QCD STUDIES AT HIGH ENERGY

How Important? How precise?
– A selection of observations & methods –

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I. Introduction: the Relevance of pQCD
II. Synopsis of pQCD methods & central results
III. Case study in resummation
IV. Power Corrections from Perturbation Theory
V. Conclusions

I. INTRODUCTION: ON THE RELEVANCE OF QCD STUDIES

Vignette: Lunar apogee in the 18th C

Sep 1747 (Euler to Clairaut):
"... the forces that act on the moon do not follow the rule of Newton."

Nov 1747 (Clairaut to French Royal Acad.)
Newton's theory of gravity false

Jun 1748 (d'Alembert to Cramer):
"... the gravitation of the moon to the sun will not explain [the] irregularities of its motion"

May 1749 (Clairaut to Fr. Royal Acad.):
Terms previously neglected show Newton's laws right all along

→ $1/r^4$ correction? (magnetism?)
→ theory of the 3-body problem
→ beginnings of pertubation theory ... Laplace ... Poincaré
→ d'Alembert and Clairaut never spoke again

(source: Jean d'Alembert, T.L. Haskins, Oxford 1990)

Back to our century...

Quantum Chromodynamics Today

How QCD was accepted

- Asymptotic freedom and scale (breaking in DIS) ← The 2-body problem of QCD
- Lattice technology for confinement; spectra
- Jets for gluons, high-p_T jet in $p\bar{p}$
- QCD Sum rules & consistency of OPE / bound states (duality)

The proliferation of phenomenological parameters

- Vacuum condensates
- Parton distributions
- Fragmentation functions
- Multi-parton distrib^s
- Spin distrib.^{s}
- Hadronization parameters

The "theories" of QCD

- pQCD • LQCD • QSDSR
- HQET • Chiral PT

In addition to higher order PT:
Improved precision from additional nonperturbative parameters or more accurate measurements of existing ones: but is it worth it?
Yes(- for backgrounds to and rates for "new physics"....
but also:)

QCD as "the" quantum field theory

- QCD vs QED: first correction to the running coupling

$$\alpha_s(Q_2^2) = \frac{\alpha_s(Q_1^2)}{1 + \frac{b_2 \alpha_s(Q_1)^2}{4\pi} \ln \frac{Q_2^2}{Q_1^2}}$$

changes by ~ 2 $Q_1 \sim 1\,\text{GeV}$
$Q_2 \sim 10^2\,\text{GeV}$

- at radius of convergence -

corresponding "Q_2" for $Q_1 = m_e$, $\alpha_s(Q_1) = 1/137$, is at "Landau pole"

$$\alpha_{EM}(Q_2^2) = \frac{\alpha_{EM}}{1 - \frac{\alpha_{EM}}{3\pi} \ln \frac{Q_2^2}{m_e^2}}$$

pQCD for $1 \lesssim \mu \lesssim 10^2$ GeV covers wide range in relative roles of high and low orders

In defence of parameters:

- Many NP parameters have direct field theoretic interpretation

$$\langle 0 | F_{\mu\nu}^2 | 0 \rangle \,, \quad \langle p | \bar{q} D^{\mu_1} \dots D^{\mu_n} q | p \rangle$$

- Economical in parameters: $g(\mu)$; $m_{u,s,d}(\mu)$. Rich in consequences at accessible energies

- Analogy to our 18th Century friends: once again formulating a theory to unify the treatment of long- and short-distances

- Toward a theory of high-order and non-perturbative corrections at high energy ...

- An unfinished program, awaits:
  - unifying principle?
  - sophisticated numerical verification?

II Synopsis: how pQCD works

Recurring Themes

- IR Safety (stability)

$$Q^2 \hat{\sigma}(Q^2, \mu^2, \alpha_s(\mu^2)) = \sum_n C_n(Q^2/\mu^2)\, \alpha_s^n(\mu^2)$$

free of long-distance dependence

$\sigma^{tot}_{e^+e^-}$

e^+e^- jets, event shapes

- Factorization (of SD, LD)
  - identify small parameters
  - effective theories

$$\sigma_{phys}(Q, m) = \hat{\sigma}_{PT}\left(\frac{Q}{\mu}, \alpha_s(\mu)\right) \otimes \phi_{NP}\left(\frac{\mu}{m}, \alpha_s(\mu)\right) + \mathcal{O}(1/Q^2)$$

(free)

DIS, HH...

