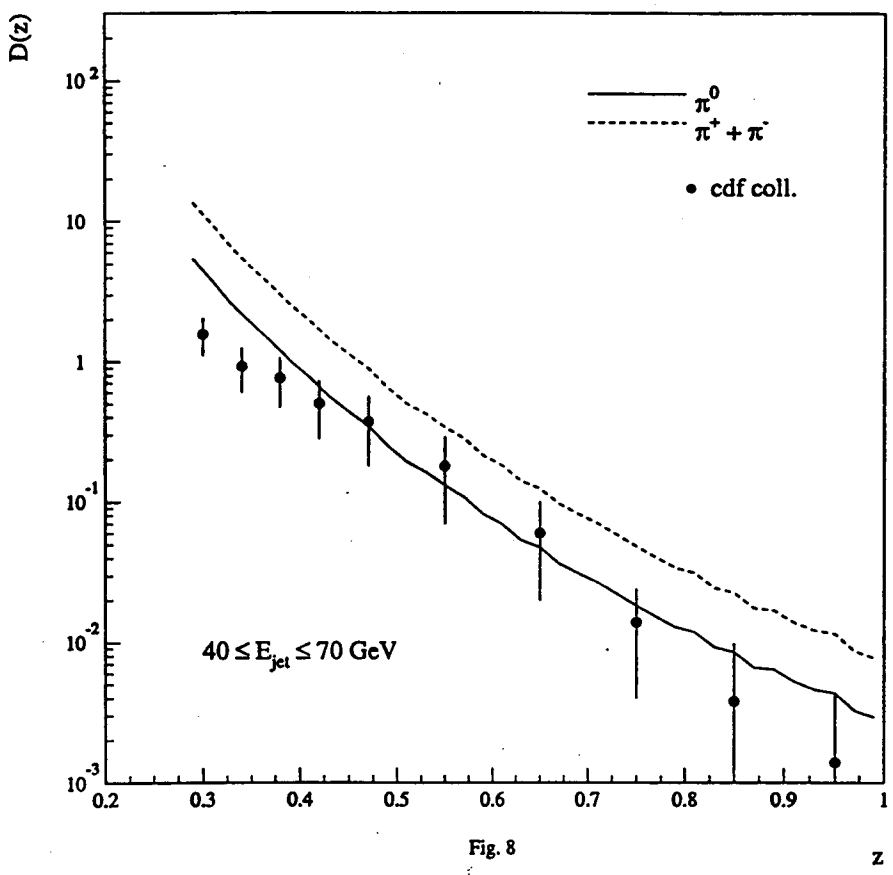


Fig. 8

S. Rolli, D.G.

Fragmentation Functions from real data (II) (charged hadrons, neutral kaons) BKK

- $D_d^{\pi^+ + \pi^-} = D_u^{\pi^+ + \pi^-}$
- $D_s^{K^+ + K^-} = D_u^{K^+ + K^-}$, $D_s^{K^0 + \bar{K}^0} = D_d^{K^0 + \bar{K}^0}$,
- $M_{f0} = \sqrt{2}$ GeV for u, d, s;
- $M_{f0} = m(\eta_c) = 2.97$ GeV for c;
- $M_{f0} = m(Y) = 9.46$ GeV for b;

$$\frac{d\sigma^{h^+ + h^-}}{dz} = [1 + f(x)]\frac{d\sigma^{\pi^+ + \pi^-}}{dz} + \frac{d\sigma^{K^+ + K^-}}{dz}$$

Fit to TCP and LEP data and cross check on e^+e^- data.

K_s^0 production

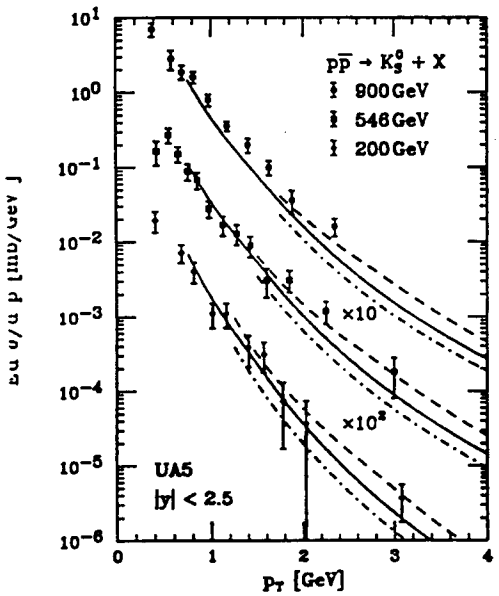


Fig. 4

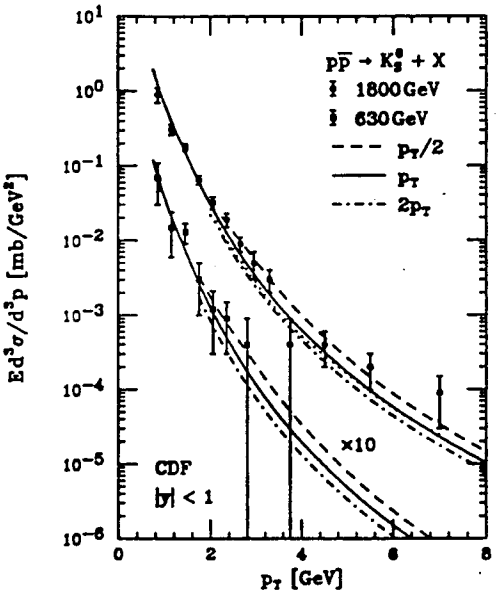
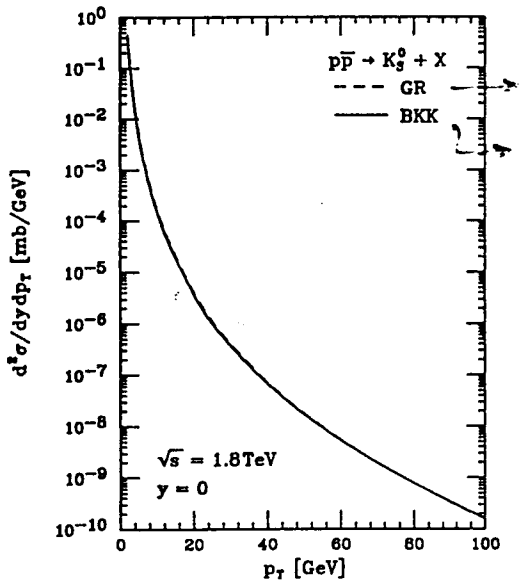


Fig. 5



HG, ROLLI

+ BINNEWIES, KNIEHL, KRAMER

Fig. 8

FF from BKK

Comparisons of the results of the two different groups: { Biannewies, Kniehl, Kramer / M.G., Rolli

- **kaons**: K_s^0 at CDF: the agreament is very good;
  $K^{\pm}$: BKK set is very different from neutral set, while this is not the case for GR. This is a point to be investigated even because for

- **pions**: BKK do not have neutral and they obtain them from charged, simply applying a $SU(2)$ transformation.

- **eta** : The only set existing is GR. Good agreament with E706.

HEAVY (CHARMED) MESON'S (D, D^*)

Heavy quark production is calculable in pQCD ($m \gg \Lambda$). m acts as cutoff of collin. divergs.

Fixed order calculs. Nason, Dawson, Ellis
Beenaker, Kniff, Van Neerven, Smith

$$d\sigma(S, m^2, \ldots) = \sum_{jk} \int dx_1\, dx_2\, F_j^A(x_1, M^2)\, F_k^B(x_2, M^2) \cdot \cdot d\hat{\sigma}_{jk}(\hat{s} = x_1 x_2 S;\ m^2, M^2)$$

$O(\alpha_s^3)$ result is limited at large p_T

Large p_T resummation. $[\alpha_s \ln(p_T^2/m^2)]$ Cacciari-M.G.

