C. Pascaud

Testing perturbative QCD with structure function measurements

Introduction

- Starts with first SLAC measurement
- Measurments getting more accurate
- Kinematical domain increase
- Theoritical calculation get more accurate
- Precise test needs a Gobal view and a precise treatment of experiment and theory.

Plan

Introduction

Theory

- Theoretical framework
- DGLAP evolution codes
- Code comparison
- Parton parametrisation

Experiment

- Experiment theory comparison
- Data sample
- Results

Conclusion

Theoretical frame work.

- Running strong coupling:

$$\frac{\partial \alpha_s(Q^2)}{\partial \log Q^2} = -\beta_0 \alpha_s^2 - \beta_1 \alpha_s^3 + O(\alpha_s^4)$$

resolution method gives $\delta\Lambda = 10$ MeV

- DGLAP equations

$$\frac{\partial f(x,Q^2)}{\partial \log Q^2} = \left[\alpha_s P_0(x) + \alpha_s^2 P_1 + O(\alpha_s^3)\right] \otimes f(x,Q^2)$$

f is either a non singlet parton distribution or a 2 component vector for gluon and singlet

structure functions are obtained from parton distribution eventually $\otimes$ with coefficient functions.

- Flavor evolution

  2 possibilities

  a) Massless evolution

  b) Photon gluon fusion for heavy quark
     LO: Collins, Hoffmann and Reya
     NLO: Riemersma, Smith and van Neerven

- Target mass correction

- Higher Twist.

DGLAP evolution Codes

- Variety of existing programs
  tested by Workshop on Future Physics at Hera

MRS	R.G. Roberts	x-space
CTEQ	W.K. Tung	x-space
H1	C. Pascaud F. Zomer	x-space
Zeus	M. Botje (M. Virchaux, A. Quadt)	x-space
—	A. Vogt	N-space
—	S. Riemersma	N-space

- <u>x-space programs</u> (ZEUS, H1)

  - Use a grid in x (and Q^2)
  - Parton distribution and convolution integrals are linearly interpolated on the x-grid

    $f \Rightarrow$ Vector

    $P \Rightarrow$ Matrix

    $\otimes \Rightarrow$ Vectorial product

DGLAP equations → set of coupled differential equations.

Zeus program linearly extrapolate to the next Q^2 grid point

H1 program uses a linearly spaced $\log x$ grid and directly exponentiate the matrix

• N space programs

uses Mellin transform to get in the momentum space and write an approximate solution

$$f_N(\alpha_s) = \left[1 - \frac{\alpha_s - \alpha_0}{\beta_0}\left({}^N P_1 - \frac{\beta_1}{\beta_0}{}^N P_0\right) + O(\alpha_s^2)\right]\left(\frac{\alpha_s}{\alpha_0}\right)^{{}^N P_0/\beta_0} f_N(\alpha_0)$$

From this one get back to x-space by "one contour integral"

- S. Riemersma program may also evolute from the original Mellin transform of DGLAP

Code validity

H1 precision test

- Change x grid and look at

  q_{NS}, Σ, g from 5 to 10^4 GeV^2

- check on momentum sum rule

Misc codes comparison

- Use same input q_{NS}, Σ, g
  - evolate from 4 GeV^2 $\Lambda^4_{\overline{MS}} = 230$ MeV
  - Mellin evolution

- Look at $Q^2 = 100$, 10000 GeV^2

- difference between x-space and N-space entirely due to "approximate" solution of DGLAP

