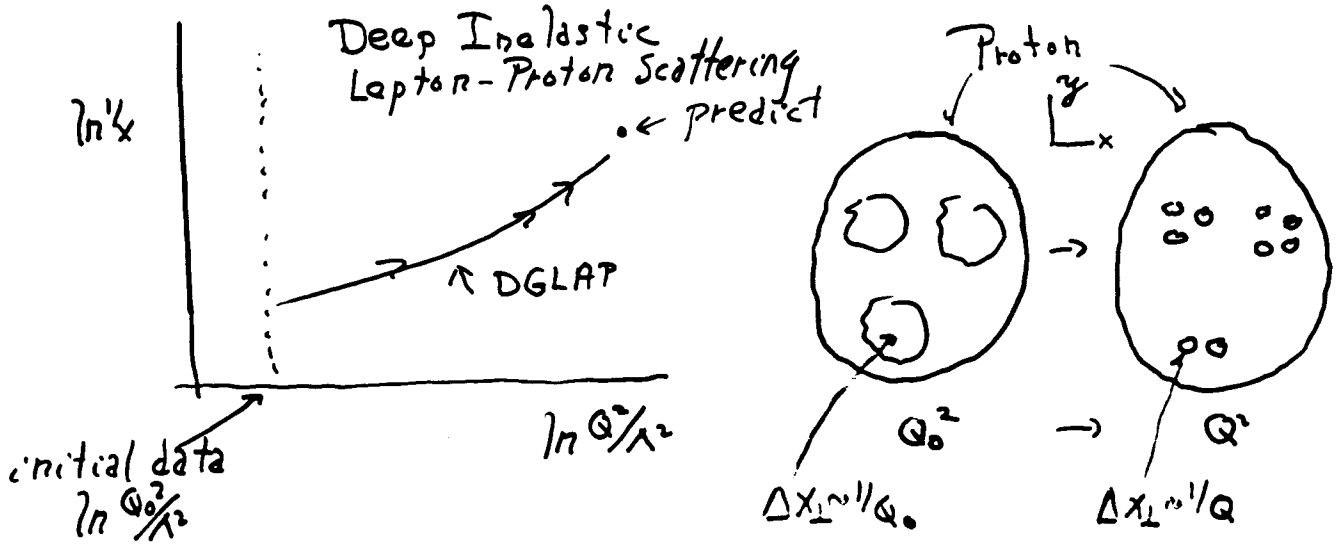
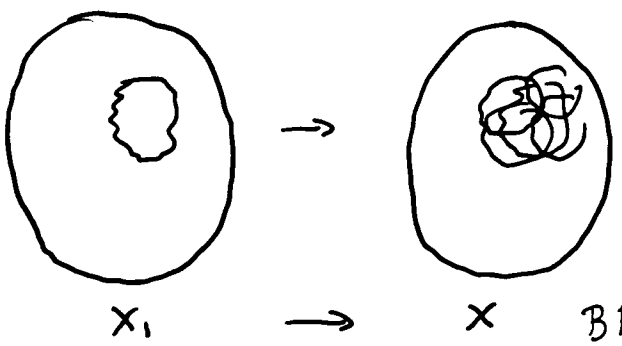
MUELLER

1. DGLAP and BFKL evolution (general comments)

DGLAP evolution is for a <u>two</u> scale problem.



BFKL evolution (in pure form) is for <u>one</u> scale problem



evolution toward lower momentum-fraction partons, on one transverse scale

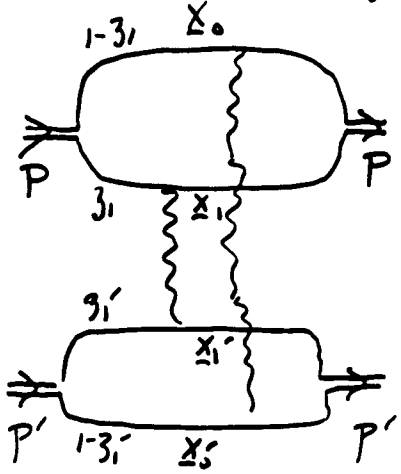
BFKL evolves toward dense parton configurations

In general DGLAP description of small-x deep inelastic scattering BFKL effects come in through high order terms in anomalous dimension and coefficient functions. (See later)

2. BFKL evolution

Pure case: Heavy onium-Heavy onium scattering

single scale is onium radius $R \ll 1/\Lambda$.