Perturbative Fragmentation Functions Mele-Nason

partonic $D_i^c(x, \mu; m)$

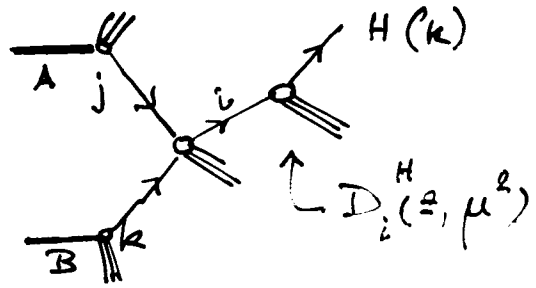
hadronic $D_i^H(z, \mu, m) = D_i^c(z, \mu, m) \otimes D_{np}^{c \to H}(z)$

$D_{np}^{c \to D, D^*}(z)$ from e^+e^- data (non singlet) Colangelo, Nason

$$D_{np}^H(z) = \langle n_H \rangle A (1-z)^\alpha z^\beta$$ Cacciari, Rolli, M.G., Frixione
(D, D^* production @ TEVATRON)

Method for resummation based on Fragmentation Functions

$A + B \to H + X$



$$\frac{E\, d\sigma^H}{d^3k} \sim \sum_{jki} \int dx_1\, dx_2\, \frac{dz}{z^2}\, F_j^A(x_1, M^2)\, F_k^B(x_2, M^2)$$

$$d\hat{\sigma}_{jk,i}\left(x_1, x_2, \frac{k}{z}; M^2, \mu^2\right) D_i^H(z, \mu^2)$$

Fragmentation F. is process independ. (Factor. Th.)
When H is <u>heavy</u> Fragm. F. is calculable in perturbation theory.

e.g. $e^+e^- \to H + X$

$$d\sigma(x, Q, m) \sim \sum_i \int dz\; d\hat{\sigma}_i\left(\frac{x}{z}, Q, \mu\right) D_i(z, \mu, m)$$

↑ short distance ↑ universal

Definition of $d\hat{\sigma}_i$ and D_i not unique - Depends on the scheme used to separate collinear sings

NLO evolution eqs. for $D_i(x,\mu,m)$

$$\frac{\partial D_i(x,\mu,m)}{\partial \ln \mu^2} = \sum_j \int \frac{dz}{z} P_{ij}\left(\frac{x}{z}, \alpha(\mu)\right) D_j(z,\mu,m)$$

$$P_{ij}(x,\alpha) = P_{ij}^{0}\left(\frac{\alpha}{2\pi}\right) + P_{ij}^{(1)}\left(\frac{\alpha}{2\pi}\right)^2 + \cdots$$

