α_s from Deep Inelastic Scattering Structure Functions

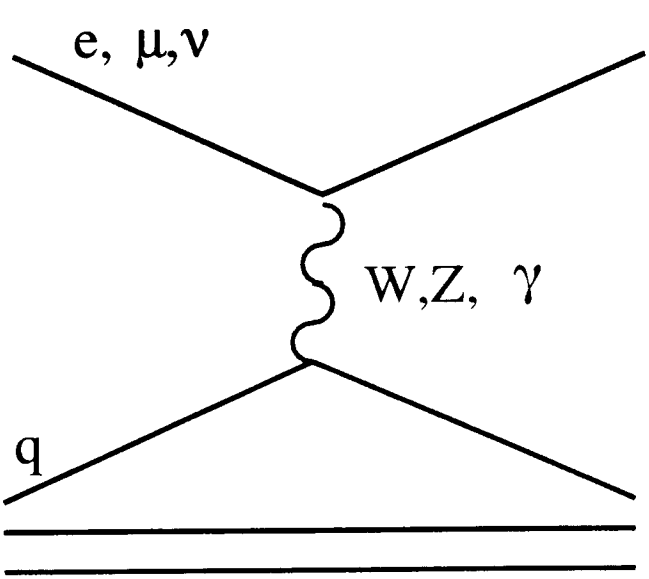
1. How α_s shows up in Structure Functions
2. Current Experiments and Measurements
3. Outstanding Issues and Caveats
4. Which Errors need the most improvement?
5. Is there a short-term future in SF α_s measurements?
6. Big Dreams...

Deborah Harris
University of Rochester

Deep Gratitude to the Authors of the following papers:

- A MEASUREMENT OF α_s AND HIGHER TWISTS FROM A QCD ANALYSIS OF HIGH STATISTICS F_2 DATA ON HYDROGEN AND DEUTERIUM TARGETS. By Marc Virchaux, Alain Milsztajn (Saclay). SACLAY-DPHPE-91-08, Aug 1991. Published in Phys.Lett.B274:221-229,1992.
- TESTING QCD AT CCFR: THREE STRUCTURE FUNCTIONS AT ONCE By CCFR Collaboration, submitted to the ICHEP96 proceedings.
- QUARK AND GLUON DISTRIBUTIONS AND α_s FROM NUCLEON STRUCTURE FUNCTIONS AT LOW x. By New Muon Collaboration (M. Arneodo et al.). CERN-PPE-93-73, Apr 1993. Published in Phys.Lett.B309:222-230,1993.
- DETERMINATION OF α_s FROM $F_2(p)$ AT HERA. By Richard D. Ball, Stefano Forte (CERN). CERN-TH-95-148, Jan 1995. Published in Phys.Lett.B358:365-378,1995.
- DEEP INELASTIC SCATTERING OF POLARIZED ELECTRONS BY POLARIZED HE-3 AND THE STUDY OF THE NEUTRON SPIN STRUCTURE. By E142 Collaboration (P.L. Anthony et al.). SLAC-PUB-7103, Sep 1996. To be published in Phys.Rev.D
- PADE APPROXIMANTS, BOREL TRANSFORMS AND RENORMALONS: THE BJORKEN SUM RULE AS A CASE STUDY. By John Ellis (CERN), Einan Gardi, Marek Karliner (Tel Aviv U.), Mark A. Samuel (Oklahoma State U.). CERN-TH-95-155, Sep 1995. Published in Phys.Lett.B366:268-275,1996.
- A MEASUREMENT OF $\alpha_s(Q^2)$ FROM THE GROSS-LLEWELLYN-SMITH SUM RULE. By CCFR-NUTEV Collaboration. FERMILAB-CONF-95-144, Mar 1995. Contributed to the 30th Rencontres de Moriond: QCD and High Energy Hadronic Interactions, Meribel les Allues, France, 19-25 Mar 1995. Published in Moriond 1995

DIS at a GLANCE



x=fractional momentum of q

Q=momentum transfer of W,Z, γ

Structure Functions

In the Quark Parton Model, to first order, the structure functions relevant to α_s measurements are:

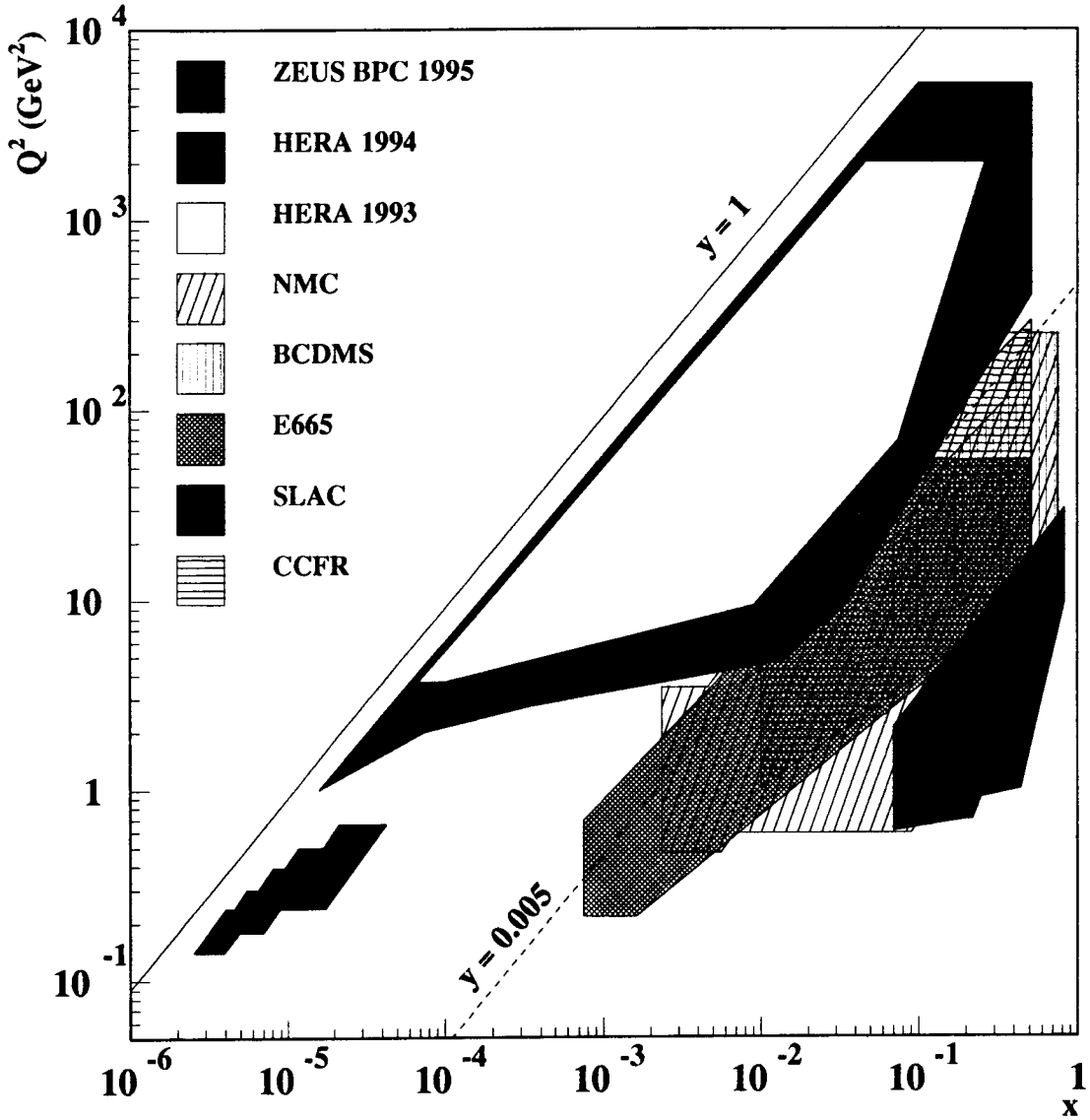
$$xF_3^{\nu N}(x,Q^2) = \sum_i xq_i - x\overline{q}_i \quad \text{valence}$$