- Evolution

$$\mu \frac{d\sigma_{phys}}{d\mu} = 0 \Longrightarrow \text{PT}$$

Corrections modify P_i form of eqn. is stable

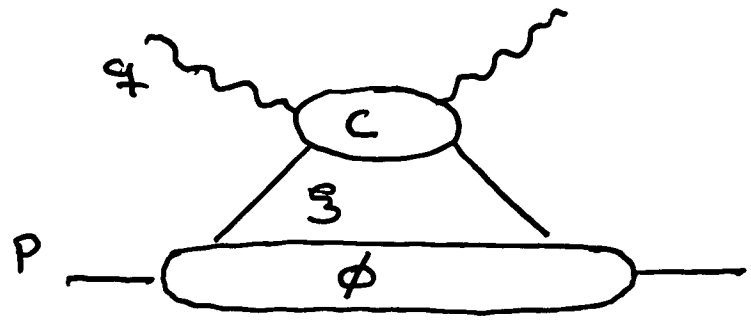
DGLAP for DIS $\rightarrow$

$$\mu \frac{d\phi_{NP}}{d\mu} = P(\alpha_s(\mu)) \otimes \phi_{NP} + \mathcal{O}(1/Q^2)$$

$$\mu \frac{d\hat{\sigma}_{PT}}{d\mu} = -P(\alpha_s(\mu)) \otimes \hat{\sigma}_{PT} + \mathcal{O}(1/Q^2)$$

- BFKL evolution

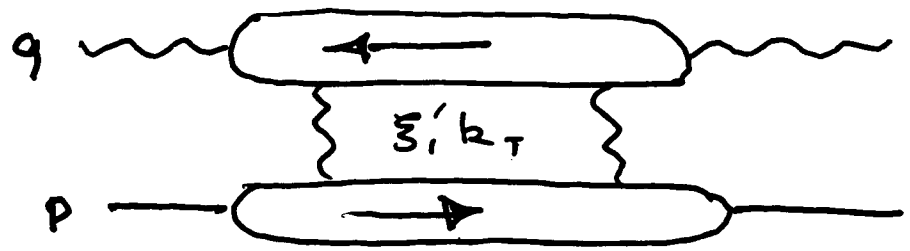
DIS version of BFKL; ξ becomes factorization scale!



$$F(x,Q^2) = \int_x^1 \frac{d\xi}{\xi} C\left(\frac{x}{\xi}, \frac{\mu^2}{Q^2}\right) \phi(\xi,\mu^2) + O\left(\frac{1}{Q^2}\right)$$

$$= \int d^2k_T \, \tilde{C}\left(\frac{x}{\xi}, Q, k_T\right) \psi(\xi', k_T)$$

$$\phi(x,\mu) = \int^{\mu} d^2k_T \, \psi(x,k_T) \qquad + \boxed{O\left(1/\ln 1/x\right)}$$



BFKL Eqn:

$$\xi \frac{d\psi(\xi,k_T)}{d\xi} = \int d^2k_T' \, K(k_T - k_T') \, \psi(\xi,k_T)$$

K: ξ-independent

- famous solution: $\sim$ 'δ'

$$\xi\psi(\xi,k_T) \sim \xi^{-4N\frac{\alpha}{\pi}\ln 2}\, k_T^{-1/2}$$

  - part of DGLAP, picks out LL $\ln\frac{1}{x}$ to all orders, any Q^2
  - but, which α? (2-loop k_T will help)
  - more important, (N)LL in $\ln\frac{1}{x}$ changes eqn
    → more k_T integrals

- BFKL in a larger context

$$\psi \to S\sigma^{tot}_{\gamma^* p} \sim \frac{1}{x^{1+\delta}} \sim \left(\frac{W^2}{Q^2}\right)^{1+\delta} \sim \left(\frac{S_{\gamma^* p}}{m^2_{\gamma^*}}\right)^{1+}$$

works for any two particles:

$$\sigma^{tot}_{AB} \sim S^{\alpha} \qquad \alpha \gtrsim 1$$

QCD Regge behavior

Regge: $\frac{d\sigma}{dt}_{AB} \sim \sum_i \beta_{Ai}\, S^{\alpha_i(t)}\, \beta_{Bi}$

$\alpha_i(t \Rightarrow m^2_{is}) = J$, spin of exchanged particl

WINDOW TO HADRONIC DOF?!

