CAN LATTICE QCD AND PERTURBATIVE QCD MEET HALFWAY?

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CTEQ SYMPOSIUM
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TOPICS

SPECTRUM - ONIA
LIGHT HADRONS

+

PERTURBATION THEORY,
RESUMMATION,
N.P. SHORT DISTANCE

$\Rightarrow \alpha_s$ WHAT ARE THE UNCERTAINTIES? PROSPECTS?

MATRIX ELEMENTS WEAK
DIS ← WHY HARDER?

PREPRINTS:

SHIGEMITSU, LATTICE '96 HEP/LAT 9608058
α_s, ONIA

GÖCKELER, HORSLEY, +A? LATTICE '96 HEP/LAT 9608046
" " " BOCKNOW HEP/LAT 9610007
DIS

IN QCD: 2 COMMON REGULATORS

	LATTICE	DIMENSIONAL REGULATION
METHODS:		
PERTURBATION THEORY	TAKES WORK	"EASY"
NONPERTURBATIVE (MONTE CARLO)	✓	X
SHORT DISTANCE CALCULATIONS:		
$q\bar{q} \to t\bar{t}g$	TAKES WORK	EASY
TEST RESUMMATION ANSÄTZE	EASY	X
SEARCH FOR POWER CORRECTIONS	TAKES WORK	X
LONG DISTANCE CALCULATIONS		
SPECTRUM	"EASY"	X
D.I.S. MOMENTS	TAKE WORK	X (PHENOMENOLOGY
FRAGMENTATION FUNCTIONS	X	X (PHENOMENOLOGY

LATTICE QUARKS LIVE ON SITES

$$\partial_\mu \psi_x \to \frac{1}{2a}\left(\psi_{x+\hat{\mu}} - \psi_{x-\hat{\mu}}\right) - \frac{1}{12a^3}\left(\psi_{x+2\hat{\mu}} - 2\psi_{x+\hat{\mu}} + 2\psi_{x-\hat{\mu}} - \psi_{x-2\hat{\mu}}\right)$$

TREELEVEL: TAYLOR'S THEOREM

IN HIGHER ORDER: WILSON R.G.

∂_μ^3 CORRECTION

GAUGE FIELDS LIVE ON LINKS

$$U_{x\mu} = e^{iaT^a A^a_{x\mu}}$$

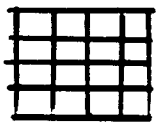
↑ SU(3) MATRIX

$$\mathcal{L} \to \square \sim UUUU \sim e^{iF_{\mu\nu}}$$

$$\sim \frac{1}{2}F_{\mu\nu}^2 + \frac{1}{24}(F^2)^2 +$$

SCHEMATIC OVERVIEW OF A MESON MASS CALCULATION

$t \to i t_{euc}$



MAKE ENSEMBLE OF GAUGE LATTICES,
$P(u) \propto e^{-S(u)}$

METROPOLS, ..

ON EACH LATTICE
MAKE Q, $\bar{Q}$ PROPAGATORS

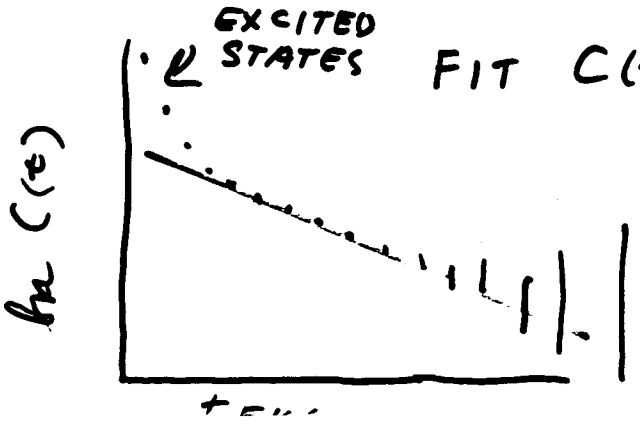
SOLVE $(\not{D} - m)G = \text{SOURCE}$

ON LATTICE, DIFF. OP. → (SPARCE) MATRIX
DIFF. EQ → SPARCE MATRIX EQ.
GAUSS-SEIDEL, MINIMUM RESIDUAL, ..

TIE SPINS+COLORS OF Q, $\bar{Q}$ TOGETHER

⇒ MESON CORRELATION FUNCTION

AVERAGE
FLUCTUATIONS → STATISTICAL ERRORS

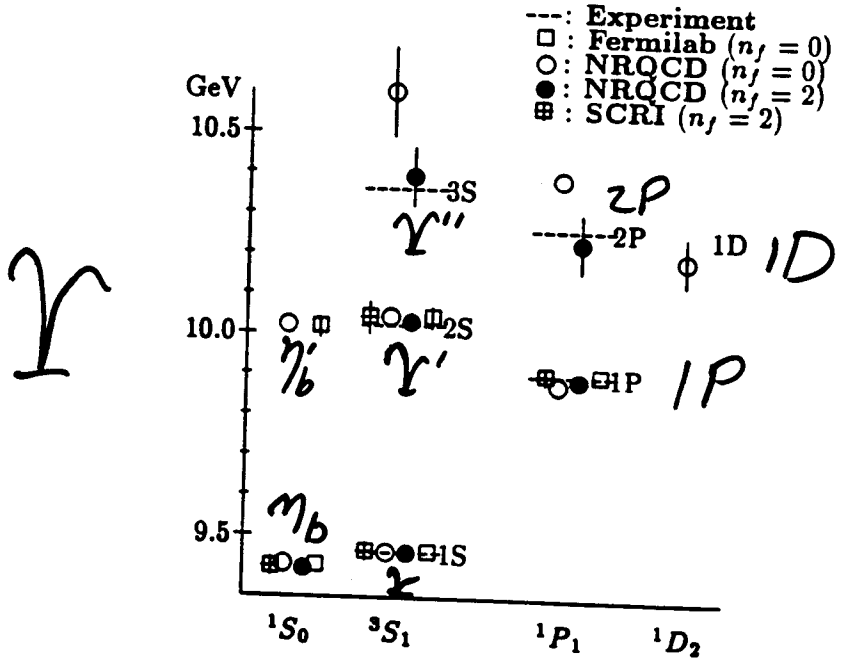


FIT $C(t) = Ae^{-mt} + \underbrace{A'e^{-m't}}_{\text{EXCITED STATES}} + \ldots$

A: HADRONIC AMPLITUDES → f_π, f_B, ..

