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The atomic nuclide with the highest mean binding energy

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It seems to be widely believed that ^{56}Fe is the most tightly bound atomic nuclide. Data are compiled showing that this is not so: Both ^{58}Fe and ^{62}Ni are more strongly bound than ^{56}Fe, with ^{62}Ni having the highest mean binding energy. Reasons for the erroneous favoring of ^{56}Fe are canvassed. The history of atomic-mass measurements does not provide an explanation, nor does the liquid-drop model; an analysis using this model shows that, if we could switch off shell effects and allow both the atomic number and the mass number to be fractional, but retain the underlying liquid-drop characteristics of real nuclei, then the most tightly bound nuclide would have $A \approx 58.3$, $Z \approx 26.6$. It seems that belief in ^{56}Fe as the most tightly bound atomic nuclide may originate from studies of stellar nucleosynthesis. © *1995 American Association of Physics Teachers.*

I. INTRODUCTION

The September 1990 issue of *Physics Today* contains an interesting article on supernovae, in which the following statement appears:[1] "As is well known, ^{56}Fe is the most strongly bound nucleus... ." Indeed, this does seem to be well known. It is widely taught. Several first-year-physics textbooks mention it.[2] It appears in the astrophysics literature[3–5] and in introductory astronomy textbooks.[6] In fact, the statement is incorrect. Both ^{58}Fe and ^{62}Ni are more strongly bound than ^{56}Fe, with ^{62}Ni having the highest average binding energy of all nuclides. The data to support this assertion are compiled in the next section. These data have been available for several decades, and indeed a note pointing out that ^{62}Ni is the most tightly bound nuclide has appeared in this journal previously.[7] Yet support for ^{56}Fe is very persistent, as the exchange between the authors of Refs. 1 and 7 in the letters column of *Physics Today* attests.[8] Why is this so? A first response to this question might be that perhaps early measurements of atomic masses pointed to ^{56}Fe as the nuclide with the highest mean binding energy. The history of mass measurements in the $A \approx 60$ mass region is outlined in Sec. II, showing that this simple answer is not correct.

Perhaps studies of the liquid-drop model of nuclei might lead one to believe that ^{56}Fe is the most tightly bound nuclide. There is some reason for exploring this possibility; for nickel has a closed proton shell, that is, its isotopes are more tightly bound than mass systematics would lead one to expect. In addition, the location of the mass parabola with respect to integer values of the atomic number Z varies slowly from one A value to the next. Perhaps this has an effect between iron and nickel. There seems to be no previously published consideration of these points, so they probably can not explain the support for ^{56}Fe. Nevertheless, Sec. III examines these effects by exploring the systematics of the $A=49$ to $A=71$ mass region using the semiempirical mass formula. This analysis shows that, if we were able to switch off shell effects and allow both A and Z to be nonintegral, then the most tightly bound nuclide would have $A \approx 58.3$, $Z \approx 26.6$. Thus the reason for the belief that ^{56}Fe is the most tightly bound nuclide is not to be found in the liquid-drop model.

Section IV suggests that the reason for the favoring of ^{56}Fe might lie in one of the successes of the theory of stellar nucleosynthesis, namely, the explanation of the relatively high stellar abundance of ^{56}Fe. The main points of the paper are summarized in Sec. V.

II. ATOMIC-MASS MEASUREMENTS IN THE REGION OF STRONGEST NUCLEAR BINDING

Figure 1 shows the average binding energy B/A of all nuclides in the mass range $49 \leq A \leq 71$ whose masses have been measured or estimated. The data come from the 1983 atomic-mass evaluation[9] and have been spread over the six panels of Fig. 1 for clarity. The curves in Fig. 1 are weighted-least-squares parabolae; these are included to guide the eye and for the purposes of Sec. III. Having compiled the data, it is a simple matter to find the nuclide with the highest B/A value for each mass number. The B/A values of these are plotted against mass number in Fig. 2. This figure shows clearly that ^{62}Ni and ^{58}Fe are both more strongly bound than ^{56}Fe, with ^{60} Ni lying fourth. The actual values of the mean binding energies of these four nuclides are given in Table I.