- Modestly back away from small-x for now
- Many high-statistics QCD cross sections measured to a few percent
  - eN, νN DIS
  - $e^+e^- \to Z \to$ hadrons

- Some predictions work at this level
- h-h cross sections

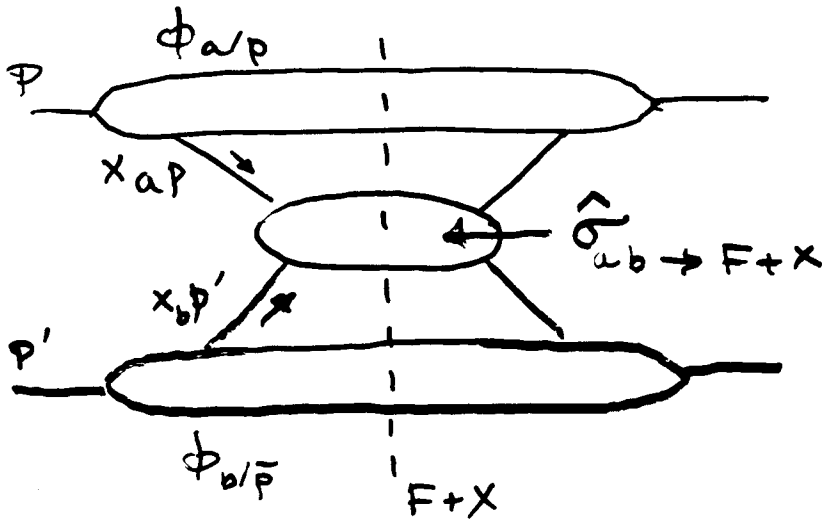
$F = 2J$, DY, NJet.... $Q \lesssim S^{1/2}$

$$\frac{d\sigma_{p\bar{p}\to F(Q)+X}}{dQ^2} = \sum_{a,b} \int_{Q^2/S}^{1} dz \int \frac{dx_a}{x_a} \frac{dx_b}{x_b} \delta\left(z - \frac{Q^2}{x_a x_b S}\right)$$

$$\cdot \phi_{a/p}(x_a, \mu^2)\, \phi_{b/\bar{p}}(x_b, \mu^2)$$

$$\cdot \hat{\sigma}_{ab\to FX}\left(z, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right)$$

$x_a x_b S \gtrsim Q^2 \hookrightarrow z \lesssim 1$



$$\hat{\sigma}_{ab \to F+X} = \sigma_{Born} + \frac{\alpha_s(\mu)}{\pi} \hat{\sigma}^{(1)} + \cdots$$

- To understand (and improve) successes and failures need:
  - theory of multiple perturbative scales in $\hat{\sigma}$
  - improved parton distribution functions

- For scales $Q_1, Q_2 \ldots$ in $\hat{\sigma}$: full cross sections:

$$\sigma(Q_i) = Q_1^{-n_\sigma} C_0\left(\frac{Q_1}{\mu}, \frac{Q_2}{\mu} \ldots \alpha_s(\mu)\right) + \sum_{I \geq 1} Q_1^{-n_\sigma - I} C_I\left(\frac{Q_1}{\mu} \ldots \alpha_s(\mu)\right)$$

Perturbation theory & PDFs:

$$Q_1^{-n_\sigma} C_0 = \sum_{ab} \phi_a \otimes \phi_b \otimes \hat{\sigma}_{ab \to FX}$$

The rest C_I : { power corrections / "higher twist"

Study: "higher" orders in C_0

(Set PDFs aside)

Will find: Perturbative analysis of multiple scales in $C_0 \rightarrow$ power corrections (C_I) and "new" nonperturbative parameters

For simplicity: study two "classic" cases from Drell-Yan cross sections

$$F = \gamma^*(Q^2), Z(Q^2) \ldots$$

III CASE STUDY:

Perturbative Corrections and Their Resummation for DY ("$x \to 1$")

(1) $\dfrac{d\sigma}{dQ^2}$ $\quad z = \dfrac{Q^2}{x_a x_b s}$ $\quad \alpha_s^n \left[\dfrac{\ln^{2n-1}(1-z)}{1-z}\right]_+$

(2) $\dfrac{d\sigma}{dQ^2 d^2Q_T}$ $\quad \alpha_s^n \left[\dfrac{\ln^{2n-1} Q_T^2/Q^2}{Q_T^2}\right]_+$

generalize factorization

$(n=1)$

(1) → "large" corrections to $d\sigma/dQ^2$ $\quad F = DY, Q\bar{Q}, 2J$ etc.

but finite order-by-order

(2) → already a problem at NLO

$$\frac{d\sigma_{Born}}{dQ^2 d^2Q_T} \propto \delta^2(Q_T)$$

Both "Sudakov" corrections...