↳ AP ↳ Curci, Furmanski, Petronzio

$$D(x,\mu_0) \xrightarrow[NLO]{} D(x,\mu)$$

Resums all powers $\alpha_s^n(\mu) \ln^n(\mu/\mu_0)$

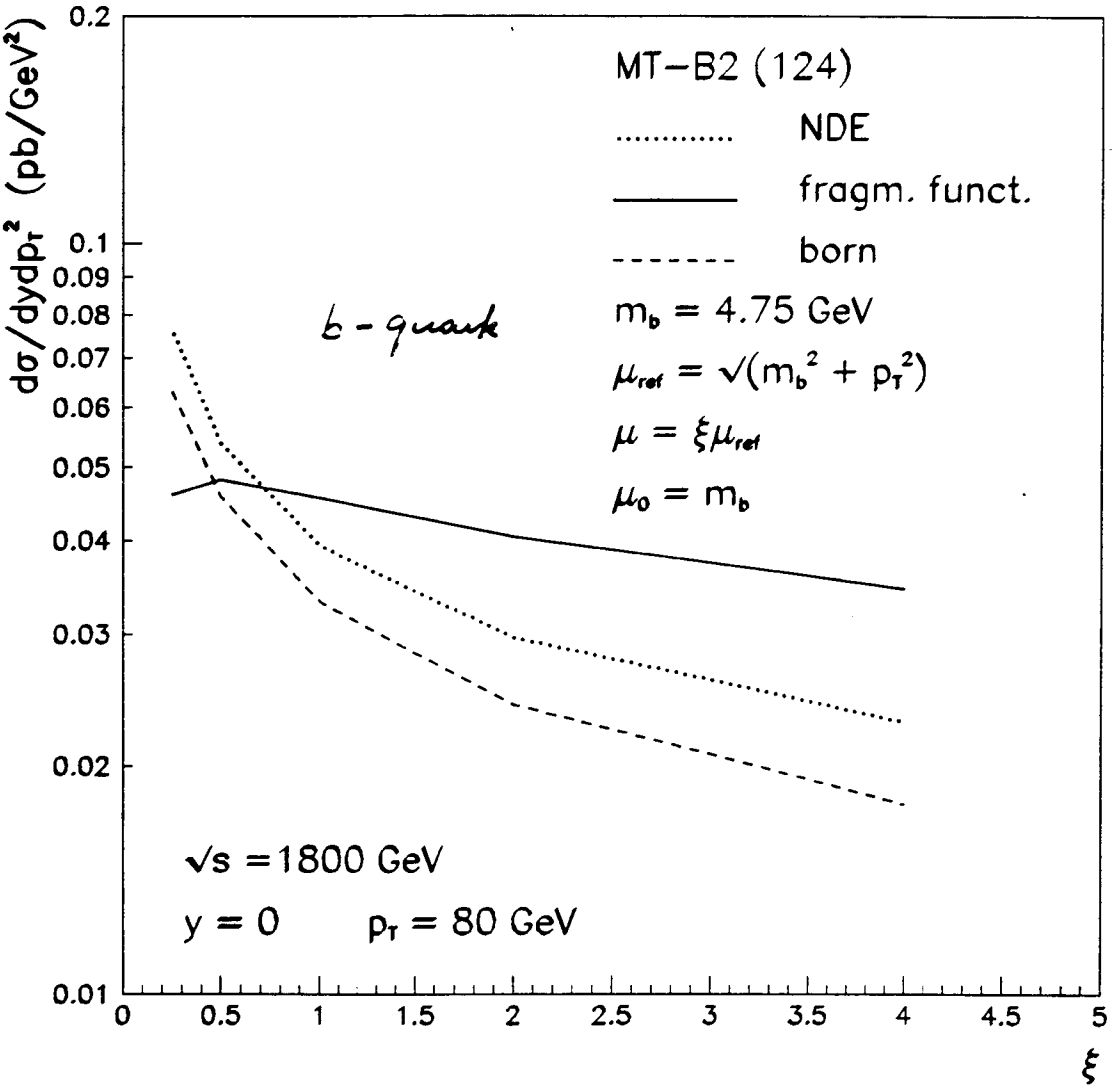
$\alpha_s^{n+1}(\mu) \ln^n(\mu/\mu_0)$

NLO initial conditions for heavy quarks perturb. fragm. funct's. Mele - Nason

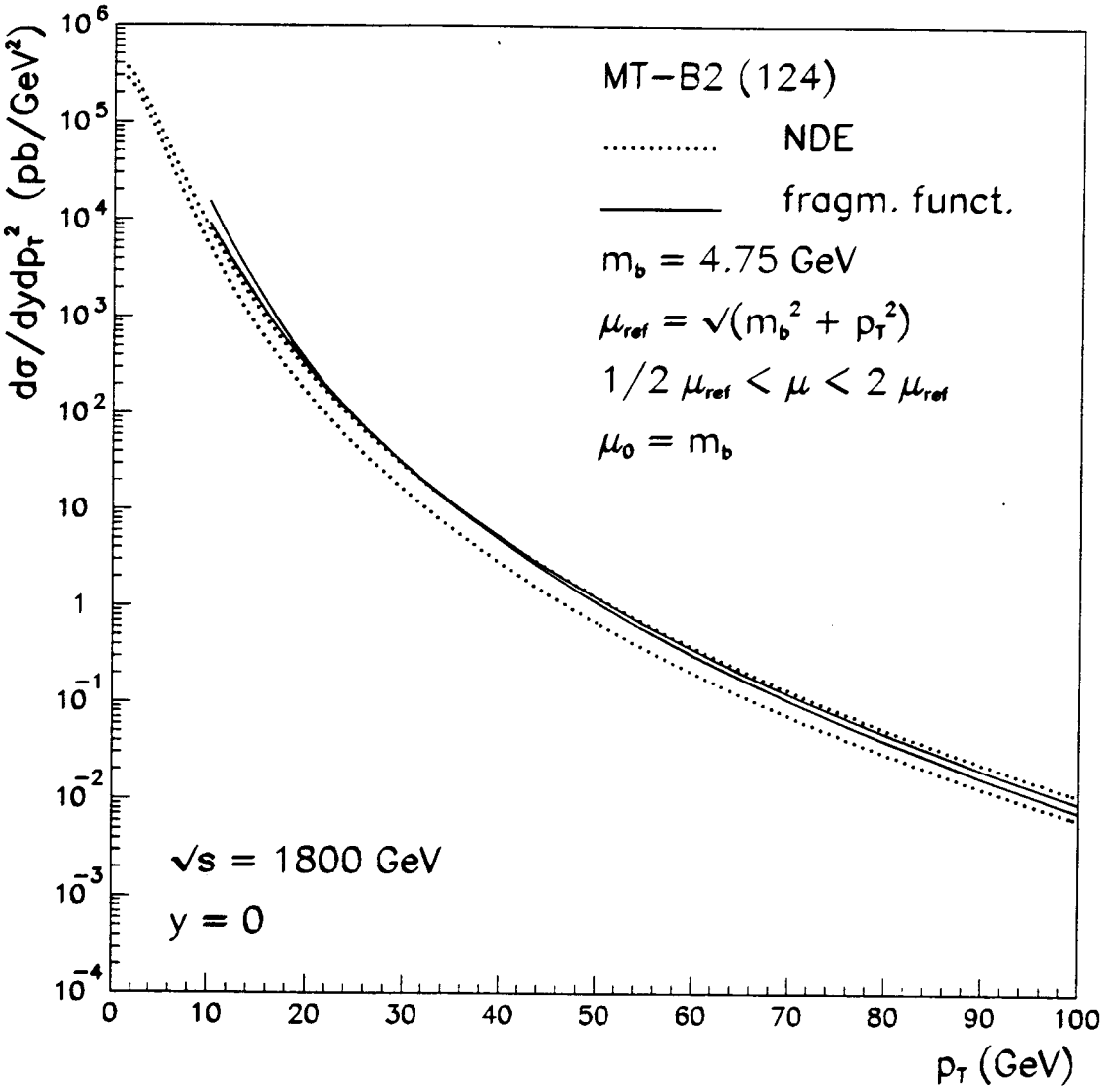
$\mu_0 \sim O(m)$

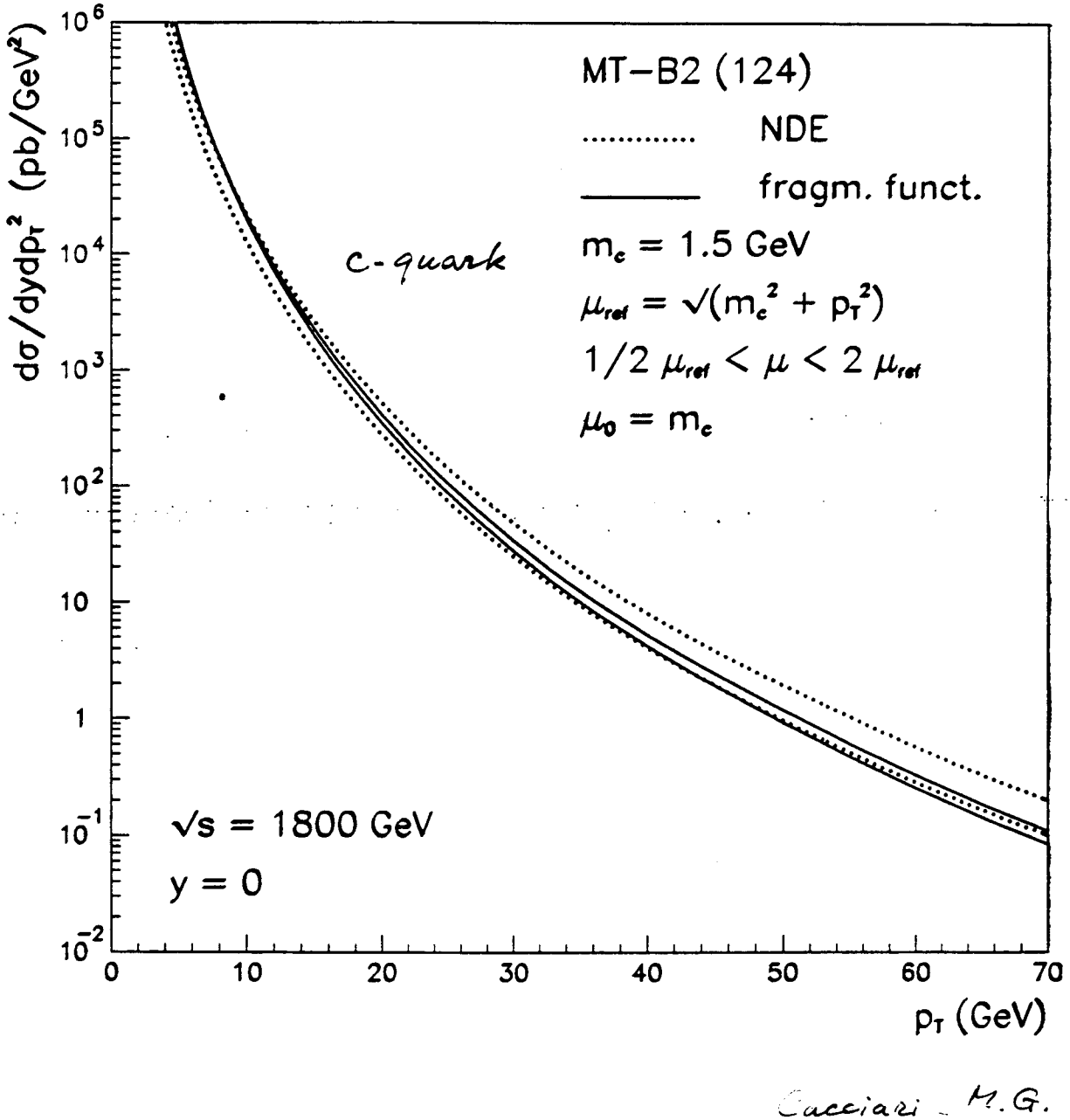
$$\begin{cases} D_q^Q = \delta(1-x) + \dfrac{\alpha_s C_F}{2\pi}\left\{\dfrac{1+x^2}{1-x}\left[\ln\dfrac{\mu_0^2}{m} - 2\ln(1-x) - 1\right]\right\}_+ \\ D_g^Q = \dfrac{\alpha_s T_f}{2\pi}\left[x^2 + (1-x)^2\right]\ln\left(\dfrac{\mu_0^2}{m^2}\right) \\ D_{q,\bar{q},\bar{q}}^Q = 0 \end{cases}$$

⇒ residual dependence on μ_0.
But $\ln(\mu_0/m)$ not large.


MT−B2 (124)
NDE
fragm. funct.
born
m_b = 4.75 GeV
$\mu_{ref} = \sqrt{(m_b^2 + p_T^2)}$
$\mu = \xi\mu_{ref}$
$\mu_0 = m_b$
b - quark
$\sqrt{s}$ = 1800 GeV
y = 0
p_T = 80 GeV
$d\sigma/dydp_T^2$ (pb/GeV^2)
0.2