SHIGEMITSU, LATTICE '96

QUARKONIA - EASIEST SPECTRUM CALCULATIONS



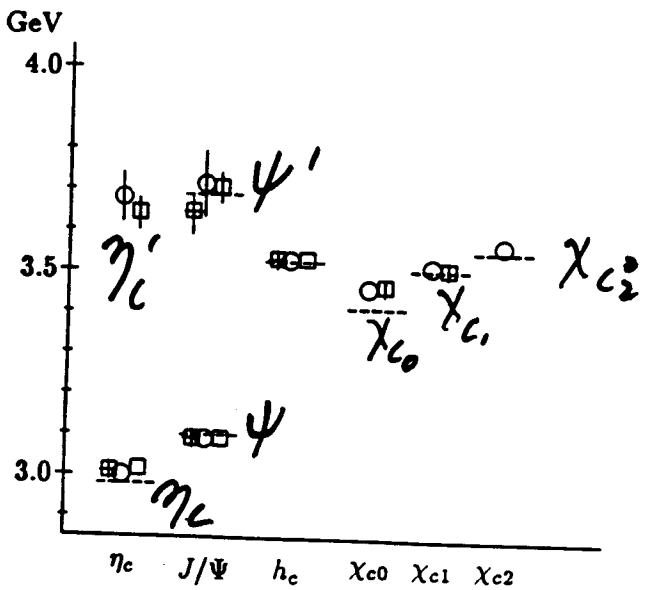
Υ

2P 1D 1P

- SMALL
- GOOD STATISTICS
- NON RELATIVISTIC METHODS

PREDICT ERRORS & CORRECTIONS IN ADVANCE

E.G. $\langle \Psi | C_c | \Psi \rangle$



ψ

LIGHT HADRONS - HARDEE

GÖCKELER +AL

BUCKNOW '96

- BIG $m_q \rightarrow m_{AWIS}$ CORRECT

χ_{PT}: $\Delta m_N \propto m_q^{3/2}$

- WORSE STATISTICS

SIGNAL ~ e^{-MT}

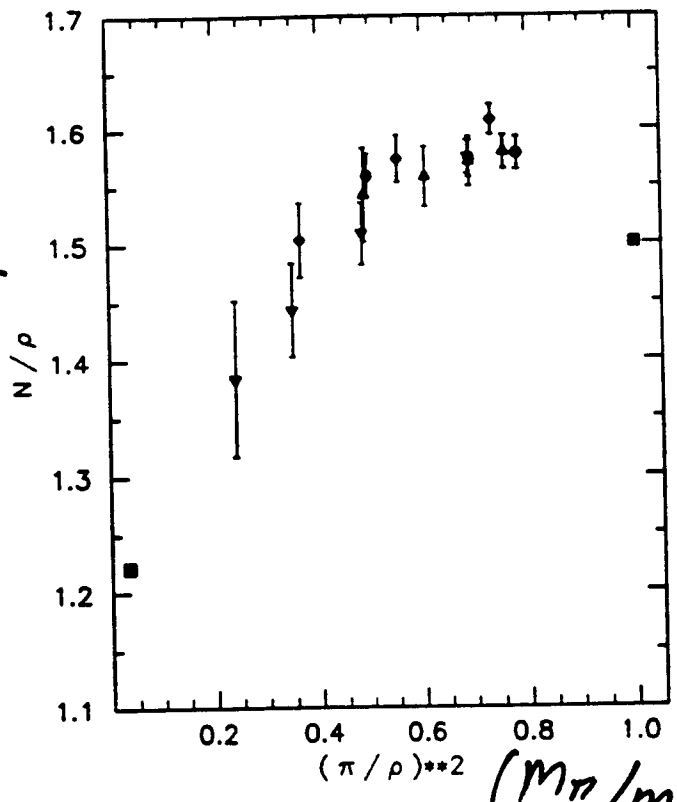
NOISE ~ $f(m_q..)$

- BIGGER VOLUMES

KEK: AT 2fm

$\Delta m_N \sim 5\%$

$\Delta m_\pi \sim 1\%$



m_N/m_ρ

$(m_\pi/m_\rho)^2 \propto m_q$

Figure 1. The APE plot m_N/m_ρ against $(m_\pi/m_\rho)^2$ for our results (filled circles, $\beta = 6.0, 16^3 \times 32$ lattice), [4]. For comparison we also show the results of [6], (triangles, $\beta = 6.0, 18^3 \times 32$ lattice), [7] (inverted triangles, $\beta = 6.0, 24^3 \times 32$ lattice), and [8] (diamonds, $\beta = 5.7, 16^3 \times 20$ lattice). The filled squares are the experimental result and heavy quark limit (1,3/2).