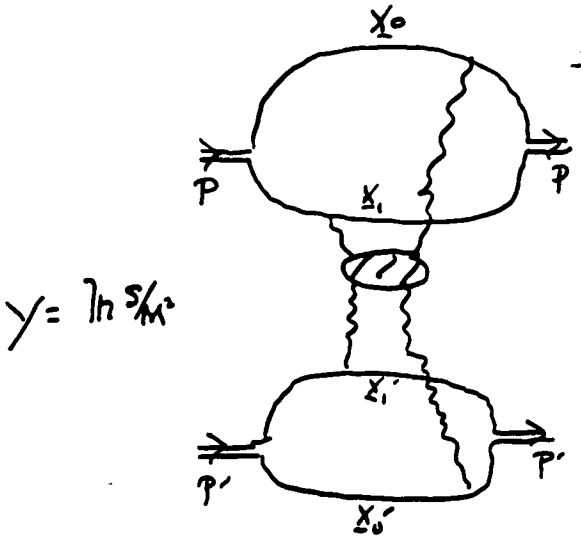


Lowest order

$$\sigma = \int d^2x_{01} \int_0^1 d\beta_1 \, \Phi(x_{01},\beta_1) \int d^2x'_{01} \int_0^1 d\beta'_1 \, \Phi(x'_{01},\beta'_1) \, \sigma^{(0)}_{dipole}$$

$$\sigma_{dipole} = 2\pi\alpha^2 x_c^2 \left(1 + \ln x_3/x_c\right)$$

$$\sigma = c\pi\alpha^2 R^2$$



Leading logs

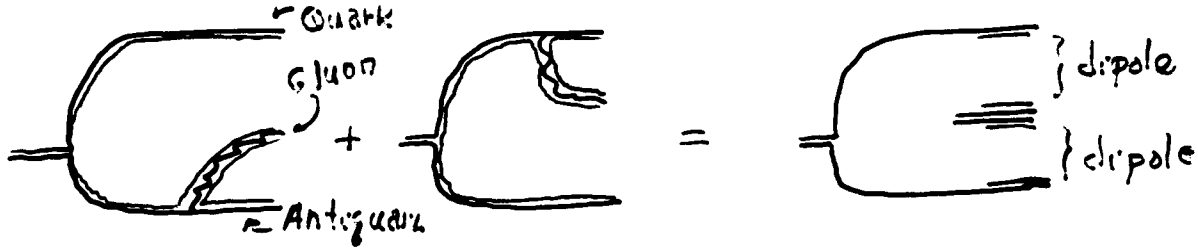
$$\sigma^{BFKL}_{dipole} = 4\pi\alpha^2 \frac{e^{(\alpha_P - 1)y}}{\sqrt{7\zeta(3) N_c \alpha y}} x_{01} x'_{01}$$

$$\sigma = \frac{16\pi\alpha^2 e^{(\alpha_P - 1)y}}{\sqrt{7\zeta(3) N_c \alpha y}} \qquad \alpha_P - 1 = (4\ln 2)\frac{\alpha N_c}{\pi}$$

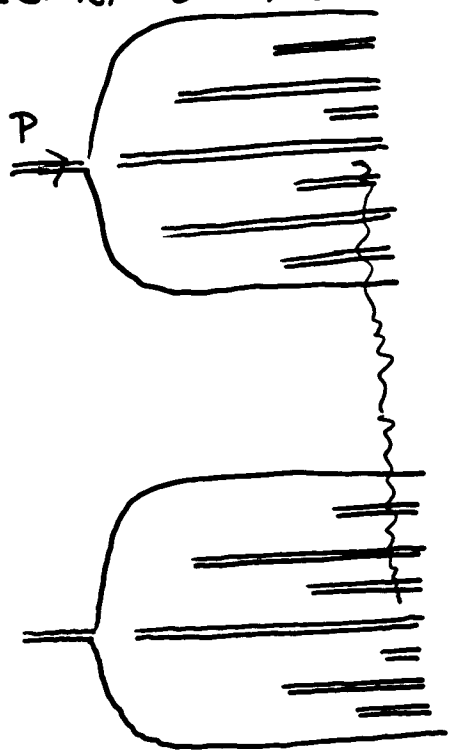
When $y \gtrsim \frac{1}{\alpha_P - 1} \ln 1/\alpha^2$ unitarity corrections will become important

