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UNITARY LOWEST WEIGHT REPRESENTATIONS OF THE NON-COMPACT SUPERGROUP $OSp(2m^*/2n)$

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Abstract

We give the oscillator construction of the unitary lowest weight representations of the non-compact supergroup $OSp(2m^*/2n)$ with the even subgroup $SO^*(2m) \times USp(2n)$. We also derive simple rules for determining the $SO^*(2m) \times USp(2n)$ decomposition of the lowest weight representations of $OSp(2m^*/2n)$. The particular examples of $OSp(4^*/2)$ with the even subgroup $SU(1,1) \times SU(2) \times SU(2)$ and $OSp(4^*/6)$ with the even subgroup $SU(1,1) \times SU(2) \times USp(6)$ are studied in greater detail.

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1 INTRODUCTION

Lie groups play a fundamental role in the formulation of physical theories based on symmetry principles. Over the last decade, Lie supergroups have come to play a comparable role in the formulation of theories based on supersymmetry. For example, they underlie all supergravity and superstring theories. The supergroups that occur in theories in which supersymmetry enters at a fundamental level are, in general, non-compact. For example, all space-time supergroups (Poincaré, anti-de Sitter, and conformal) are non-compact. In general for physical applications, one is interested in positive energy representations of these supergroups. The positive energy unitary representations of space-time supergroups correspond to their lowest weight unitary representations.

A general method for the oscillator construction of the unitary lowest weight representations of non-compact groups and non-compact supergroups was developed in references [1] and [2], respectively. This method was further developed and applied to supergravity and superstring theories in references [3-6]. In [3], the positive energy unitary representations of the seven dimensional, $N = 4$ anti-de Sitter supergroup $OSp(8^*/4)$ were constructed. The spectrum of the S^4 compactification [7] of the eleven dimensional supergravity [8] was shown to form an infinite set of short supermultiplets of $OSp(8^*/4)$. In [4], the positive energy unitary representations of the four dimensional, $N = 8$ anti-de Sitter supergroup $OSp(8/4, \Re)$ were constructed using the oscillator method and applied to the S^7 compactification of the eleven dimensional supergravity. In [5], the spectrum of the S^5 compactification of the chiral $N = 2$ supergravity in ten dimensions was first obtained and fitted into unitary supermultiplets of the five dimensional, $N = 8$ anti-de Sitter supergroup $SU(2, 2/4)$. A complete classification of finite dimensional conformal superalgebras in two dimensions was given in [6] and the lowest weight unitary representations of a large class of these superalgebras were studied [6]. Somewhat later, a detailed study of the unitary lowest weight representations of $OSp(2n/2m, \Re)$ and their decomposition with respect to the even subgroup $SO(2n) \times Sp(2m, \Re)$ was given in [9].

In the works quoted above, the lowest weight unitary representations of non-compact superalgebras g are constructed over the super Fock space of a set of bosonic and fermionic oscillators which transform in the fundamental representation of a maximal compact subsuperalgebra g^0. The Lie superalgebra g has a Jordan decomposition (3-grading) with respect to g^0. The oscillator method gives directly the construction of the lowest weight UIR's of g in terms of the finite dimensional irreducible representations of the maximal compact subsuperalgebra g^0. All non-compact Lie algebras have a Jordan structure (3-grading) with respect to their maximal compact subalgebras. This turns out not to be the case for non-compact Lie superalgebras. In [10], the oscillator method has been generalized to non-compact Lie superalgebras that have a Kantor structure (5-grading) with respect to a compact subsuperalgebra of maximal rank. Within this generalization, the oscillator method can be used to construct unitary lowest weight representations of all simple non-compact supergroups of the classical type as classified by Kac [11]. These include the exceptional and strange superalgebras [11].

In this paper we shall extend the results of [3] on $OSp(8^*/4)$ to the construction of the unitary lowest weight representations of the general non-compact Lie superalgebra $OSp(2m^*/2n)$ with the even subgroup $SO^*(2m) \times USp(2n)$. As mentioned, the oscillator method we shall use readily yields the decomposition of the unitary lowest weight representations of $OSp(2m^*/2n)$ in terms of the finite dimensional irreducible representations (irreps) of its maximal compact subsuperalgebra $U(m/n)$. We shall also derive simple rules for determining the decomposition of a unitary representation of $OSp(2m^*/2n)$ with respect to its even subgroup $SO^*(2m) \times USp(2n)$.

The plan of our paper is as follows: In section 2, we give the oscillator construction of the representations of the compact group $USp(2n)$ and the rules for determining the Dynkin labels of irreps of $USp(2n)$ from their lowest weight states. In section 3, we study the lowest weight representations of $SO^*(2m)$ following references [1] and [3]. In section 4, we give the oscillator construction of the unitary lowest weight representations of $OSp(2m^*/2n)$ and give the rules for their decomposition with respect to $SO^*(2m) \times USp(2n)$. Finally in section 5, we give a detailed application of the method to $OSp(4^*/2)$ with even subgroup $SU(2) \times SU(2) \times SU(1,1)$ and to $OSp(4^*/6)$ with even subgroup $SO^*(4) \times USp(6)$. Of these, the superalgebra $OSp(4^*/2)$ occurs as the finite dimensional subsuperalgebra of certain $N = 4$ conformal superalgebras [12].

2 UNITARY REPRESENTATIONS OF THE COMPACT GROUP $USp(2n)$

One can construct all the unitary representations of the compact group $USp(2n)$ over the Fock space of a set of fermionic oscillators transforming in the fundamental representation of its maximal subgroup $U(n)$. The Lie algebra g_F of $USp(2n)$ has a Jordan structure (3-graded decomposition) with respect to its maximal subalgebra $g_F^0 = U(n)$:

$$g_F = g_F^{-1} \oplus g_F^0 \oplus g_F^{+1} \tag{2-1}$$

In other words, the elements belonging to various graded subspaces obey the formal commutation rules

$$[g_F^{(m)}, g_F^{(n)}] \;\subseteq\; g_F^{(m+n)} \qquad\qquad m,n = 0, \mp 1 \tag{2-2}$$

and

$$g_F^{(m)} = 0 \qquad\qquad \text{for } \; |m| > 1. \tag{2-3}$$

Consider f pairs of fermionic oscillators $\alpha^\mu(r),\ \alpha_\mu(r),\ \beta^\mu(r)$, and $\beta_\mu(r)$ ($r = 1, \ldots, f$ and $\mu = 1, \ldots, n$) that satisfy the anti-commutation relations

$$\begin{aligned}
&\{\alpha_\mu(r), \alpha^\nu(s)\} = \delta_\mu^\nu \delta_{rs} && r, s, \ldots = 1, \ldots, f \\
&\{\beta_\mu(r), \beta^\nu(s)\} = \delta_\mu^\nu \delta_{rs} && \mu, \nu, \ldots = 1, \ldots, n \\
&\{\alpha_\mu(r), \beta_\nu(s)\} = \{\alpha_\mu(r), \beta^\nu(s)\} = \{\alpha_\mu(r), \alpha_\nu(s)\} = \{\beta_\mu(r), \beta_\nu(s)\} = 0
\end{aligned} \tag{2-4}$$

We shall choose the Fock vacuum $|0\rangle$ to be the state that is annihilated by all the $\alpha_\mu(r)$ and $\beta_\mu(r)$:

$$\alpha_\mu(r)|0\rangle \;=\; 0 \;=\; \beta_\mu(r)|0\rangle \tag{2-5}$$

Denoting the generators of $USp(2n)$ belonging to the grade -1, 0, and $+1$ spaces as $A_{\mu\nu}$, M^μ_ν, and $A^{\mu\nu}$, we can realize them in terms of the bilinears of the fermionic oscillators:

$$\begin{aligned}
M^\mu_\nu &= \vec{\alpha}^\mu \cdot \vec{\alpha}_\nu - \vec{\beta}_\nu \cdot \vec{\beta}^\mu \\
A_{\mu\nu} &= \vec{\alpha}_\mu \cdot \vec{\beta}_\nu + \vec{\alpha}_\nu \cdot \vec{\beta}_\mu \\
A^{\mu\nu} &= (A_{\mu\nu})^\dagger = \vec{\beta}^\mu \cdot \vec{\alpha}^\nu + \vec{\beta}^\nu \cdot \vec{\alpha}^\mu
\end{aligned} \tag{2-6}$$

where $\vec{\alpha}_\mu \cdot \vec{\beta}_\nu \equiv \sum_{r=1}^{f} \alpha_\mu(r)\beta_\nu(r)$, etc.