The mean binding energy of ^{62}Ni exceeds that of ^{56}Fe by about 4 keV/A. This is so large compared with the uncertainties that earlier atomic-mass evaluations must surely have told the same story. And indeed they did. Table II gives the

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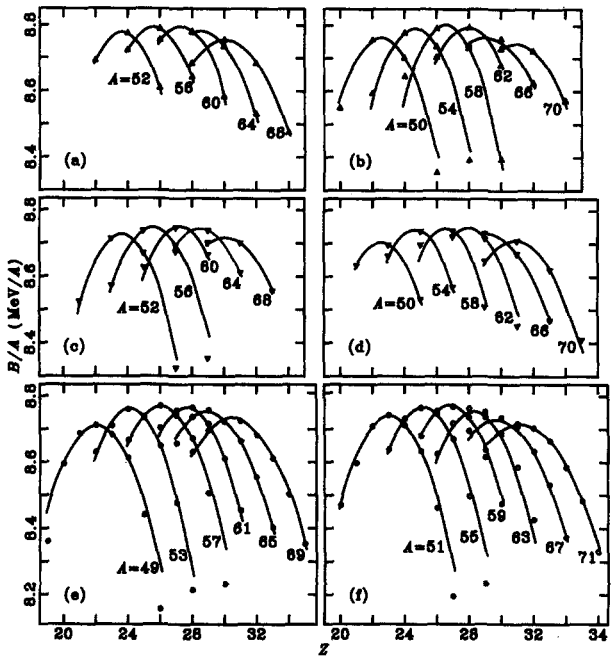

Fig. 1. Average binding energies B/A of nuclides with masses $49 \leq A \leq 71$ against atomic number Z. The data are spread over six panels for clarity: (a) $A=4n$, even Z, (b) $A=4n+2$, even Z, (c) $A=4n$, odd Z, (d) $A=4n+2$, odd Z, (e) $A=4n+1$, (f) $A=4n+3$, where n is an integer. Filled and open symbols alternate with A value. The data are from Wapstra and Audi (Ref. 9); all uncertainties are smaller than the size of the symbols. The curves are fitted parabolae described in Sec. III.

results from the 1955 and 1977 mass evaluations.[10,11] It is true that, in the 1955 evaluation, the uncertainties of the values for ^{62}Ni and ^{58}Fe overlap, and that the value for ^{62}Ni is high by 1.6 standard deviations. Nevertheless, the consistent message since about 1955 has been that ^{62}Ni is the most tightly bound of all nuclides.

Prior to 1955, the picture was somewhat more equivocal. The first published compilation of atomic masses was Aston's famous packing-fraction curve.[12] This appeared in 1927, before the discovery of the neutron, so the concept of binding energy did not then have its modern form. The packing fraction f of a nucleus is defined as

$$f=(M-A)/A,$$

where M is the atomic mass in suitable units. The average binding energy and f are closely related.[13] The masses of nuclides around $A=60$ were not available in 1927, and Aston speculated that the most strongly bound nuclide lay[12] "in the region of mass number 80."

During the next decade, many of the gaps in Aston's curve were filled in. Dempster's updated packing-fraction curve of

Table I. Average binding energy B/A of the four most tightly bound nuclides according to the 1983 atomic-mass evaluation (Ref. 9).

Nuclide	B/A (keV/A)
^{62}Ni	8794.60±0.03
^{58}Fe	8792.23±0.03
^{56}Fe	8790.36±0.03
^{60}Ni	8780.79±0.03

Table II. Average binding energy B/A of the four most tightly bound nuclides according to earlier atomic-mass evaluations.