TO DETERMINE α_s OR m_q, V_{CKM}

1) DETERMINE BARE PARAMETERS

- FIX NONRENORMALIZABLE CORRECTION OPERATOR (IF INCLUDED) TO APPROACH CONTINUUM. (E.G., RECOVER ROTATIONAL INVARIENCE)
- FIX RENORMALIZABLE OPS. (α_s, m_q) TO REPRODUCE PHYSICS

2) GET RENORMALIZED PARAMETERS (E.G. $\overline{MS}$)

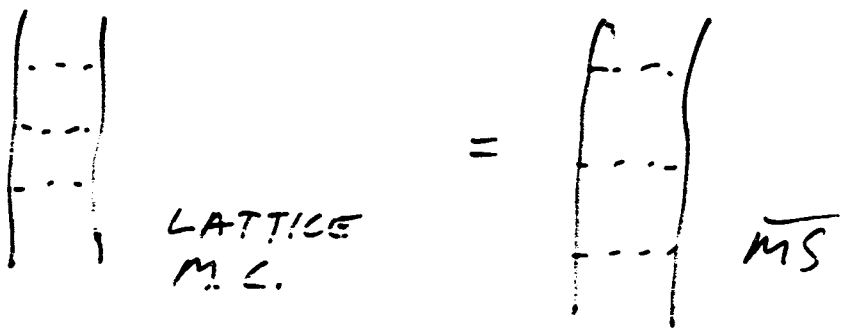
FROM SHORT DISTANCE LATTICE PHYSICS,

EITHER PERTURBATIVELY,

FROM α_0, m_0, ..

$$\mathcal{O}\Big|_{\substack{\text{LATTICE}\\ \text{P.T.}}} = \mathcal{O}\Big|_{\overline{MS}}$$

OR NON PERTURBATIVELY, WHEN POSSIBLE



WITH LATTICE, WE GET $\alpha_s\left(\frac{\#}{a}\right)$

NEED a^{-1} IN GeV.

FIT $\exp\left((ma)\frac{t}{a}\right)$.

EASIEST TO DO ACCURATELY:

SPIN AVERAGED -ONIA SPLITTINGS

E.G. $\Delta M = M_{h_c} - \frac{1}{4}\left(3M_\psi + M_{\eta_c}\right) = 458.6\,(4)\,M_e$

EXP. ERROR IN ΔM = EXP. ERROR IN Λ_{QCD}

$= 10^{-3}$.

$\Rightarrow$ EXP. ERROR IN $\alpha_s(M_Z) = 10^{-4}$.

CALCULATIONAL ERRORS IN ΔM $(+\Lambda_{QCD})$

STATISTICS

VOLUME

a

...

$< 10-20\%$

$\Rightarrow$ 1-2% IN $\alpha_s(M_Z)$

MOST SERIOUS UNCERTAINTIES: "PERT THEORY"

QUENCHED APPR.

LATTICE PERTURBATION THEORY

PUZZLES:

1) SHORT DISTANCE M.C. RESULTS > BARE 1 LOOP P.T. BY FACTOR OF 2

2) LARGE, POSITIVE, UNIFORM 2nd ORDER CORRECTIONS

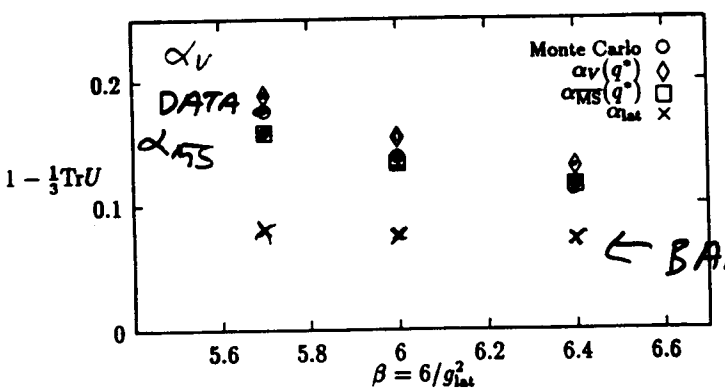
EXPLANATION:

- LIKE $\overline{MS}$ VS. MS
- BARE COUPLING MAKES REGULATOR LOOK SIMPL BUT TERRIBLE EXPANSION PARAMETER.

SOLUTION:

USE RENORMALIZED COUPLINGS

α_V, $\alpha_P = -\frac{3}{4\pi}\ln\langle \mathrm{l(P)} \rangle$, $\alpha_{\overline{MS}}$ ...



← BARE P.T.

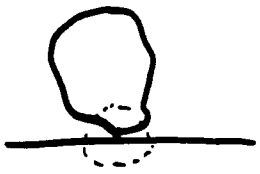
$1-\mathrm{Tr}\langle U_{xy}\rangle$
LANDAU GAUGE
(LIKE $\langle A^2_\mu \rangle$)

$\beta = 6/g_0^2$

LEPAGE
PM

SOURCE OF PATHOLOGY:

$$\mathcal{L} \propto \mathrm{Tr}\, U_P \sim \mathrm{Tr}\, e^{iF_{\mu\nu}} = 1 - \tfrac{1}{2}F_{\mu\nu}^2 + \tfrac{1}{24}F_{\mu\nu}^4 + \ldots$$



↖ $F_{\mu\nu}^4$ NEW VERTICES

$\Rightarrow$ TADPOLES

SUGGESTS RESUMMATION ANSATZ FOR HIGHER ORDERS

$$\alpha_{MS} = \frac{\alpha_{BARE}}{\underbrace{\langle \mathrm{Tr}\, U_P \rangle_{MC}}_{\text{PARISI}}} + \Sigma \alpha^2$$