They satisfy the commutation relations

$$
\begin{aligned}
&[M^\mu_\nu, M^\rho_\sigma] = \delta^\rho_\nu M^\mu_\sigma - \delta^\mu_\sigma M^\rho_\nu \\
&[A_{\mu\nu}, M^\rho_\sigma] = \delta^\rho_\nu A_{\mu\sigma} + \delta^\rho_\mu A_{\nu\sigma} \\
&[A^{\mu\nu}, M^\rho_\sigma] = -\delta^\mu_\sigma A^{\nu\rho} - \delta^\nu_\sigma A^{\mu\rho} \\
&[A_{\mu\nu}, A^{\rho\sigma}] = -\delta^\sigma_\mu M^\rho_\nu + \delta^\rho_\mu M^\sigma_\nu - \delta^\rho_\nu M^\sigma_\mu + \delta^\sigma_\nu M^\rho_\mu
\end{aligned}
\tag{2-7}
$$

For the construction of the irreps of $USp(2n)$ over the Fock space of the fermionic oscillators we need to identify those states that transform in a definite representation of $U(n)$ and are annihilated by the operators $A_{\mu\nu}$. They are in general linear combinations of the states of the form

$$
[\alpha^\mu(1)]^{n_1}\cdots[\alpha^\nu(r)]^{n_r}[\beta^\lambda(r+1)]^{n_{r+1}}\cdots[\beta^\rho(f)]^{n_f}|0\rangle \tag{2-8}
$$

where $[\alpha^\mu(r)]^{n_r}$ stands for $\alpha^{\mu_1}(r)\alpha^{\mu_2}(r)\cdots\alpha^{\mu_r}(r) \qquad (n_r \le n)$. Clearly the states of this form can have at most f columns in their $U(n)$ Young Tableaux.

Consider now a set of states, denoted as $|\Omega\rangle$, that transforms irreducibly under the maximal subgroup $U(n)$ generated by M^μ_ν and is annihilated by the operators $A_{\mu\nu}$ belonging to the subspace g_F^{-1} of g_F. Then by acting repeatedly on $|\Omega\rangle$ with the operators $A^{\mu\nu}$ belonging to the subspace g_F^{+1} we generate a finite set of states:

$$
\{|\Omega\rangle,\ A^{\mu\nu}|\Omega\rangle,\ A^{\mu\nu}A^{\rho\lambda}|\Omega\rangle, \ldots\} \tag{2-9}
$$

(That this set is finite follows from the fact that $A^{\mu\nu}$ are built as bilinears of fermionic oscillators and $(A^{\mu\nu})^{2nf+1} = 0$). Then the above set of states forms an irrep of $USp(2n)$. The irreducibility is a consequence of the irreducibility of $|\Omega\rangle$ under $U(n)$. We shall refer to $|\Omega\rangle$ as the lowest weight "vector" of the irrep of $USp(2n)$.

Every irrep of $USp(2n)$ can be constructed this way by letting f take on all possible values ($f = 1, 2, \ldots, n, \ldots$). One can label the irreps of $USp(2n)$ by the Young-Tableaux labels of the corresponding lowest weight states $|\Omega\rangle$ and the number f. In Table I we give the Young-Tableaux labels of the lowest weight states $|\Omega\rangle$ and the Dynkin labels of the corresponding irreps of $USp(2n)$.

Lowest Weight State $\|\Omega\rangle$:	The Irrep of $USp(2n)$:
$(m_1, \ldots, m_n)_{Y-T}$	$(m_{n-1} - m_n, m_{n-2} - m_{n-1}, \ldots, m_1 - m_2, f - m_1)_D$

Table I: Above we give the Young-Tableaux label of the lowest weight state $|\Omega\rangle$ and the Dynkin labels of the corresponding irrep of $USp(2n)$. Note that $(m_1, \ldots, m_n)_{Y-T}$ denotes a Young Tableau of $U(n)$ which has m_i boxes in its i'th row.

We should stress that for a different number f of pairs of oscillators a lowest weight vector $|\Omega\rangle$ with a given $U(n) = SU(n) \times U(1)_F$ tableau leads to different irreps of $USp(2n)$. This is because the $U(1)_F$ charge of the lowest weight vector depends on f. The $U(1)_F$ generator is simply

$$\begin{aligned} Q_F &= M^\mu_\mu = \vec{\alpha}^\mu \cdot \vec{\alpha}_\mu - \vec{\beta}_\mu \cdot \vec{\beta}^\mu \\ Q_F &= \vec{\alpha}^\mu \cdot \vec{\alpha}_\mu + \vec{\beta}^\mu \cdot \vec{\beta}_\mu - nf \\ Q_F &= N_F - nf \end{aligned} \tag{2-10}$$

where N_F is the fermion number operator of all the oscillators. In other words, the $U(1)_F$ charge of $|\Omega\rangle$ is given by the number of boxes in its $U(n)$ tableau minus nf.

3 THE NON-COMPACT GROUP $SO^*(2m)$

The non-compact group $SO^*(2m)$ has a Jordan structure with respect to its maximal compact subgroup $U(m)$. Thus its algebra g_B has a 3-graded decomposition with respect to its subalgebra g_B^0 of $U(m)$:

$$g_B = g_B^{-1} \oplus g_B^0 \oplus g_B^{+1} \tag{3-1}$$

The unitary lowest weight representations of $SO^*(2m)$ can be simply obtained via the oscillator method [1, 3]. To do so we consider bosonic annihilation and creation operators $a_i(r),\ b_i(r)$ and $a^i(r),\ b^i(r)$ that satisfy the commutation relations:

$$\begin{aligned} &[a_i(r), a^j(s)] = \delta_i^j \delta_{rs} \qquad\qquad [b_i(r), b^j(s)] = \delta_i^j \delta_{rs} \\ &[a_i(r), a_j(s)] = [a_i(r), b_j(s)] = [a_i(r), b^j(s)] = [b_i(r), b_j(s)] = 0 \end{aligned} \tag{3-2}$$

where $i, j, ... = 1, 2, ...m$ and $r, s, ... = 1, 2, ...f$. Note that the creation operators are denoted with the upper indices $a^i(r) = a_i^\dagger(r)$. The generators of the $SO^*(2m)$ subalgebra are realized by the bilinears:

$$\begin{aligned} M_i^j &= \vec{a}^j \cdot \vec{a}_i + \vec{b}_i \cdot \vec{b}^j \\ A_{ij} &= \vec{a}_i \cdot \vec{b}_j - \vec{a}_j \cdot \vec{b}_i \\ A^{ij} &= \vec{a}^i \cdot \vec{b}^j - \vec{a}^j \cdot \vec{b}^i = (A_{ij})^\dagger \end{aligned} \tag{3-3}$$

where we defined $\vec{a}_i \cdot \vec{b}_j = \sum_{r=1}^f a_i(r) b_j(r)$. The generators $A_{ij},\ M_j^i,\ A^{ij}$ correspond to the subspaces g_B^{-1}, g_B^0 and g_B^{+1}, respectively. They satisfy the commutation relations:

$$
\begin{aligned}
&[M^i_j, M^k_l] = \delta^k_j M^i_l - \delta^i_l M^k_j \\
&[A_{ij}, M^k_l] = \delta^k_j A_{il} - \delta^k_i A_{jl} \\
&[A^{ij}, M^k_l] = \delta^i_l A^{jk} - \delta^j_l A^{ik} \\
&[A_{ij}, A^{kl}] = -\delta^k_j M^l_i + \delta^l_j M^k_i - \delta^l_i M^k_j + \delta^k_i M^l_j
\end{aligned}
\tag{3-4}
$$