Dipole picture (Large N_c QCD)

Wave Function looks like collection of dipoles



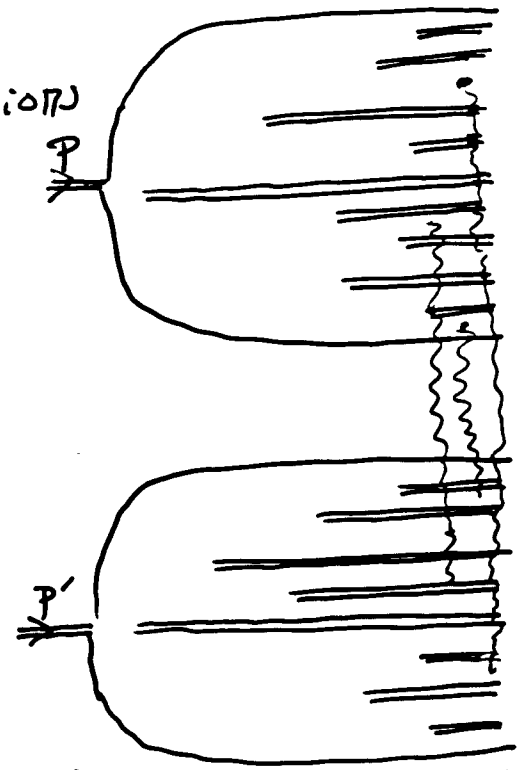
In center of mass



BFKL (one gluon exchange in amplitude)

Unitarity corrections

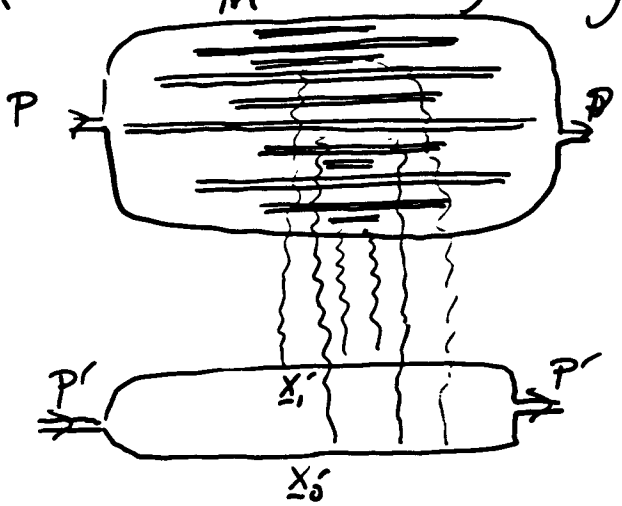
CM



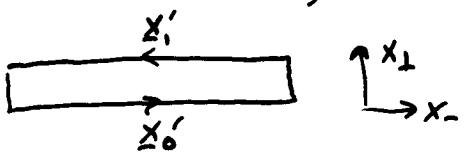
Numerical calculations of unitarity have been done. Salam

Unitarity corrections

$P/M \gg 1$ P'/M not very large



$$= \langle P | e^{ig\int_C A_\mu(x)\,dx_\mu} | P \rangle_{\text{average in } P}$$



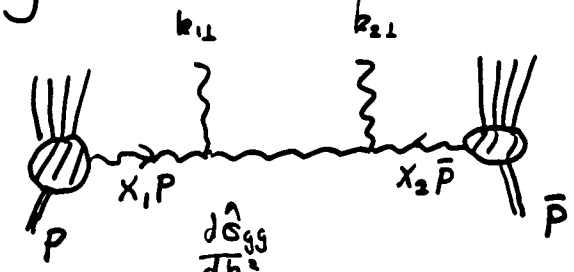
Unitarity occurs when $A_\mu \sim 1/g$. Strong field configurations. BFKL leads to $A \gg 1/g$