SHORT DISTANCE M.C.
⇒ LAB FOR PQCD

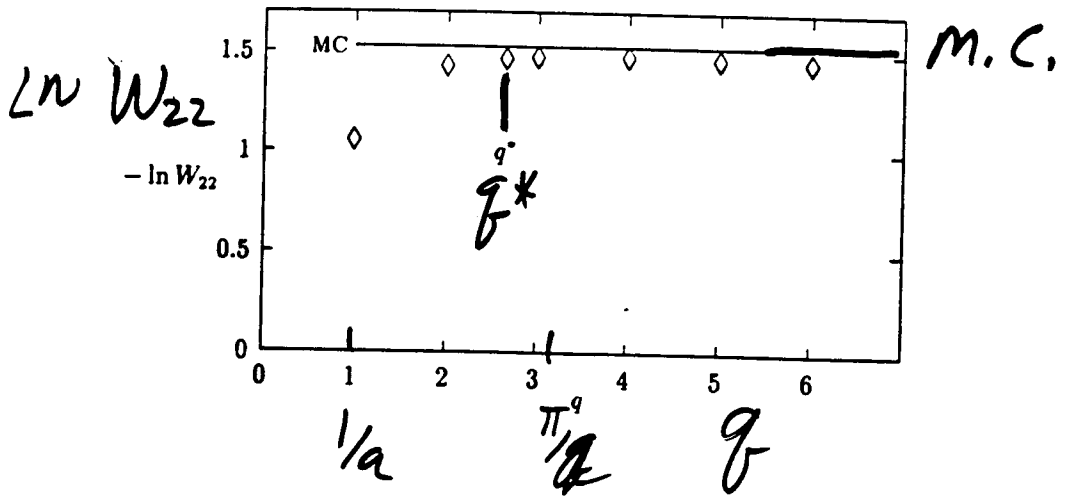
EXAMPLE I. TEST RESUMMATION ANSÄTZE

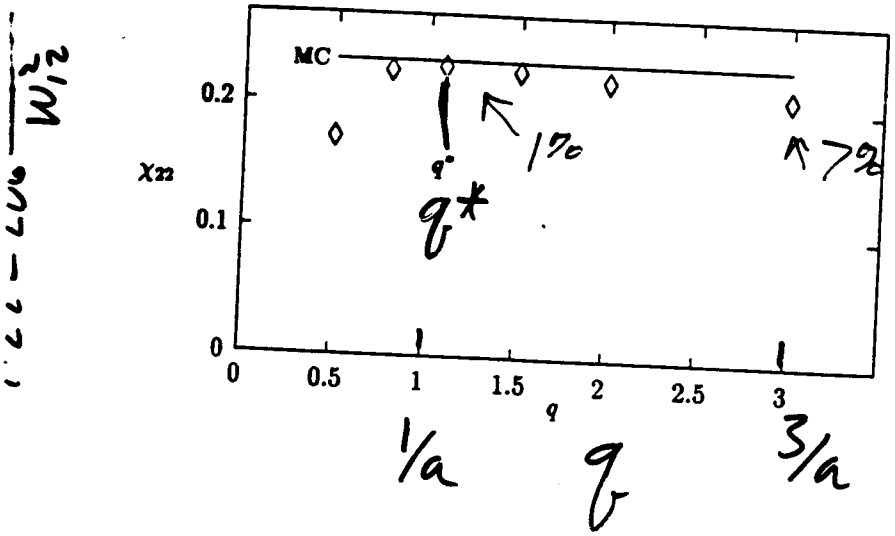
1) SCALE CHOICE

LESS THEOLOGICAL ON LATTICE — CAN DO ALL ORDERS TEST

DEFINE $\ln(q^*) \equiv \dfrac{\int d^4q\, f(q) \ln q}{\int d^4q\, f(q)}$

↑ LATTICE VERSION OF BLM





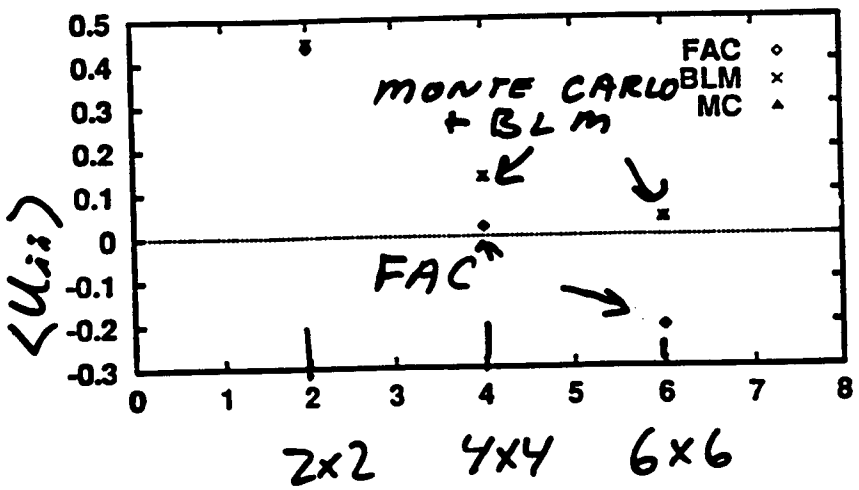
BLM RESUMS ONLY CORRECTIONS ASSOCIATED WITH U.V. LOGS + SCALE CHOICE.

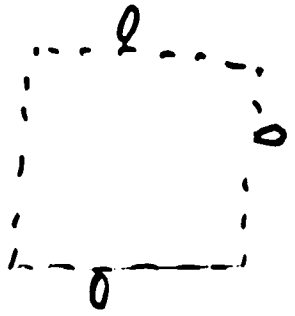
ANY OTHER LARGE CORRECTIONS LEFT ALONE.
(INDICATES MORE WORK IS REQUIRED.)

E.G. LARGE WILSON LOOPS NOT WELL-BEHAVED,
" " EXPONENTIALS OF GOOD SERIES.

