

Measurement Error

Introduction

We have confidence that a particular physics theory is telling us something interesting about the physical universe because we are able to test **quantitatively** its predictions by making measurements. For example, 2.1 meters. Notice **both** the **number** (2.1) and the **units** (meters) are **required to specify a measured quantity**.

Precision

The **precision of a measurement** is indicated by the amount of **significant figures** given for the measurement. (Note: the word ‘digit’ is used interchangeably with the word ‘figure’ – both mean one of the symbols 0,1,2,3,4,5,6,7,8,9.) The technique to determine the amount of significant figures in any given number is as follows:

1. Find the leftmost non-zero digit. This is the most significant digit
2. If there is no decimal point, the rightmost non-zero digit is the least significant.
3. If there is a decimal point, the rightmost digit is the least significant, even if it is zero.
4. All digits between the least significant and the most significant (inclusive) are themselves significant.

For example, the following numbers each have 4 significant figures (shown in bold):
2,314 **2,314,000** **2.314** **9009** **9.009** **0.00009009** **9.000**

Units do not affect the determination of significant figures. It may be difficult to comprehend that 0.0002 grams is less precise than 120.317 grams. The first number has 1 significant figure the second has 6.

Accuracy

Accuracy of a measurement is the quality of **nearness to the true value**. In mathematical terms **accuracy** is **measured value** minus **true value**. Note: if the subtraction yields a negative number we change the sign, so accuracy is always positive; it is quoted to the same number of **decimal places** as the measurement.

	<u>Measurement</u>	<u>Measurement Precision</u>	<u>True Value</u>	<u>Accuracy</u>
1)	11.9990	6 Sig Digits	12.000000	0.0010
2)	11.999	5 Sig Digits	12.000000	0.001

Random Error – (Variation)

Random error (better called random variation) is the natural variation in multiple measurements where each measurement may be high or low compared to the true value and have no fixed or predictable pattern. The accuracy of each individual measurement will appear random. Such random errors occur for various reasons, in particular because of our inability to completely isolate the experiment from (random) external influences. It is not really an ‘error’ on our part.

Multiple Measurements

Why make multiple measurements of the same thing? Why not just one and be done with it? When two measurements are done, typically you will have two different readings because of random errors. Neither will be the true value, but by taking multiple measurements one can take an average with the reasoning that the average measurement will be close to the true value. More measurements will yield an average closer to the true value because random errors will tend to cancel out in taking the average.

Systematic Error

Measurements may sometimes display good precision but yet be very inaccurate. Often, you may find that your measurements are usually *consistently* too high or too low compared to the accurate (true) value. The measurements are “systematically” wrong. For example, weighing yourself with a scale that has a bad spring inside of it will yield weight measurements that are inaccurate, either always too high or always too low, even though the weight measurements may seem reasonable. It is often very difficult to catch systematic errors; how do you know when your measurement is wrong? This kind of error really is an ‘error’ on our part since we have overlooked or misunderstood something.

Uncertainty

No physical measurement is ever completely accurate or precise. Since we don’t normally know the true value of the quantity we are measuring, (otherwise why would we need to measure it?), we need some way to estimate the reliability of our measurements. The **Uncertainty** of a measurement will tell you with confidence that the true "actual" value is within a particular range of your measured value. The better the uncertainty, the closer your measured value is to the actual value. **The uncertainty will also tell you the precision of your measurements.** For example, it would be silly to measure something with a toy ruler and then announce that the object was 9.3756283 centimeters long. It would be more reasonable to say the uncertainty in the toy ruler measurement was 0.3 cm. That means you would report your reading as **9.3 +/- 0.3 cm. REPORT THE MEASUREMENT WITH THE SAME UNITS AND SAME NUMBER OF DECIMAL PLACES AS THE UNCERTAINTY.** See how the uncertainty tells you the actual value is between 9.6 cm and 9.0 cm and determines the precision of the measurement (2 sig. digits in this case)..

In general four items make up the Uncertainty:

1. **Instrument** - the instrument used to make the measurement is not perfectly precise. THIS WILL ALWAYS BE THE MINIMUM THE UNCERTAINTY CAN BE.
2. **Process** - how you use the instrument and make the measurement. A different process may yield more consistent measurements.
3. **Object** - the object will not have a true value of the quantity being measured. Think of measuring the diameter of an orange. The orange is not perfectly round, so it has no exact diameter.
4. **Person** - each person will have their own way of making the measurement or reading the instrument.