Nuclide	B/A (keV/A) 1955[a]	1977[b]
^{62}Ni	8801±4	8794.63±0.04
^{58}Fe	8796±3	8792.28±0.04
^{56}Fe	8796±3	8790.39±0.04
^{60}Ni	8781±4	8780.82±0.04

[a]Reference 10.
[b]Reference 11.

1938 identified ^{48}Ti as the most strongly bound nuclide,[14] despite a measurement by Aston indicating that both ^{52}Cr and ^{58}Ni are more strongly bound.[15] Measurements during the 1940's seem to have consistently underestimated the binding energy of the iron and nickel isotopes. Consequently, during this period ^{52}Cr was regarded as the most strongly bound nuclide. Table III presents some of these early results. (Several of the entries in Table III require the calculation of B/A from packing fractions. For the sake of consistency this was done using data of the time: the neutron and hydrogen masses quoted by Flügge and Mattauch[17] and the mass-energy ratio in Evans' book.[21])

The principle point arising from Table III is that at no time did mass spectroscopy clearly identify ^{56}Fe as the nuclide with the highest mean binding energy. As Table III shows, it might have been possible to believe this for a brief period in 1951 following the publication of Wapstra's paper[19] advocating a substantial increase in the accepted binding energy of ^{56}Fe. That period ended with the report[20] of the remeasurement of the masses of the nickel isotopes.

Although the difference in the mean binding energies of ^{62}Ni and ^{56}Fe is very large compared with the precision of modern mass measurements, it amounts to less than 0.05%. This is presumably of negligible importance to the theory of supernovae.[1,7,8] Rather, it is interesting to wonder how errors like this survive and even flourish. For, although references are given in Sec. I to statements in print that ^{56}Fe is the most strongly bound nuclide,[2–6] in fact textbook authors often avoid making a definite identification. Many of the popular first-year-physics texts are in this category,[22] as are some of the standard undergraduate nuclear-physics texts.[23] Some texts contain statements to the effect that nuclear binding energies peak near $A=60$.[21,24,25] This is, of course, perfectly

Table III. Values of the average binding energies B/A of selected nuclides from early compilations and measurements.

Nuclide	B/A (keV/A) 1938[a]	1943–8[b]	1951[c]	1952–5[d]
^{48}Ti	8650±30	8690±20		8721±4
^{52}Cr	8740±30	8800±30	8774±6	8773±4
^{56}Fe	8630±50	8690±40	8790±5	8788±4
^{58}Fe				8785±9
^{60}Ni		8750±20	8786±5	8792±7
^{62}Ni		8740±20		8811±4

[a]References 15 and 16.
[b]References 17 and 18.
[c]Reference 19.
[d]References 20 and 21.

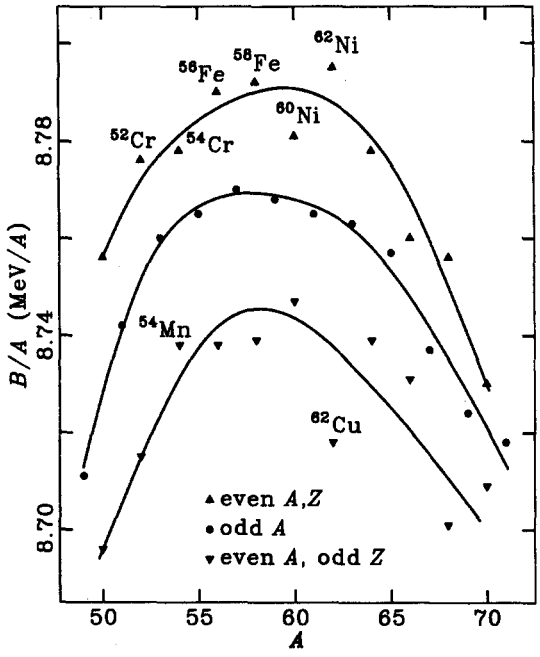

Fig. 2. Average binding energies B/A of nuclides lying closest to the apexes of the curves of Fig. 1 against mass number A. Nuclides mentioned in the text are identified. The curves are freehand interpolations to guide the eye.