E.G. $\langle U \rangle = \exp[-L\,m] = \exp[-L\,(\underbrace{c_1 g^2 + c_2 g^4 + \ldots}_{\text{WELL BEHAVED}})]$

BADLY BEHAVED





2) TADPOLE RESUMMATION

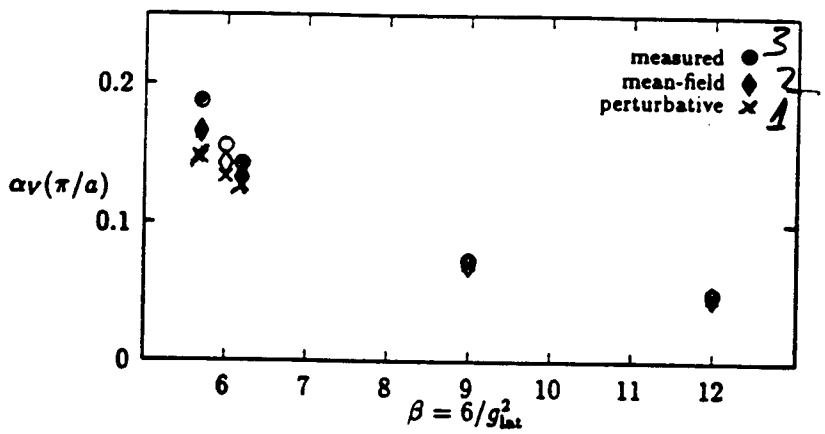
TO OBTAIN RENORMALIZED α_s

1) $\alpha_V = \alpha_0 + C\alpha^2$ ← BIG

2) $\alpha_V = \frac{\alpha_0}{\langle Tr U_p \rangle} + C'\alpha^2$ ← SMALL — TADPOLE IMPROVE

3) $-\frac{3}{4\pi} \ln \langle Tr U_p \rangle = \alpha_V + C''\alpha^2$ — NONPERTURBATIVELY CALCULATED

↑ OR ANY OTHER SMALL LOOP



UPSHOT FOR α_s EXTRACTION:

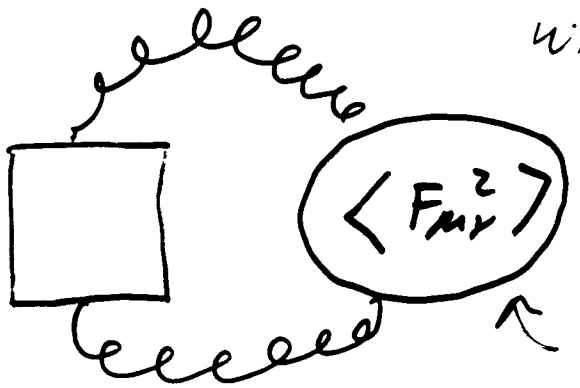
- FORGET BARE α_s
- PULL OUT RENORMALIZED α NONPERTURBATIVELY
  - HEAVY QUARK POTENTIAL
  - SMALL WILSON LOOPS
  - SCHRODINGER FUNCTIONAL — LUSCHER + AL

AGREE TO $\sim (\pm 1)\alpha^n$

EXAMPLE II. LOOK FOR SHORT DISTANCE POWER LAW EFFECTS

RENORMALONS WHATEVER

E.G. $\langle U_P \rangle$



FIT $\langle U_P \rangle_{MC} = 1 - \frac{1}{3} g_0^2 + c' g^4 + \ldots$

$+ C a^4 \langle F_{\mu\nu}^2 \rangle$

↑ MEASURE CONDENSATE

HARD SO FAR TO DISTINGUISH a^4 FROM α_s^P <...