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
ξ

Cacciari - M. G.


MT−B2 (124)
NDE
fragm. funct.
m_b = 4.75 GeV
$\mu_{ref} = \sqrt{(m_b^2 + p_T^2)}$
$1/2\ \mu_{ref} < \mu < 2\ \mu_{ref}$
$\mu_0 = m_b$
$\sqrt{s}$ = 1800 GeV
y = 0
$d\sigma/dydp_T^2$ (pb/GeV^2)
p_T (GeV)
10^6
10^5
10^4
10^3
10^2
10
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0
10
20
30
40
50
60
70
80
90
100


MT−B2 (124)
..... NDE
_____ fragm. funct.
m_c = 1.5 GeV
$\mu_{ref} = \sqrt{(m_c^2 + p_T^2)}$
$1/2\ \mu_{ref} < \mu < 2\ \mu_{ref}$
$\mu_0 = m_c$
c-quark
$\sqrt{s}$ = 1800 GeV
y = 0
$d\sigma/dydp_T^2$ (pb/GeV^2)
p_T (GeV)
10^6
10^5
10^4
10^3
10^2
10
1
10^{-1}
10^{-2}
0
10
20
30
40
50
60
70

Cacciari - M.G.

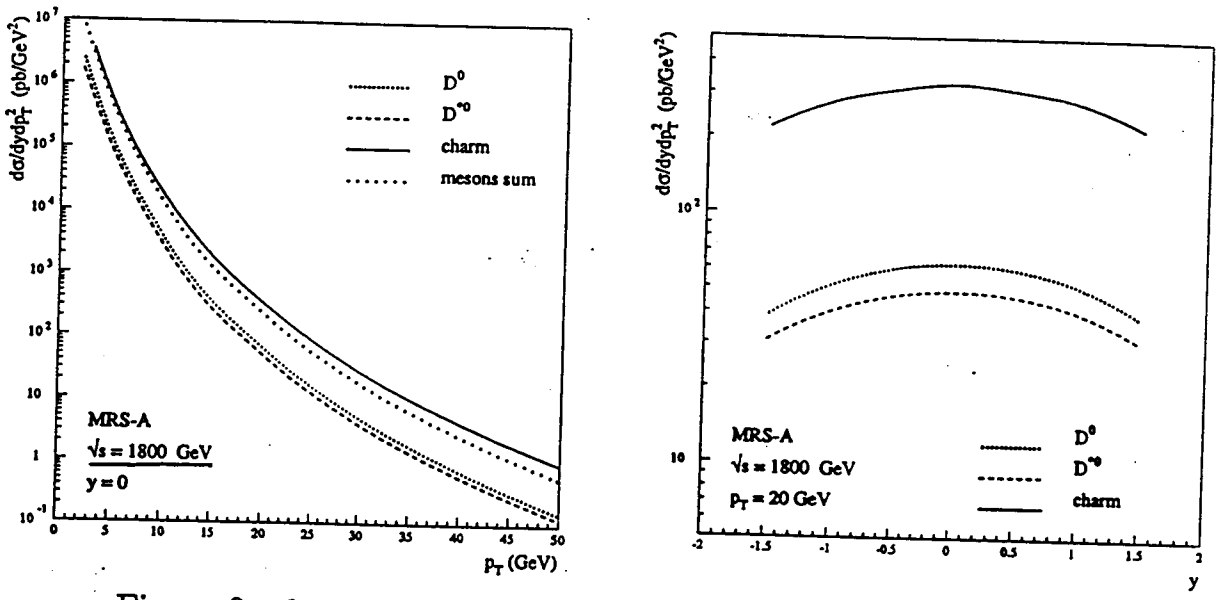


Figure 2: Cross sections for mesons production at the Tevatron, as predicted by our fragmentation functions approach. Comparison with the pure charm quark cross section is also shown. The mesons sum also includes D^+ and D^{*+} contributions.