$$F_2^{\nu N}(x,Q^2) = \sum_i xq_i + x\overline{q}_i \quad \text{valence + sea}$$

$$F_2^{\mu,eN}(x,Q^2) = \frac{5}{18}F_2^{\nu N} - \frac{1}{3}(x\overline{s} - x\overline{c})$$

$$g_1^{n,(p)}(x,Q^2) = \frac{1}{2}\sum_i e_i^2 \Delta q_i \quad \text{spin averaged}$$

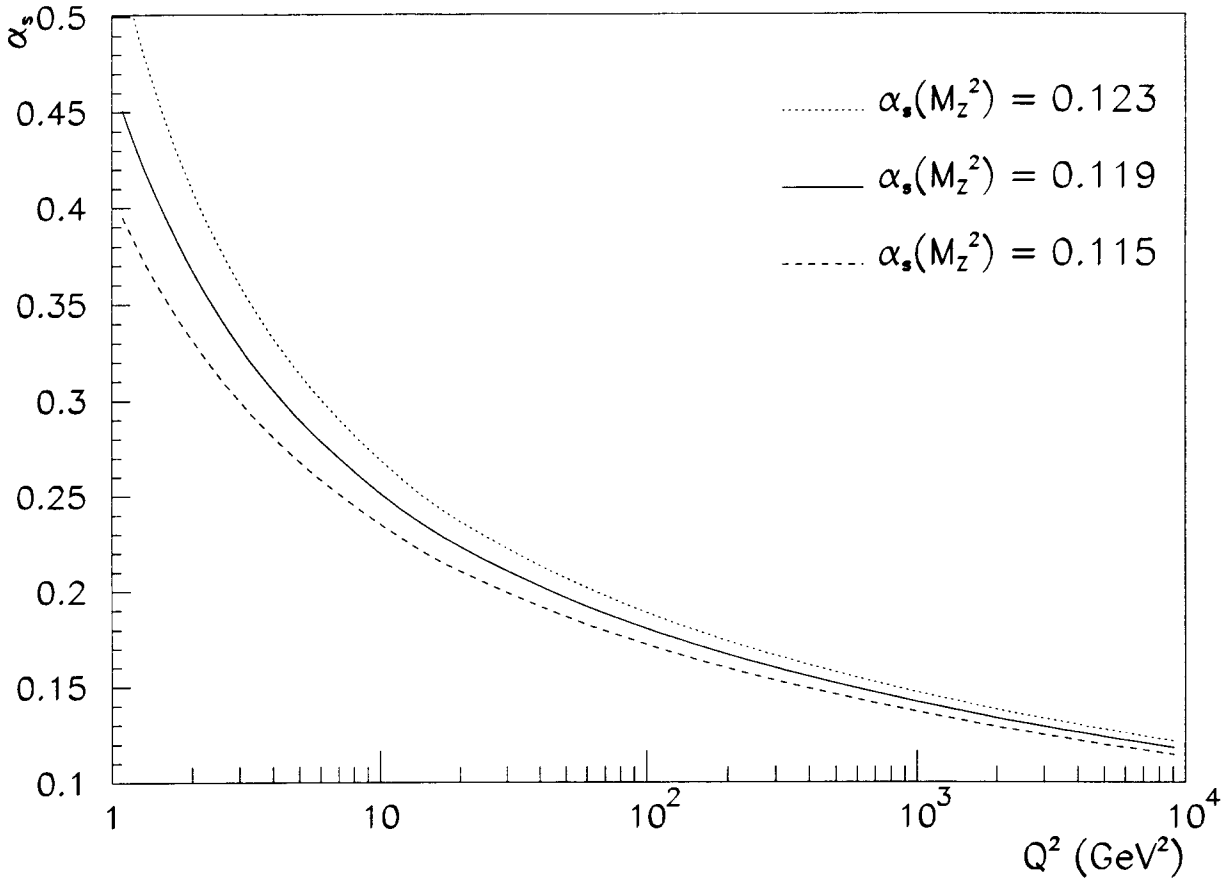
$$\Delta q_i = q_i^{\uparrow} - q_i^{\downarrow}$$



(Plot from R.D.Ball and A. De Roeck DESY-96-185)

- CCFR νN scattering measures xF_3, F_2 separately
- "SLAC" many experiments ep,en, scattering (F_2)
- SLAC E142,E143,E154 measure eHe, $(g_1^n)(g_1^p)$ ep,en
- BCDMS: μp,μd measures F_2
- NMC: μp,μd, measures F_2
- HERA: ep, measures F_2

But not all Q^2's are created equal...



The lower you go in Q^2, the more non-perturbative effects start entering...

The higher you go in Q^2, the smaller an effect you're trying to measure and α_s is changing less rapidly.

How does α_s show up in Structure Functions?

DGLAP Evolution Equations

$$\frac{dF_{NS}}{d\log Q^2} = \frac{\alpha_s(Q^2)}{\pi}\int_x^1 P_{qq}\left(\frac{x}{z},Q^2\right)F_{NS}(z,Q^2)dz$$

$$\frac{dF_S}{d\log Q^2} = \frac{\alpha_s(Q^2)}{\pi}\int_x^1 \left(P_{qq}\left(\frac{x}{z},Q^2\right)F_S(z,Q^2)+P_{qG}\left(\frac{x}{z},Q^2\right)F_G(z,Q^2)\right)dz$$

$$\frac{dF_G}{d\log Q^2} = \frac{\alpha_s(Q^2)}{\pi}\int_x^1 \left(P_{GG}\left(\frac{x}{z},Q^2\right)F_G(z,Q^2)+P_{Gq}\left(\frac{x}{z},Q^2\right)F_S(z,Q^2)\right)dz$$