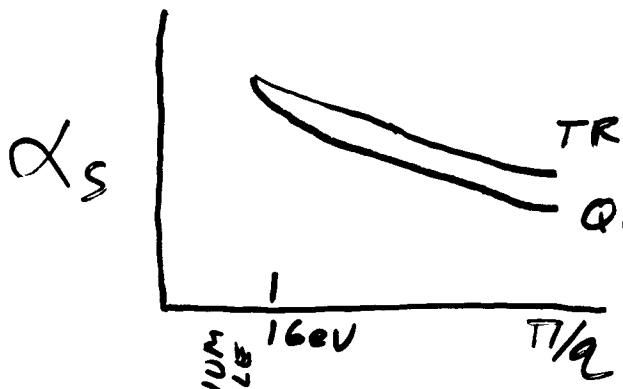
BUT

NONPERTURBATIVE SHORT DISTANCE CALCULATIC

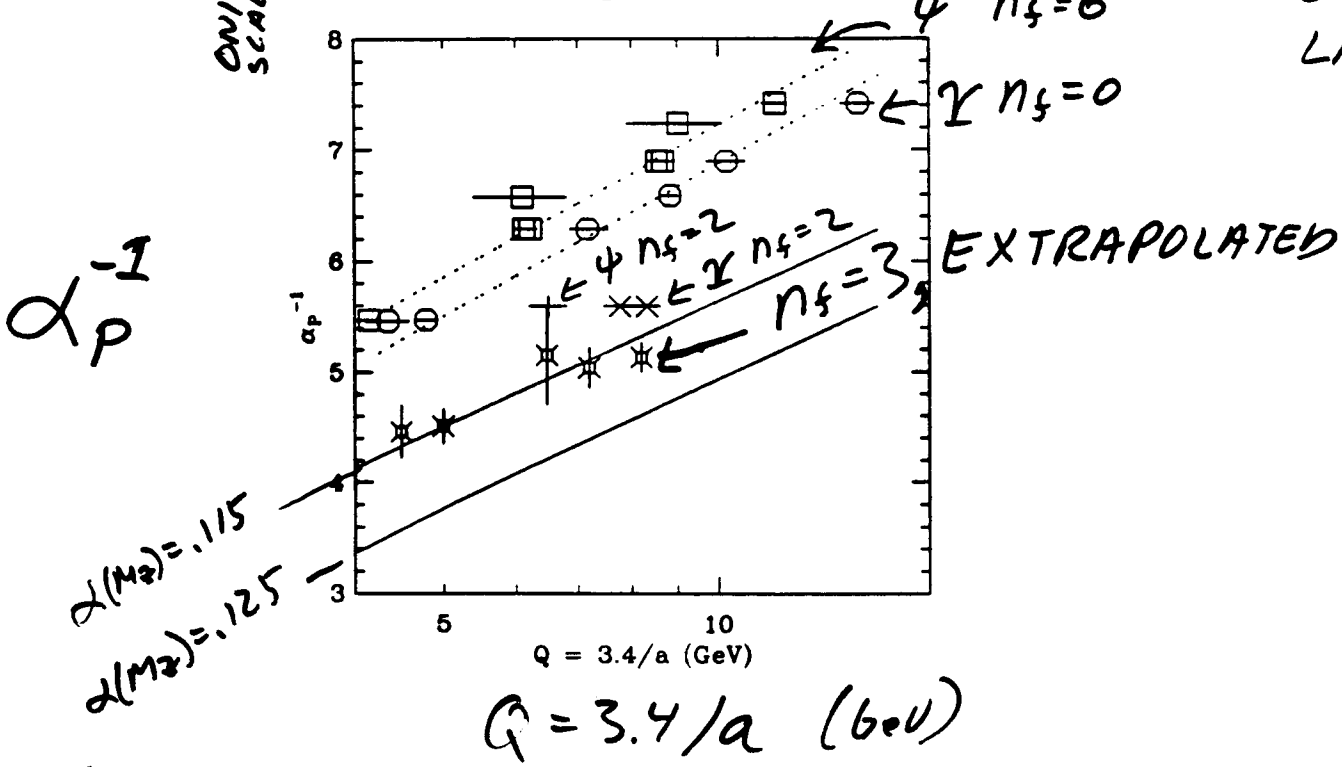
HOT AREA AT PRESENT

QUENCED APPROXIMATION + α_s

- LITTLE EFFECT ON ONIA SPECTRA, BUT
- α EVOLUTION TO SHORT DISTANCE TOO FAST



(HIGEMI, LATTICE)



$Q = 3.4/a$ (GeV)

HIGEMITSU SUMMARY

NRQCD $n_f=0$ / $n_f=2$

$$\alpha_s(M_Z) = \begin{cases} 0.1175\,(11)(13)(19) & \text{S-P} \\ 0.1180\,(14)(14)(19) & \text{1S-2S} \end{cases}$$

- (19): P.T. $\sim\alpha^3$
- (14): n_f EXTRAPOLATION, m_q DYNAMICAL
- (14): EVERYTHING ELSE — STATISTICS, DISCRETIZATION, RELATIVISTIC CORRECTIONS
- (13): P.T. $\sim\alpha^3$

FNAL $n_f=0$
SCRI $n_f=2$

$$\alpha_s(M_Z) = 0.1159\,(19)(13)(19) \quad \text{S-P } \Upsilon$$

AIMING FOR TOTAL ERROR OF 1%,
0.5% FROM EACH SOURCE, IS FEASIBLE

REQUIRES:

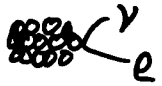
- 3rd ORDER P.T. — IN PROGRESS FROM LUESCH + A.L
  TESTS OF MORE WAYS TO EXTRACT α
- TESTS WITH 3 LIGHT FLAVORS, m_q DEPENDENCE.

WEAK MATRIX ELEMENTS

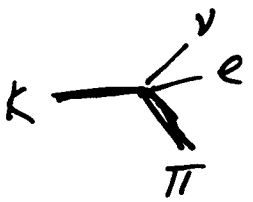
BLACK,
- LATTICE NO HELP
- COLORED,
  LATTICE HELPS

CKM MATRIX

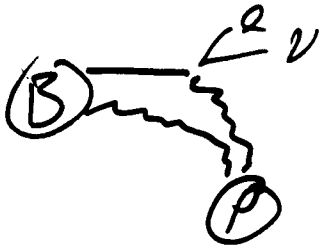
$|V_{ud}|$
0.974
B DECAY



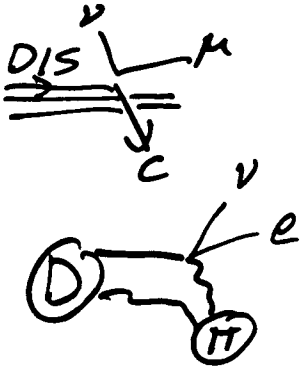
$|V_{us}|$
0.220



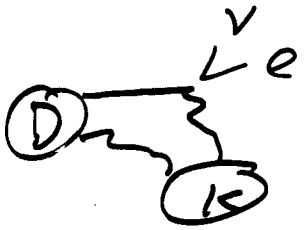
$|V_{ub}|$
0.002 - 0.007



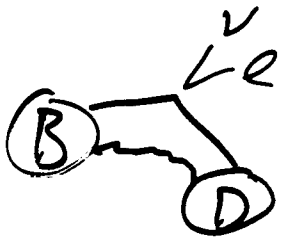
$|V_{cd}|$
0.179 - 0.228



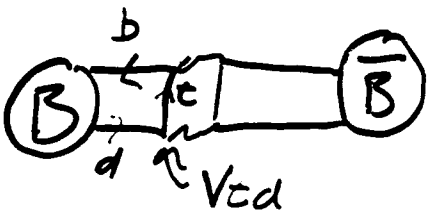
$|V_{cs}|$
0.864 - 0.975



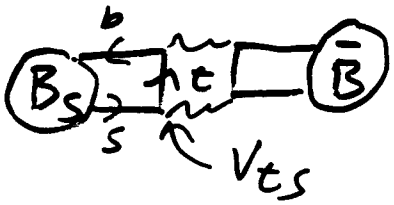
$|V_{cb}|$
0.032 - 0.054



$|V_{td}|$
0.0 - 0.14



$|V_{ts}|$
0.0 - 0.45



$|V_{tb}|$
0.0 - 0.9995

B_K $\langle K|\bar{K}\rangle$ (CP)