3. Measuring the BFKL Pomeron at Fermilab and HERA

A. Fermilab inclusive

Look at two jets inclusively (2-jet correlations) with jets widely separated in rapidity.

leading order



$\int d\phi$ ← az. angle between jets

$$\frac{x_1 x_2\, d\sigma}{dx_1 dx_2\, dk_{1\perp}^2\, dk_{2\perp}^2} = \left(\frac{3\alpha}{\pi}\right)^2 \frac{\pi^3}{2(k_{1\perp}^2 k_{2\perp}^2)}\, x_1\left[G(x_1,k_{1\perp}^2) + \tfrac{4}{9}\left(q(x_1,k_{1\perp}^2)+\bar{q}(x_1,k_{1\perp}^2)\right)\right]\cdot$$

$$\cdot\, x_2\left[G(x_2,k_{2\perp}^2) + \tfrac{4}{9}\left(q(x_2,k_{2\perp}^2)+\bar{q}(x_2,k_{2\perp}^2)\right)\right]\cdot\ \delta(k_{1\perp}^2 - k_{2\perp}^2)$$

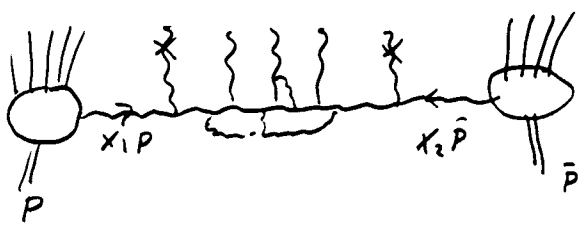
Leading order Formalism (2nd order Formalism complete and used.)

leading logs in αy. ($y = \ln \frac{x_1 x_2 s}{4 k_\perp^2}$)

$$\delta(k_{1\perp}^2 - k_{2\perp}^2) \to \frac{1/4}{\sqrt{k_{1\perp}^2 k_{2\perp}^2}}\ \frac{e^{(d_p-1)y}}{\sqrt{7/2\,\alpha C_A\, 3(3)\, y}}$$

$k_{1\perp}^2/k_{2\perp}^2$ not too far from 1

$\alpha_P = 1 + 12\frac{\alpha}{\pi}\ln 2 \approx \frac{8}{3}\alpha(k_\perp^2) +$ ← BFKL intercept



Take ratio of 1800 and 630 GeV data keeping x_1, x_2 and $k_\perp$ cuts fixed. DØ analysis under way

$$\frac{\sigma_{2jet}^{1800}}{\sigma_{2jet}^{630}} = \mathcal{R}^{(\alpha_p-1)\ln\left(\frac{1800}{630}\right)^2} \sqrt{\frac{Y(630)}{Y(1800)}}$$

gives estimate/measurement of $\alpha_p - 1$!

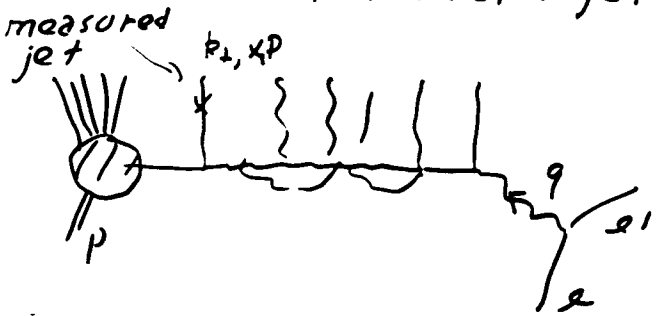
If $Y(630) \approx 4$ then $Y(1800) \approx 6$