correct. Unfortunately, many authors betray a bias for ^{56}Fe by going on to imply that $A=60$ corresponds to iron,[26] whereas nickel would be a more reasonable identification. Nevertheless, when the collection of textbooks is viewed as a whole, it cannot be claimed that textbook authors are responsible for perpetuating the notion that ^{56}Fe is the most tightly bound nuclide. Those doing so form a minority. On the other hand, the statements of this minority go almost unchallenged; I know of only three textbooks identifying ^{62}Ni as the most tightly bound nuclide.[27] All three are first-year texts. One is a new book (first published in 1994), and the other two are new editions of long-standing first-year texts. In each of these, the text pointing to ^{62}Ni is a recent amendation.[28] Perhaps we are seeing the beginning of a trend.

III. SYSTEMATICS OF THE $A \approx 60$ MASS REGION

The maximum mean binding energies shown in Fig. 2 are scattered about a general trend. Some of this fluctuation is undoubtably due to shell effects, but there is an additional effect. Close examination shows that the fluctuations are particularly pronounced in the even-A mass chains and that the fluctuations in the even-A, even-Z data are correlated with those in the even-A, odd-Z data. For example, ^{54}Cr lies below the general trend and ^{54}Mn lies above, whereas ^{62}Ni lies above the trend and ^{62}Cu below. These correlated fluctuations are due to the gradual drift from one A value to the next of the location of the apexes of the parabolae with respect to the integers. They are particularly marked in the even-A chains because the odd–even staggering means that the discretization in Z is to every second integer rather than to every integer.

The correlated fluctuations due to discretization in Z can be removed, and the shell effects minimized, by taking the average binding energy at the apex rather than that at the integer nearest to the apex (or nearest appropriate integer, in the case of even A). For this purpose, parabolae were fitted to the data, since, according to the semiempirical mass equation, the variation of mass with Z should be parabolic.[25,29,30] The actual function used to fit the data is

$$f(Z)=aZ^2+bZ+c+p,$$

where the pairing term p has the form

$$p=\begin{cases} \delta & A \text{ even, } Z \text{ even} \\ 0 & A \text{ odd} \\ -\delta & A \text{ even, } Z \text{ odd} \end{cases}$$

Thus the fitting function has three parameters (a,b,c) for odd-A mass chains and four parameters (a,b,c,δ) for even-A mass chains. There are between six and eight binding energies available for each A value. Since these have widely different uncertainties, weighted least-squares fits were performed, with the weights being taken as the inverse squares of the uncertainties quoted by Wapstra and Audi.[9] No attempt was made to constrain the parameters to a smooth variation with A.[31]

The results of the fitting are summarized as the curves in Fig. 1. It may be noted that most of the mass chains show evidence of a component with a quartic or higher-power dependence on Z. This is clearest in the odd-A mass chains: the fits consistently overestimate the binding energy of nuclides away from the apex or underestimate that of those near the apex. This was not investigated further; for a study covering a much wider mass range than that investigated here concludes that quartic terms are unimportant.[30]

Figure 3 shows the variation with mass number of the fitted binding energies $f(A)$ at the apex and of the location Z_A of the apex of the parabolae. (The first is shown for even-A even-Z nuclides only.) It is interesting that the variation of Z_A with A seems to be linear, but with a change of slope at about $A=57.3$. The fluctuations in the data of Fig. 3 are considerably less than those in Fig. 2, indicating that the fitting has largely removed the Z-discreteness effect and perhaps also smoothed out some shell effects. In this model, the most strongly bound system has $A=58.3$, $Z=26.6$. This is something like ^{58}Fe. Thus in a liquid-drop world, the most tightly bound nuclide would be approximately an iron isotope (the pairing term would favor $Z=26$ over $Z=27$), but it would not have mass 56. It is interesting that the effect of the drift in the parabola-apex location favors ^{62}Ni and disadvantages ^{56,58}Fe. This is in addition to the shell effect favoring the nickel isotopes.