$\varepsilon \rightarrow$ CKM

$\varepsilon = B_K \times f[\text{CKM ANGLES}]$

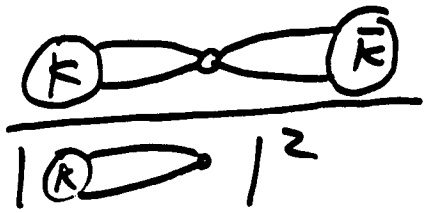
EASIER THAN SOME LESS INTERESTING CALCULATIONS

- USE $m_q = m_s/2$

  EXTRAPOLATE IN $m_q - m_d$
  3% INSTEAD OF ~20%

- PSEUDOSCALARS
  - GOOD STATISTICS,
    VOLUME ERRORS

- $B_K = \dfrac{\langle \bar{K}|O|K\rangle}{\frac{8}{3} m_K^2 f_K^2}$



RATIO OF 2 SIMILAR AMPLITUDES
SOME ERRORS CANCEL
- QUENCHED
- P.T

STAGGERED FERMIONS + χ SYM. HELP

WILSON

χ SYM BROKEN BY $\bar{\Psi} D^2 \Psi$

O_{LL} MIX WITH O_{LR} ETC

AMESS TO DISENTANGLE

PECULIAR FEATURE—
a DEPENDENCE CALCULATED AS a^2
NOT MUCH SIGN IN NUMERICAL RESULTS.

JLQCD

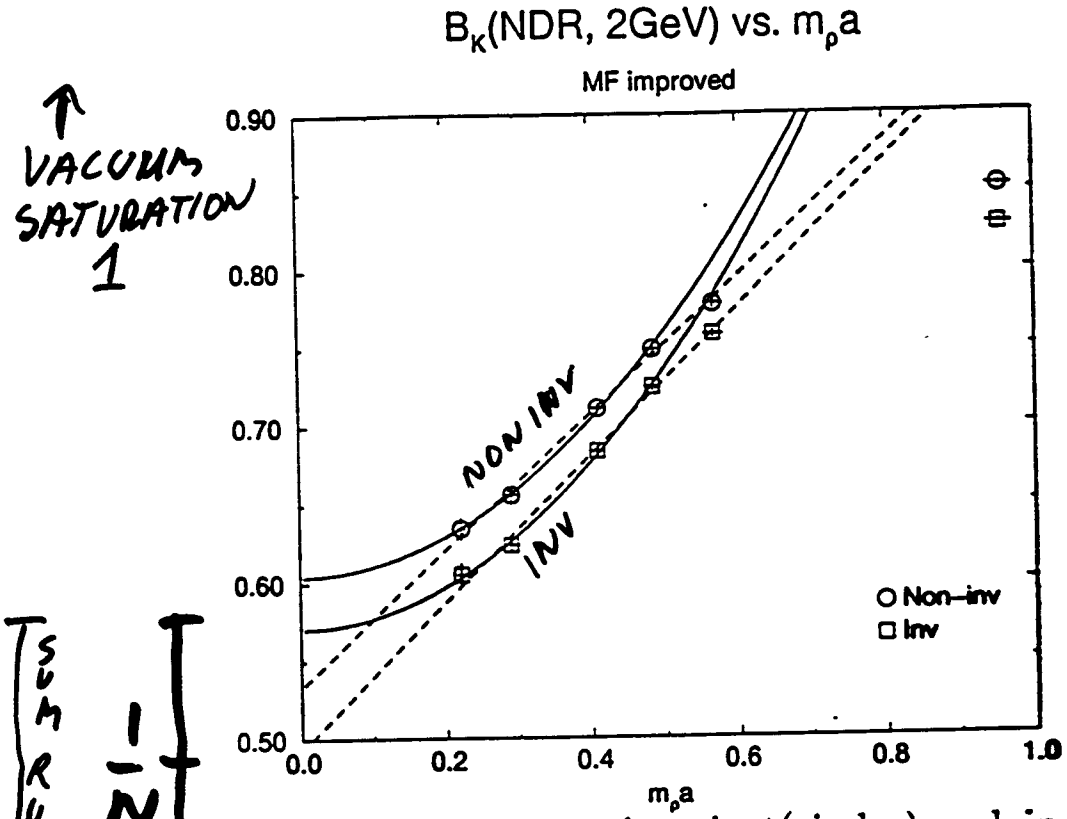


Figure 1. Gauge non-invariant(circles) and invariant(squares) B_K(NDR, 2GeV) as a function of $m_\rho a$, together with a quadratic fit to four data points on the left (solid lines) and a linear fit to five points (dashed lines).

IMPROVED WEAK OPERATORS
LARGE + COMPLICATED
—FUNNY P DEPENDENCE?

DIS MOMENTS HARDER THAN THESE
(BUT NOT IMPOSSIBLE)