- $\varepsilon \sim 0.05\%$ except for $x \simeq 0.1$

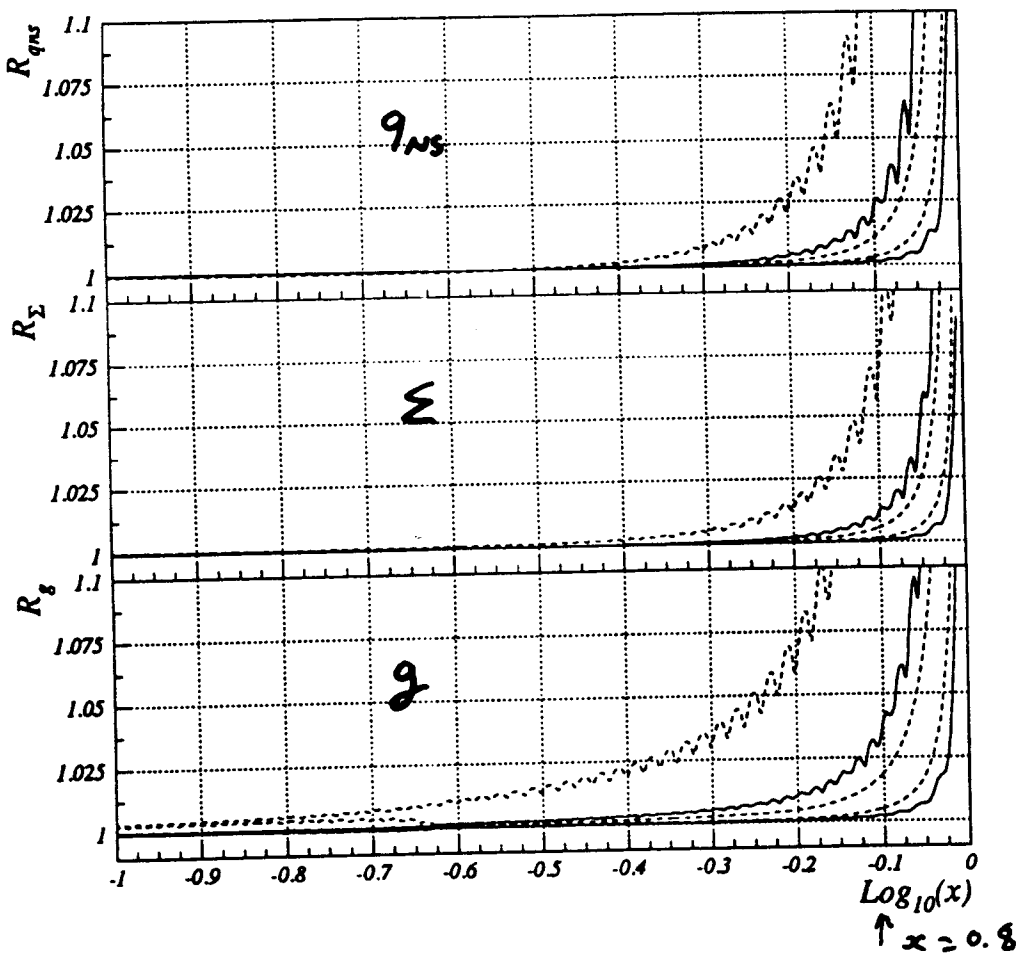


Figure 2: *a)* : $R'_{q_{NS}}(x, n, n')$; *b)* $R'_\Sigma(x, n, n')$; *c)* : $R'_g(x, n, n')$. *See eq. (16) for definitions. The full lines correspond to* $n = 700$ *and the dashed lines correspond to* $n = 250$. *The less accurate curve (far from the line* $R = 1$*) corresponds to* $n' = 1$. *The next one to* $n' = 4$. *The last one (defined only in the case* $n = 250$*) to* $n' = 8$.

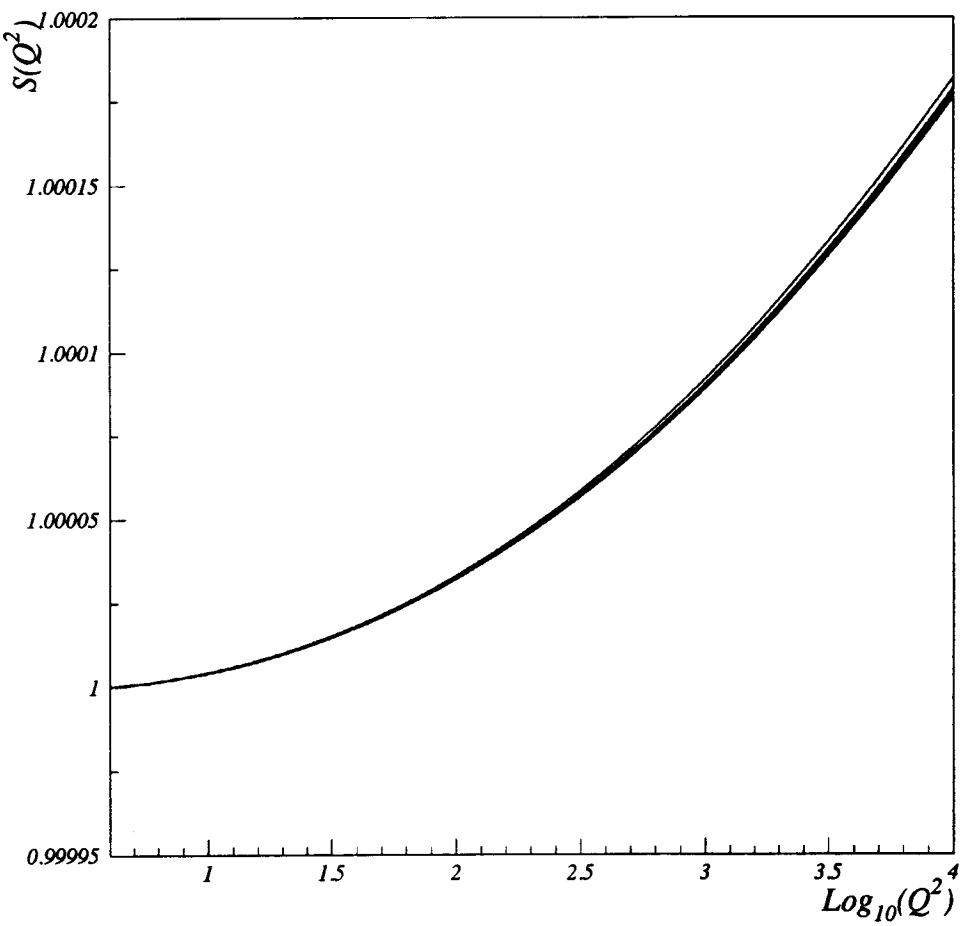


Figure 3: *Momentum sum-rule as function of Q^2. The curves described in the text superpose almost exactly.*