The unitary irreducible representations (UIR's) of the lowest weight type of $SO^*(2m)$ can be constructed over the Fock space of the bosonic oscillators. One chooses a set of states $|\Omega\rangle$ (referred to as the "ground" state or the lowest weight state) that transforms irreducibly under the maximal compact subgroup $U(m)$ and is annihilated by the operators A_{ij} belonging to the g_B^{-1} subspace:

$$
A_{ij}|\Omega\rangle \;=\; 0 \tag{3-5}
$$

Then acting on $|\Omega\rangle$ repeatedly with the operators A^{ij} belonging to the g^{+1} subspace, we generate an infinite set of states that forms the basis of a UIR of $SO^*(2m)$:

$$
\{|\Omega\rangle,\ A^{ij}|\Omega\rangle,\ A^{ij}A^{kl}|\Omega\rangle,\ \ldots\} \tag{3-6}
$$

The irreducibility of the resulting representation of $SO^*(2m)$ follows from the irreducibility of $|\Omega\rangle$ under $U(m)$. Thus we can label the lowest weight UIR's of $SO^*(2m)$ by the $U(m)$ labels of the corresponding lowest weight state $|\Omega\rangle$ and the number f of pairs of oscillators. The states in the Fock space that are annihilated by the operators A_{ij} are of the general form:

$$
[a^i(1)]^{n_1}\cdots[a^j(r)]^{n_r}[b^k(r+1)]^{n_{r+1}}\cdots[b^l(f)]^{n_f}|0\rangle \tag{3-7}
$$

The lowest weight states $|\Omega\rangle$ of the UIR of $SO^*(2m)$ are obtained by projecting out the irreducible representations (irreps) of $U(m)$ from the above set of states. Clearly, the possible irreps of $U(m)$ one can obtain depends on the number of pairs of oscillators. For $f = 1$, the possible lowest weight states $|\Omega\rangle$ transform in the symmetric tensor representation of $U(m)$. The corresponding unitary lowest weight representations (ULWR's) are called the "doubleton irreps" [3]. Again for different f, a lowest weight vector $|\Omega\rangle$ with a given $U(m) = SU(m) \times U(1)_B$ tableau leads to different UIR's of $SO^*(2m)$. This is because the $U(1)_B$ charge of $|\Omega\rangle$ depends on f. The generator Q_B of $U(1)_B$ is defined as

$$\begin{aligned} Q_B &= M_i^i = \vec{a}^i \cdot \vec{a}_i + \vec{b}_i \cdot \vec{b}^i \\ Q_B &= \vec{a}^i \cdot \vec{a}_i + \vec{b}^i \cdot \vec{b}_i + mf \\ Q_B &= N_B + mf \end{aligned} \tag{3-8}$$

where N_B is the number operator of all the bosonic oscillators. The possible UIR's one can construct by this method by letting $f = 1, 2, \ldots, m$ exhaust the list of irreducible unitary lowest weight representations of $SO^*(2m)$.

4 UNITARY LOWEST WEIGHT REPRESENTATIONS OF $OSp(2m^*/2n)$

The non-compact Lie supergroup $OSp(2m^*/2n)$ has a Jordan structure with respect to its maximal compact subsupergroup $U(m/n)$. The even subgroup $U(m) \times U(n)$ of $U(m/n)$ is the subgroup with respect to which the even subgroup $SO^*(2m) \times USp(2n)$ of $OSp(2m^*/2n)$ has a Jordan structure.

$$g = g^{-1} \oplus g^0 \oplus g^{+1} \tag{4-1}$$

Denoting the generators of the grade -1, 0, and $+1$ subspaces as A_{AB}, M^A_B, and A^{AB} we can write the commutation relations of $OSp(2m^*/2n)$ as

$$\begin{aligned}
&[M^A_B, M^C_D] = \delta^C_B M^A_D - (-1)^{(\deg A+\deg B)(\deg C+\deg D)}\delta^A_D M^C_B \\
&[M^A_B, A_{CD}] = -\delta^A_C A_{BD} - \delta^A_D A_{CB} \\
&[M^A_B, A^{CD}] = \delta^C_B A^{AD} + \delta^D_B A^{CA} \\
&[A_{AB}, A^{CD}] = \delta^C_B M^D_A + \ 3 \text{ terms}
\end{aligned} \tag{4-2}$$

The lower and upper indices A, B, C, etc. correspond to the covariant and contravariant fundamental representation indices of $U(m/n)$ generated by M^A_B. One can realize the generators of $OSp(2m^*/2n)$ in terms of the superoscillators $\xi_A(r)$ and $\eta_A(r)$ whose first m components are bosonic and the remaining n components are fermionic:

$$\begin{aligned}
\xi_A(r) &= \begin{pmatrix} a_i(r) \\ \alpha_\mu(r) \end{pmatrix} \qquad & \xi^A(r) &= \begin{pmatrix} a^i(r) \\ \alpha^\mu(r) \end{pmatrix} \\
\eta_A(r) &= \begin{pmatrix} b_i(r) \\ \beta_\mu(r) \end{pmatrix} \qquad & \eta^A(r) &= \begin{pmatrix} b^i(r) \\ \beta^\mu(r) \end{pmatrix}
\end{aligned} \tag{4-3}$$

We have $\deg A = 0$ for a bosonic index ($A = 1, 2, \ldots, m$) and $\deg A = 1$ for a fermionic index ($A = m+1, \ldots, m+n$). The superoscillators satisfy the supercommutation relations:

$$\begin{aligned}
&\{\xi_A(r), \xi^B(s)] = \delta^B_A \delta_{rs} \qquad && A, B = 1, 2, \ldots, m, m+1, \ldots, m+n \\
&\{\eta_A(r), \eta^B(s)] = \delta^B_A \delta_{rs} \qquad && r, s = 1, \ldots, f \\
&\{\xi_A(r), \eta^B(s)] = \{\xi_A(r), \xi_B(s)] = \{\eta_A(r), \eta_B(s)] = 0 &&
\end{aligned} \tag{4-4}$$

where { ,] represents an anti-commutator among any two fermionic oscillators and a commutator otherwise. In terms of the superoscillators the generators take the form

$$
\begin{aligned}
A_{AB} &= \vec{\xi}_A \cdot \vec{\eta}_B - \vec{\eta}_A \cdot \vec{\xi}_B \\
A^{AB} &= (A_{AB})^\dagger = \vec{\eta}^B \cdot \vec{\xi}^A - \vec{\xi}^B \cdot \vec{\eta}^A \\
M^A_B &= \vec{\xi}^A \cdot \vec{\xi}_B + (-1)^{(\deg A)(\deg B)} \vec{\eta}_B \cdot \vec{\eta}^A
\end{aligned}
\tag{4-5}
$$

where $\vec{\xi} \cdot \vec{\eta} \equiv \sum_{r=1}^{f} \xi(r)\eta(r)$ etc.

The unitary lowest weight representations of $OSp(2m^*/2n)$ can be constructed over the super Fock space of these oscillators. Again one chooses a set of states transforming irreducibly under the maximal compact subsupergroup $U(m/n)$ and annihilated by the operators in the g^{-1} subspace:

$$A_{AB}|\Omega\rangle = 0$$

Then acting on $|\Omega\rangle$ repeatedly by the operators A^{AB} belonging to the g^{+1} subspace we generate an infinite set of states that form the basis of an UIR of the lowest weight type:

$$\{|\Omega\rangle, A^{AB}|\Omega\rangle, A^{AB}A^{CD}|\Omega\rangle, \ldots\}$$

Since all the irreps of $USp(2n)$ and all the lowest weight UIR's of $SO^*(2m)$ can be obtained by the oscillator method, we expect to obtain all the lowest weight UIR's of $OSp(2m^*/2n)$ by the above procedure by letting f take on all possible values. The possible lowest weight states are of the general form

$$[\xi^A(1)]^{n_1} \cdots [\xi^B(r)]^{n_r} [\eta^C(r+1)]^{n_{r+1}} \cdots [\eta^D(f)]^{n_f} |0\rangle$$

where $[\xi^A(1)]^n$ denotes $\xi^{A_1}(1)\xi^{A_2}(1) \cdots \xi^{A_n}(1)$. If we denote the contragradient fundamental representation of $U(m/n)$ by the supertableau $\boxslash$ then the state $[\xi^A(1)]^n|0\rangle$

transforms in the supersymmetric tensor representation of rank n:

$$[\xi^A(1)]^n|0\rangle \approx \underbrace{\boxslash\boxslash\boxslash\,\cdots\,\boxslash}_{n} \tag{4-6}$$

Thus the possible lowest weight states can have at most f rows in their supertableaux.