IV. STELLAR NUCLEOSYNTHESIS

It is easy for an outsider to the field of astrophysics to form the impression that some of the most persistent supporters of ^{56}Fe as the most tightly bound nuclide are astrophysicists.[1,3–5,8] This partiality has even found its way into that most conservative repository of knowledge, the *Encyclopædia Britannica*, where one reads, in an article on the origins of the chemical elements,[32] "...iron, the element with the highest fractional binding energy..." An explanation of this favoring of ^{56}Fe possibly lies in one of the more remarkable achievements of modern astrophysics: the explanation of the observed stellar abundances of the elements.

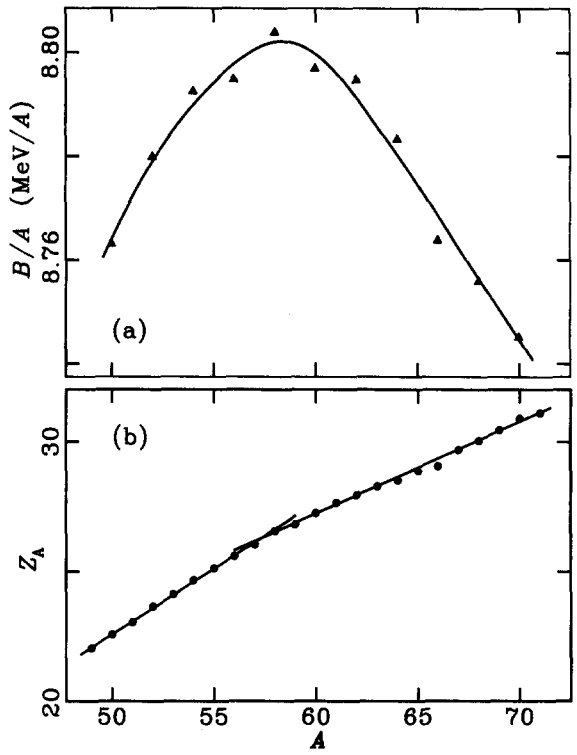

Fig. 3. Variation with mass number A of (a) the fitted average binding energy $f(Z_A)$ at the apexes of the parabolae in Figs. 1(a) and 1(b) and (b) the location Z_A of the apexes of the fitted parabolae for all mass chains. The curve in (a) is a freehand interpolation. In (b), the trend is shown as two straight lines.

A striking feature of the elemental abundances is the relatively high abundance of nuclides in what astrophysicists almost universally refer to as "the iron group." Of these nuclides, ^{56}Fe is by far the most abundant, with an abundance about an order of magnitude greater than those of its neighbors.[33] Among the light elements leading up to the iron group, the stable alpha-conjugate nuclides are particularly abundant. These are nuclides with mass numbers divisible by four and with equal numbers of protons and neutrons. Indeed, below the iron group, nuclides with A divisible by four are significantly more abundant than their neighbors, but this is not so for nuclides with $A>60$.

During the 1940's and 1950's, Hoyle, Fowler, and others showed that the abundances of elements in the iron group could be explained in detail by a process of statistical equilibrium among nuclear reactions occurring in the centers of very massive stars.[34–36] The relatively high abundances of mass chains with A divisible by four is attributed to another process: a sequence of alpha-capture reactions that is the main route from the region of ^{16}O toward the iron group. These two processes were dubbed the e process and the α process, respectively. Hoyle's early work on the e process[34] was carried out before the 1955 compilation of atomic masses,[10] but the seminal paper by Burbridge, Burbridge, Fowler, and Hoyle[35] on stellar nucleosynthesis used essentially modern masses, except for the masses of some unstable nuclides. The most important of these was the mass of ^{56}Ni. This is the alpha-conjugate nuclide with the highest mean binding energy, and so is the natural end point of the α process. A sufficiently accurate value of its mass was available by the early 1960's.[37,38]