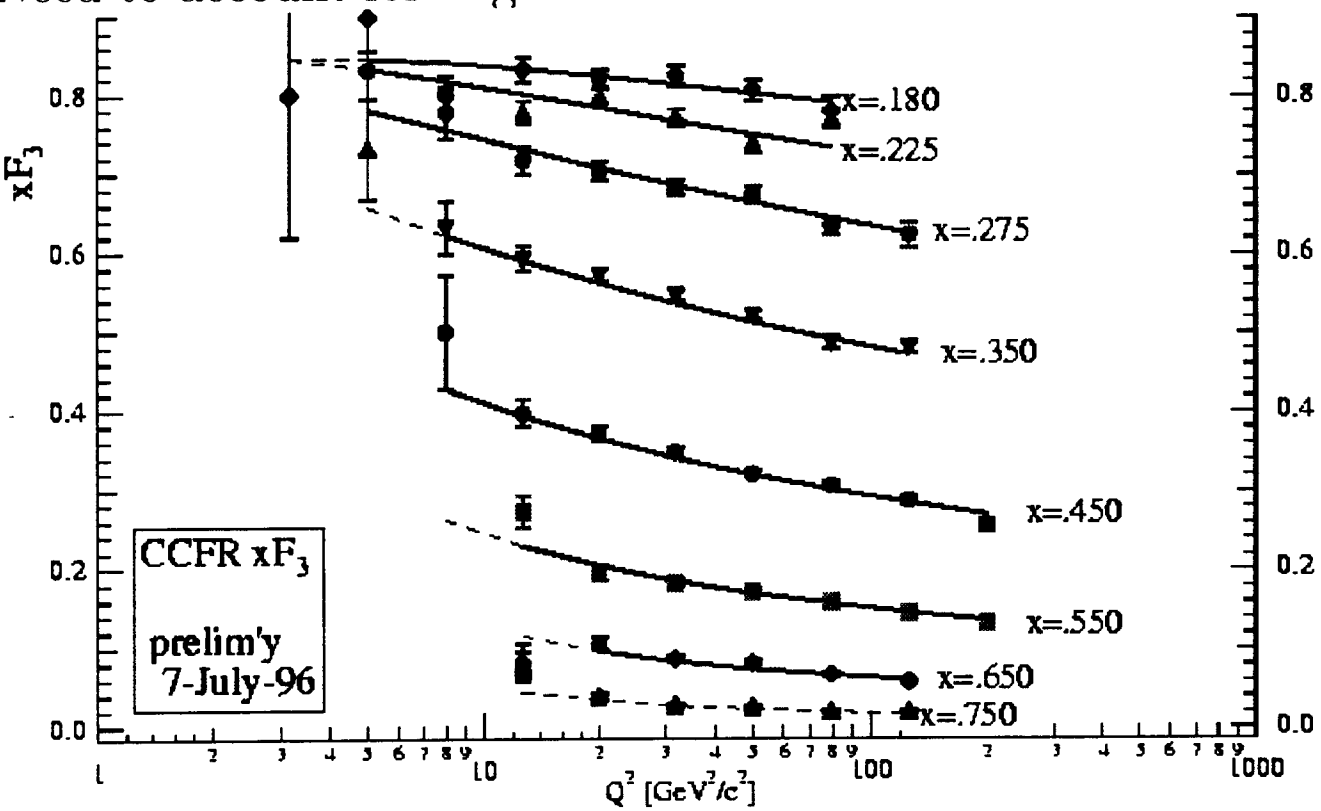
Only F_{NS} evolution is independent of gluons!