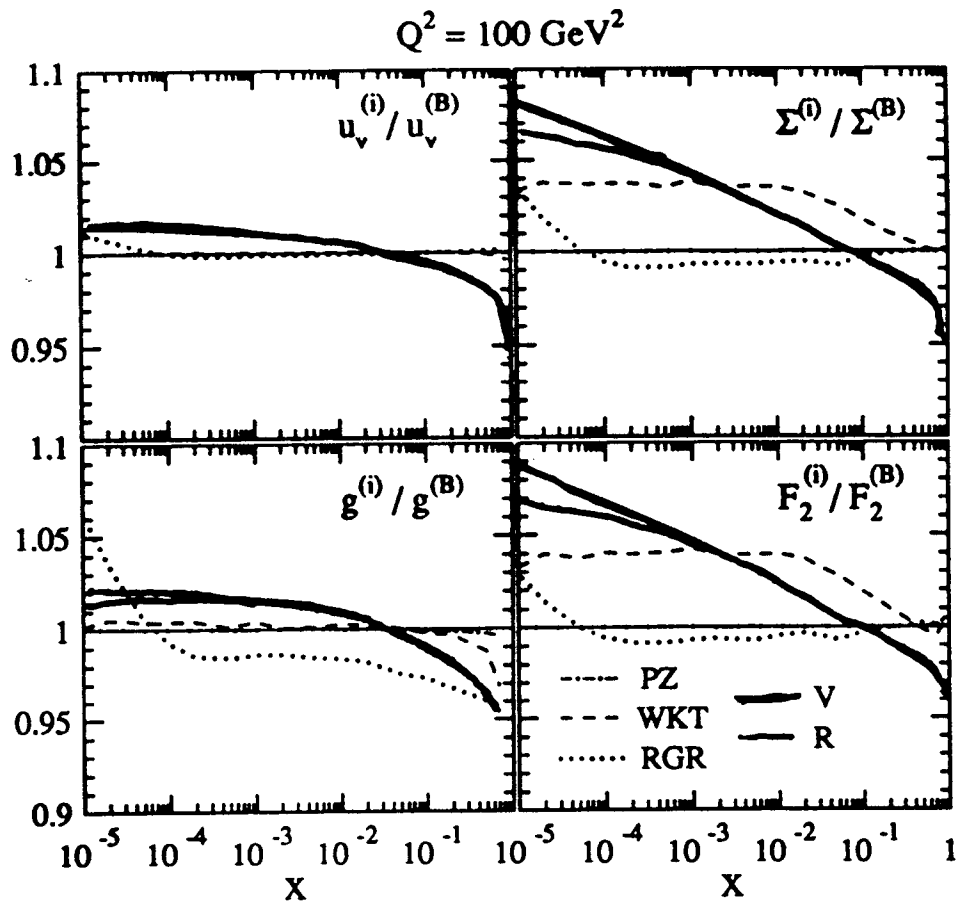


Figure 1: The differences between the up-valence, singlet quark and gluon densities, u_v, Σ and g, and the proton structure functions F_2, as obtained from evolving the input (7) with various NLO evolution programs [9, 10, 14, 15, 16, 17] to $Q^2 = 100$ GeV^2. All results have been normalized to those of ref. [9]. For a detailed discussion see the text.

Initial parton parametrisation

g, u_v, d_v, $\bar{u}$, $\bar{d}$, $\bar{s}$

$\Sigma = u_v + d_v + 2(\bar{u}+\bar{d}+\bar{s})$

$q^+ = q + \bar{q} - \frac{\Sigma}{3}$

$(u^+ + d^+ + s^+ = 0)$

$$F_2^D = \frac{F_2^p + F_2^n}{2} = \frac{2}{9}\Sigma - \frac{1}{6}s^+$$

$$F_2^p - F_2^n = \frac{1}{3}(u^+ - d^+) = \frac{1}{3}(u_v - d_v) + \frac{2}{3}(\bar{u} - \bar{d})$$

measuring Pond D $\rightarrow g$ Σ $\bar{s}$ and $u^+ - d^+$

most common parametrisation is

$$f(x, Q_0^2) = A\, x^{\delta}(1-x)^{\eta}(1 + \gamma x + \varepsilon\sqrt{x})$$

- Zeus parametrisation

$x g$ $\quad \varepsilon = 0$ $\quad Q_0^2 = 7\,GeV^2$

$x \Sigma$

$x(u^+ - d^+)$ $\quad \gamma = \varepsilon = 0$

$x s^+ = x\bar{s} - \frac{1}{3} x \Sigma$

$\uparrow_{0.2}\ (\Sigma - u_v - d_v)$

$\downarrow$ from MRS D$^-$

- H1 parametrisation

In the following study, same parametrisation than Zeus except

$x(u^+ - d^+)$ 5 parameters

$x\, s^+$ 5 parameters

$Q_0^2 = 1.5\ \mathrm{GeV}^2$

Experiment - theory comparison

$$\chi^2 = \sum_{data} \left(\frac{F_2^{th} - F_2^{m}}{\sigma} \right)^2$$

- $\sigma = \sigma_{stat}$

  Wrong - χ^2 is too high -
  goodness of fit impossible to define

- Systematics exist

  $\sigma^2 = \sigma_{stat}^2 + \sigma_{syst}^2$

  Wrong - χ^2 is too small usually
  goodness of fit ?

- Full error matrix

  Very big error matrix ($N_{data} \geq 1000$)
  Inversion questionable

- Systematic sources method

- triggered by the way systematics are calculated by the experiments.
"Known" causes or sources.
usually classified as uncorrelated or fully correlated.

For each correlated source a study is made of the change in F_2^m induced by a 1 σ or -1 σ of the source (either by M. C. or on the data itself)

- Uncorrelated sources are added quadratically to statistics

- Correlated source
define δ $\delta = 0$ measurement
$\delta = \pm 1$ for ± 1 sigma

define modified theoretical expected value:

$$\widetilde{F_2} = F_2^{th}(x, Q^2)_i \cdot \left[1 - \sum C_{ik} \delta_k\right]$$

$$\chi^2 = \sum_{data} \left(\frac{F_2^{th}\left[1 - \Sigma C_k \Delta_k\right]^{*} - F_2^{m}}{\sigma_{stat}} \right)^2$$

$$+ \sum_k \Delta_k^2$$

* Normalisation is just one source
* quadratic terms may be considered

- equivalent to full error matrix
  (if matrix indeed composed from sources)
- No correlation problems
  (even sources with identical effects may coexist)
- May give the actual value of the sources

C Pescara

Data Samples

- **SLAC on P and D**

$0.07 < x < 0.85$ $\qquad$ $0.59 < Q^2 < 29.$

- accurate results
- point to point systematical errors not available (?)
- Several experiments are grouped
- difficult to understand χ^2