For the decomposition of the supertableaux of $U(m/n)$ in terms of the tableaux of its even subgroup $U(m)\times U(n)$ we refer to [13]. Here we give a few examples.

$$\begin{aligned}
U(m/n) \quad & \supset U(m)\times U(n) \\
\boxslash \quad & = (\square, 1) + (1, \square) \\
\boxslash\boxslash \quad & = (\square\square, 1) + (\square, \square) + \left(1, \begin{array}{c}\square\\ \square\end{array}\right) \\
\begin{array}{c}\boxslash\\ \boxslash\end{array} \quad & = \left(\begin{array}{c}\square\\ \square\end{array}, 1\right) + (\square, \square) + (1, \square\square) \\
\begin{array}{ll}\boxslash\boxslash\\ \boxslash\end{array} \quad & = \left(\begin{array}{ll}\square\square\\ \square\end{array}, 1\right) + (\square\square, \square) + \left(\square, \begin{array}{c}\square\\ \square\end{array}\right) \\
& \quad + \left(1, \begin{array}{ll}\square\square\\ \square\end{array}\right) + \left(\begin{array}{c}\square\\ \square\end{array}, \square\right) + (\square, \square\square)
\end{aligned} \tag{4-7}$$

The generators A^{AB} transform in the super-anti-symmetric tensor representation $\begin{array}{c}\boxslash\\ \boxslash\end{array}$ of $U(m/n)$. Since $\{A^{AB}, A^{CD}] = 0$ we find that the products of these operators transforms in the supersymmetrized product of the representation $\begin{array}{|c|}\hline \diagup\\ \hline \diagup\\ \hline\end{array}$.

$$(A^{AB}A^{CD}) \approx \left(\begin{array}{|c|}\hline \diagup\\ \hline \diagup\\ \hline\end{array} \otimes \begin{array}{|c|}\hline \diagup\\ \hline \diagup\\ \hline\end{array} \right)_S = \begin{array}{|c|c|}\hline \diagup&\diagup\\ \hline \diagup&\diagup\\ \hline\end{array} \oplus \begin{array}{|c|}\hline \diagup\\ \hline \diagup\\ \hline \diagup\\ \hline \diagup\\ \hline\end{array}$$

$$(A^{AB}A^{CD}A^{EF}) \approx \left(\begin{array}{|c|}\hline \diagup\\ \hline \diagup\\ \hline\end{array} \otimes \begin{array}{|c|}\hline \diagup\\ \hline \diagup\\ \hline\end{array} \otimes \begin{array}{|c|}\hline \diagup\\ \hline \diagup\\ \hline\end{array} \right)_S = \begin{array}{|c|c|c|}\hline \diagup&\diagup&\diagup\\ \hline \diagup&\diagup&\diagup\\ \hline\end{array} \oplus \begin{array}{|c|c|}\hline \diagup&\diagup\\ \hline \diagup&\diagup\\ \hline \diagup\\ \cline{1-1} \diagup\\ \cline{1-1}\end{array} \oplus \begin{array}{|c|}\hline \diagup\\ \hline \diagup\\ \hline \diagup\\ \hline \diagup\\ \hline \diagup\\ \hline \diagup\\ \hline\end{array} \tag{4-8}$$

$$(A^{AB}A^{CD}A^{EF}\cdots A^{GH}) \approx \left(\begin{array}{|c|}\hline \diagup\\ \hline \diagup\\ \hline\end{array} \otimes \begin{array}{|c|}\hline \diagup\\ \hline \diagup\\ \hline\end{array} \otimes \begin{array}{|c|}\hline \diagup\\ \hline \diagup\\ \hline\end{array} \otimes \cdots \otimes \begin{array}{|c|}\hline \diagup\\ \hline \diagup\\ \hline\end{array} \right)_S$$

The possible irreps of $U(m/n)$ one can construct over the super Fock space can have at most $2f$ rows. Therefore depending on f some of the supertableaux may not occur in the products as given above. For example if $f = 1$ then we have

$$A^{AB} \approx \begin{array}{|c|}\hline \diagup\\ \hline \diagup\\ \hline\end{array}$$

$$A^{AB}A^{CD} \approx \begin{array}{|c|c|}\hline \diagup&\diagup\\ \hline \diagup&\diagup\\ \hline\end{array} \tag{4-9}$$

$$A^{AB}A^{CD}\cdots A^{EF} \approx \begin{array}{|c|c|c|c|c|}\hline \diagup&\diagup&\diagup&\cdots&\diagup\\ \hline \diagup&\diagup&\diagup&\cdots&\diagup\\ \hline\end{array}$$

To obtain the $U(m/n)$ decomposition of a lowest weight UIR of $OSp(2m^*/2n)$ we need to simply tensor the supertableau of the lowest weight state $|\Omega\rangle$ with the supertableaux corresponding to the products of the operators A^{AB} belonging to the space g^{+1} of g.

For $f = 1$ possible lowest weight states are of the form

$$\underbrace{\xi^A\xi^B\cdots\xi^C}_{k}|0\rangle \approx |\underbrace{\begin{array}{|c|c|c|c|c|}\hline \diagup&\diagup&\diagup&\cdots&\diagup\\ \hline\end{array}}_{k}\rangle$$

which transform irreducibly under $U(m/n)$ for a given k. The $U(m/n)$ decomposition of the corresponding lowest weight UIR of $OSp(2m^*/2n)$ is obtained by tensoring the supertableaux given in (Eq. 4-9) with that of $|\Omega\rangle$:

$$
\begin{aligned}
|\Omega\rangle &= \xi^A \cdots \xi^B |0\rangle = \Big|\; \underbrace{\boxed{\diagup}\boxed{\diagup}\boxed{\diagup}\cdots\boxed{\diagup}}_{k} \;\Big\rangle \\
A^{AB}|\Omega\rangle &= \left|\; \begin{array}{l} \boxed{\diagup}\boxed{\diagup}\boxed{\diagup}\cdots\boxed{\diagup} \\ \boxed{\diagup} \end{array} \;\right\rangle \\
A^{AB}A^{CD}|\Omega\rangle &= \left|\; \begin{array}{l} \boxed{\diagup}\boxed{\diagup}\boxed{\diagup}\cdots\boxed{\diagup} \\ \boxed{\diagup}\boxed{\diagup} \end{array} \;\right\rangle \qquad\qquad (4\text{-}10) \\
&\vdots \qquad\qquad \vdots \\
\underbrace{A^{AB}A^{CD}\cdots A^{EF}}_{n}|\Omega\rangle &= \left|\; \begin{array}{l} \underbrace{\begin{array}{l}\boxed{\diagup}\boxed{\diagup}\boxed{\diagup}\cdots\boxed{\diagup} \\ \boxed{\diagup}\boxed{\diagup}\boxed{\diagup}\cdots\boxed{\diagup}\end{array}}_{n}\begin{array}{l}\underbrace{\boxed{\diagup}\cdots\boxed{\diagup}}_{k} \\ \end{array} \end{array} \;\right\rangle
\end{aligned}
$$

These representations of $OSp(2m^*/2n)$ are called the doubleton supermultiplets.