$$\alpha_p - 1 = 12\frac{\alpha}{\pi}\ln 2 + O(\alpha^2) \approx 1/2$$

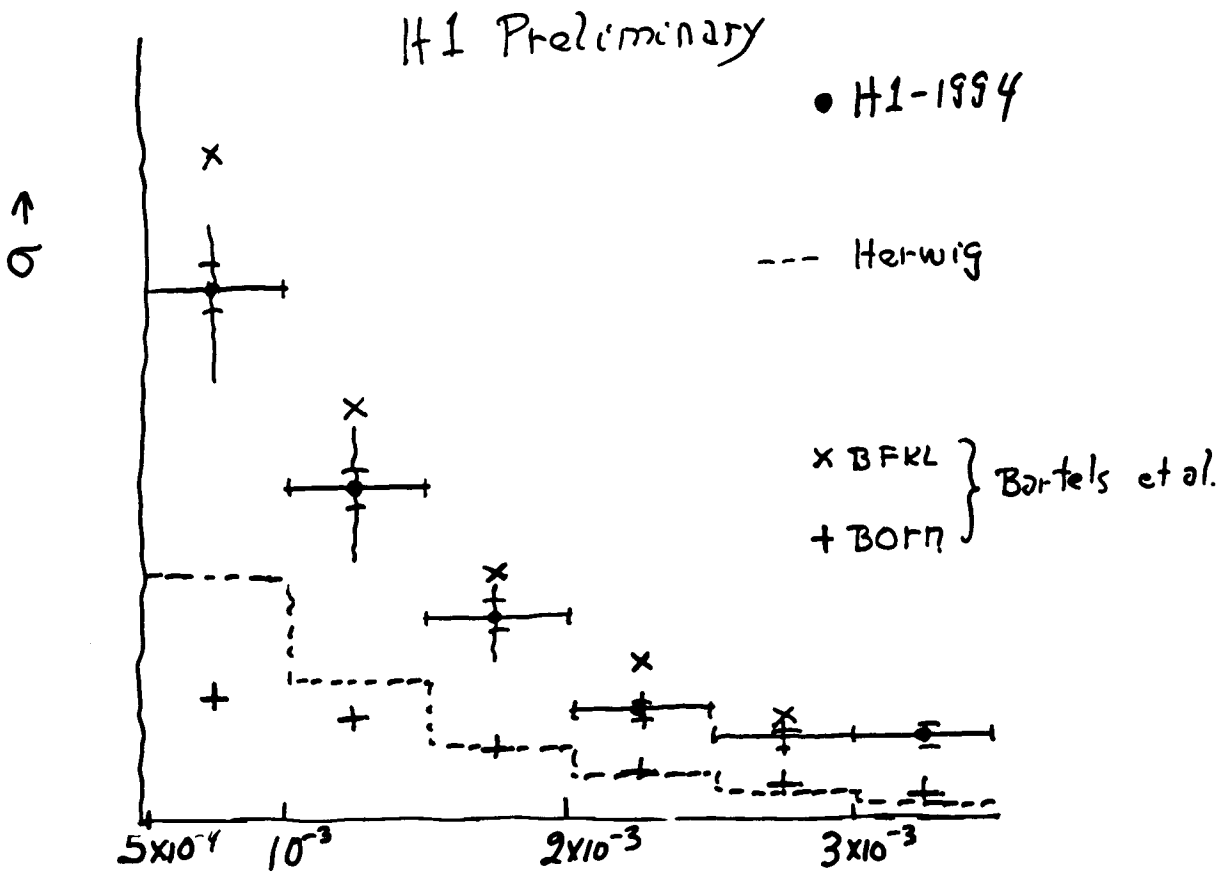
$$\frac{\sigma^{1800}}{\sigma^{630}} \approx 2$$

B. HERA inclusive

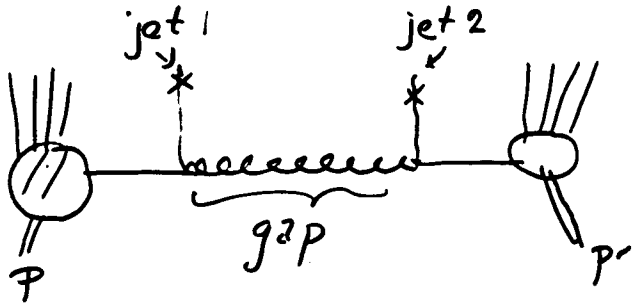
Look at Forward jet inclusively



$$\frac{\frac{k_\perp^2 x_1 d(\nu W_2)}{dk_\perp^2 dx_1}}{\left(\frac{k_\perp^2 x_1 d(\nu W_2)}{dk_\perp^2 dx_1}\right)_{LO}} = \sqrt{\frac{Q^2}{k_\perp^2}}^{(\alpha_p-1)\ln x_1/x} \frac{e}{\sqrt{\frac{7}{2}\alpha N_c S(3)\ln x_1/x}}$$