- **E665 on P and D**

$0.0009 < x < 0.387$ $\qquad$ $0.228 < Q^2 < 64.$

- 7 sytematic sources
- most ~~[illegible]~~ low Q^2

- **BCDMS on P and D**

$0.07 < x < 0.32$ $\qquad$ $7.5 < Q^2 < 230$

- 1 main systematic source

C. Prescott

- NMC on P and D

  $0.0045 \leq x \leq 0.5$ $\quad 0.5 < Q^2 < 75$

- Last analysis with 4 beam energies

- 5 sources of systematics

  Beam energy calibration
  Scattered μ energy calibration
  Acceptance corrections
  Radiative corrections
  Reconstruction efficiency

- Total of 12 sources

- Zeus on P

  $0.00003 < x < 0.32$ $\quad 1.5 < Q^2 < 5000$

- 6 sources of systematics (sometime groupements)

  Positron finding efficiency
  Position angle
  Position energy scale
  Hadronic energy scale

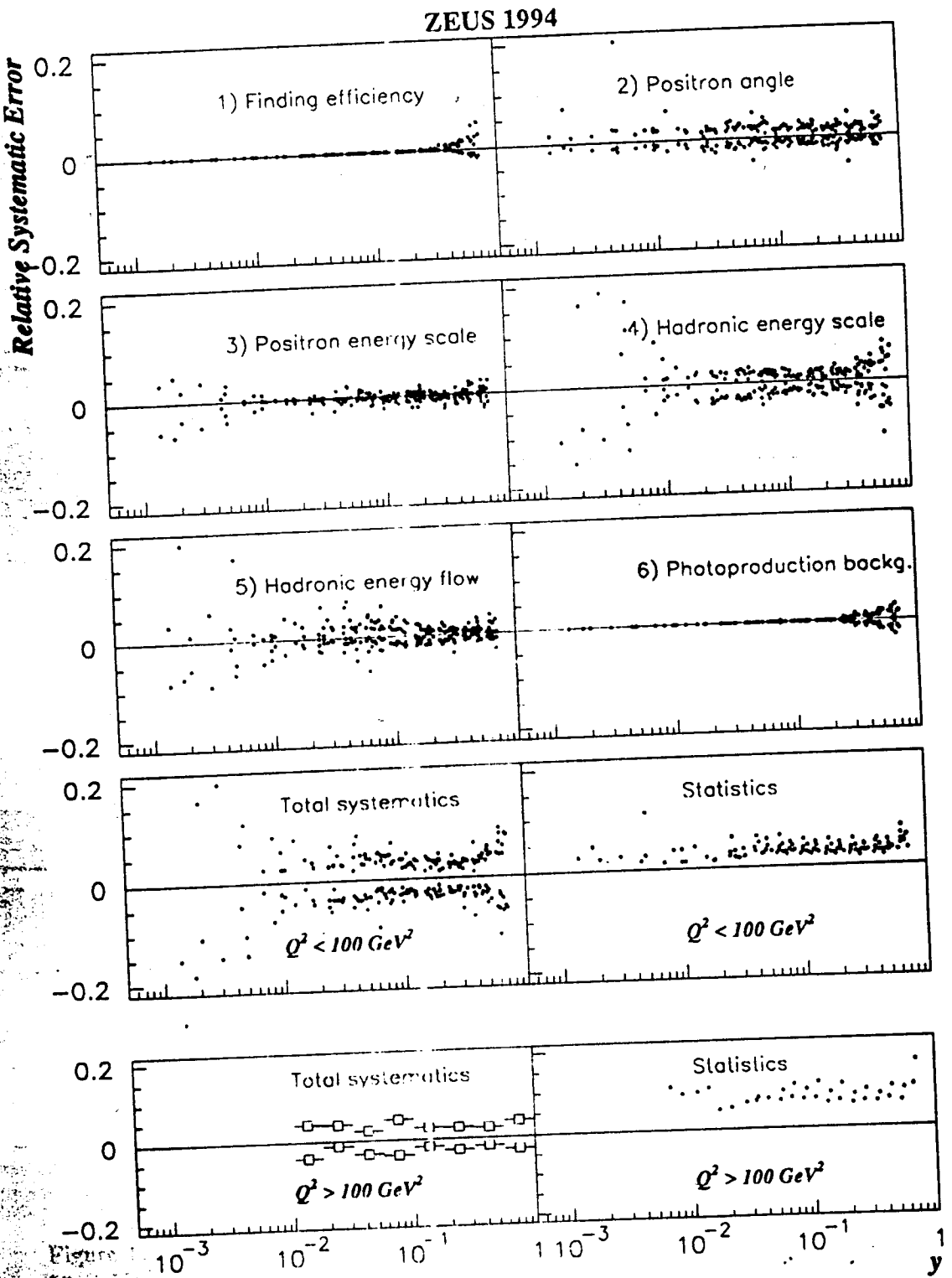


Figure 10: *Relative systematic errors as a function of y for different categories of systematic uncertainties for bins with $Q^2 < 100$ GeV^2. The total systematic and statistical errors are also given. The bottom two plots show total systematic and statistical errors for bins with with $Q^2 > 100$ GeV^2. In categories 4) and 5), and the total systematic error plot for $Q^2 < 100$ GeV^2, the four lowest y points are off-scale and are not shown.*