"Universal" Sudakov Resummation

DDT, PP
Collins, Soper
Altarelli, Ellis, Greco
...
Contopanago
Laenen
G.S.

DIS	$x \to 1$	EW-induced
DY	$1 - Q_T/Q \to 1$	
e^+e^-	$z \to 1$	
	$T \to 1$	
$Q\bar{Q}$	$z \to 1$	QCD induced
JJ	$z \to 1$	

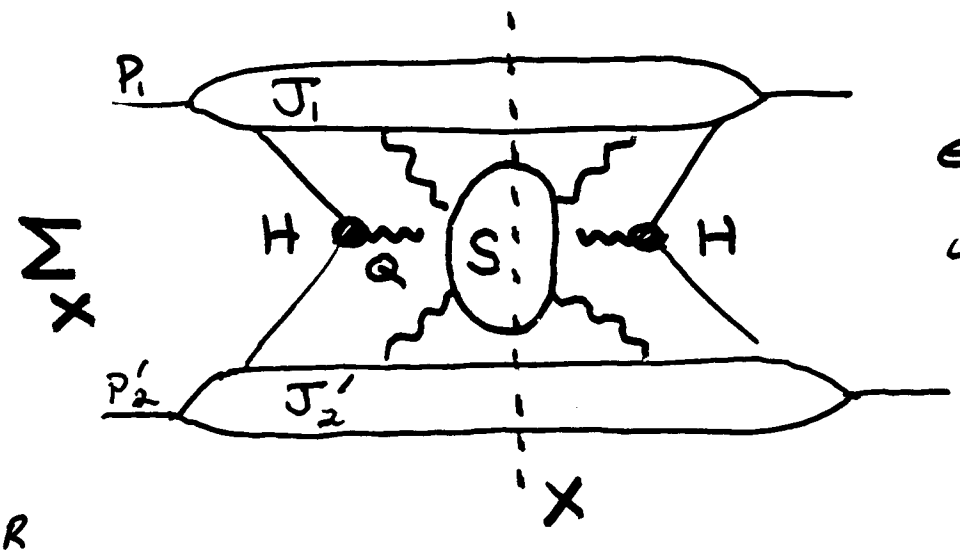
all share kinematic features:

(1) All radiation forced to be soft <u>except</u>

(2) Fast lines in jets of collinear particles <u>or</u>

(3) Nearly on-shell heavy quarks

yet another factorization/evolution scenario:



example:

$\omega = 1 - \frac{Q^2}{S}$,

OR

Q_T/Q

OR

$\omega = \frac{Q_T}{\sqrt{Q^2}}$ (DY or 2γ), $1-x$ (DIS), $1-T$ (e^+e^-

Factorization:

$$\frac{d\sigma}{d\omega} = H\left(\frac{p_1}{\mu}, \frac{p_2}{\mu}, s_1, s_2\right) \cdot \int \frac{d\omega_1}{\omega_1}\frac{d\omega_2}{\omega_2}\frac{d\omega_s}{\omega_s}\,\delta(\omega - \omega_1 - \omega_2 - \omega_s) \cdot J_1(p_1\cdot s_1, \omega_1 Q^2, \mu)\, J_2(p_2\cdot s_2, \omega_2 Q^2, \mu) \cdot S(\omega_s Q/\mu, \hat{p}_1, \hat{p}_2, s_1, s_2)$$

$$= H\{J_1 \otimes J_2 \otimes S\}$$

$\hat{p}_i^\mu$ velocities (viz. HQET)

s_j^μ analogous to μ

(Catani et al. Collins, Ellis.)