Both the α process and the e process require very high temperatures, such as are found only in the cores of very massive stars. The ^{4}He nuclei necessary for the α process are produced by nuclear photodisintegration. The (γ,α) reaction on ^{20}Ne, the first to start, provides sufficient alphas at a temperature of about 10^9 K for the α process to begin.[39] Clearly, there is competition between alpha capture and photodisintegration. At the start of the α process, alpha capture dominates, driven by the release of binding energy. This released energy raises the temperature in the core of the star. The α process requires ever higher temperatures to overcome the increasing Coulomb barrier as the charge on the capturing nuclei rises, but higher temperature also means higher rates of photodisintegration. It has been estimated that a temperature of 8×10^9 K would be sufficient to produce very significant photodisintegration of iron nuclei, almost regardless of their mass.[40] However, this is also about the temperature required for an α-capture reaction on iron and is within an order of magnitude of the temperature at which the α process begins.

Thus the α and e processes yield mainly ^{56}Fe rather than ^{62}Ni not because ^{56}Fe is the most strongly bound nuclide, but because the competition between photodisintegration and charged-particle capture starts to favor photodisintegration at iron. This is a central result of the studies of the e process.[34–38, 41] Because of this, the original workers on the e process were not concerned to identify any particular nuclide as most tightly bound; for, by the time that the processes of stellar nucleosynthesis reach iron, binding-energy differences are no longer as important as they were earlier. Burbridge, Burbridge, Fowler, and Hoyle state simply[42] "...^{56}Fe lies near minimum of packing fraction curve."

The statement that ^{56}Fe is the most tightly bound nuclide enters the astrophysics literature in the mid 1960's.[4,5] The inference is that this explains why ^{56}Fe is the most abundant nuclide in the iron group. Truran, Cameron, and Gilbert state this explicitly.[4]

> "At temperatures $>3\times10^9$ K the photodisintegration of silicon will proceed rapidly, releasing protons, neutrons and alpha particles. The capture of these light particles on nuclei remaining in this region will result in the buildup of nuclei in the vicinity of iron. ^{56}Fe is favored in this instance by the fact that it has the maximum binding energy per nucleon."

This 1966 paper is the earliest reference that I could find to ^{56}Fe as the most tightly bound nuclide. The origin of the notion is perplexing, for two reasons. First, Clifford and Tayler had, about two years earlier in a major article on the e process, published a table of mean binding energies explicitly identifying ^{62}Ni as the nuclide with the highest mean binding energy.[38] Second, many authors at about this time, including Truran, Cameron, and Gilbert, emphasize that the most abundant product of the e process depends principally on the density, temperature, and mean ratio of protons to neutrons in the core of the star.[43] For reasonable values of these parameters, the e process can result in any of ^{54,56,58}Fe as the most abundant product. It requires some careful choices of parameter values to obtain ^{56}Fe as the most abundant, in agreement with observation.

Have we therefore traced the favoring of ^{56}Fe to works on stellar nucleosynthesis? It may seem so, although the reasons why authors in this field should have come to this conclusion are not entirely clear.

V. SUMMARY

It is again pointed out that ^{56}Fe is **not** the most tightly bound nuclide; both ^{58}Fe and ^{62}Ni have higher average binding energy, with ^{62}Ni having the highest B/A value of any nuclide. The measurements showing this were reported prior to the 1955 atomic-mass evaluation, yet many recent textbooks, including first-year-physics, undergraduate nuclear-physics and introductory astronomy texts, point to ^{56}Fe or to an iron isotope as the most tightly bound nuclide.