Hadronic energy flow

Background substraction

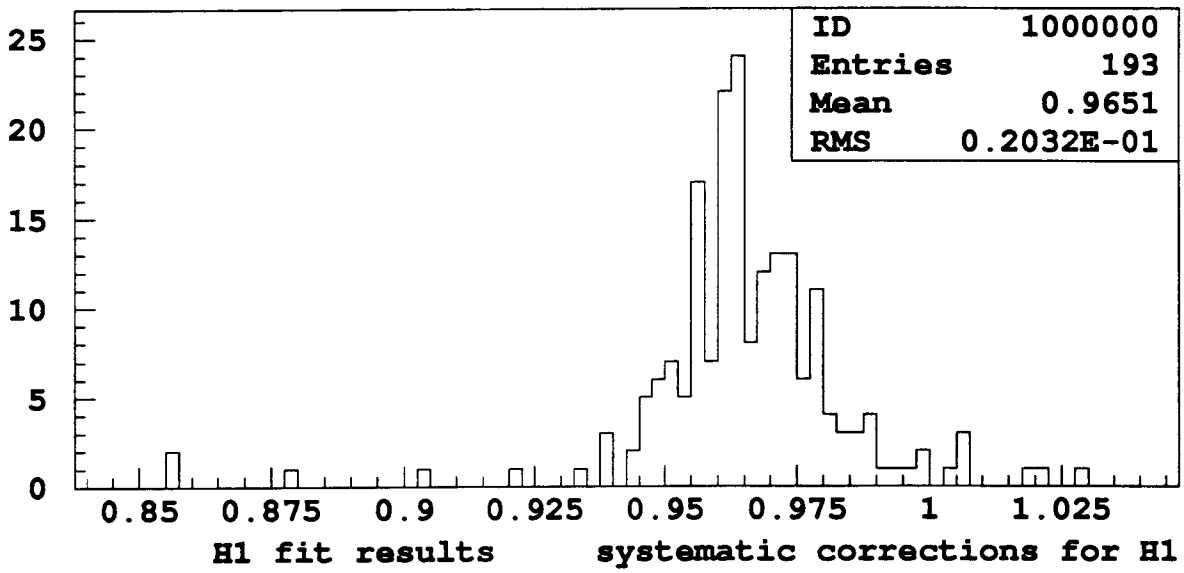
- Use parametrisation of the sources
- Plan to define uncorrelated contribution

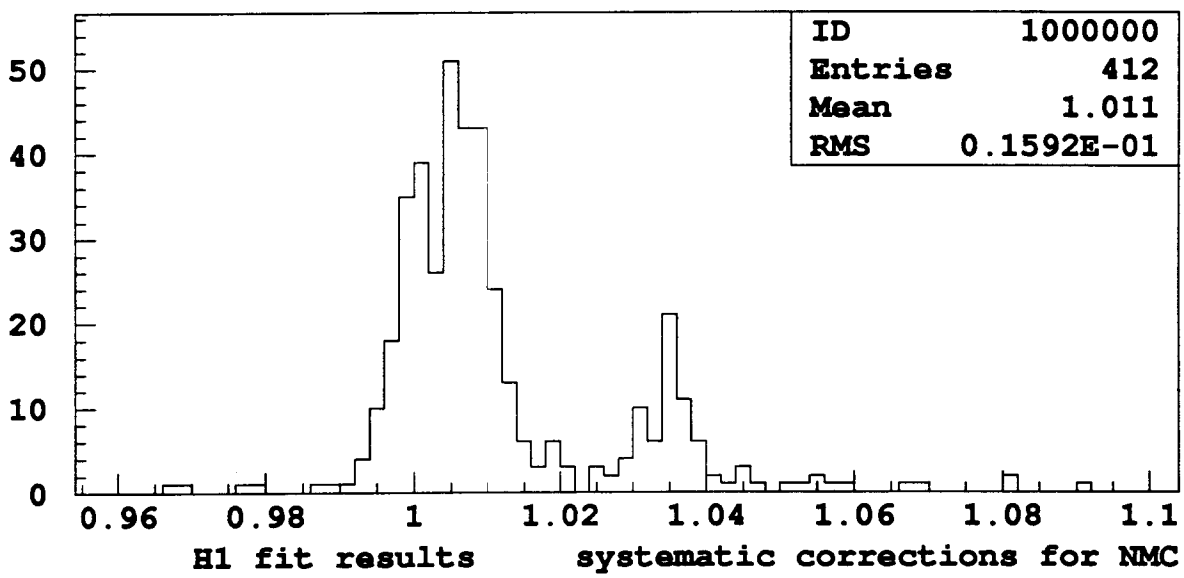
- **H1 on P**

$0.00003 < x < 0.32$ $\quad 1.5 < Q^2 < 5000.$

- 8 sources

e^+ energy scale (BEMC)
e^+ angle (BEMC
e^+ energy scale (Lar)
e^+ angle (Lar)
Hadronic energy scale
Vertex efficiency
Radiative correction
Background substraction


ID 1000000
Entries 193
Mean 0.9651
RMS 0.2032E-01
25
20
15
10
5
0
0.85 0.875 0.9 0.925 0.95 0.975 1 1.025
H1 fit results systematic corrections for H1


ID 1000000
Entries 412
Mean 1.011
RMS 0.1592E-01
50
40
30
20
10
0
0.96 0.98 1 1.02 1.04 1.06 1.08 1.1
H1 fit results systematic corrections for NMC

Results

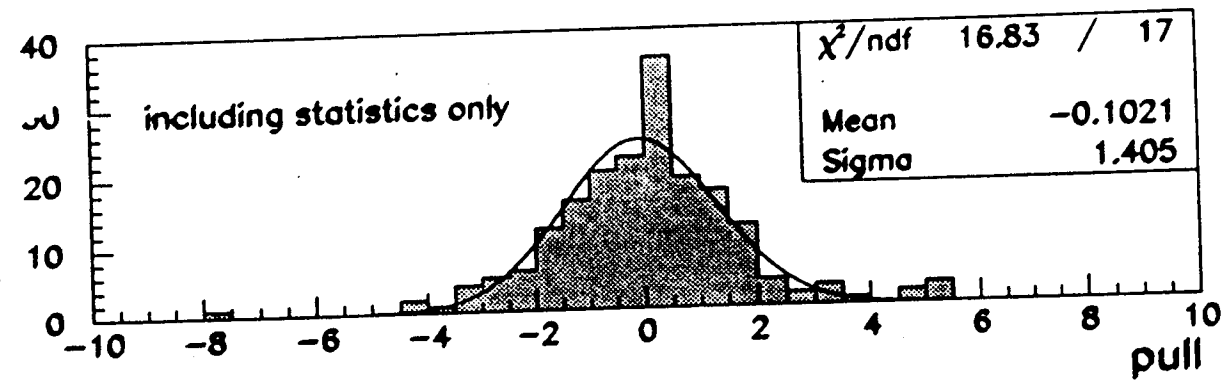
- Use only NMC as fixed target Data
- Cut $Q^2 > 1.5$ Hera
  $Q^2 > 4$ NMC
  higher Twist effect
- $\Lambda_{QCD} = 263$ MeV
- $m_c = 1.5$ $m_b = 5$ (4.75 for H1)
- NLO heavy quark as standard procedure

