10. Starting with the non-linear Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi - D|\psi|^2\psi$$

show that there is a "soliton solution"

$$\psi(x,t) = Ae^{-i\Omega t}e^{imvx/\hbar}\operatorname{sech}\left(\frac{x-vt}{\Delta}\right)$$

where v is the wave-packet velocity,  $\hbar\Omega$  is the soliton energy, D is the potential, and  $\Delta$  is the soliton width.

- (a) Determine A by requiring  $\int_{-\infty}^{+\infty} dx \ |\psi(x,t)|^2 = 1$
- (b) Find  $\Omega$  and  $\Delta$  from the differential equation.
- 11. A current i(t) flows in a circular ring of radius a in the xy-plane. The current has time dependence

$$i(t) = \frac{i_0}{1 + \left(\frac{t}{\tau}\right)^2}$$

where  $i_0$  and  $\tau$  are constants. Convince yourself that the coordinate-space current density is

$$\vec{J}(\vec{r},t) = i(t) \ \hat{e}_{\phi} \ \frac{\delta(r-a)}{a} \ \delta\left(\theta - \frac{\pi}{2}\right)$$

Choose a coordinate system such that the field point is in the xz-plane. Find an expression for the total energy radiated by the system. Do as many of the integrals as you can. You may find the following formula for the Bessel function of order zero useful:

$$J_0(u) = \frac{1}{2\pi} \int_{-\pi}^{+\pi} d\phi \ e^{iu\cos\phi}$$

12. Consider a charge q undergoing simple harmonic motion in the z-direction.

$$z(t) = z_0 \cos \omega t$$

Find the Fourier coefficient  $\vec{J}_{\vec{k}_n,\omega_n}$  and simplify the factor

$$\left[\vec{J}_{\vec{k}_n,\omega_n}^*\vec{J}_{\vec{k}_n,\omega_n} - \left(\hat{n}\cdot\vec{J}_{\vec{k}_n,\omega_n}^*\right)\left(\hat{n}\cdot\vec{J}_{\vec{k}_n,\omega_n}\right)\right]$$