The infinite set of irreps of $U(m/n)$ that make up a lowest weight UIR of $OSp(2m^*/2n)$ corresponds to a finite number of irreps of the even subgroup $SO^*(2m) \times USp(2n)$. For most physical applications it is important to have a decomposition of the UIR of a supergroup with respect to the even subgroup. Here we shall give the simple rules for obtaining such a decomposition for the lowest weight UIR's of $OSp(2m^*/2n)$.

The unitary irreps of $SO^*(2m)$ that occur in a lowest weight UIR of $OSp(2m^*/2n)$ are all of the lowest weight type. Thus to obtain a decomposition of a lowest weight UIR with respect to the even subgroup we need simply to identify all the lowest weight vectors of $SO^*(2m) \times USp(2n)$ in the infinite set of states obtained by the repeated action of A^{AB} on the lowest weight vector $|\Omega\rangle$. Then from these simultaneous lowest weight vectors of $SO^*(2m)$ and $USp(2n)$ we can read off the full $SO^*(2m) \times USp(2n)$ content of the lowest weight UIR's of $OSp(2m^*/2n)$. First the lowest weight vector $|\Omega\rangle$ of $OSp(2m^*/2n)$ is also a simultaneous lowest weight vector of both $SO^*(2m)$

and $USp(2n)$. In general $|\Omega\rangle$ corresponds to a lowest weight vector of a reducible representation of $SO^*(2m) \times USp(2n)$. To obtain the decomposition into the irreps of $SO^*(2m) \times USp(2n)$ one needs to decompose the irrep $|\Omega\rangle$ of $U(m/n)$ into the irreps of its even subgroup $U(m) \times U(n)$. The additional simultaneous lowest weight vectors of $SO^*(2m)$ and $USp(2n)$ can be found by the action of the odd generators $A^{i\mu}$ belonging to the subspace g^{+1}. The other operators A^{ij} and $A^{\mu\nu}$ belonging to the g^{+1} subspace act within a given irrep of the subgroups $SO^*(2m)$ and $USp(2n)$, respectively. The operators $A^{i\mu}$ transform in the (m, n) representation of the $U(m) \times U(n)$ subgroup of $U(m/n)$ with the Young Tableau $(\,\Box\,,\,\Box\,)$. Since $\{A^{i\mu}, A^{j\nu}\} = 0$ the product of the operators $A^{i\mu}$ transforms in the anti-symmetrized powers of the representation $(\,\Box\,,\,\Box\,)$ of $U(m) \times U(n)$:

$$
\begin{aligned}
A^{i\mu} &\approx (\,\Box\,,\,\Box\,) \\
A^{i\mu}A^{j\nu} &\approx [(\,\Box\,,\,\Box\,) \otimes (\,\Box\,,\,\Box\,)]_A = (\,\Box\Box\,,\,\begin{smallmatrix}\Box\\\Box\end{smallmatrix}\,) + (\,\begin{smallmatrix}\Box\\\Box\end{smallmatrix}\,,\,\Box\Box\,) \\
A^{i\mu}A^{j\nu}A^{k\rho} &\approx (\,\Box\Box\Box\,,\,\begin{smallmatrix}\Box\\\Box\\\Box\end{smallmatrix}\,) + (\,\begin{smallmatrix}\Box\Box\\\Box\;\;\end{smallmatrix}\,,\,\begin{smallmatrix}\Box\Box\\\Box\;\;\end{smallmatrix}\,) + (\,\begin{smallmatrix}\Box\\\Box\\\Box\end{smallmatrix}\,,\,\Box\Box\Box\,) \qquad (4\text{-}11) \\
&\;\;\vdots \qquad\qquad \vdots \\
\underbrace{A^{i\mu}\cdots A^{\ell\lambda}}_{k \text{ copies}} &\approx (\,\Box\,,\,\Box\,)^k_A
\end{aligned}
$$

In taking the anti-symmetrized products of the representation $(\,\Box\,,\,\Box\,)$ one has to keep in mind that $U(m)$ and $U(n)$ tableaux with more than m and n rows vanish, respectively. Furthermore, if $2f$ is less than m (or n), then the states whose $U(m)$ (or $U(n)$) tableaux have more than $2f$ rows (or $2f$ columns) do not occur in the super Fock space.

Consider now the case $f = 1$. The lowest weight states $|\Omega\rangle$ of $OSp(2m^*/2n)$ all have $U(m/n)$ supertableaux with one row as explained above. Under the even subgroup $U(m) \times U(n)$ they decompose as

$$
\begin{aligned}
U(m/n) \quad & \supset U(m) \times U(n) \\
|\underbrace{\boxed{\diagup}\boxed{\diagup}\boxed{\diagup}\cdots\boxed{\diagup}}_{k}\rangle \quad & = |(\underbrace{\square\square\square\cdots\square}_{k},1)\rangle \oplus |(\underbrace{\square\square\square\cdots\square}_{k-1}, \square)\rangle \\
& \oplus |(\underbrace{\square\square\square\cdots\square}_{k-2}, \begin{smallmatrix}\square\\ \square\end{smallmatrix})\rangle \oplus \cdots \oplus |(1, \left.\begin{smallmatrix}\square\\ \square\\ \square\\ \vdots\\ \square\end{smallmatrix}\right\} k)\rangle
\end{aligned}
\tag{4-12}
$$

The irreps of $U(m) \times U(n)$ that occur in the decomposition above correspond to the lowest weight states of $SO^*(2m) \times USp(2n)$. To obtain all such lowest weight states one needs to tensor the anti-symmetric powers of the supersymmetry operators $A^{i\mu} = (\,\square,\ \square\,)$ with the above states and identify the resulting $SO^*(2m) \times USp(2n)$ lowest weight vectors. Now since $f = 1$

$$
\begin{aligned}
A^{i\mu}A^{j\nu} \quad & \approx (\,\square\square,\ \begin{smallmatrix}\square\\ \square\end{smallmatrix}\,) + (\,\begin{smallmatrix}\square\\ \square\end{smallmatrix},\ \square\square\,) \\
A^{i\mu}A^{j\nu}A^{k\rho} \quad & \approx (\,\square\square\square,\ \begin{smallmatrix}\square\\ \square\\ \square\end{smallmatrix}\,) + (\,\begin{smallmatrix}\square\square\\ \square\end{smallmatrix},\ \begin{smallmatrix}\square\square\\ \square\end{smallmatrix}\,) \\
\vdots \quad & \qquad\qquad \vdots
\end{aligned}
\tag{4-13}
$$

The states with more than two rows (columns) in their $U(m)$ ($U(n)$) Young Tableaux do not occur in the super Fock space. Furthermore for $f = 1$ possible lowest weight vectors of $SO^*(2m)$ ($USp(2n)$) have only one row (one column) in their $U(m)$ ($U(n)$) tableaux. Under this constraint it is straightforward to identify all the simultaneous lowest weight vectors of $SO^*(2m)$ and $USp(2n)$ that occur in the tensor product of the states in (Eq. 4-13) and (Eq. 4-12). In Table II we give the $SO^*(2m) \times USp(2n)$ decomposition of the lowest weight UIR's of $OSp(2m^*/2n)$ for $f = 1$.