Zeus analysis

Data	Zeus	NMC (old)	
# pt	136	246	$\Rightarrow$ ndf = 394

χ^2/ndf (statistic only) = 2.3

statistic ⊕ systematics = 1.2



$\sigma = 1.4$

$\sigma = 1.0$

- Massless / Heavy quark comparison

  Favors Heavy quark prescription

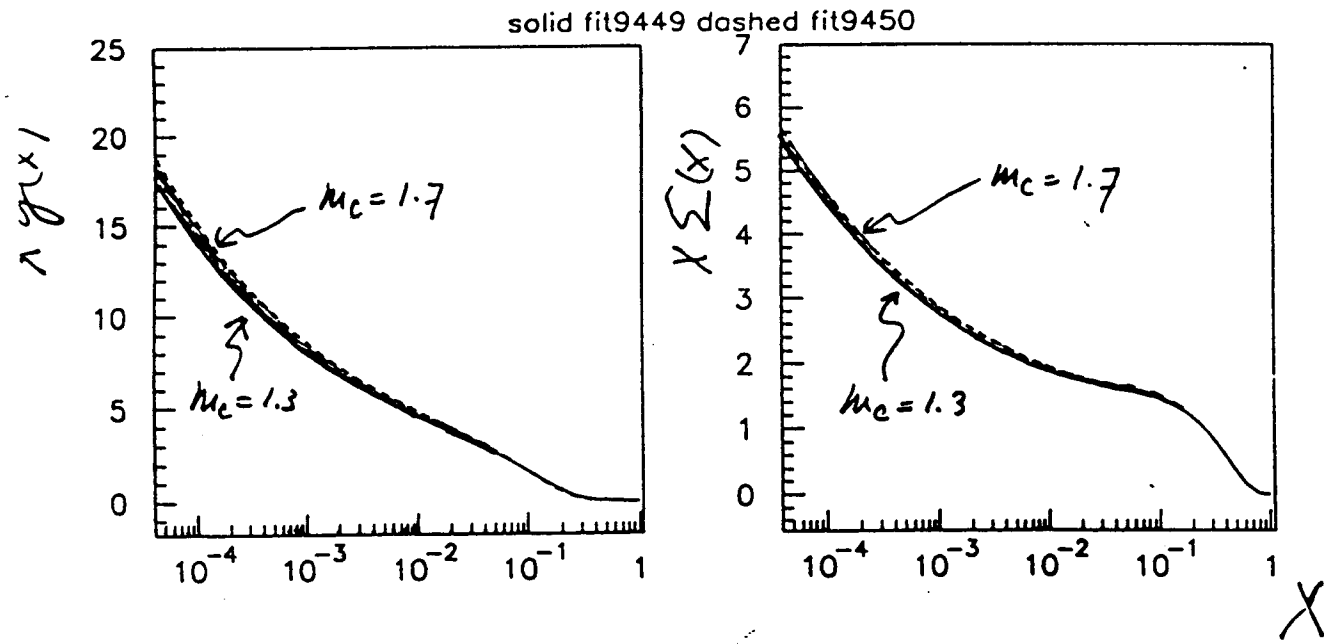
  $\Delta\chi^2 = 11$ stat ⊕ syst

Sensitivity to charm mass

$m_c = 1.3$ GeV → $\chi^2 = 459/408-14$

$m_c = 1.5$ GeV → $\chi^2 = 463/408-14$

$m_c = 1.7$ GeV → $\chi^2 = 469/408-14$



- H1 analysis (Very preliminary)