H1 Preliminary
• H1-1994
--- Herwig
× BFKL
+ Born
Bartels et al.
↑ σ
5x10⁻⁴
10⁻³
2x10⁻³
3x10⁻³

C. Gaps bounded by jets



$$\frac{k_{jet1}}{p} = x_1 \qquad \frac{k_{jet2}}{p'} = x_2$$

Fix $x_1, x_2, b_{1\perp}, k_{2\perp}$

$$\frac{\sigma^{2jet}_{gap}(1800)}{\sigma^{2jet}_{gap}(630)} = e^{2(\alpha_P - 1)[y(1800) - y(630)]} \left[\frac{y(630)}{y(1800)}\right]^3 \frac{\langle S^2\rangle_{1800}}{\langle S^2\rangle_{630}}$$

eg $y(630) = 4$ $\quad \alpha_P - 1 \approx \frac{1}{2}$

$y(1800) = 6$

$$\frac{\sigma(1800)}{\sigma(630)} = \underbrace{e^{2\ln\frac{1800}{630}}}_{2-3} \left(\frac{2}{3}\right)^3 \frac{\langle S^2\rangle_{1800}}{\langle S^2\rangle_{630}}$$

With some luck maybe an increase. DØ analysis under way

A significant increase would be interesting

4. "Breakdown" of operator product expansion at small values of x

Try to describe BFKL in terms of DGLAP (Catani, Hautmann Bali Forte Thorne--)

BFKL amplitude (Fixed coupling)

$$F(Y, \ln Q^2/Q_0^2) = \int \frac{d\lambda}{2\pi i} e^{\frac{2\alpha N_c}{\pi}\chi(\lambda)Y + \lambda \ln Q^2/Q_0^2} = \int \frac{dn}{2\pi i} e^{\gamma_n \ln Q^2/Q_0^2 + (n-1)Y} C_n$$

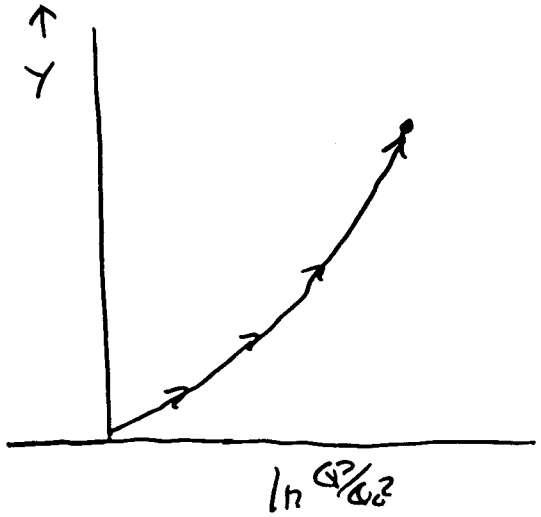
(γ_n: anomalous dimension; C_n: coefficient function)

$$\chi(\lambda) = 2\psi(1) - \tfrac{1}{2}\psi(1-\lambda) - \tfrac{1}{2}\psi(\lambda) \qquad \psi = \frac{\Gamma'}{\Gamma}$$

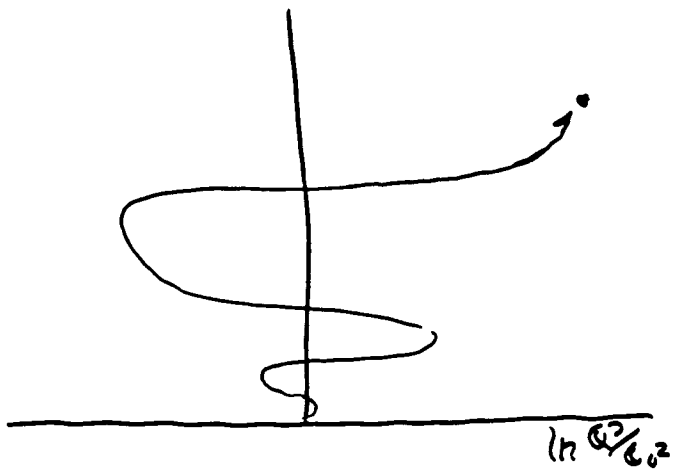
$$F(Y, \ln Q^2/Q_0^2) = \int_0^Y dY' A(Y', \ln Q^2/Q_0^2)\, C(Y-Y', \alpha)\, \frac{e^{(\alpha_P - 1)(Y-Y')}}{\sqrt{56\alpha N_c \chi(3)(Y-Y')}}$$