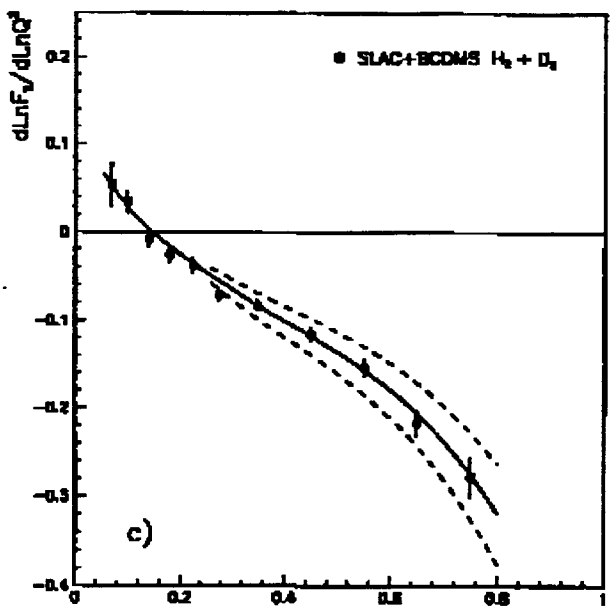
xF_3 is purely non-singlet

$F_2 \approx xF_3$ at high x and so is also non-singlet

NLO DGLAP equations have been computed

Need to account for Higher Twist

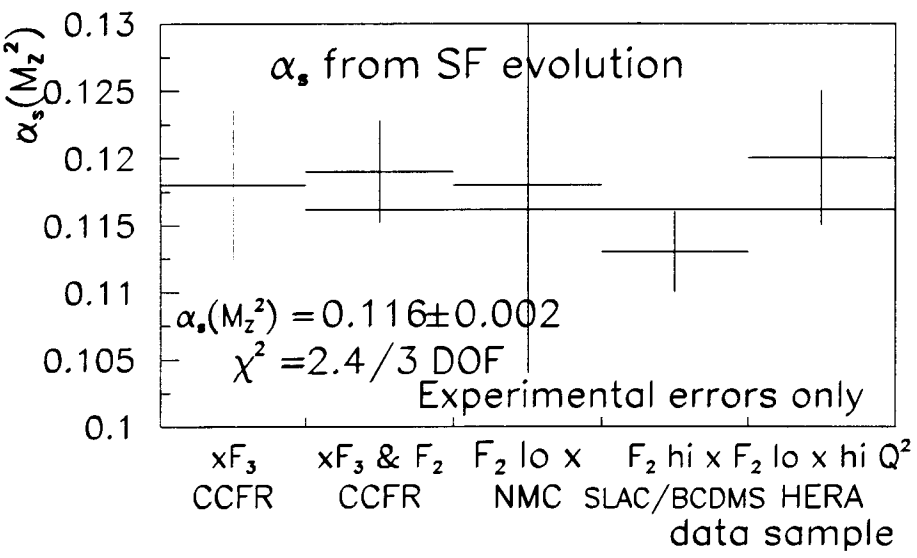




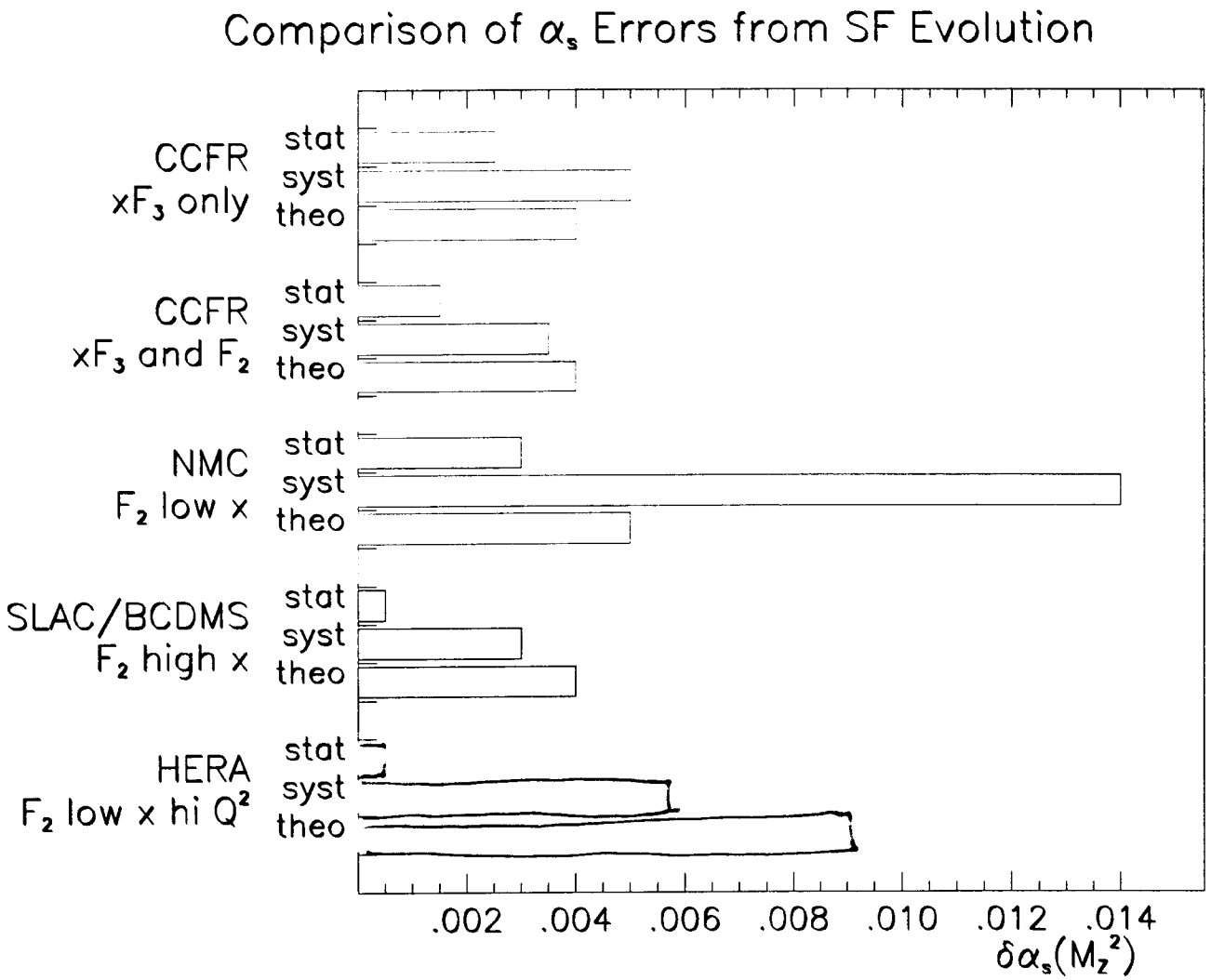
From Virchaux & Milsztajn fit of SLAC/BCDMS data $x \geq 0.25$

Structure Function Evolution Measurements

Method	Experiment	$Q^2(\text{GeV}^2)$	$\alpha_s(Q^2)$	$\alpha_s(M_Z^2)$
xF_3 only	CCFR	25	$.215 \pm .024$	$.118 \pm .007$
xF_3 and F_2	CCFR	25	$.217 \pm .018$	$.119 \pm .0055$
F_2 low x	NMC	7	$.264 \pm .073$	$.118 \pm .015$
F_2 high x	SLAC & BCDMS	50	$.180 \pm .013$	$.113 \pm .005$
F_2 low x high Q^2 (double scaling)	HERA	4-100	-	$.120 \pm .010$



How do the different errors in these measurements compare?



Experiment	Dominant Systematic Error
CCFR	Energy Scale
NMC	Spectrometer Acceptance & Energy Calibration
SLAC/BCDMS	Beam Energy Calibration
	Spectrometer Calibration
	Spectrometer Resolution
HERA	x distribution parameterization

Biggest Theoretical Error: Scale Uncertainties

Since DGLAP is known only to NLO,
what happens at higher order of α_s?

Factorization Scale:
Scale at which you relate the measured
Structure Functions to Parton Distribution Functions

$$F(x,Q^2) = C\int_x^1 q(y,k_1Q^2)\times \left[\delta\left(\frac{x}{y}-1\right)+\frac{\alpha_s(k_1Q^2)}{2\pi}\left(f\left(\frac{x}{y}\right)-P_0\left(\frac{x}{y}\right)\ln k_1\right)\right]\frac{dy}{y}$$

k_1Q^2 is scale at which PDF's *really* should be evaluated for a measurement at Q^2.

Renormalization Scale:
Scale at which PDF's are evolved

$$\frac{dq(x,Q^2)}{d\ln Q^2} = \frac{\alpha_s(k_2Q^2)}{2\pi}\times \int_x^1 q(z,Q^2)\left[P_0\left(\frac{x}{z}\right)+\alpha_s(k_2Q^2)\left(P_1\left(\frac{x}{z}\right)+\frac{\beta_0}{4\pi}P_0\left(\frac{x}{z}\right)\ln k_2\right)\right]\frac{dz}{z}$$

k_2Q^2 is scale at which PDF's *really* should be evolved for a measurement of $\alpha_s(Q^2)$.

How can the scale uncertainty be estimated?

- Compare difference between NLO and LO DGLAP equations
- Redo QCD fits setting both k_1 and k_2 to 0.10 and 4.0, quote difference in α_s as the scale difference. (Done by Virchaux and Milsztajn for SLAC/BCDMS data to get $\Delta\alpha_s(M_Z^2) = .004$)
- Add k_1 and k_2 as two free parameters to the "QCD fits" and see what the data allows, between χ^2_{min} and $\chi^2_{min} + 1$. Beware–this is dangerous: What if k_1, k_2 were pulled far away from 1 for these fits?

  - Is this new physics?
  - Is this a signal that the higher order terms are large and so these measurements aren't sensitive to α_s?
  - Is it fair to let the statistical precision in the experiment tell you what k_1 or k_2 can be?

Warning of things to come:
Similar game is played for experimental systematics in SF evolution analyses...

What do you do when you have high statistics but correlated systematics?

1. One thing not to do: add systematic error in quadrature with statistical error for each bin and fit...
Example: in CCFR bin by bin: $\delta(\text{stat}) >> \delta(\text{syst})$ but $\delta\alpha_s(\text{stat}) << \delta\alpha_s(\text{syst})$

2. Re-extract structure functions for each different effect so you can see all the correlations. Re-evaluate α_s for each SF, and add all the $\delta(\alpha_s)$ in quadrature.

Uncertainty	xF_3 Only $\alpha_s(M_Z^2)$	xF_3 & F_2 $\alpha_s(M_Z^2)$
Statistical Only	0.0025	0.0015
CCFR Experimental Uncertainties		
Energy Calibration	0.005	0.003
Flux Measurement	0.002	0.001
External Input Uncertainties		
σ^ν,$\sigma^{\overline{\nu}}/\sigma^\nu$,$R$	0.001	< 0.001
Heavy Quark Effects	0.002	0.001
Total Systematic	0.0055	0.0035
QCD Theoretical Uncertainties		
Parameterization of $xG(x)$	???	???
Scale (Virchaux, Milsztajn)	0.004	0.004
TOTAL SO FAR	.007	.005

From CCFR, ICHEP'96 talk–Preliminary

3. Do a global fit for α_s plus another parameter for each systematic effect you want to vary–add one term to the χ^2 for each systematic parameter.

PROS:

- you can get a really tiny total experimental error on α_s within the framework of QCD

CONS:

- "Using QCD to determine experiment's energy scale"
- Is there enough freedom in the fits for any real effect?
- It's not really a test of QCD–more like a cross-check
- To Test QCD, compare result with others
- How to find new physics?

Summary

Where we are in Evolution Measurements:
$\alpha_s(M_Z^2) = 0.116 \pm .002(\text{exp}) \pm .004(\text{scales})$
$\chi^2 = 2.4/3$ DOF
See no difference between
non-singlet and singlet determinations.