Cacciari, Rolli, Tauzini, MF.

Similar analysis by Kniehl, Kramer, Spira for D^* mesons at HERA

- Perturbative F.F.
- F.F. à la Peterson et al.

⇒ Non-perturbative effects are important

Data at Tevatron are needed

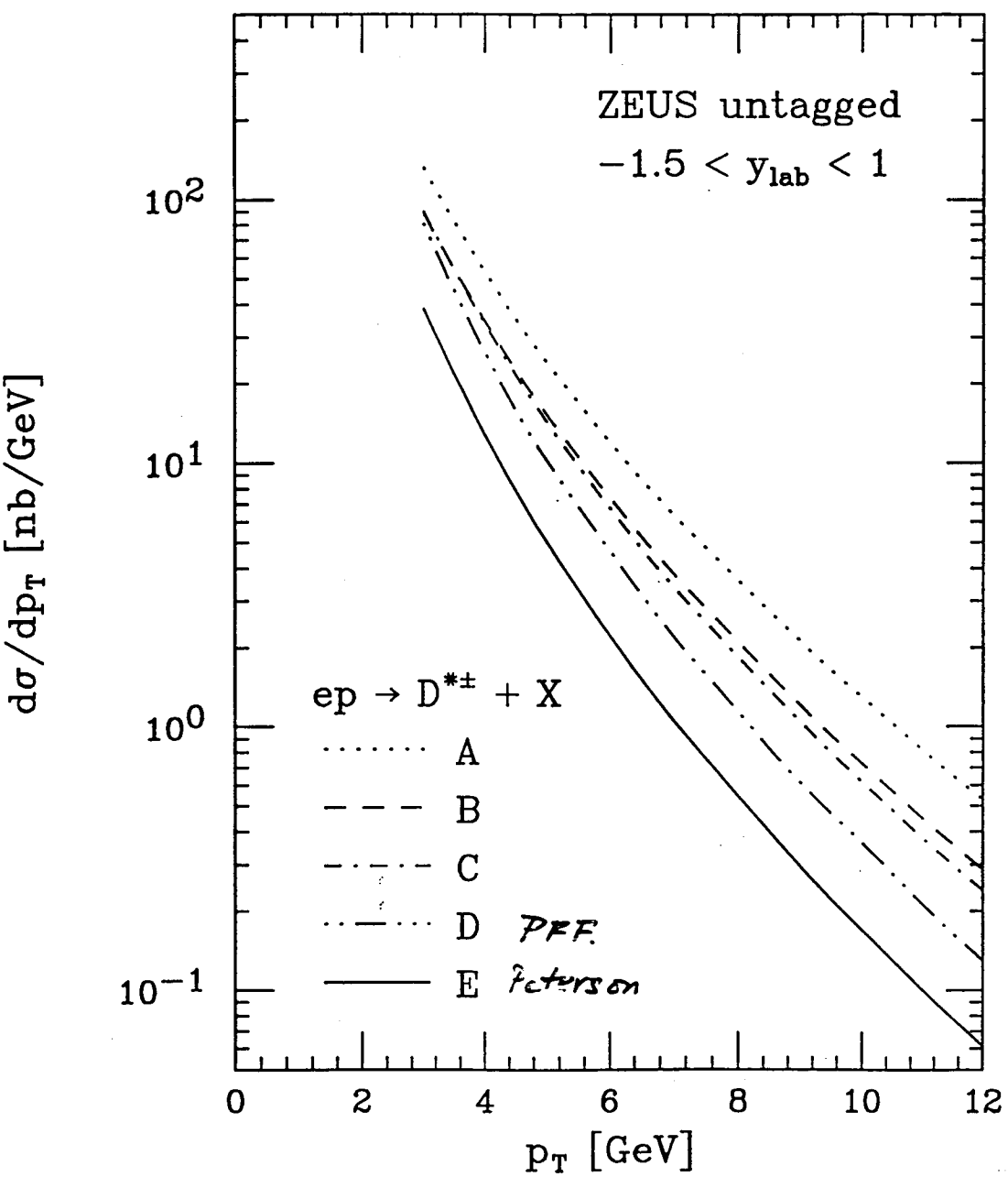


Fig. 1

Aniel, kramer, Spira

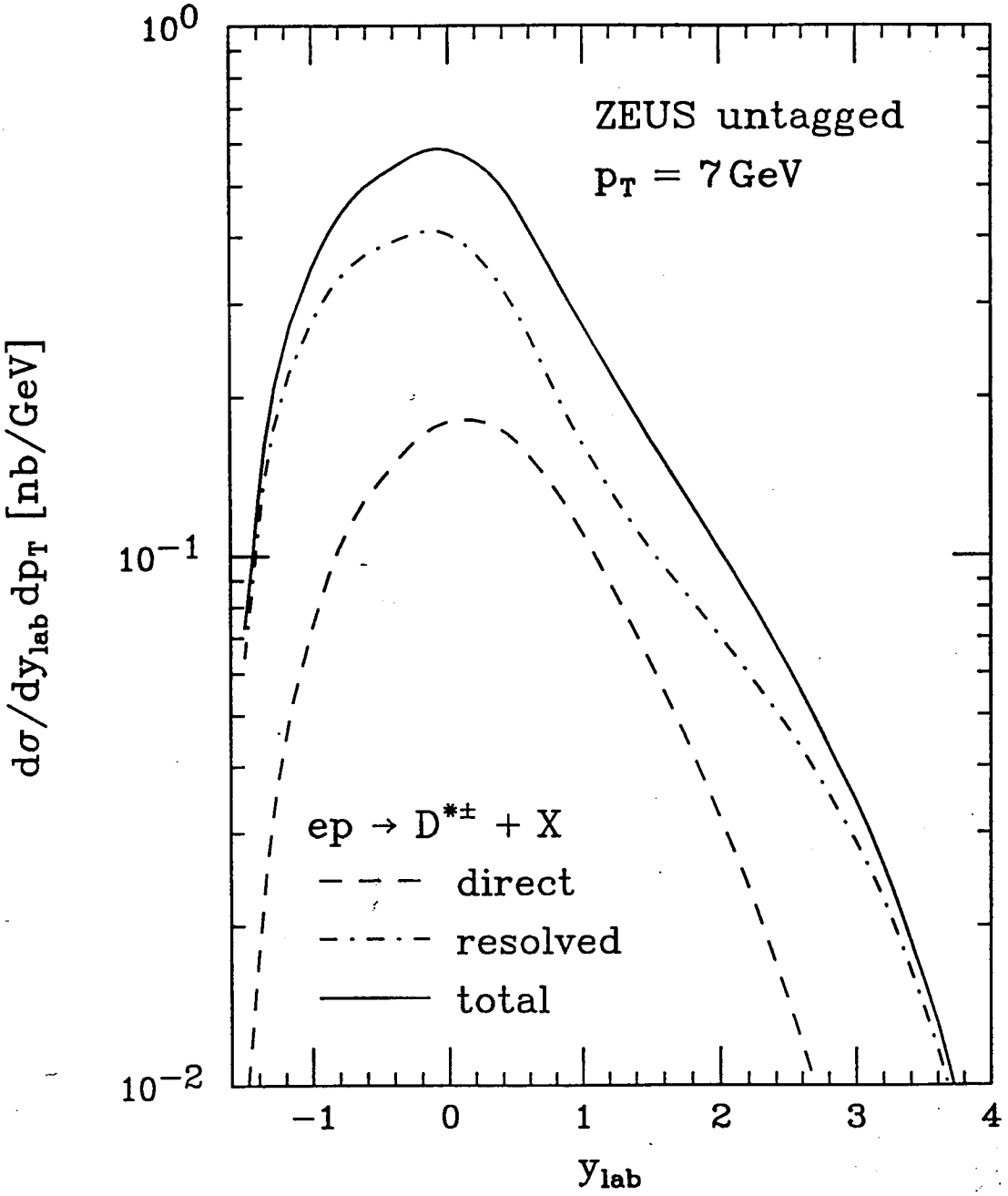

ZEUS untagged
$p_T = 7\,GeV$
$ep \to D^{*\pm} + X$
direct
resolved
total
$d\sigma/dy_{lab}\,dp_T$ [nb/GeV]
y_{lab}
10^0
10^{-1}
10^{-2}
−1
0
1
2
3
4