For $\omega_i \sim \frac{q_{iT}}{m}$ a generalization of standard factorization

Moments:

$$\tilde{\sigma}(N) = \int dw\, e^{-Nw} \frac{d\sigma}{dw}$$

$$= H\left(\frac{p_i}{\mu}, \xi_i\right) \tilde{J}_1\left(\frac{p_1\cdot\xi_1}{\mu}, \frac{Q^2}{N\mu^2}\right) \tilde{J}_2\left(\frac{p_2\cdot\xi_2}{\mu}, \frac{Q^2}{N}\right)$$
$$\cdot \tilde{S}\left(\frac{Q}{N\mu}, \hat{p}_i\cdot\xi_j\right) + \theta\left(\frac{1}{N}\right)$$

Evolution from factorization:

$$\frac{\partial}{\partial \ln p_1\cdot\xi_1} J_1\left(\frac{p_1\cdot\xi_1}{\mu}, \frac{Q^2}{N\mu^2}\right)$$
$$= -\frac{\partial}{\partial \ln p_1\cdot\xi_1} H\left(\frac{p_i}{\mu}, \xi_i\right)$$
$$- \frac{\partial}{\partial \ln p_1\cdot\xi_1} \tilde{S}\left(\frac{Q}{N\mu}, \hat{p}_i\cdot\xi_j\right) + \theta\left(\frac{1}{N}\right)$$

Solution: (typical)

$$\ln \tilde{\sigma}(N) = D(\alpha_s(\mu), \alpha_s(Q/N))$$
$$+ 2\int_{Q/N^{1/2}}^{Q} \frac{d\lambda}{\lambda}^{\text{IR}} \left[\int_{Q/N}^{\lambda} \frac{d\xi}{\xi}^{\text{collinear}} A(\alpha_s(\xi^2)) + B(\alpha_s(\lambda^2))\right]$$
$$+ \theta\left(\frac{1}{N}\right)$$

↑ a sort of "walking BLM"

$$= D(\alpha_s(Q), \alpha_s(Q/N))$$
$$\left(\begin{array}{l}\text{no } \mu_{F,R} \\ \text{for } N\to\infty\end{array}\right) \to + 2\int_{Q/N}^{Q} \frac{d\xi}{\xi}^{\text{IR}} \left[\ln\frac{Q^2}{\xi^2}\cdot A(\alpha_s(\xi^2)) + B(\alpha_s(\xi^2))\right]$$

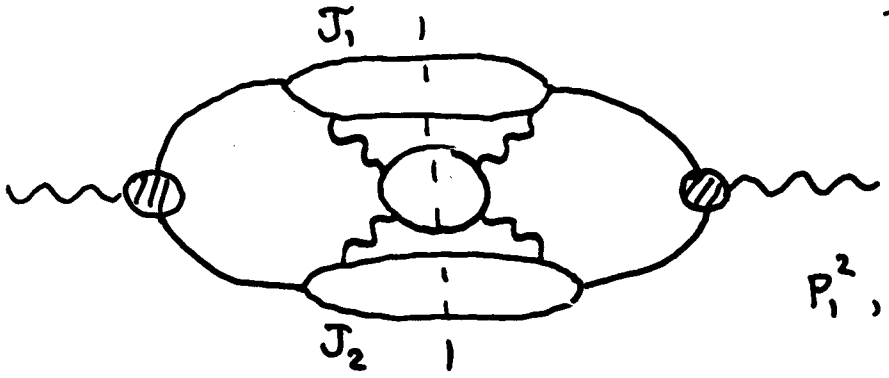
For Q_T - distribution:

$$N = ibQ$$

$$\ln \tilde{\sigma}_{DY}(b, \alpha_s(Q^2)) = -2 \int_{1/b}^{Q} \frac{d\xi}{\xi} \ln\left(\frac{Q}{\xi}\right) A(\alpha_s(\xi^2)) + \dots$$

Similarly for thrust $T \to 1$

Catani Turnock Treutadue Webber



$$\ln \tilde{\sigma}_{1-T}(N, \alpha_s(Q^2)) = -2 \int_{Q/\sqrt{N}}^{Q} \frac{d\lambda}{\lambda} \left[\int_{Q^2/\lambda N}^{\lambda} \frac{d\xi}{\xi} A(\alpha_s(\xi^2)) - B_{1-T}(\alpha_s(\lambda^2)) \right] + D_{1-T}(\alpha_s(Q^2), \alpha_s(Q^2/N^2))$$

A: universal

Physical cross section: invert moments (energy conservation)

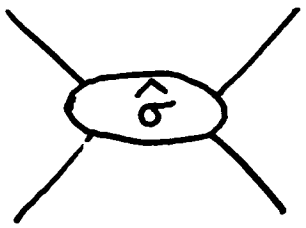
Singularity at $1/b$, $Q^2/\lambda N = \Lambda_{QCD} \to$

$\alpha_s^n \beta_2^n n!$ (IR renormalon)

To resum . . . NLLs in QCD Hard Scat.