For Theoretical Improvement:

Scale Uncertainties–need those higher order calculations!

For Experimental Improvement:

Energy scale of experiment.
Where's the "mass peak" you can calibrate fixed target experiments on?
Better Calibration
Better Calorimetry
Better Spectrometers

So, is there any other way to measure α_s from structure functions with smaller theoretical errors?

Sum Rules–may be an up and coming field

Gross Llewellyn Smith Sum Rule

$$\int_0^1 xF_3(x,Q^2)\frac{dx}{x} = 3(1-\tfrac{\alpha_s}{\pi}-a_2(n_f)(\tfrac{\alpha_s}{\pi})^2-a_3(n_f)(\tfrac{\alpha_s}{\pi})^3)-\frac{\Delta HT_{GLS}}{Q^2}$$

Three Valence Quarks in the Nucleon

Bjorken Sum Rule

$$\int_0^1 (g_1^n(x)-g_1^p(x))dx = \frac{1}{6}g_A(1-\tfrac{\alpha_s}{\pi}-b_2(n_f)(\tfrac{\alpha_s}{\pi})^2-b_3(n_f)(\tfrac{\alpha_s}{\pi})^3)-\frac{\Delta HT_{Bj}}{Q^2}$$

- Fundamental Predictions of QCD
- Theoretically Robust low Q^2 Predictions
- Independent of Gluon Distribution
- Calculated to $\mathcal{O}({\alpha_s}^3)$, estimated to $\mathcal{O}({\alpha_s}^4)$

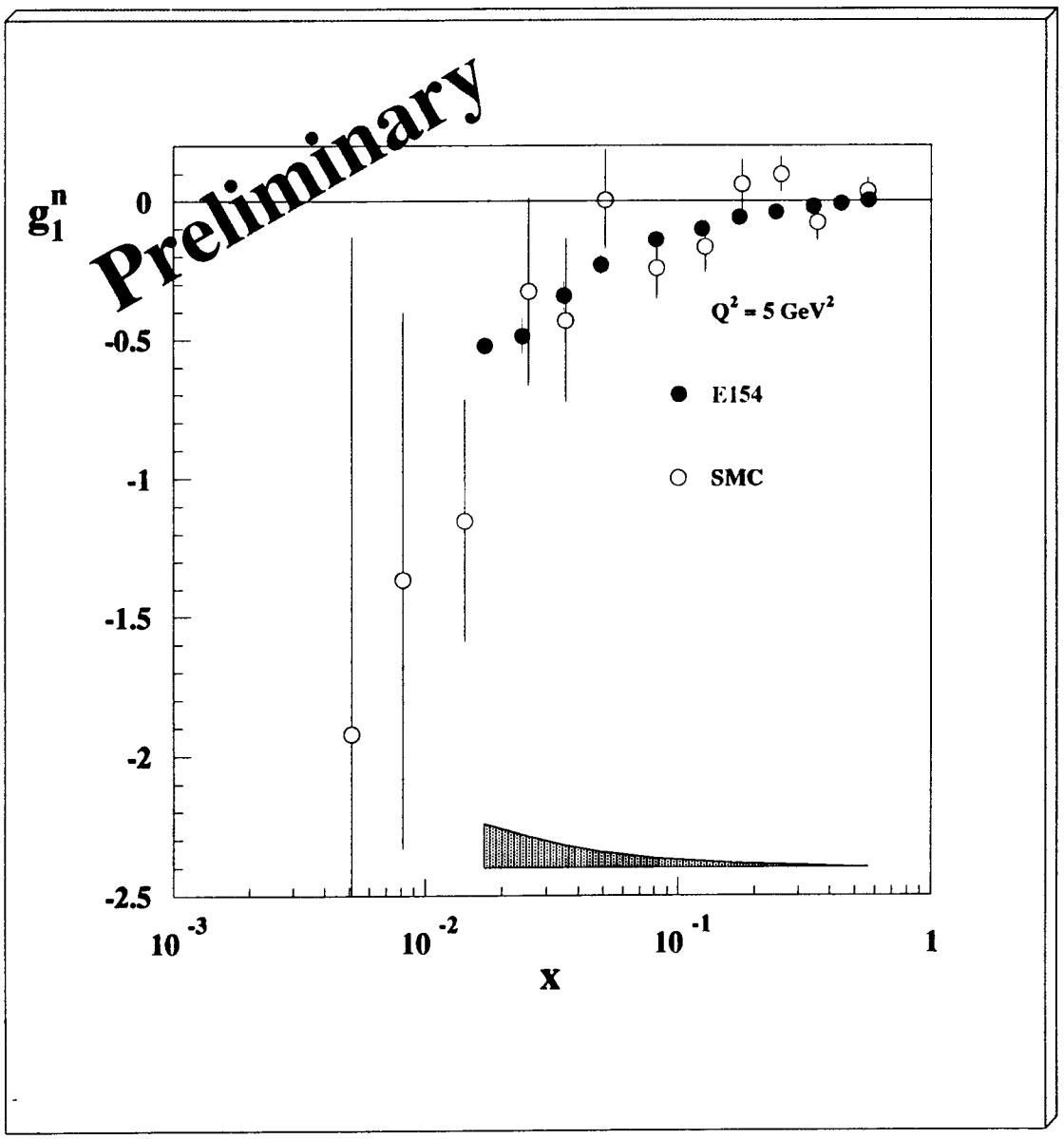
Caveats–experimental problems

- Need to be at low Q^2 for low x: Higher Twist Effects only Predicted, not Measured
  Different theories agree for GLS, not for BJorken SR.
- Low x extrapolation: Again, only theory predicts (Regge, quark counting)–impossible to measure to $x = 0$.

Lesson from HERA: you never know what's going on at small x until you measure it.