The reason for the favoring of ^{56}Fe was sought in the history of atomic-mass measurements and in the liquid-drop model of the nucleus. The history of mass measurements does not provide the answer. Fits of the semiempirical mass formula to the measured average binding energies were used to remove, as far as possible, shell effects and the effects of restricting Z and A to be integers. This analysis shows that, as concerns strength of binding, the underlying liquid-drop behavior of atomic nuclei does indeed tend to favor iron over nickel. However, the average liquid-drop binding energy peaks near $A=58$ rather than $A=56$. Not only does this analysis rule out the liquid-drop model as a possible explanation, but also there does not seem to have been any prior consideration of this point.

Finally, the early literature on stellar nucleosynthesis was examined. The statement that ^{56}Fe is the most tightly bound nuclide first appears in this field in the mid 1960's. Its origins are unclear, but it may well have been from here that the idea spread into the astrophysics literature generally and from there to introductory textbooks.

To a determinedly practical person, this whole question of whether ^{56}Fe or ^{62}Ni is the more tightly bound may seem unimportant: The difference in mean binding energy between these two is so small that it has no effect even on stellar nucleosynthesis. It is certainly irrelevant in other areas such as explanations of nuclear-power processes. Nevertheless, statements purporting to be fact ought to be fact, and the claim that ^{56}Fe is the most tightly bound nuclide does not pass this test.

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Determination of the half-life of ^{212}Po

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A gas lantern mantle provides a safe radioactive thorium source in an undergraduate laboratory. We describe its use in an experiment to measure the 300 ns half-life of ^{212}Po which occurs in the natural thorium decay chain. © *1995 American Association of Physics Teachers.*

I. INTRODUCTION

Many advanced undergraduate laboratories include a measurement of the muon lifetime by observing the pulses produced when cosmic-ray muons stop in a scintillator and subsequently β-decay. In this note we draw attention to a similar experiment which requires only a very small scintillator and yields gratifyingly accurate results within a short time: a measurement of the 300 ns lifetime of the short-lived isotope ^{212}Po which occurs in the natural decay chain of ^{232}Th. This experiment has been part of the junior/senior laboratory at the University of Minnesota for several years.

The radioactive ^{232}Th, in the form of thorium oxide, is provided by a gas lantern mantle,[1] which has a very low activity (<0.1 μCi). ^{232}Th has a half-life of 1.41×10^{10} years and its decay chain, which ends in stable ^{208}Pb, consists of six α and four β decays, all of which have lifetimes much shorter than the ^{232}Th decay. By far the shortest of these lifetimes is that for the α decay of ^{212}Po to the final state ^{208}Pb with a half-life of 300 ns.

The decay of interest can be identified by a pair of pulses generated in a scintillator within a short time interval. The first pulse corresponds to the β^- decay of ^{212}Bi to form ^{212}Po, with a β endpoint energy of 2.25 MeV. The second is the α decay of the ^{212}Po ($E_\alpha=8.78$ MeV) the half-life of which is to be measured. This pair of decays is responsible for 66.3% of the ^{212}Bi decays; the remaining 33.7% decays are via an α decay to ^{208}Tl, followed by beta decay to ^{208}Pb.

The principle of the method is to start a clock with any pulse from the scintillator, and to allow the subsequent pulse to stop it. If this second pulse does not arrive within some long time interval (1 or 2 μs, say) then the clock is started again. The distribution of time intervals between the pulse pairs is the characteristic exponential decay curve of the ^{212}Po isotope.

II. APPARATUS

For the lifetime measurement, we have used a piece of 1/4 or 1/2 in. thick plastic scintillator (NE102) placed directly on a photomultiplier tube. The results we show here are for a 2 in. diameter RCA 8575 photomultiplier and standard NIM electronics: a discriminator and a Lecroy model 3001 qVT multichannel analyzer, which can be used to digitize time intervals up to 1000 ns in 4 ns intervals. We also used a Lecroy model 3157 qVT interface unit with associated printer to get a hard copy of the data. Any laboratory with a muon decay experiment will have appropriate electronics for this time digitization. Some institutions may have access to surplus nuclear physics instrumentation. In that case, a standard time-to-pulse-height converter (often called a TAC for time-to-amplitude converter) could be used with a pulse

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