In this lab, you will be asked to write the precision of each measurement. This typically means $\frac{1}{2}$ of the smallest division on the instrument's measuring scale – the precision of the measuring instrument. For example, a meter stick may have 1 mm for its smallest divisions. The precision of the instrument would be ± 0.5 mm. The meter stick is can of distinguish between two items which differ by 0.5 mm.

Also you will be asked to estimate the total uncertainty of your measurements. This will be at least as big as the imprecision of the instrument on one measurement. Take the diameter of the orange again. After making several measurements you notice there is a maximum and a minimum to the diameter.

You can estimate the uncertainty from the scatter in the data as

$$\text{max} - \text{average} \quad \text{or} \quad \text{average} - \text{min}.$$

Objectives

1. To see how measurements and error analysis are a fundamental part of experimental science.
2. To make some actual measurements and analyze the errors in them.
3. To observe and understand the difference between accuracy and precision.
4. To understand the nature of random and systematic errors.

Equipment & Procedure

1. Measure the length of the metal rods.
 - a) Get a “special ruler” marked SR, a “normal ruler” and one rod.
 - b) Find the precision of both rulers and write this down. See the last paragraph in the section on uncertainty.
 - c) Using the **special ruler**, measure the length of the rod 5 times independently (rotate the rod, change the person measuring, etc..) Average the values.
 - d) Next, use the **normal** plastic ruler to measure your pieces again. Measure 5 times independently as before. Average the values to get a better measure of the piece’s true length.
 - e) Estimate the total uncertainty of your measurements.

2. Measure the height of the candle flame.
 - a) Obtain a candle and use the normal ruler
 - b) Find the precision of the ruler.
 - c) Light your candle and let the flame burn steadily for a minute or so.
 - d) Use the plastic ruler to measure the height of the flame. Make 5 measurements and try not to melt the ruler. Hold the ruler a small distance away from the flame. Record your measurements. **Remember to write only a sensible number of significant figures.**
 - e) Estimate the total uncertainty.

3. Measure your reaction time.

Your reaction time is the time that passes between some external stimulus and your first action. We will use an old method to measure your reaction time. A falling ruler will suffice.

 - a) This is what to do. Have your partner hold the normal ruler vertically, holding it by the top and having the zero point toward the bottom.
 - b) Place your thumb and forefinger at the ruler’s bottom, surrounding the zero point. Be prepared to pinch the ruler as if it were to fall. **Rest your forearm on the lab table to steady your hand.**
 - c) Your partner will drop the ruler without warning.
 - d) Pinch and grab the falling ruler as fast as you can; record how far it fell. Make 5 independent measurements like this, and then repeat for your partner's reactions.
 - e) Compute your reaction time using Galileo’s formula: $s = \frac{1}{2}gt^2$.
s = distance, t = time, and g is acceleration. Ask the instructor if you need help.
 - f) Record the corresponding reaction times for each distance measurement.
 - g) Compute the average and estimate the uncertainty on your reaction time.

Results

1. Rod length

	Special Rule	Ruler Precision	Plastic Ruler	Ruler Precision
Measurement 1	_____	_____	_____	_____
Measurement 2	_____		_____	
Measurement 3	_____		_____	
Measurement 4	_____		_____	
Measurement 5	_____		_____	
Average Value	_____		_____	
Uncertainty	_____		_____	

2. Candle Flame

	Flame height	Ruler Precision
Measurement 1	_____	_____
Measurement 2	_____	
Measurement 3	_____	
Measurement 4	_____	
Measurement 5	_____	
Average Value	_____	
Uncertainty	_____	

3. Your reaction time

	Distance	Time	Ruler Precision
Measurement 1	_____	_____	_____
Measurement 2	_____	_____	
Measurement 3	_____	_____	
Measurement 4	_____	_____	
Measurement 5	_____	_____	
Average value	_____	_____	
Uncertainty		_____	

Questions

1. Rod length
 - a. Explain the possible sources of error in this measurement.
 - b. How well did your measurements with the special ruler agree with those done with the plastic ruler? If there was a disagreement, what kind of error was it? Random or systematic? What caused this error?

2. Candle flame
 - a. Describe the possible sources of error in this measurement.
 - b. What might you do to get a better measurement of the flame's height?

3. Your reaction time
Describe the possible sources of error in this measurement and explain their relevance.

Conclusions

Write a paragraph summarizing the main things you learnt from **your data** in this lab about the topic of the lab. Quote actual data to support your conclusions.

(Do not write about the procedure, what you learnt how to do, what you learnt from background reading, etc.)