Pinch of Salt: Low x extrapolation

Neutron

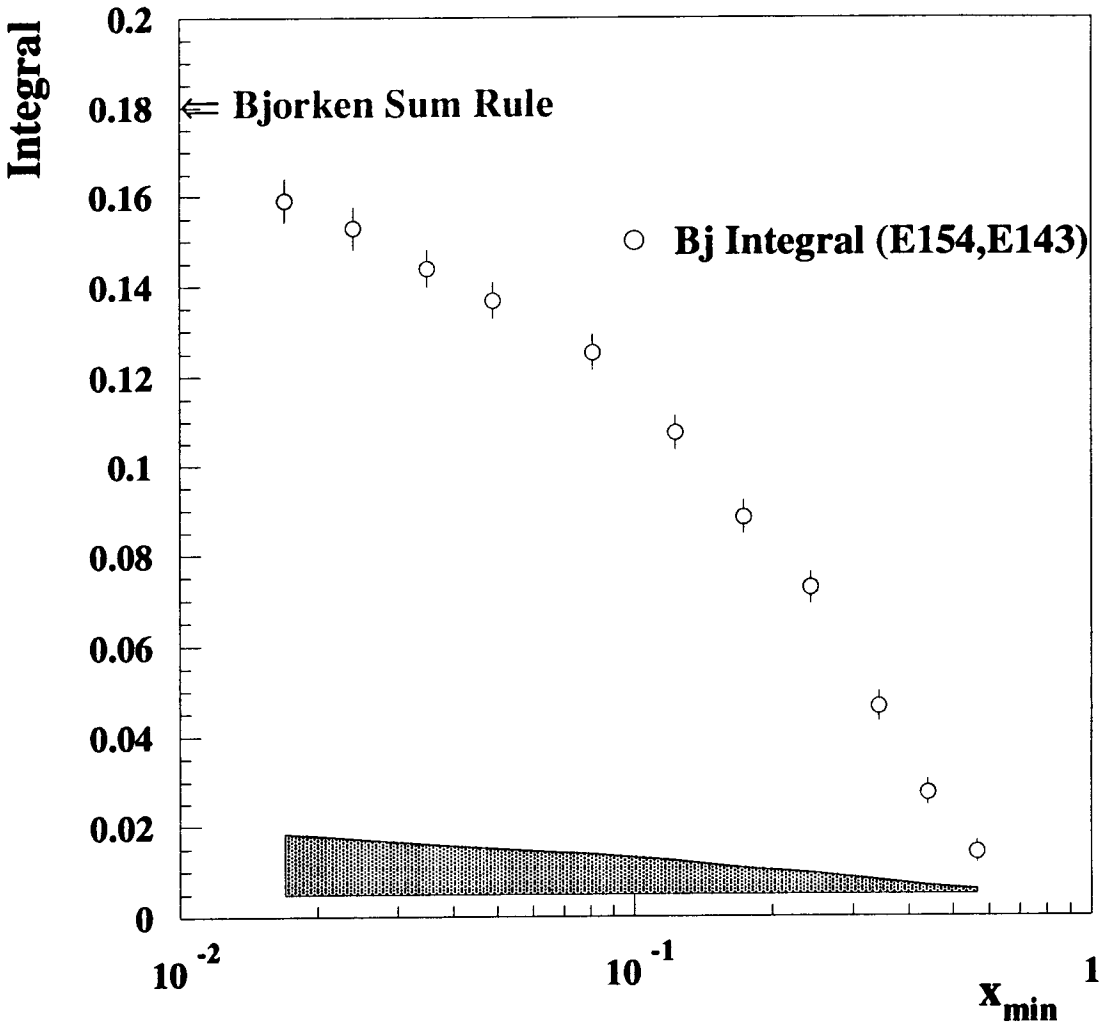


From Emlyn Hughes, E154, ICHEP'96

Regge Theory prediction: $g_1(x \to 0) \propto x^{-\alpha_1}$

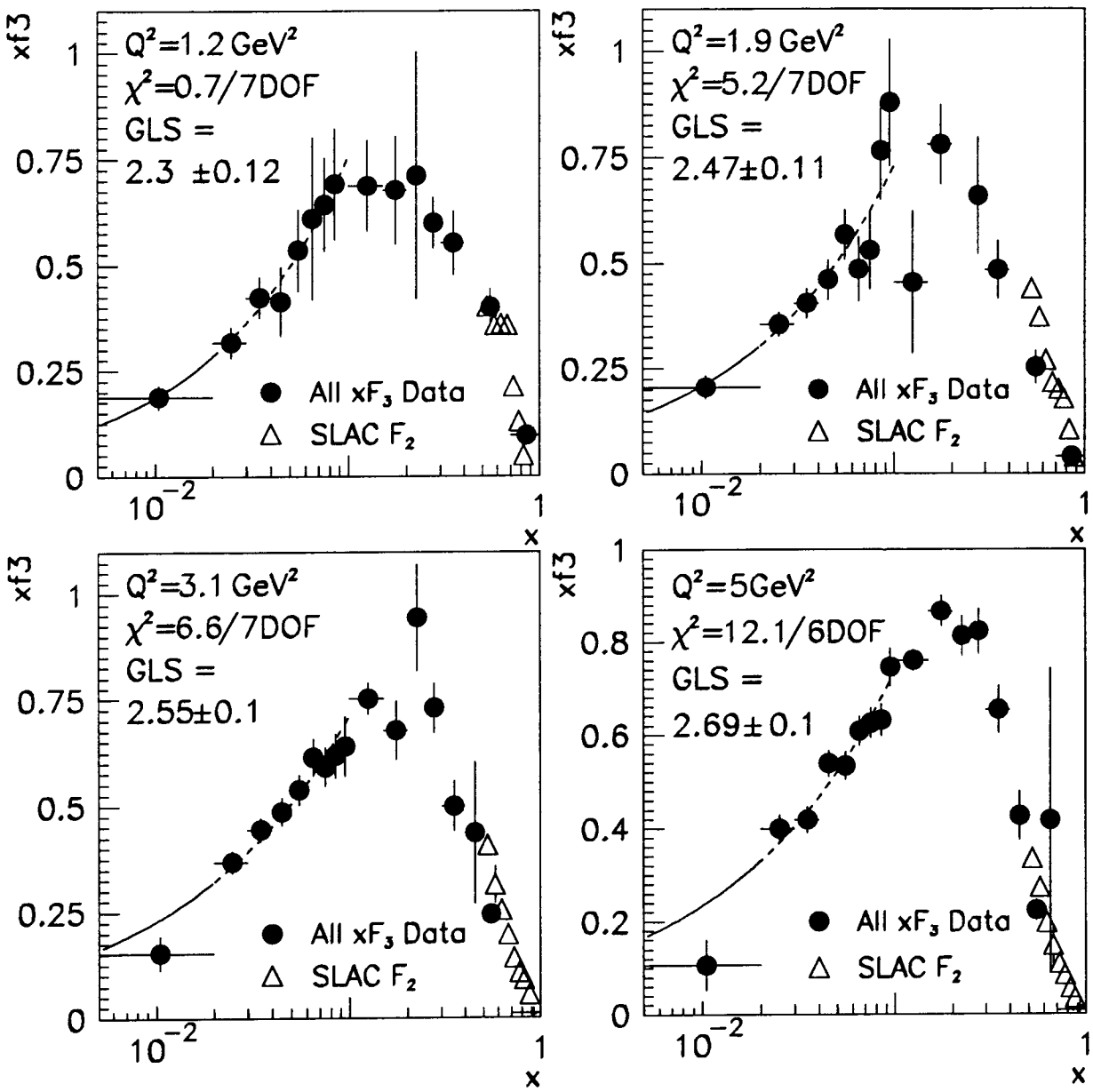
But have to require $-.5 < \alpha_1 < 0$ so integral converges. Use $\alpha_1 = 0$ up to $x_0 = 0.1$ from proton behavior. Old result: E142,E143 combined gave reasonable results, but new E154 data don't converge. $\to$ no new measurement, brings into question the previous ones.

Bjorken Sum Rule



From Emlyn Hughes, E154, ICHEP'96

What does it mean when the data doesn't fit a function that converges?
Have to rely on theory, or wait for more low x data.



From CCFR, Moriond'95–Preliminary

GLS integral–valence quarks only!

Quark counting arguments: low x behavior is $\propto x^{-.5}$

Data below $x = 0.1$ support this

Integrate up to $x = .02$ then the rest is real data.