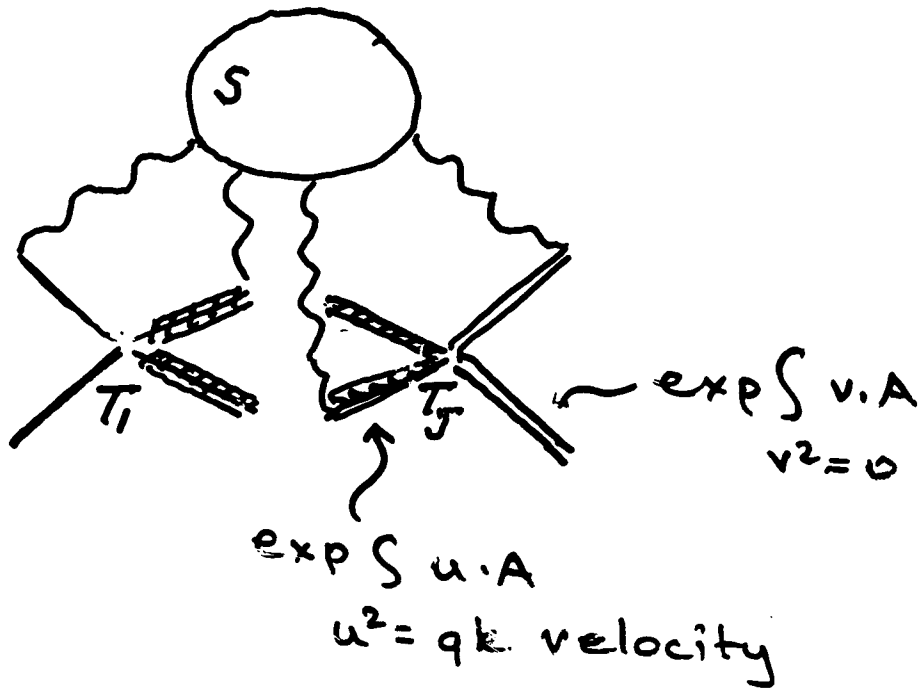
e.g. $Q\bar{Q}$ (Kidonakis, GS 96)

Color Flow Essential



$= h_I(Q)\, h_J^*(Q)\, S_{IJ}$

S_{IJ} =



T_I, T_J : composite vertices between Wilson lines with color exchange

Renormalize these vertices → renormalization at NLL

Botts, St.; Sotiropoulos, St.
Korchemsky, Korchemskaya

close relation to onium production
Braaten, Bodwin, Lepage

Korchemsky
Oderda
G.S.

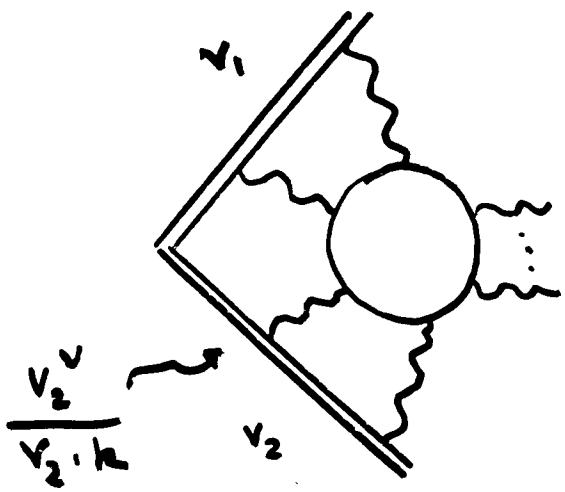
II Power Corrections from PT

(Analogy to OPE for $\sigma^{tot}_{e^+e^-}$, Mueller $F^{DIS}_{(x,Q^2)}$)

Replace regions with strong coupling by nonperturbative matrix elements

- Example: soft-gluon contribution S to $\frac{d\sigma_{DY}}{dQ^2 d^2Q_T}$

S is "Two-jet effective theory" (2JET)



$$\frac{1}{Q_T^2} S(v_1, v_2, \underline{Q}_T) = \sum_n \delta^2(\underline{Q}_T - \underline{Q}_n) \cdot |\langle 0 | T \bar{\Phi}_{v_1}(\infty,0)\, \bar{\Phi}_{v_2}(0,\infty) | 0 \rangle|^2$$

$$\bar{\Phi}_v(\infty,0) = \mathcal{P} \exp\left(-ig \int_0^\infty dx'\, v\cdot A(\lambda v)\right)$$