$U(m) \times U(n)$	$SO^*(2m) \times USp(2n)$
$\sum_{\mu=0}^{n} (k+\mu, 0, \ldots, 0)_{Y-T} \otimes (\underbrace{1, 1, \ldots, 1}_{\mu \text{ times}}, 0, \ldots, 0)_{Y-T}$	$\sum_{\mu=0}^{n} (k+\mu, 0, \ldots, 0)_{Y-T} \otimes (0, \ldots, 0, 1, \underbrace{0, \ldots, 0}_{\mu})_D$
$\oplus \sum_{\mu=0}^{n-1} (k+\mu-1, 0, \ldots, 0)_{Y-T} \otimes (\underbrace{1, 1, \ldots, 1}_{\mu+1}, 0, \ldots, 0)_{Y-T}$	$\oplus \sum_{\mu=0}^{n-1} (k+\mu-1, 0, \ldots, 0)_{Y-T} \otimes (0, \ldots, 0, 1, \underbrace{0, \ldots, 0}_{\mu+1})_D$
$\oplus \sum_{\mu=0}^{n-2} (k+\mu-2, 0, \ldots, 0)_{Y-T} \otimes (\underbrace{1, 1, \ldots, 1}_{\mu+2}, 0, \ldots, 0)_{Y-T}$	$\oplus \sum_{\mu=0}^{n-2} (k+\mu-2, 0, \ldots, 0)_{Y-T} \otimes (0, \ldots, 0, 1, \underbrace{0, \ldots, 0}_{\mu+2})_D$
$\oplus \ \ldots$	$\oplus \ \ldots$
$\oplus \sum_{\mu=0}^{n-k} (\mu, 0, \ldots, 0)_{Y-T} \otimes (\underbrace{1, 1, \ldots, 1}_{\mu+k}, 0, \ldots, 0)_{Y-T}$	$\oplus \sum_{\mu=0}^{n-k} (\mu, 0, \ldots, 0)_{Y-T} \otimes (0, \ldots, 0, 1, \underbrace{0, \ldots, 0}_{\mu+k})_D$

Table II: Above we give the $SO^*(2m) \times USp(2n)$ decomposition of the lowest weight UIR of $OSp(2m^*/2n)$ defined by the lowest weight vector with the $U(m/n)$ supertableau $|\underbrace{\boxed{\diagup}\boxed{\diagup}\boxed{\diagup}\boxed{\cdots}\boxed{\diagup}}_{k}\rangle$ and $f = 1$. The first column gives the $U(m) \times U(n)$ Young Tableaux of the lowest weight states and the second column gives the corresponding $SO^*(2m) \times USp(2n)$ irreps. $SO^*(2m)$ irreps are denoted by the Young Tableaux of their lowest weight vectors and f which is one in this case. $USp(2n)$ irreps are denoted by their Dynkin labels. These UIR's of $OSp(2m^*/2n)$ are called the doubleton supermultiplets.

The $SO^*(2m) \times USp(2n)$ content of a general lowest weight UIR of $OSp(2m^*/2n)$ can similarly be obtained. The simple rules for this can be summarized as follows:

1. Decompose the irrep of $U(m/n)$ corresponding to the lowest weight vector $|\Omega\rangle$ into its irreducible components under the even subgroup $U(m) \times U(n)$. This is

done simply by decomposing the supertableau of $|\Omega\rangle$ into the Young Tableaux of $U(m) \times U(n)$. The $U(1)$ charges associated with the two $U(1)$ factors in $U(m) \times U(n)$ depend on f. Two lowest weight vectors with the same $U(m) \times U(n)$ Young Tableaux but different f will lead to inequivalent UIR's of $SO^*(2m) \times USp(2n)$.

2. Act on the lowest weight state $|\Omega\rangle$ with the supersymmetry generator $A^{i\mu}$ repeatedly and identify the additional lowest weight vectors of $SO^*(2m) \times USp(2n)$ thus generated. Here recall that the lowest weight vectors of $SO^*(2m)$ and $USp(2n)$ have at most f rows and f columns respectively.

3. From the lowest weight vectors thus obtained one can read off the corresponding $SO^*(2m) \times USp(2n)$ content of the UIR of $OSp(2m^*/2n)$ using Table I and the results of Section 3.

5 THE APPLICATION OF THE METHOD TO $OSp(4^*/2)$ AND $OSp(4^*/6)$

The Lie superalgebra $OSp(4^*/2)$ has a Jordan decomposition with respect to its maximal compact subsupergroup $U(2/1)$. Its generators can be written in a $U(2/1)$ covariant manner as follows:

$$OSp(4^*/2) = A_{AB} \oplus M^A_B \oplus A^{AB} \tag{5-1}$$

where M^A_B are the generators of $U(2/1)$ corresponding to grade zero subspace and A^{AB} and A_{AB} are the grade one and minus one subspaces, respectively. In terms of the superoscillators transforming in the fundamental representation of $U(2/1)$ the generators of $OSp(4^*/2)$ can be realized as

$$\begin{aligned}
A_{AB} &= \vec{\xi}_A \cdot \vec{\eta}_B - \vec{\eta}_A \cdot \vec{\xi}_B \\
A^{AB} &= (A_{AB})^\dagger = \vec{\eta}^B \cdot \vec{\xi}^A - \vec{\xi}^B \cdot \vec{\eta}^A \\
M^A_B &= \vec{\xi}^A \cdot \vec{\xi}_B + (-1)^{(\deg A)(\deg B)} \vec{\eta}_B \cdot \vec{\eta}^A
\end{aligned} \tag{5-2}$$

where

$$\begin{aligned}
&\vec{\xi}_A = \begin{pmatrix} \vec{a}_i \\ \\ \vec{\alpha} \end{pmatrix} \qquad && \vec{\xi}^A = \begin{pmatrix} \vec{a}^i \\ \\ \vec{\alpha}^\dagger \end{pmatrix} \qquad && i = 1, 2. \\
&\vec{\eta}_A = \begin{pmatrix} \vec{b}_i \\ \\ \vec{\beta} \end{pmatrix} && \vec{\eta}^A = \begin{pmatrix} \vec{b}^i \\ \\ \vec{\beta}^\dagger \end{pmatrix} &&
\end{aligned} \tag{5-3}$$

and $\vec{\xi}_A \cdot \vec{\eta}_B \equiv \sum_{r=1}^{p} \xi_A(r) \eta_B(r)$, etc.

The fermion bilinears generate the $USp(2) = SU(2)_S$ subalgebra:

$$\begin{aligned}
&F^- = \vec{\alpha} \cdot \vec{\beta}, \qquad F^+ = \vec{\beta}^\dagger \cdot \vec{\alpha}^\dagger, \qquad F^0 = \tfrac{1}{2}(\vec{\alpha}^\dagger \cdot \vec{\alpha} - \vec{\beta} \cdot \vec{\beta}^\dagger) \\
&[F^-, F^+] = -2\ F^0, \qquad [F^0, F^\pm] = \pm F^\pm
\end{aligned} \tag{5-4}$$

The bosonic bilinears generate the $SO^*(4)$ subalgebra. Since the Lie algebra $SO^*(4)$ is isomorphic to $SU(1,1) \oplus SU(2)_L$ we define:

$$\begin{aligned}
B^- &= \epsilon^{ij} \vec{a}_i \cdot \vec{b}_j = \vec{a}_1 \cdot \vec{b}_2 - \vec{a}_2 \cdot \vec{b}_1 \\
B^+ &= \epsilon_{ij} \vec{a}^i \cdot \vec{b}^j = \vec{a}^1 \cdot \vec{b}^2 - \vec{a}^2 \cdot \vec{b}^1 \\
B^0 &= \tfrac{1}{2}(\vec{a}^i \cdot \vec{a}_i + \vec{b}_i \cdot \vec{b}^i) \\
I^i_j &= \vec{a}^i \cdot \vec{a}_j + \vec{b}^i \cdot \vec{b}_j - \tfrac{1}{2}\delta^i_j(\vec{a}^k \cdot \vec{a}_k + \vec{b}^k \cdot \vec{b}_k)
\end{aligned} \tag{5-5}$$

The operators B^-, B^+, and B^0 generate the $SU(1,1)$ subalgebra and I^i_j generate the $SU(2)_L$ subalgebra of $SO^*(4)$. We have

$$\begin{aligned}
&[B^-, B^+] = 2\ B^0, \qquad && [B^0, B^\mp] = \mp B^\mp \\
&[I^i_j, I^k_\ell] = \delta^k_j I^i_\ell - \delta^i_\ell I^k_j, && i, j, k, \ell = 1, 2
\end{aligned} \tag{5-6}$$

For $f=1$ the possible lowest weight states of $OSp(4^*/2)$ are of the form

$$\xi^A\xi^B\cdots\xi^C|0\rangle \tag{5-7}$$

which corresponds to the irrep of $U(2/1)$ with the supertableaux $\boxed{\diagup}\boxed{\diagup}\boxed{\diagup}\boxed{\;\cdots\;}\boxed{\diagup}$.