Data	H1	NMC	
# pt	193	412	⇒ ndf = 585

χ^2/ndf (statistic only) = 1.3

stat ⊕ syst = 0.78

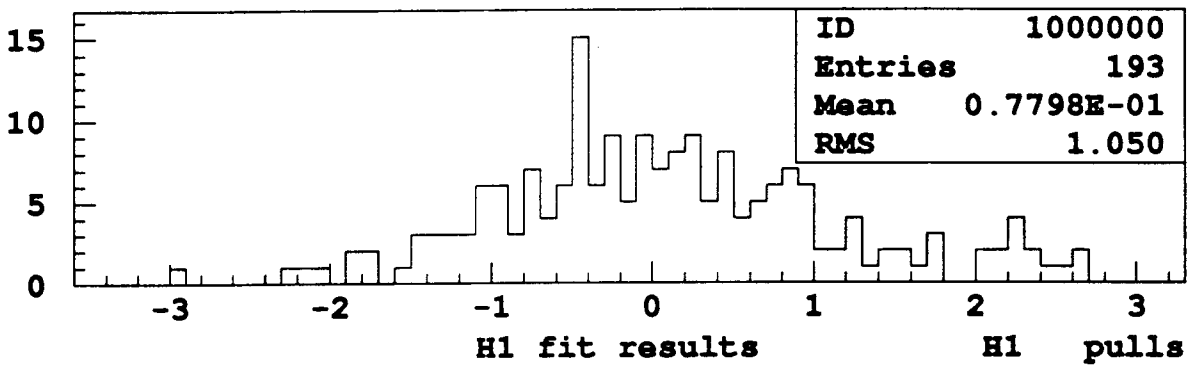
stat fitti = 1.2 $\Delta\chi^2 = 40$

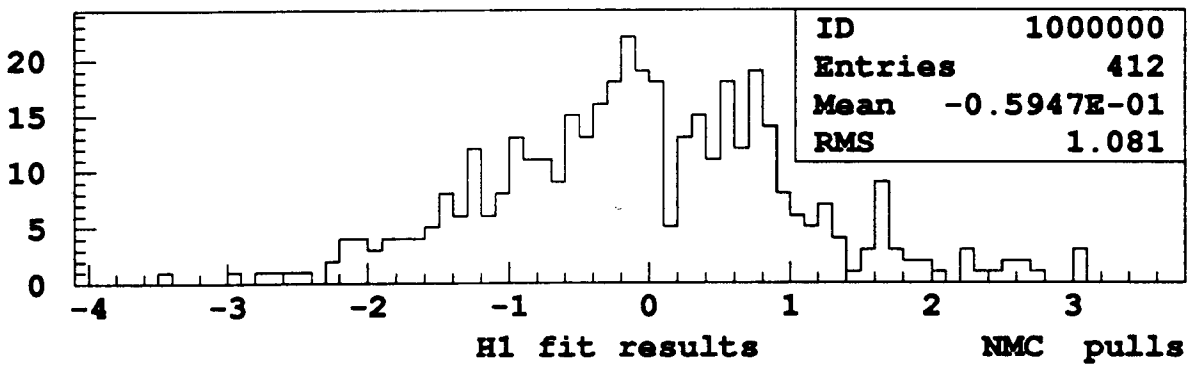
- Massless / Heavy quark Comparison

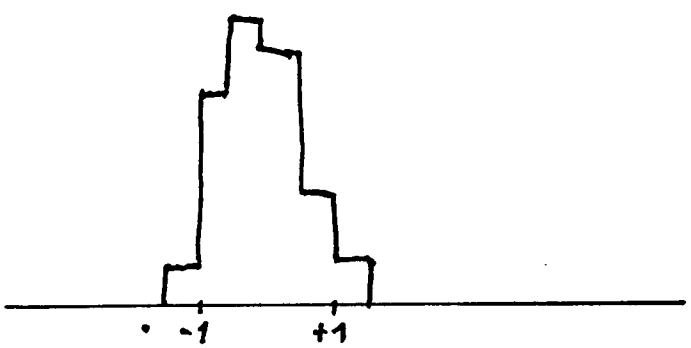
also in favor of Heavy quark

$\Delta\chi^2 = 31$ stat ⊕ syst

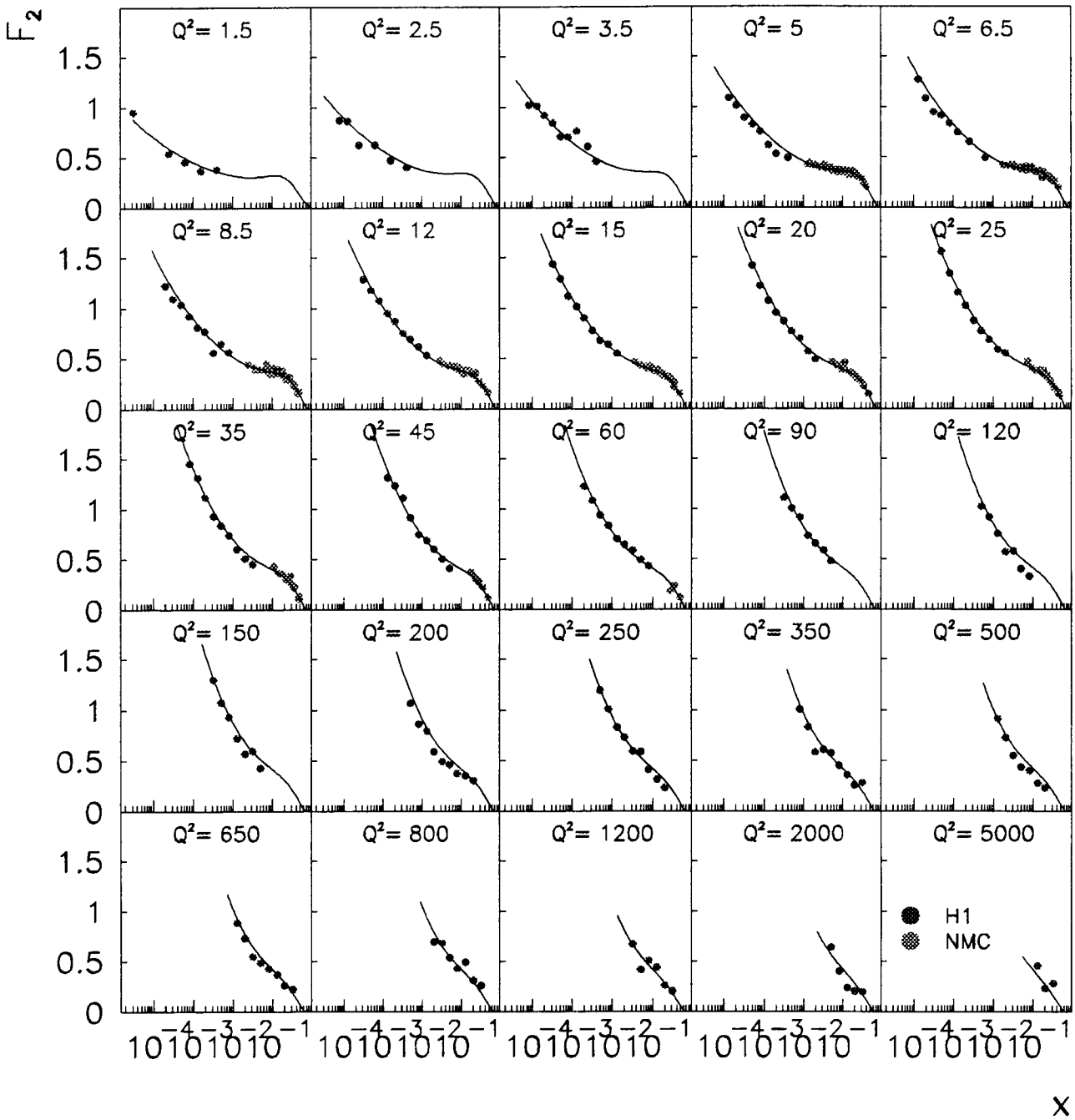
66 syst fittiv



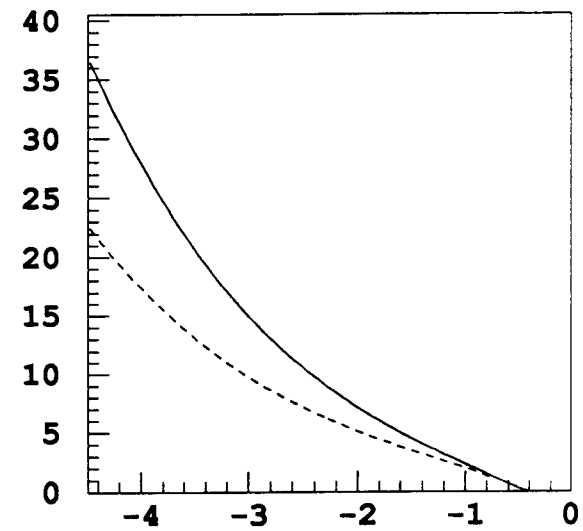


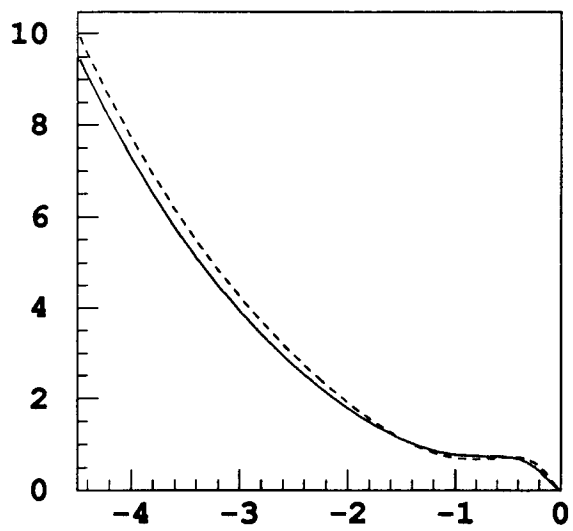


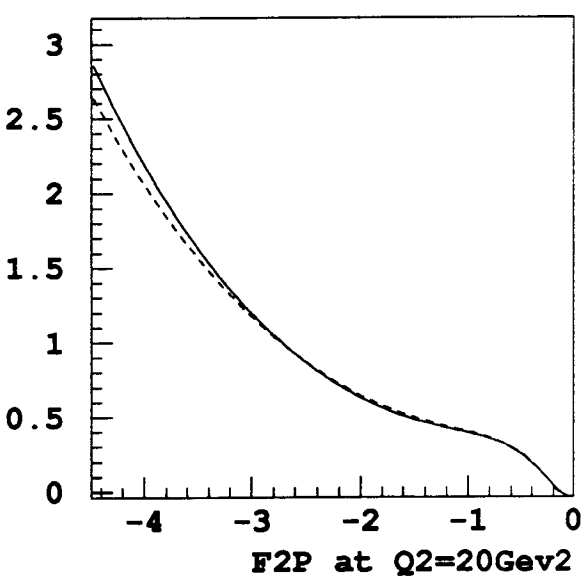