Wilson line ⟷ eikonal approximation

Moments can now be reexpressed as

$$\ln S(b, \alpha_s(Q^2)) = \int_0^{Q^2} \frac{dk_\perp^2}{k_\perp^2} \ln\left(\frac{Q^2}{k_\perp^2}\right) A(\alpha_s(k_\perp^2)) \cdot \left(e^{-i\underline{k}_\perp \cdot \underline{b}} - 1\right) + \cdots$$

Collins-Soper (simplified)

$$= \int_{1/b^{*2}}^{Q^2} \frac{dk_\perp^2}{k_\perp^2} \ln\left(\frac{Q^2}{k_\perp^2}\right) A(\alpha_s(k_\perp^2)) \left(e^{-i\underline{k}_\perp \cdot \underline{b}} - 1\right)$$

$$+ E_{1R}(b^*, Q, \alpha_s(Q^2))$$

in PT:

$$E_{1R} \cong \int_0^{1/b^{*2}} \frac{dk_\perp^2}{k_\perp^2} \ln\left(\frac{Q^2}{k_\perp^2}\right) A(\alpha_s(k_\perp^2 \cdot \cdot (-k_\perp^2 b^2) + \cdots$$

reinterpret as

$$S_{1R} = b^{*2}\left[\ln(Q^2 b^{*2}) g_2 + g_1\right]$$

"replace μ" → new NP parameters

dim = (m^2)

$$\ln S = E_{PT}(b^*, Q) + E_{NP}(b^*, Q)$$ ← "substitution

$b^2 g_2 \ln Q$ "universal"

g_1 not (jet contributions:
no $\ln Q$ but $\ln x_i$;
OK (Yuan, Ladinsky))

$$\tilde{\sigma}(Q,b) = \tilde{\sigma}^{(reg)}_{p_T}(b)\, e^{-b^2(g_2 \ln Q + g_1)}$$

inverse transform for any $\tilde{\sigma}^{reg}$

$$\frac{d\sigma}{dQ^2 d^2Q_T} = \int \frac{d^2k_T}{(2\pi)^2} \frac{d\sigma^{reg}_{p_T}}{dQ^2 dQ_T}(Q_T - k_T) \cdot e^{-\frac{k_T^2}{g_2 \ln Q + g_1}}$$

power-suppressed but potentially significant → Q-dpdt k_T smearing
Extend to single-particle

practical (but soluable) issue:
double counting

other applications:

- B decay l enpoint — Korchemsky, G.S
- Perturb. Regge limit — Korchemskaya
- Elastic form factors

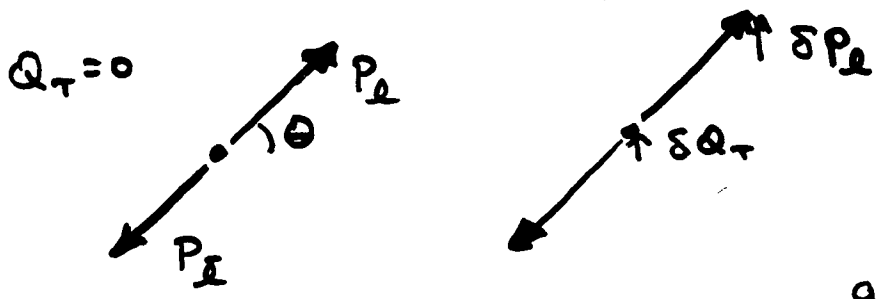
shrinkage s^{-bt} (!)

- Observations on Single-particle p_T Spectra

  (lots of phenomenological interest)

- $\frac{d\sigma}{dp_T^\ell}$ in DY for $Q_T \approx 0$

  boost $1+\cos^2\theta^*$. Power suppressed since $\Delta\theta \sim Q_T/Q$



- Case of direct γ in Compton process $q+G \to \gamma+q'$

  1) Factorize $\frac{d\sigma}{dp_T}^{D\gamma} = \frac{d\hat{\sigma}}{dp_T}^{Born}_{ab} \otimes J_1 \otimes J_2 \otimes S$

  $\frac{d\sigma}{dQ_T}^{Born} \sim \delta^2(Q_T)$ $\quad Q_T = p_T^\gamma + p_T^{q'}$