In terms of the even subgroup $U(1)_B \times SU(2)_L \times U(1)_F$ the above supertableaux decomposes as

$$\begin{aligned} |\underbrace{\boxed{\diagup}\boxed{\diagup}\boxed{\diagup}\boxed{\;\cdots\;}\boxed{\diagup}}_{n}\rangle &= |h=\tfrac{n}{2}+1, \underbrace{\boxed{}\boxed{}\boxed{}\boxed{\;\cdots\;}\boxed{}}_{n}, s_3=-\tfrac{1}{2}\rangle \\ &\oplus |h=\tfrac{n+1}{2}, \underbrace{\boxed{}\boxed{}\boxed{}\boxed{\;\cdots\;}\boxed{}}_{n-1}, s_3=0\rangle \end{aligned} \tag{5-8}$$

where h and s_3 are the eigenvalues of B^0 and F^0, respectively. Denoting the spin of the irreps of $SU(2)$ generated by I^i_j as ℓ we can rewrite the above decomposition as

$$|\underbrace{\boxed{\diagup}\boxed{\diagup}\boxed{\diagup}\boxed{\;\cdots\;}\boxed{\diagup}}_{n}\rangle = |\tfrac{n}{2}+1, \ell=\tfrac{n}{2}, s_3=-\tfrac{1}{2}\rangle \oplus |\tfrac{n+1}{2}, \ell=\tfrac{n-1}{2}, s_3=0\rangle \tag{5-9}$$

Both states in the above decomposition are simultaneous lowest weight states of both $SO^*(4)$ and $SU(2)$. The irreps generated from them by the action of B^+ and F^+ are

$$\begin{aligned} |\tfrac{n}{2}+1, \ell=\tfrac{n}{2}, s_3=-\tfrac{1}{2}\rangle &\implies |h=\tfrac{n}{2}+1, \ell=\tfrac{n}{2}, s=\tfrac{1}{2}\rangle \\ |\tfrac{n+1}{2}, \ell=\tfrac{n-1}{2}, s_3=0\rangle &\implies |h=\tfrac{n+1}{2}, \ell=\tfrac{n-1}{2}, s=0\rangle \end{aligned} \tag{5-10}$$

where the irreps of $SU(1,1)$ were labelled by the eigenvalue h of B^0 on their lowest weight vectors. Both irreps of $SU(1,1)\times SU(2)_L\times SU(2)_S$ belong to the supermultiplet of $OSp(4^*/2)$ generated by the lowest weight vector given above.

To find the other irreps of $SU(1,1)\times SU(2)_L\times SU(2)_S$ in the same supermultiplet we have to act with supersymmetry generators on the above irreps. This is readily achieved by identifying the lowest weight vectors of $SU(1,1)\times SU(2)_L\times SU(2)_S$ generated by the action of the supersymmetry operators $\vec{a}^i\cdot\vec{\beta}^\dagger + \vec{b}^i\cdot\vec{\alpha}^\dagger$ on the lowest weight vectors given in (Eq. 5-10). For $f = 1$ we find the following additional lowest weight vector

$$|\tfrac{n}{2} + \tfrac{3}{2}, \ell = \tfrac{n}{2} + \tfrac{1}{2}, s_3 = 0\rangle \tag{5-11}$$

Thus the full $SU(1,1) \times SU(2)_L \times SU(2)_S$ content of the unitary supermultiplet of $OSp(4^*/2)$ with the lowest weight vector (Eq. 5-7) is

$$\begin{aligned} &|h = \tfrac{n}{2} + 1, \ell = \tfrac{n}{2}, s = \tfrac{1}{2}\rangle \quad \oplus \\ &|h = \tfrac{n-1}{2}, \ell = \tfrac{n-1}{2}, s = 0\rangle \quad \oplus \\ &|h = \tfrac{n+3}{2}, \ell = \tfrac{n+1}{2}, s = 0\rangle \end{aligned} \tag{5-12}$$

For $f = 2$ the possible lowest weight vectors are of the form

$$\begin{gathered} \xi^A(1) \cdots \xi^B(1) \xi^C(2) \cdots \xi^D(2)|0\rangle \\ \text{or of the form} \\ \xi^A(1) \cdots \xi^B(1) \eta^C(2) \cdots \eta^D(2)|0\rangle \end{gathered} \tag{5-13}$$

The irreps of $U(2/1)$ given by the states of the form (Eq. 5-13) can be represented by the supertableaux with two rows

$$\left| \begin{array}{|c|c|c|c|c|c|c|} \hline \diagup & \diagup & \diagup & \cdots & \diagup & \cdots & \diagup \\ \hline \diagup & \diagup & \diagup & \cdots & \diagup \\ \cline{1-5} \end{array} \right\rangle \tag{5-14}$$

The lowest weight states whose supertableaux have one row for $f = 2$ lead to different unitary supermultiplets of $OSp(4^*/2)$ than those for $f = 1$. For $f = 2$, the lowest weight vector $|\,\begin{array}{|c|c|c|c|c|}\hline \diagup & \diagup & \diagup & \cdots & \diagup \\ \hline\end{array}\rangle$ has the decomposition

$$\begin{aligned} |\,\underbrace{\begin{array}{|c|c|c|c|c|}\hline \diagup & \diagup & \diagup & \cdots & \diagup \\ \hline\end{array}}_{n}\,\rangle &= |h = \tfrac{n}{2} + 2, \underbrace{\begin{array}{|c|c|c|c|c|}\hline \; & \; & \; & \cdots & \; \\ \hline\end{array}}_{n}, s_3 = -1\rangle \\ &\oplus |h = \tfrac{n+3}{2}, \underbrace{\begin{array}{|c|c|c|c|c|}\hline \; & \; & \; & \cdots & \; \\ \hline\end{array}}_{n-1}, s_3 = -\tfrac{1}{2}\rangle \end{aligned} \tag{5-15}$$

By the action of the supersymmetry generators on the above states we generate additional lowest weight vectors of both $SO^*(4)$ and $USp(2)$. The full content of the corresponding irrep of $OSp(4^*/2)$ for $f = 2$ is then as follows:

$$
\begin{aligned}
&|h = \tfrac{n+4}{2}, \ell = \tfrac{n}{2}, s = 1\rangle && \oplus \\
&|h = \tfrac{n+5}{2}, \ell = \tfrac{n+1}{2}, s = \tfrac{1}{2}\rangle && \oplus \\
&|h = \tfrac{n+5}{2}, \ell = \tfrac{n-1}{2}, s = \tfrac{1}{2}\rangle && \oplus \\
&|h = \tfrac{n+3}{2}, \ell = \tfrac{n-1}{2}, s = \tfrac{1}{2}\rangle && \oplus \\
&|h = \tfrac{n+4}{2}, \ell = \tfrac{n}{2}, s = 0\rangle && \oplus \\
&|h = \tfrac{n+4}{2}, \ell = \tfrac{n-2}{2}, s = 0\rangle && \oplus \\
&|h = \tfrac{n+6}{2}, \ell = \tfrac{n}{2}, s = 0\rangle
\end{aligned}
\tag{5-16}
$$