$$\frac{Q}{Q_0} \frac{\exp\left\{(\alpha_P-1)Y - \frac{\ln^2 Q^2/Q_0^2}{56\frac{\alpha}{\pi}N_c\chi(3)Y}\right\}}{\sqrt{56\alpha N_c \chi(3) Y}} \qquad \frac{Q}{\mu} \frac{\exp\left\{(\alpha_P-1)Y' - \frac{\ln^2 Q^2/Q_0^2}{56\frac{\alpha}{\pi}N_c\chi(3)Y'}\right\}}{\sqrt{56\alpha N_c \chi(3) Y}} \frac{\ln Q^2/Q_0^2}{Y}$$

Conflicting pictures



DGLAP



BFKL Picture (Diffusion)

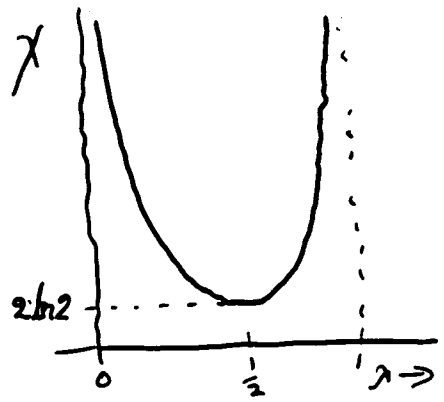
To see BFKL diffusion

$$F(Y, \ln Q^2/Q_0^2) = \frac{Q}{Q_0} e^{(\alpha_P - 1)Y} \tilde{F}(Y, \ln Q^2/Q_0^2)$$

$$\tilde{F}(Y, \ln Q^2/Q_0^2) = \int \frac{d\lambda}{2\pi i} e^{\frac{2\alpha N_c}{\pi}(\chi(\lambda) - \chi(\frac{1}{2}))Y + (\lambda - \frac{1}{2})\ln Q^2/Q_0^2}$$

$$\frac{\partial \tilde{F}}{\partial Y} = \frac{2\alpha N_c}{\pi} \int \frac{d\lambda}{2\pi i} (\chi(\lambda) - \chi(\tfrac{1}{2})) e^{\frac{2\alpha N_c}{\pi}(\chi(\lambda) - \chi(\frac{1}{2}))Y + (\lambda - \frac{1}{2})\ln Q^2/Q_0^2}$$

$$\frac{1}{2}\chi''(\tfrac{1}{2})(\lambda - \tfrac{1}{2})^2$$



$$\left(\frac{\partial}{\partial Y} - \frac{\alpha N_c}{\pi} \chi''(\tfrac{1}{2}) \frac{\partial^2}{\partial \ln^2 Q^2/Q_0^2} \right) \tilde{F}(Y, \ln Q^2/Q_0^2) = 0$$

BFKL can be described by DGLAP because γ and C also have diffusion

But operator product expansion breaks down when γ and C diffuse into $K^2 \approx \Lambda^2$.

OPE ok when

$$\ln Q_0^2/\Lambda^2 \gg \sqrt{56 \frac{\alpha N_c}{\pi} \chi(3)}\, Y/2$$

BFKL accounted for in DGLAP without diffusing into dangerous domain

Technically the problem shows up as large coefficients in running coupling corrections in γ and C.

(i) Different from renormalons. Diffusion, not running coupling, driven. No n!'s.

(ii) This is leading twist effect.

Limitations on operator product expansion similar to limitations on use of BFKL in general?

$y \ll \frac{1}{\alpha^3(Q)} \frac{2}{56 N_c \zeta(3) b^2}$, fixed coupling $b = \frac{11 N_c - 2 N_f}{12\pi}$

$y \lesssim \frac{1}{\alpha^3(Q)} \frac{2}{56 N_c \zeta(3) b^2}$, general applicability

numerical studies by Bartels and Lotter