  2) Exponentiate S' including IS and FS interactions

  3) $k_T(S') = k_T(S')^{reg}_{p_T} + k_T(S')_{NP}$

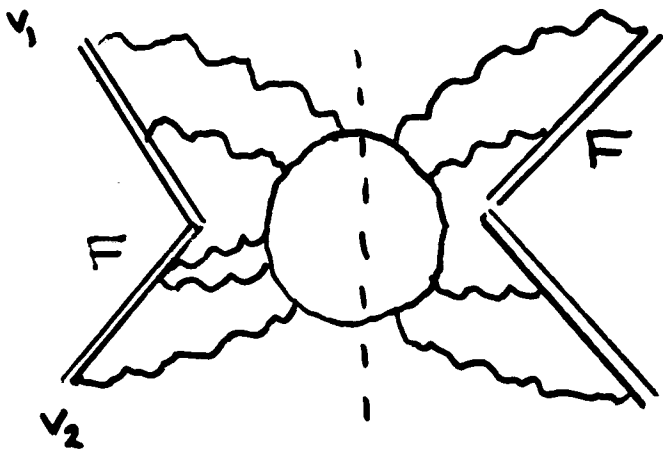
  4) $Q_T \approx -k_T$ (+ jet contributions)

  5) Boost as for p_T^ℓ in DY

- The operator? (DY)
- *"Nonlocal square of field strengths"*
- *Operators on a one-dimensional manifold:*

$$\langle 0 \Big| \Phi^\dagger_{v_2}(0,-\infty) \left(\vec{\mathcal{F}}_{v_1}(0) - \vec{\mathcal{F}}^\dagger_{v_2}(0) \right) \Phi_{v_1}(0,-\infty) \Big|^2 |0\rangle ,$$

$$\mathcal{F}^\alpha_v(x) = -ig \int_{-\infty}^0 ds \; \Phi_v(x, x+sp) \; v_\mu F^{\mu\alpha}(sp+x) \; \Phi_{-v}(x+ps, x)$$



"minimal" = 1 elim. nonlocality

Similarly, for thrust

- *Substitution:* $\ln \tilde{\sigma}_{1-T}(Q,N) = \ln \tilde{\sigma}^{reg} + \ln \tilde{\sigma}^{(1)}$

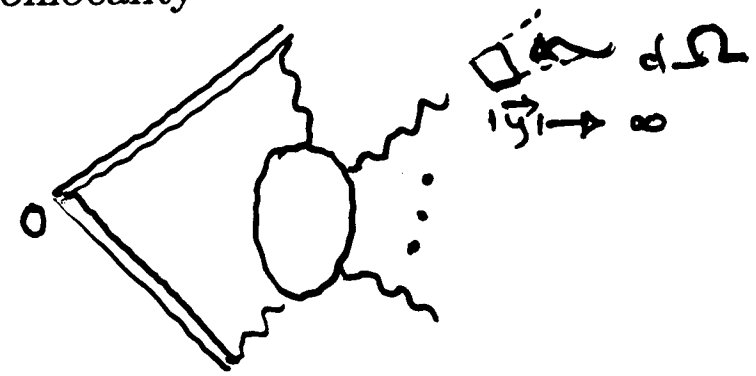
$$\ln \tilde{\sigma}_0(Q,N)^{(1)} \to \frac{N}{Q}\int_{-1}^{1} d\cos\theta\, f(\cos\theta)\, \mathcal{E}(\cos\theta)$$

$$\mathcal{E}(\cos\theta) = \langle 0 \mid W^{\dagger}_{v_1 v_2}(0)\, \Theta(\cos\theta)\, W_{v_1 v_2}(0) \mid 0\rangle .$$

- *W product of Wilson lines.*

$$\Theta(\cos\theta) = \lim_{|\vec{y}|\to\infty} \int_0^{\infty} dy_0 \epsilon_{ijk} \frac{d\Omega_{ij}(\theta,\vec{y})}{d\cos\theta}\, \theta_{0k}(y^{\mu}) .$$

- *$1/Q$ corrections for this class of w's from "universal" $\mathcal{E}(\cos\theta)$*
- *"maximal" nonlocality*



- *Relation to* sum of *1PI cross sections:*

$$\sum_{\substack{\text{particle}\\ \text{type}f}} \int_{-1}^{1} \frac{d\sigma_f(z,\theta)}{d\cos\theta}\, z dz .$$

Conclusions

- Special Demands of QCD $\leftrightarrow$ Opportunities to explore Quantum Field Theory
- Interconnection of Higher Orders with Power Corrections
- Nonlocal Operators
- Gaussian p_T's "natural"
- Just a speculative beginning

  (still looking for the "QCD apogee problem")