Fig. 3

KKS

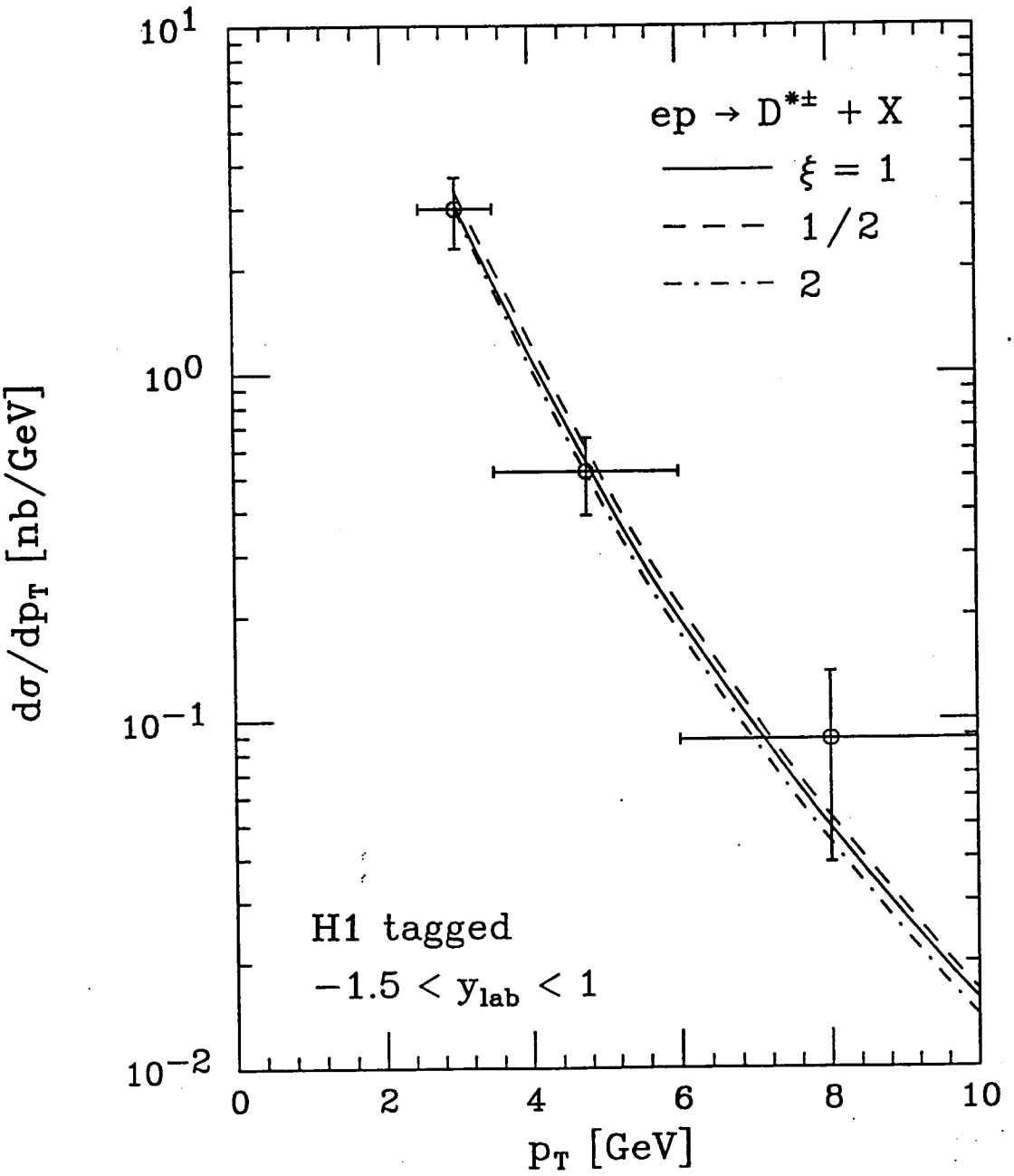

10^1
10^0
10^{-1}
10^{-2}
$d\sigma/dp_T$ [nb/GeV]
$ep \to D^{*\pm} + X$
$\xi = 1$
1/2
2
H1 tagged
$-1.5 < y_{lab} < 1$
0
2
4
6
8
10
p_T [GeV]

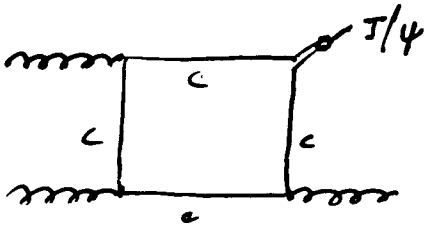
Fig. 8a

KKS

J/ψ Production (hadron colls.)

- Charmonium product. measured by UA1-CDF
- Been studied within QCD for long time
  LO calculations only. Berger-Jones (81) ..... Glover-Martin-Stirling (88)
  Theory is not solid as $p_t \to 0$:
  COLOR-SINGLET MODEL
- Two mechanisms: (B-decays later)

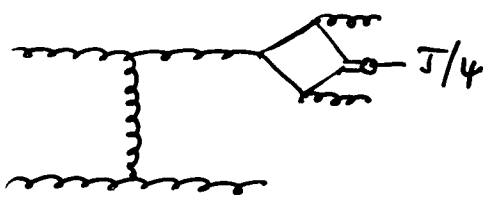
(i) Gluon fusion (direct production)



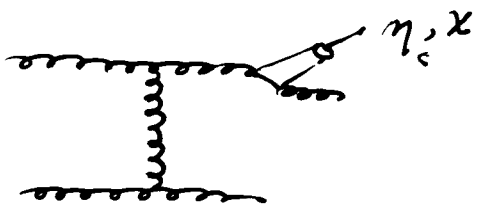
$$\frac{d\sigma}{dp_t} \sim \frac{\alpha_s^3}{p_t^{(8)}}$$

large suppression at large p_t.
Transverse separation $(Q\text{-}\bar{Q})$ is $O(1/p_t)$.

(ii) Fragmentation Braaten-Yuan (93)



$$\sim \frac{\alpha_s^5}{p_t^{(4)}}$$



$$\sim \frac{\alpha_s^4}{p_t^{(4)}}$$

Transverse separation $(Q\text{-}\bar{Q})$ is $O(1/m_Q)$.
Fragmentation dominates at large p_t.

Fragmentation Functions for charmonium prod. from g and c recently calculated in perturbation th.

Braaten - Yuan (93)
Doncheski - Fleming - Maltoni
Braaten - Cheung - Yuan
Y.-Q. Chen
.....

For J/ψ production, contribution from fragmentation of χ states ($\chi \to J/\psi\gamma$) relevant.

Standard formula: Caccari, M.G. (94)
Braaten, Doncheski, Fleming, Maltoni
Roy, Sridhar
Cho, Wise, Trivedi

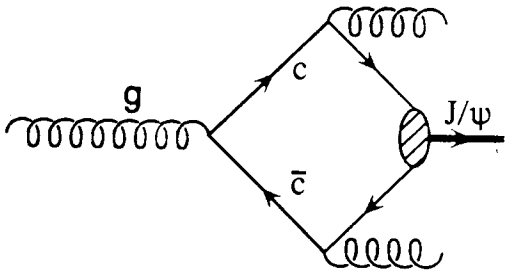
$$d\sigma(p\bar{p} \to J/\psi, \chi + X) \sim \sum_{i} \int_{(p_\perp)} dz \; d\sigma\left[p\bar{p} \to i(p_\perp/z) + X, \mu\right] * D_i^{\psi,\chi}(z,\mu)$$

$i \sim c, \bar{c}, g$

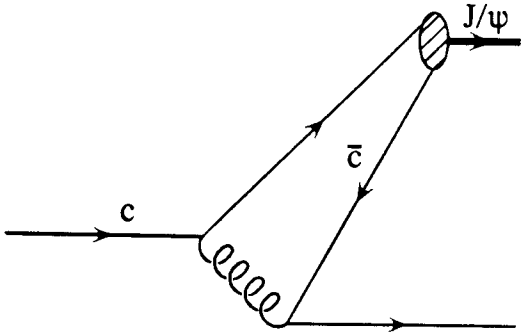
$d\sigma(p\bar{p} \to i + X)$: (LO + NLO) $d\hat{\sigma}_{jk,i}$

$D_i(z,\mu)$: NLO evolution

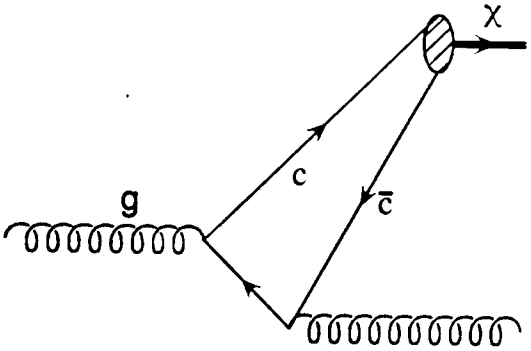
Contribution from rec. $B \to \psi + X$ measured in CDF.