→ PROTONS
- WORSE STATISTICS
- WORSE CHIRAL BEHAVIOR
- FATTER

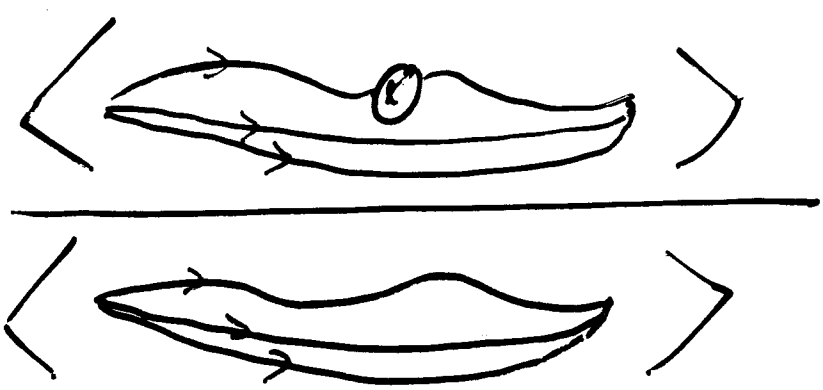
→ DIS OPERATORS

$$\bar{\psi} \gamma_\mu D_\nu D_\lambda \ldots \psi$$

DERIVATIVES SUBTRACTIONS
→ WORSE STATISTICS

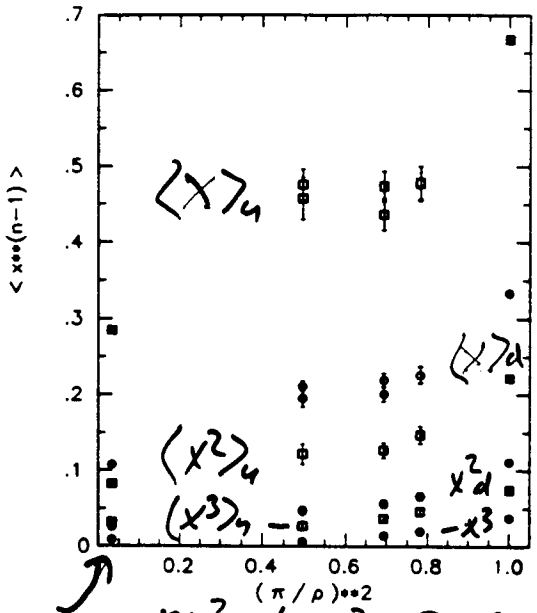
HORRIBLE TO NORMALIZE WHEN
$n > 4$, +TRACES ARE REQUIRED

MANY DERIVATIVES
→ NON LOCAL
→ WORSE DISCRETIZATION ERRORS



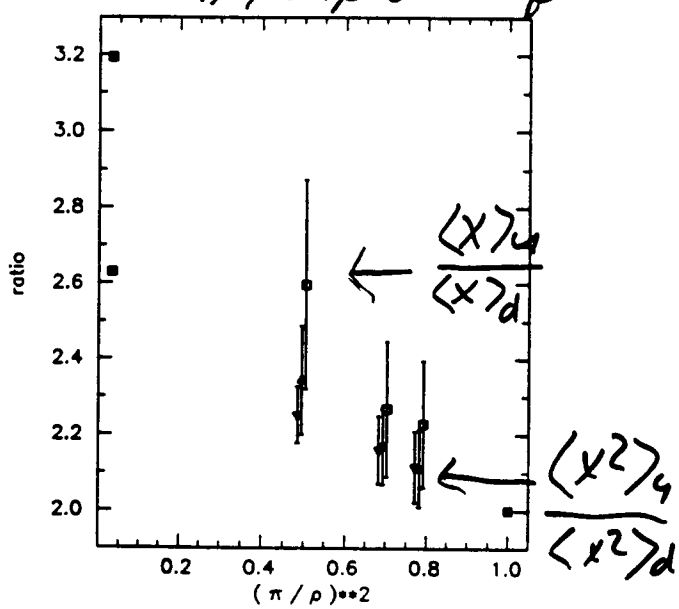
GOCKELER ET AL
BUCKNOW 96
ERRORS
QUENCHED

$\langle x^{n-1} \rangle_q$

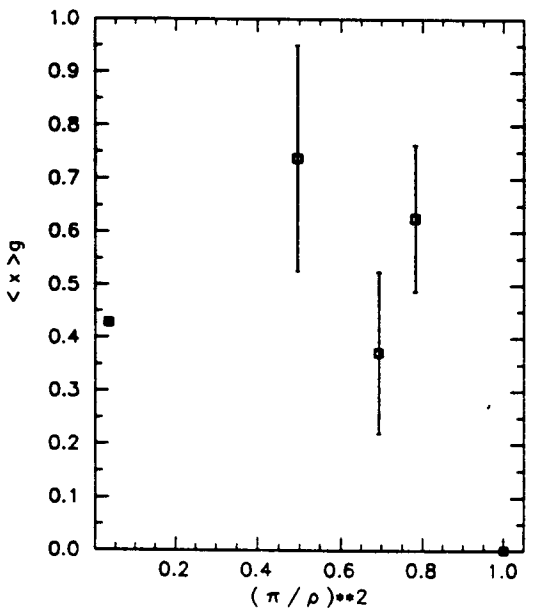


DATA (MRS)

$m_\pi^2 / m_\rho^2 \propto m_q$



$\langle x \rangle_q$



$m_q \to m_{phys}$ EXTRAPOLATION

	LATTICE (GOCKELER ET AL)		EXPERIMENT (MRS)	
	u	d	u	d
$\langle x \rangle$	0.369(26)	0.169 (13)	0.284	0.102
$\langle x^2 \rangle$	0.108 (16)	0.036 (8)	0.083	0.025
$\langle x^3 \rangle$	0.020 (10)	-0.001 (6)	0.032	0.008

- STATISTICS ONLY
- MANY OTHER ERRORS, NOT WELL ANALIZED

NOTE IMPRESSIVE EXPERIMENTAL ACCURACY

NEEDED:

STATEMENT OF UNCERTAINTIES IN EXP. MOMENT

HOW GOOD MUST LATTICE BE TO CONTRIBUTE?

FRAGMENTATION FUNCTIONS

NO RESULTS - PROBLEM OF PRINCIPE

NEED - FORMULATION WITH EUCLIDEAN M.E. (IMPOSSIBLE)

OR - NEW, LESS EUCLIDEAN LATTICE METHOD (LIGHT CONE QUANT.)

EXCLUSIVE PROCESS M.E.'S

FEW RESULTS, BUT SHOULD BE AS "EASY" AS DIS

LESS KNOWN FROM EXP.

CAN LQCD + PQCD MEET HALF WAY?

α_s ALREADY HAVE
CURRENT ERRORS - FEW %
ERROR < 1% - PAINFUL BUT STRAIGHTFORWARD

NONPERTURBATIVE SHORT DISTANCE CALCULATIONS
TEST RESUMMATIONS ✓
SEARCH FOR POWER EFFECTS HOPEFUL
RENORMALONS ...

STRUCTURE FUNCTIONS
- FOR $n \leq 4$,
DECENT ERRORS SHOULD BE POSSIBLE
5 YEARS?

- WE NEED AN UNCERTAINTY ANALYSIS
FROM YOU, TOO.