H1 fit results Systematics pulls

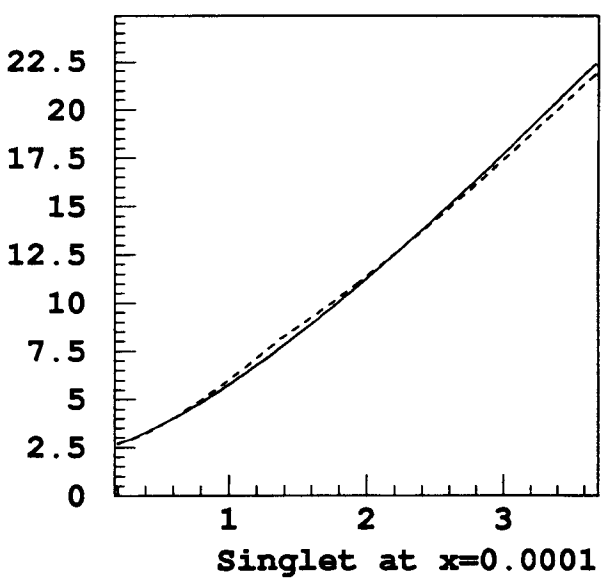

F2
Q²= 1.5
Q²= 2.5
Q²= 3.5
Q²= 5
Q²= 6.5
Q²= 8.5
Q²= 12
Q²= 15
Q²= 20
Q²= 25
Q²= 35
Q²= 45
Q²= 60
Q²= 90
Q²= 120
Q²= 150
Q²= 200
Q²= 250
Q²= 350
Q²= 500
Q²= 650
Q²= 800
Q²= 1200
Q²= 2000
Q²= 5000
H1
NMC
1.5
1
0.5
0
10101010
−4 −3 −2 −1
x

Heavy quarks ——————
Massless quarks - - - -









Conclusions

- New results are coming
- They are better and better presented
- Proper handling of systematics should bring better and more reliable results.