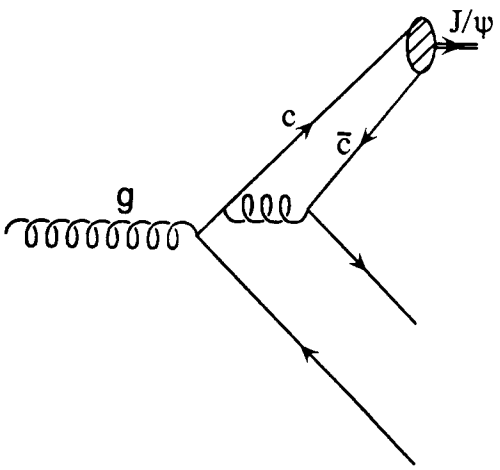
$D_g^{J/\psi}(2m_c)$



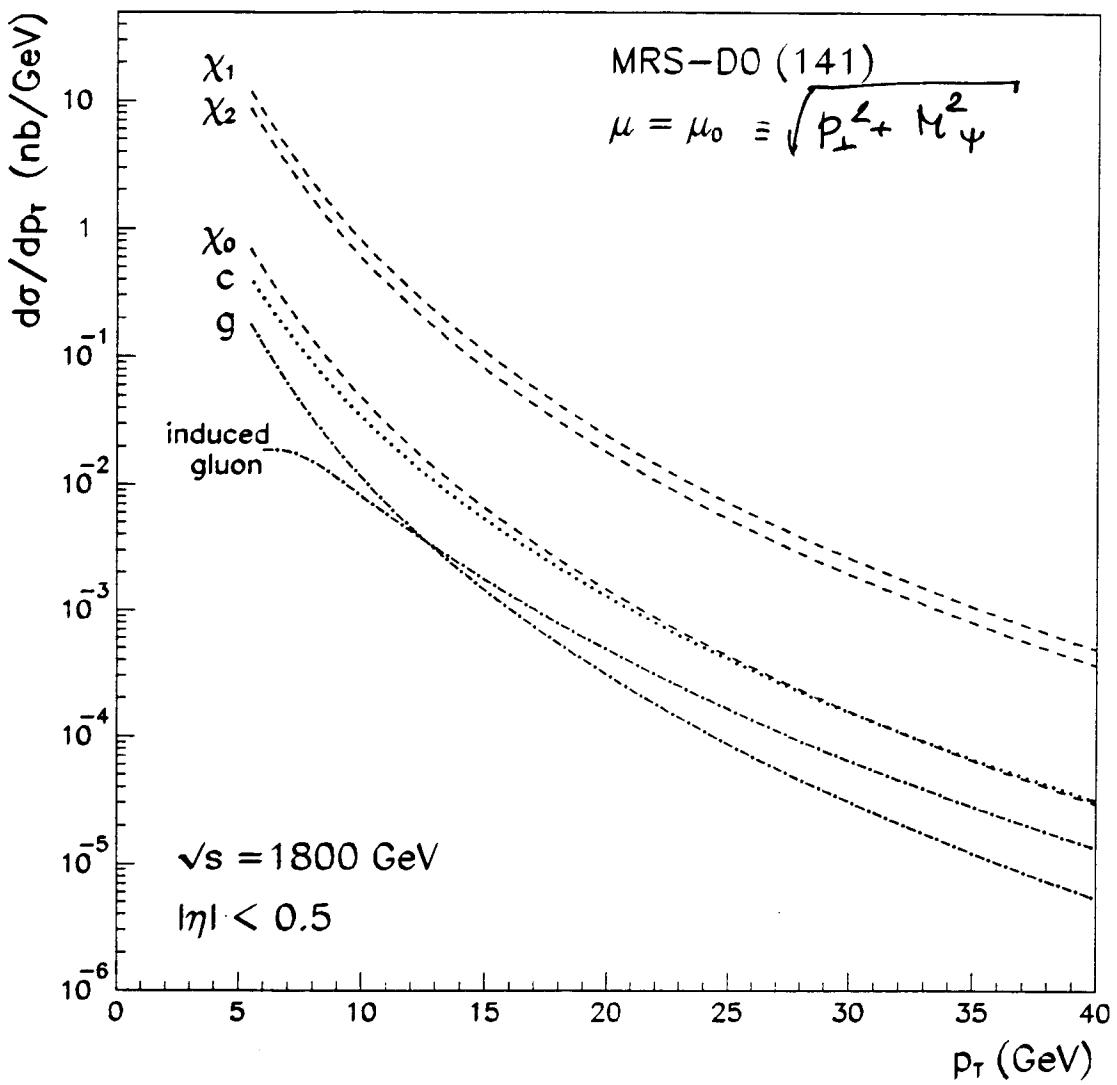
$D_c^{J/\psi}(3m_c)$



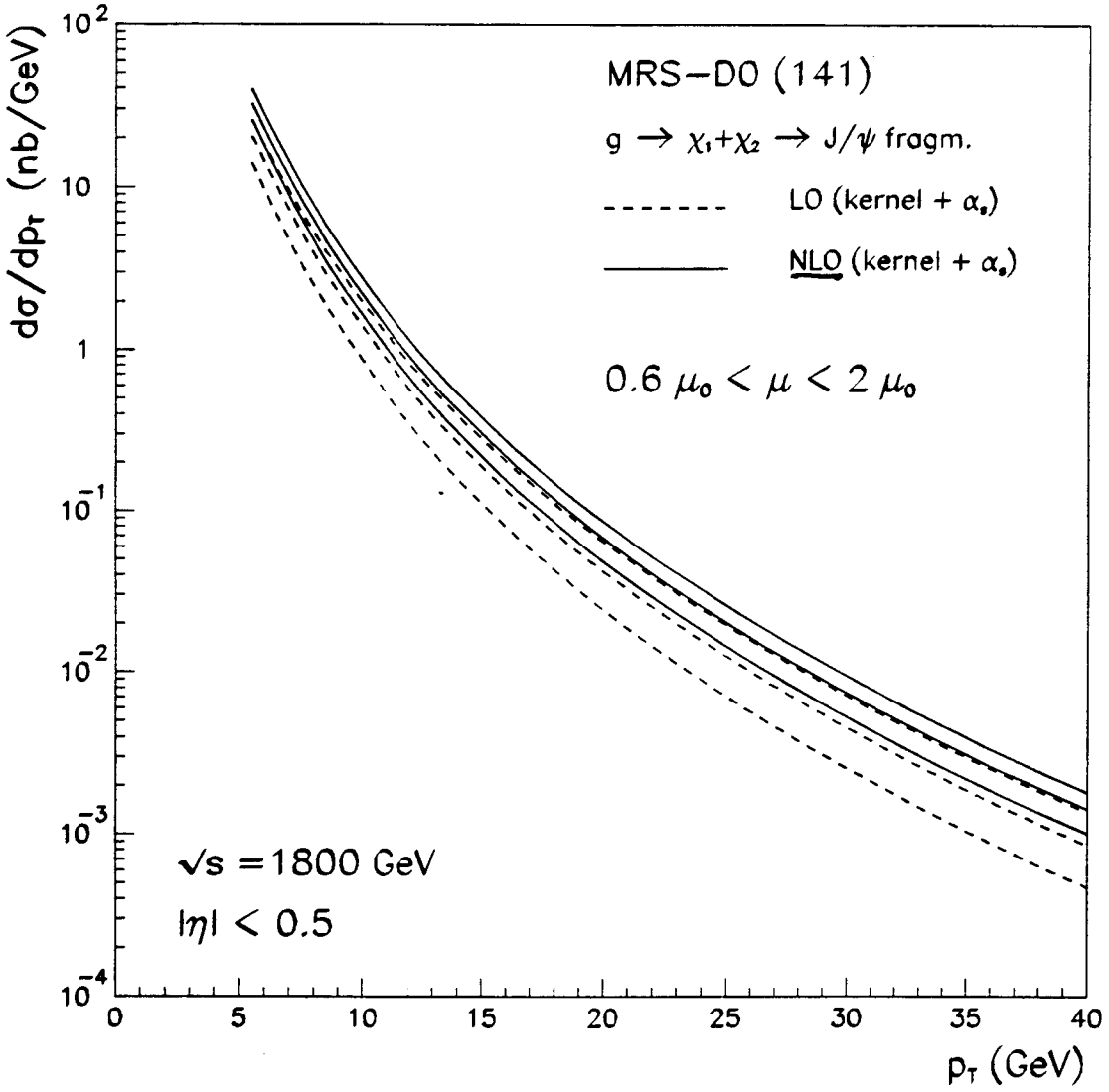
$D_g^{\chi}(2m_c)$

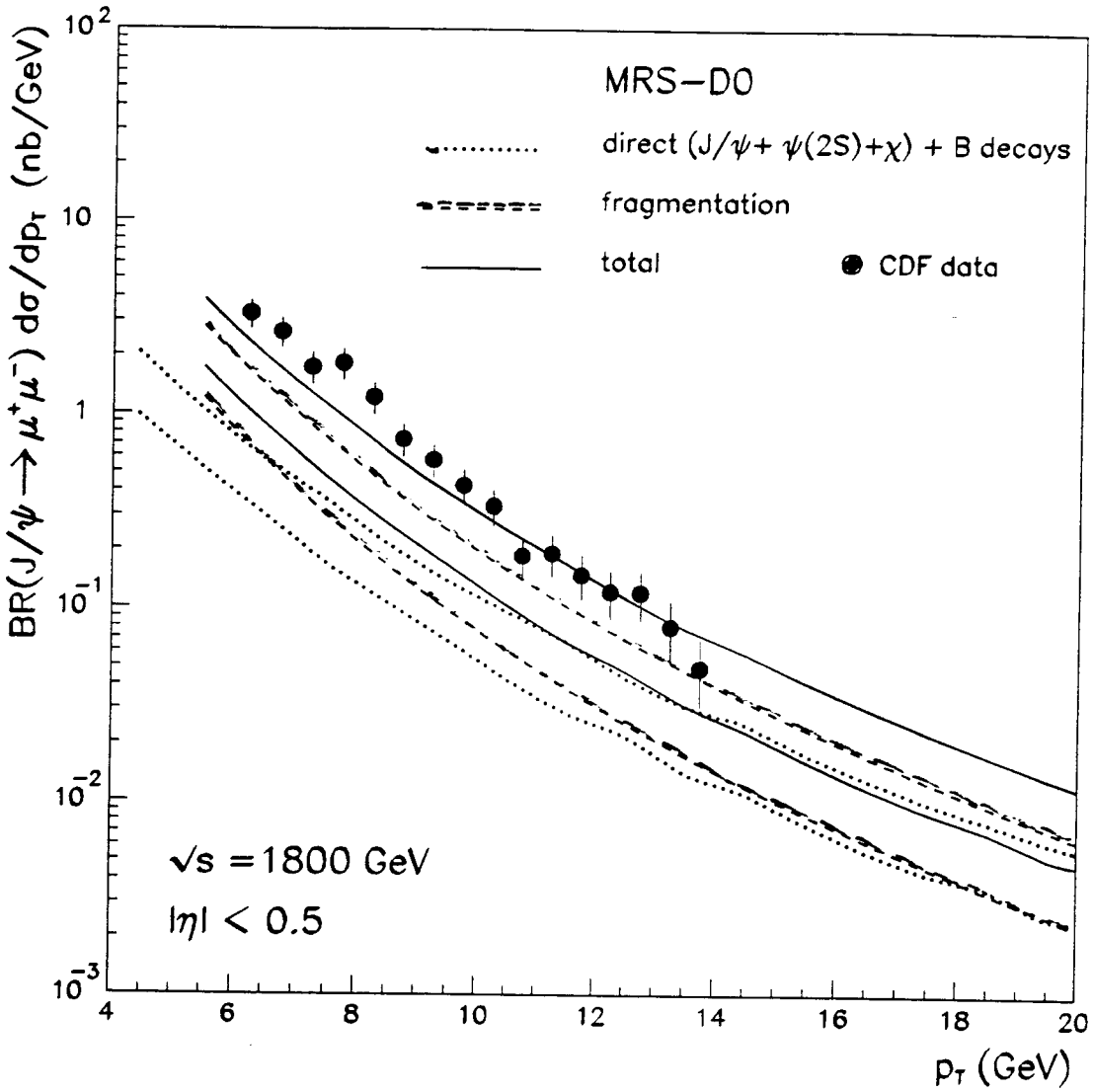


$D_{g,\,ind}^{J/\psi}(4m_c)$



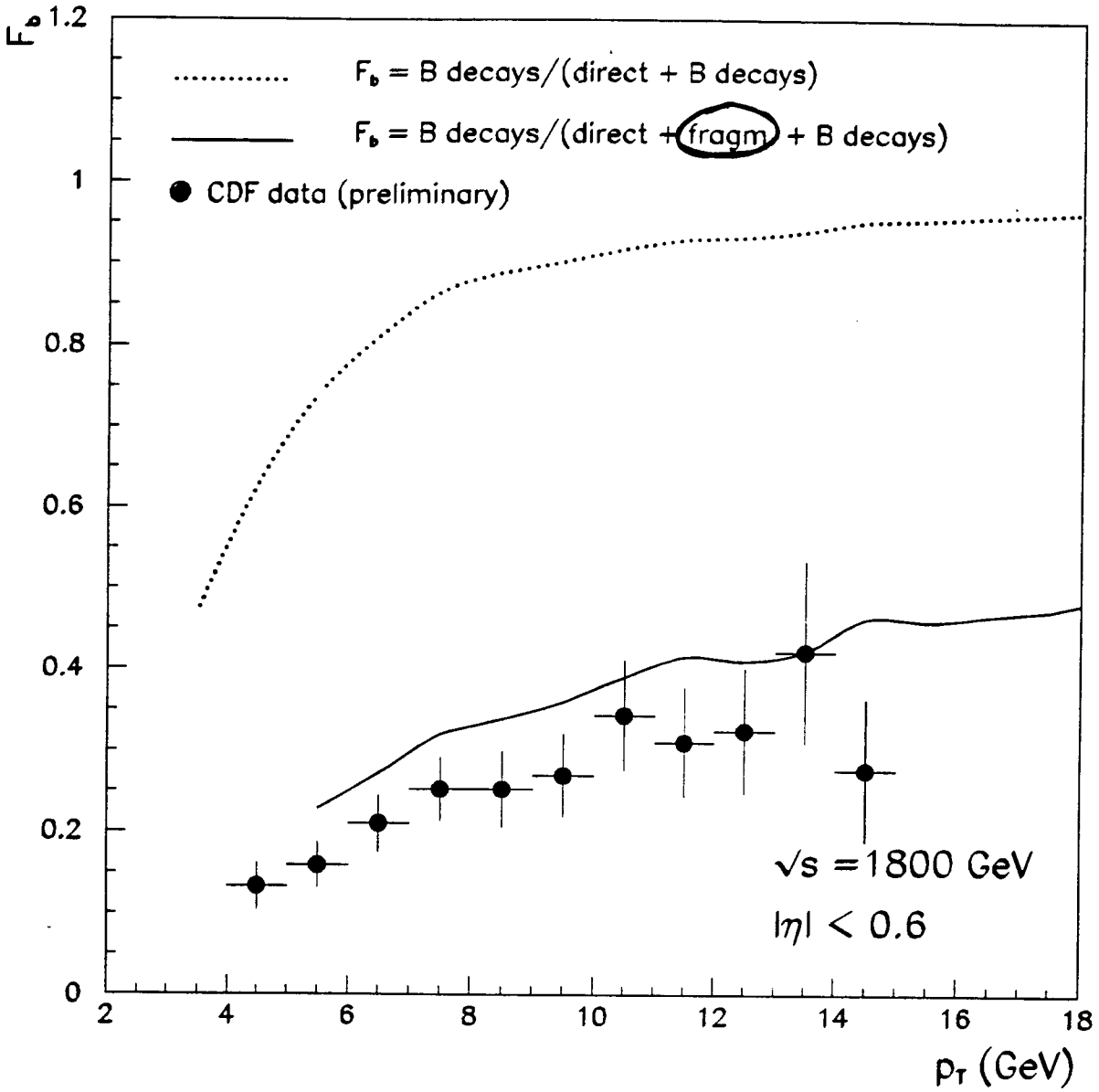
- [illegible], Phys. Rev. Lett. '[illegible]
- E. Braaten, M.A. Doncheski, S. Fleming and M.L. Mangano Phys. Lett. B
- D.P. Roy and K. Sridhar [illegible] '94