Error Comparison for Sum Rules*

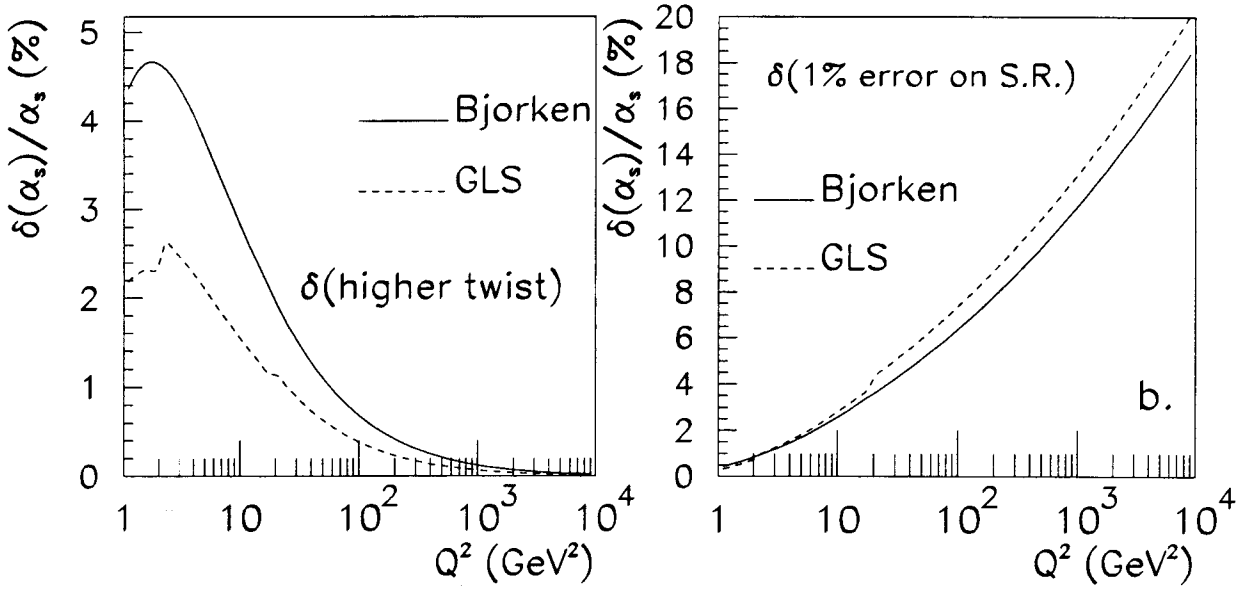
Error Source	$\delta\alpha_s(M_Z^2)$ GLS	Bjorken
Statistical	.004	$< .001$
Low x extrapolation	.002	.005†
Overall Normalization	.003	.002
Experimental Systematics	.004	.006
Higher Twist	.005	.003-.008**
Scale Dependence	.001	.002

* Errors either from Ellis & Karliner, CCFR, or E142 Result

** Range quoted for both Ellis & Karliner and E142

† From E142 result, E154 claims should be higher

In general, these errors are > Evolution Results. Experimental Errors: Detector acceptance at low x (GLS) and Polarization Uncertainties (Bj). For the future of this measurement: go to higher Q^2? Can we?



Sum Rule Summary:

GLS:

- CCFR $\alpha_s(M_Z^2) = 0.108 \pm^{.006}_{.009}$
- Badly needs to be updated with new CCFR Structure Functions!

Bjorken:

- E142& E143: $\alpha_s(M_Z^2) = .119^{+.013}_{-.018}$
- Ellis & Karliner $\alpha_s(M_Z^2) = .116^{+.004}_{-.006}$
- E154 Results: Can't Quote α_s results because of low x extrapolation issues.

Challenges here to Theorists and Experimentalists:

Higher Twist–refine predictions or just measure it!
Low x extrapolation–refine predictions or just measure it!

For experimentalists only: for higher Q^2 measurements at both high and low x, need ν data. $\rightarrow$ Will need better overall normalization
($\sigma^{\nu N}$, $\sigma^{\bar{\nu} N}$)

Short Term Future of these Measurements:

NuTeV–Now running here!

- Separate ν and $\overline{\nu}$ Beams
- Simultaneous Continuous Calibraion–expect to reduce largest systematic in evolution error by factor of 3!
- Statistics similar to CCFR
- Kinematics reach similar to or slightly extended to that of CCFR
- Goal: The most precise statistically limited measurement of α_s.

SLAC E155–Running this January

- $1 < Q^2 < 17 GeV^2$, $.15 < x < .85$
- Precision en, ep polarized structure function (g_1)
- Search for higher twist contributions
- Polarized Beam and Targets (NH_3 and ND_3).

COSMOS (and **CHORUS** & **NOMAD**)

- COSMOS to run in Main Injector Era at Fermilab
- CHORUS & NOMAD running now
- Kinematics Similar to SLAC electron experiments
- High x, and over a broad range of Q^2.

  Can go from Higher Twist to perturbative region.
- COSMOS–more statistics than CCFR + NuTeV

If I Ran the Zoo...
ν experiment

Go to higher Q^2, but still low or lower$x$. (GLS)
Tagged ν beam for lower normalization error (GLS)
Energy scale known to a tee

For fun–use a light target (not Iron) to understand nuclear effects
(not associated with α_s measurements per se)
F_2^ν versus F_2^ℓ at low x?
$xF_3^{\nu N}$ versus $xF_3^{\nu Fe}$?
Need oodles of ν's–muon collider?

ep or μp experiment:

Look at ℓ^+ AND ℓ^- and extract non-singlet structure functions for evolution measurements (at low x even) without gluon worries
Enormous luminosity needed, beyond HERA's wildest dreams

Conclusions

α_s in structure function evolution

- Energy Scale is a Killer
- NNLO Terms in DGLAP equations sorely needed!

α_s from Sum Rules

- Higher Twist
- Low x extrapolation
- Theory well understood

Future:

We can't just forge ahead with
higher statistics or higher energy–
Time to think about reducing systematics.
Still much to get out of the
currently accessible kinematic range!

α_s Results

xF_3 Only:

$$\alpha_s(M_Z^2) =$$
$$0.118 \pm 0.0025(\text{stat}) \pm 0.0055(\text{syst}) \pm 0.004(\text{scale})$$
$$\chi^2/DOF = 85/81$$

xF_3 and F_2:

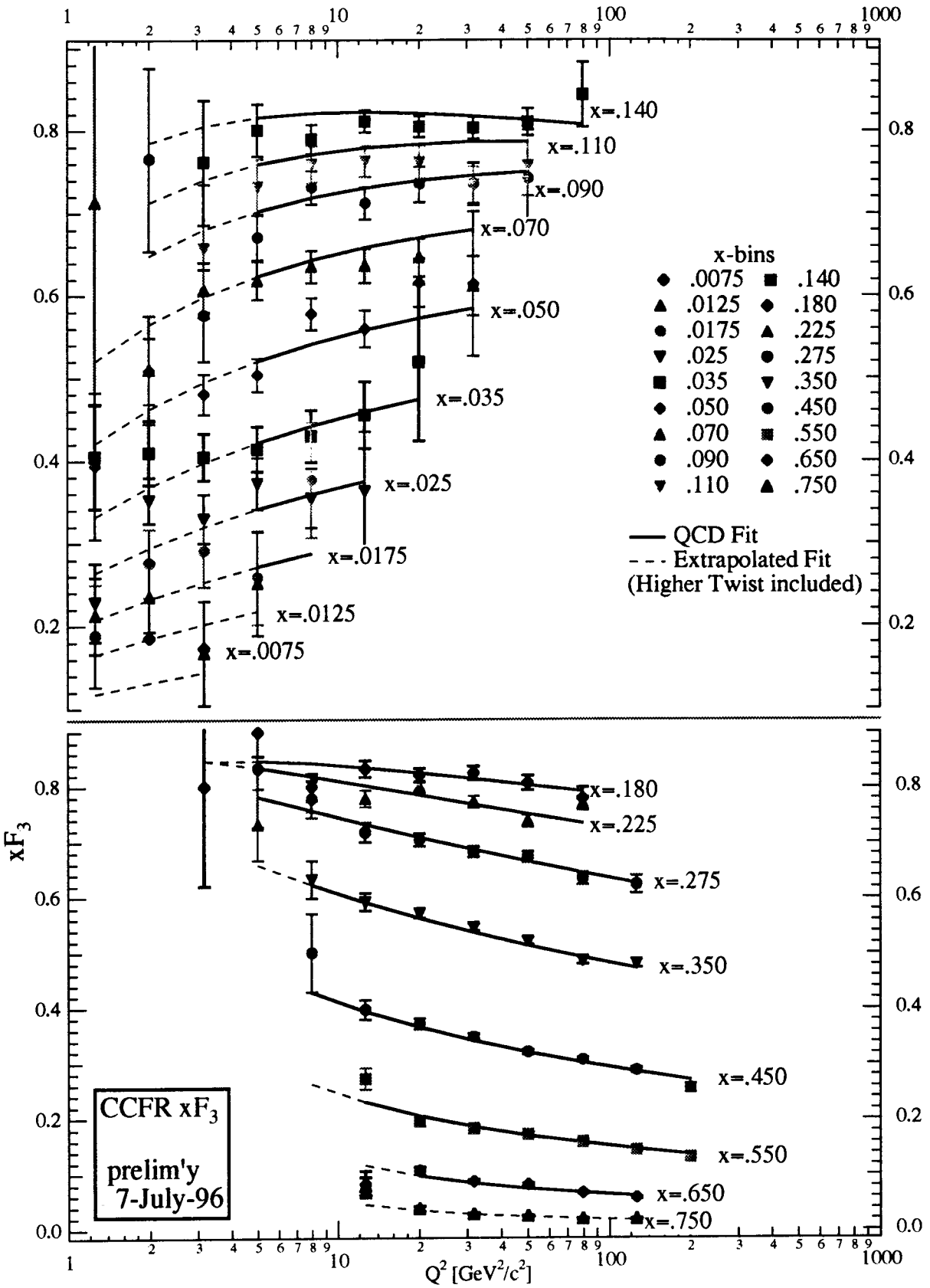
$$\alpha_s(M_Z^2) =$$
$$0.119 \pm 0.0015(\text{stat}) \pm 0.0035(\text{syst}) \pm 0.004(\text{scale})$$
$$\chi^2/DOF = 200/169$$

Compare with Old CCFR Result:

xF_3 and F_2:
$\alpha_s(M_Z^2) = .111 \pm 0.003(\text{exp}) \pm 0.004(\text{scale})$
$\chi^2/DOF = 61.6/56$, Ref: Quintas *et al* PRL71:1993

What changed? (In order of importance)

- 4 years of working on energy calibration
- Better Treatment of muon energy loss in detector
- Remove and correct for events with two muons
- Many other small effects, including:
  - Bardin Electroweak Radiative Corrections
  - Strange Sea from CCFR 2-muon data
  - Improved flux extraction technique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QCD Fit
Extrapolated Fit
(Higher Twist included)
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CCFR xF3
prelim'y
7-July-96
xF3
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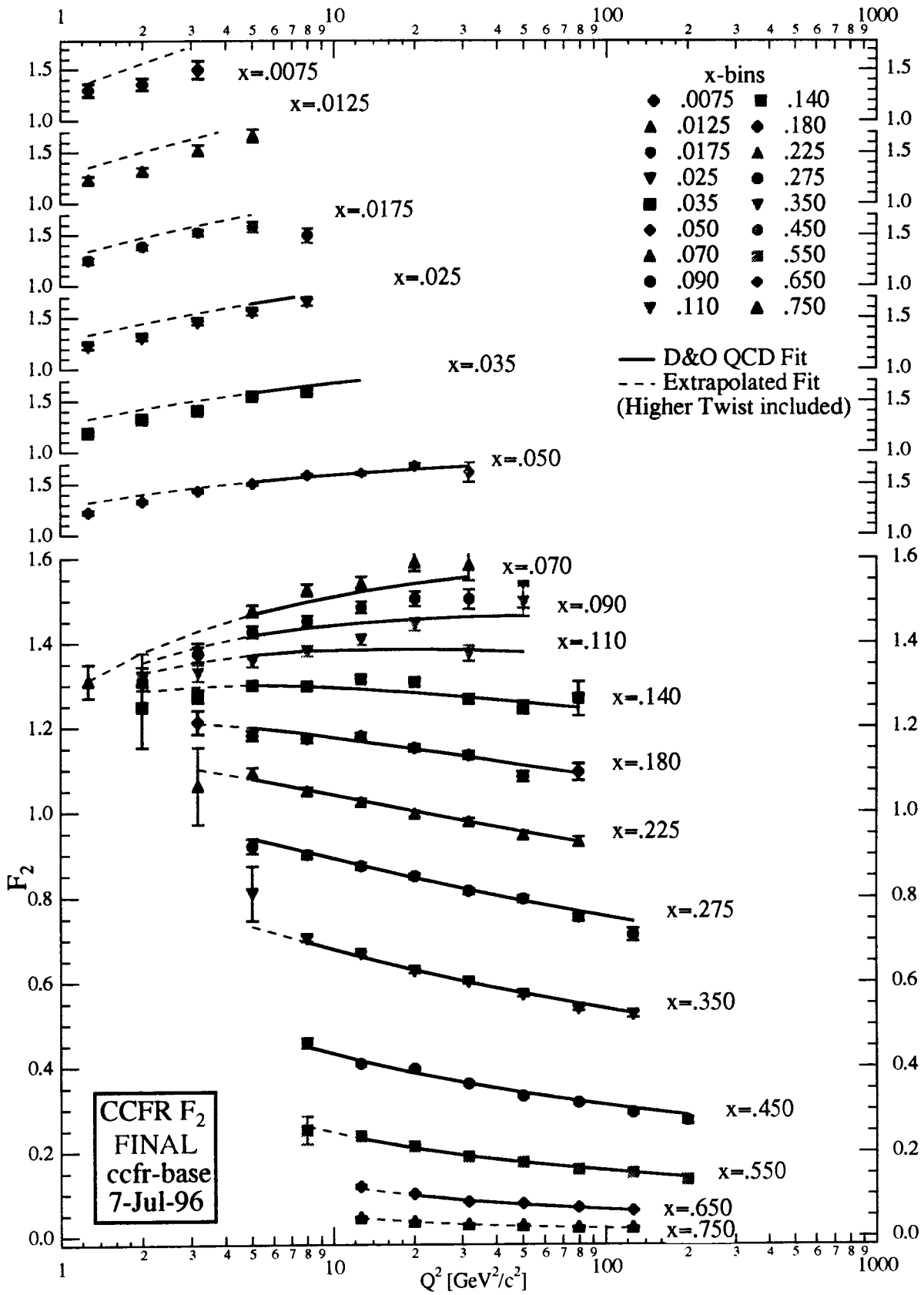
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