For $f > 2$, one finds that the UIR with the lowest weight state, whose $U(2/1)$ supertableau has one row with n boxes, has the following $SO^*(4) \times SU(2)_S = SU(1,1) \times SU(2)_L \times SU(2)_S$ decomposition

$$
\begin{aligned}
&|h = \tfrac{1}{2}(n + 2f), \ell = \tfrac{n}{2}, s = \tfrac{f}{2}\rangle && \oplus \\
&|h = \tfrac{1}{2}(n + 2f - 1), \ell = \tfrac{n-1}{2}, s = \tfrac{f-1}{2}\rangle && \oplus \\
&|h = \tfrac{1}{2}(n + 2f + 1), \ell = \tfrac{n+1}{2}, s = \tfrac{f-1}{2}\rangle && \oplus \\
&|h = \tfrac{1}{2}(n + 2f + 1), \ell = \tfrac{n-1}{2}, s = \tfrac{f-1}{2}\rangle && \oplus \\
&|h = \tfrac{1}{2}(n + 2f), \ell = \tfrac{n}{2}, s = \tfrac{f-2}{2}\rangle && \oplus \\
&|h = \tfrac{1}{2}(n + 2f), \ell = \tfrac{n-2}{2}, s = \tfrac{f-2}{2}\rangle && \oplus \\
&|h = \tfrac{1}{2}(n + 2f + 2), \ell = \tfrac{n}{2}, s = \tfrac{f-2}{2}\rangle && \oplus \\
&|h = \tfrac{1}{2}(n + 2f + 1), \ell = \tfrac{n-1}{2}, s = \tfrac{f-3}{2}\rangle
\end{aligned}
\tag{5-17}
$$

For $n = 0, 1, 2$ some of the ℓ values are negative and the corresponding states do not occur in the super Fock space. They lead to so-called shortened supermultiplets [3, 4, 14]. If we call the states with even h bosonic and odd h fermionic we find that the number d_B of bosonic degrees of freedom is equal to the number d_F of fermionic ones.

$$d_B \;=\; d_F \;=\; 4nf \;+\; 2(f-n) \tag{5-18}$$

One can similarly find the $SU(1,1)\times SU(2)_L\times SU(2)_S$ decomposition of a general UIR of $OSp(4^*/2)$ whose lowest weight state has more than one row in its supertableau.

Let us now apply the oscillator method to the construction of the so-called short supermultiplets [3, 4, 14] of $OSp(4^*/6)$ by taking as the lowest weight vector the Fock vacuum $|0\rangle$. For $f=1$ the action of the supersymmetry operator $A^{i\mu}$ ($i=1,2;\ \mu=1,2,3$) on $|0\rangle$ leads to states whose $U(2)\times U(3)$ Young Tableaux are shown below:

$$\begin{aligned}
&|0\rangle \\
A^{i\mu}|0\rangle &= |\,\yng(1)\,,\ \yng(1)\,\rangle \\
A^{i\mu}A^{j\nu}|0\rangle &= |\,\yng(1,1)\,,\ \yng(2)\,\rangle + |\,\yng(2)\,,\ \yng(1,1)\,\rangle \\
A^{i\mu}A^{j\nu}A^{k\lambda}|0\rangle &= |\,\yng(2,1)\,,\ \yng(2,1)\,\rangle + |\,\yng(3)\,,\ \yng(1,1,1)\,\rangle \\
A^{i\mu}A^{j\nu}A^{k\lambda}A^{\ell\rho}|0\rangle &= |\,\yng(2,2)\,,\ \yng(2,2)\,\rangle + |\,\yng(3,1)\,,\ \yng(2,1,1)\,\rangle \\
A^{i\mu}A^{j\nu}A^{k\lambda}A^{\ell\rho}A^{m\delta}|0\rangle &= |\,\yng(3,2)\,,\ \yng(2,2,1)\,\rangle \\
A^{i\mu}A^{j\nu}A^{k\lambda}A^{\ell\rho}A^{m\delta}A^{n\epsilon}|0\rangle &= |\,\yng(3,3)\,,\ \yng(2,2,2)\,\rangle
\end{aligned} \tag{5-19}$$

By the rules given in the previous sections, not all the states that occur above correspond to lowest weight vectors of $SO^*(4)\times USp(6)$. Since $f=1$, the lowest weight vectors of $SO^*(4)$ and $USp(6)$ can have at most one row and one column in their Young Tableaux, respectively. With this constraint we can easily read off the $SO^*(4) \times USp(6)$ decomposition of the unitary supermultiplet which we give in Table III.

Lowest Weight State of $SO^*(4) \times USp(6)$	$(h, \ell) \times (m_1 m_2 m_3)_D$
$\vert 0\rangle$	$(1, 0) \times (001)_D$
$\vert \,\square\, , \,\square\, \rangle$	$(\frac{3}{2}, \frac{1}{2}) \times (010)_D$
$\vert \,\square\square\, , \begin{smallmatrix}\square\\ \square\end{smallmatrix} \rangle$	$(2, 1) \times (100)_D$
$\vert \,\square\square\square\, , \begin{smallmatrix}\square\\ \square\\ \square\end{smallmatrix} \rangle$	$(\frac{5}{2}, \frac{3}{2}) \times (000)_D$

Table III: Above we give the $SO^*(4) \times USp(6)$ decomposition of the ultra-short supermultiplet of $OSp(4^*/6)$ defined by the lowest weight vector $|0\rangle$. The first column gives the $U(2) \times U(3)$ Young Tableaux and the second column gives the corresponding irreps of $SO^*(4) \times USp(6)$. The unitary lowest weight representations of $SO^*(4)$ are labelled by the $U(1)_{B_0} \times SU(2)_L$ labels (h, ℓ) of the lowest weight vector. $USp(6)$ irreps are designated by their Dynkin labels.

For $f = 2$, the lowest weight vectors of $SO^*(4)$ and $USp(6)$ can have Young Tableaux with two rows and two columns, respectively. Therefore by action of the supersymmetry generator $A^{i\mu}$ we create lowest weight vectors of $SO^*(4) \times USp(6)$ corresponding to every Young Tableaux given above in (Eq. 5-19). Since the Young Tableaux of $U(2)$ can have at most two rows, one does not generate any new $U(2) \times U(3)$ Young Tableaux beyond those given in (Eq. 5-19). However for different f they lead to different irreps of $SO^*(4) \times USp(6)$. In Table IV we give the $SO^*(4) \times USp(6)$ decomposition of the irreps of $OSp(4^*/6)$ defined by the lowest weight vector $|0\rangle$ for arbitrary $f \geq 2$.

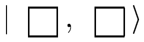
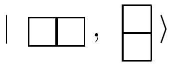
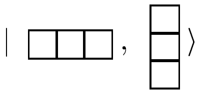
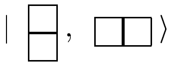
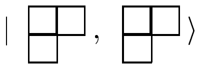
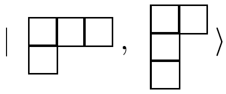
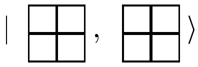
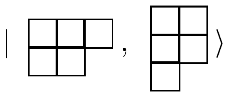
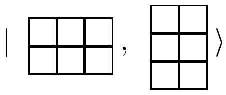
Lowest Weight State of $SO^*(4) \times USp(6)$	$(h, \ell) \times (m_1 m_2 m_3)_D$
$\|0\rangle$	$(f, 0) \times (0, 0, f)_D$
	$(f + \frac{1}{2}, \frac{1}{2}) \times (0, 1, f - 1)_D$
	$(f + 1, 1) \times (1, 0, f - 1)_D$
	$(f + \frac{3}{2}, \frac{3}{2}) \times (0, 0, f - 1)_D$
	$(f + 1, 0) \times (0, 2, f - 2)_D$
	$(f + \frac{3}{2}, \frac{1}{2}) \times (1, 1, f - 2)_D$
	$(f + 2, 1) \times (0, 1, f - 2)_D$
	$(f + 2, 0) \times (2, 0, f - 2)_D$
	$(f + \frac{5}{2}, \frac{1}{2}) \times (1, 0, f - 2)_D$
	$(f + 3, 0) \times (0, 0, f - 2)_D$

Table IV: Above we give the $SO^*(4) \times USp(6)$ decomposition of the short supermultiplets of $OSp(4^*/6)$ defined by the lowest weight vector $|0\rangle$. The first column gives the $U(2) \times U(3)$ Young Tableaux and the second column gives the corresponding irreps of $SO^*(4) \times USp(6)$. $SO^*(4)$ unitary irreps are labelled by the $U(1)_{B_0} \times SU(2)_L$ labels (h, ℓ) of their lowest weight vector. $USp(6)$ irreps are designated by their Dynkin labels.

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