MRS−D0 (141)
g → χ1+χ2 → J/ψ fragm.
LO (kernel + αs)
NLO (kernel + αs)
0.6 μ0 < μ < 2 μ0
√s = 1800 GeV
|η| < 0.5
dσ/dpT (nb/GeV)
pT (GeV)
10^2
10
1
10^-1
10^-2
10^-3
10^-4
0
5
10
15
20
25
30
35
40

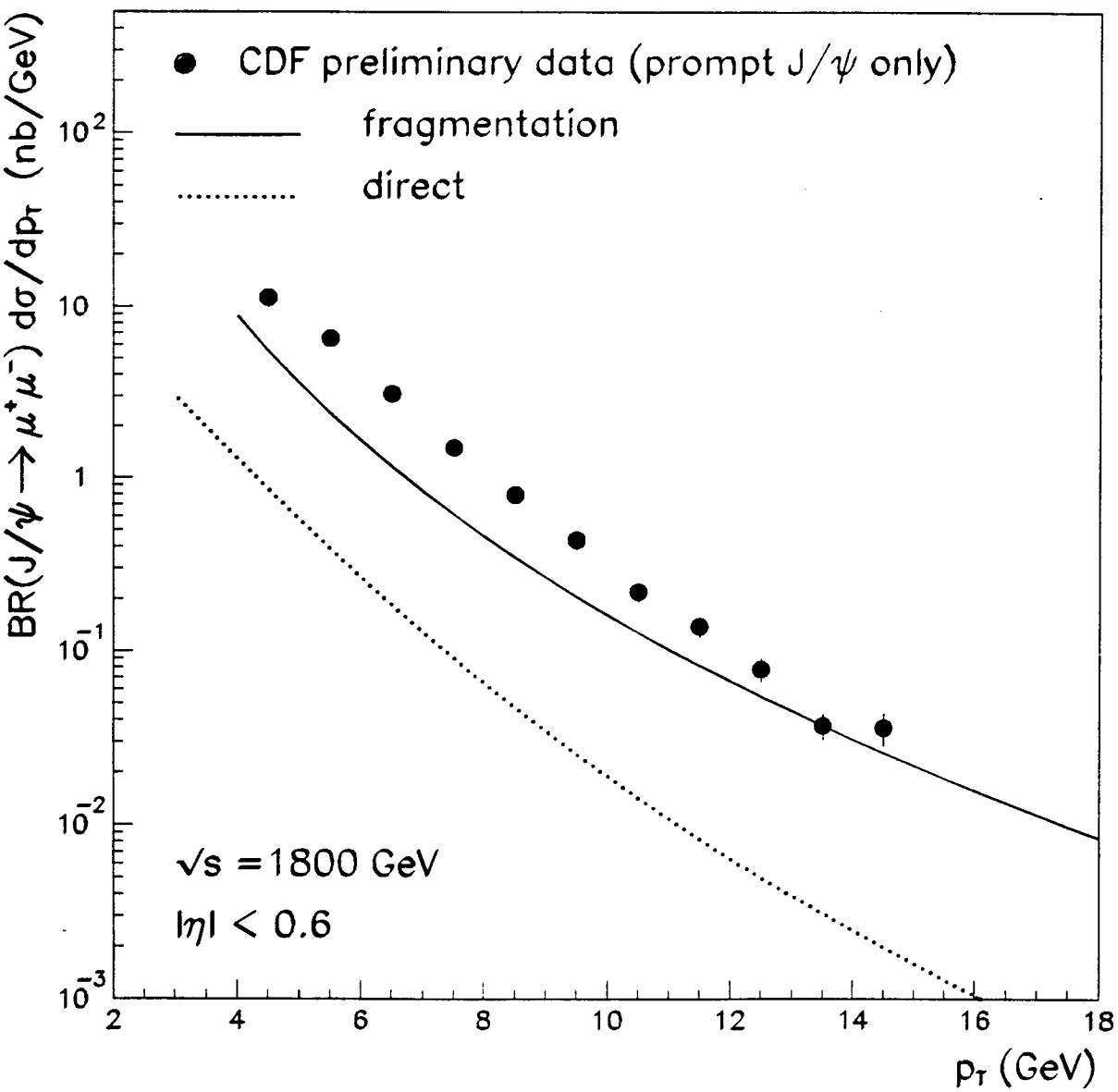


J/ψ (prompt + B decays)

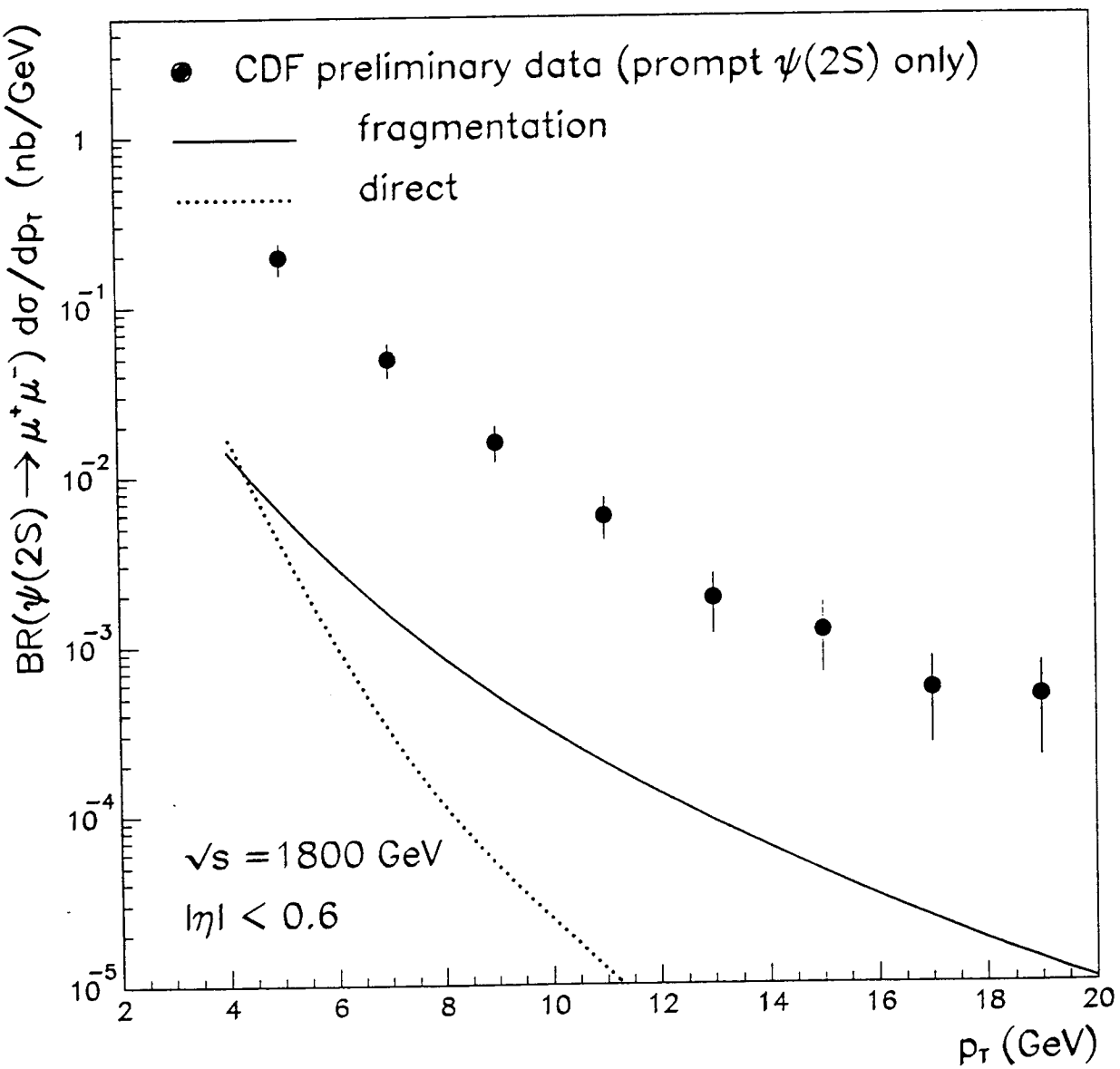
F_B


F_b
1.2
1
0.8
0.6
0.4
0.2
0
F_b = B decays/(direct + B decays)
F_b = B decays/(direct + fragm + B decays)
CDF data (preliminary)
$\sqrt{s}$ = 1800 GeV
$|\eta| < 0.6$
2
4
6
8
10
12
14
16
18
p_T (GeV)

J/ψ (prompt only)



4'


CDF preliminary data (prompt ψ(2S) only)
fragmentation
direct
√s = 1800 GeV
|η| < 0.6
BR(ψ(2S) → μ⁺μ⁻) dσ/dp_T (nb/GeV)
p_T (GeV)
1
10⁻¹
10⁻²
10⁻³
10⁻⁴
10⁻⁵
2
4
6
8
10
12
14
16
18
20