More with Special Relativity

PHYS 1301

Introduction:

This lab will continue the illustration of important relativistic effects. We will solve two important "paradoxes" that sometimes arise when special relativity is discussed. By understanding why these apparent paradoxes are not really paradoxes, you will better understand special relativity.

The first of these apparent paradoxes is the so-called "Tsao paradox." (It's named after the guy who thought it up.) This "paradox" is not particularly difficult to crack and will help to clarify the notion of relative reference frames. The setting is this. A train travels at very high speed so that it has an appreciable beta and time dilation effects are large. A runner on the train sprints toward the back of the train with the same speed (with respect to the train), as the train moves forward (with respect to the ground). The so-called "paradox" is this. We know that clocks on the train run slow compared to ground clocks. We saw this many times in the special relativity introduction lab. We know also that the runner's clock runs slow compared to the train's clock. Therefore, the runner's clock should run "doubly slow" with respect to the ground. But wait, the runner is not moving with respect to the ground! Therefore the runner's clock should run at the same rate as the ground clocks. So, how can it be that the runner's clock runs both "doubly slow" with respect to the ground clock <u>and</u> also runs at the same rate as the ground clock?

The second "paradox" is related but a bit subtler. Its solution emphasizes the importance of keeping reference frames straight when discussing special relativity. If you compare two objects, one moving with respect to the other, then you must take into account any change from one reference frame to another that an object might make in its motion. There is nothing to guarantee that during the motion of an object that object always remains in the <u>same</u> reference frame. To properly record clock times and rod lengths, you must account for all changes between reference frames.

The setting for our second, so-called "twin paradox" is this. Twins are born on earth. One of them decides to travel to a distance star, take some photographs and return home to earth. Her twin stays at home. When the traveling twin returns home, both twins stand side by side and compare ages. The traveled twin is younger than the stay-at-home twin. This is <u>not</u> a paradox. This is a true statement. (If the Earth bound observer says she has aged by 10 years and that her twin traveled with a speed 9/16 *c* during the trip to the star, the star bound twin will have aged only 8 years when the twins stand side by side and compare ages.)

The supposed paradox is sometimes stated this way. Relativity says that all motion is relative. Therefore, who is to say who stayed still and who did the traveling? From the point of view of the star bound twin, the Earth twin zooms away and later returns. Hence, the Earth twin should be younger. But when the twins stand side by side, both cannot be younger. We have a paradox, so goes the (faulty) reasoning.

We will crack both paradoxes by using Spacetime comparing clock times in different reference frames.

Procedure:

1. Locate the folder Spacetime. The instructor will help you if you can't locate it. Double click (left mouse button) on the icon Spacetime. Click OK when queried.

2. To refresh your memory, play for 10-15 minutes or so. Create rods and clocks at different values of β (beta) and at different x positions. Make sure you can transform from one reference frame to another. Refer to the documentation in the lab Introduction to Special Relativity if necessary.

3. We will solve the Tsao non-paradox first. First, create a moving clock string to represent the train. (Select clock string from the Objects menu.) Make this clock string have a β=0.9. Think of the various clocks as being located at the car ends.

4. Advance the reference clock (that's the clock along the β=0 strip and at x=0.00) until this clock reads t=7.0. **Question 1:** What time does the central train clock now read? (This clock is labelled B and will have moved from its original position.)

5. Create a new clock, to represent the runner, at the x-position of the central train clock. Do this by first jumping to the train's reference frame. (Do you see why?)

6. Now gradually step the reference clock forward. You will probably have to select it using the Select menu. **Question 2:** How fast does the runner's clock step forward by comparison with the reference clock? Don't worry about the fact that they show different times - that is because they were created at different positions - the important question is how fast they tick relative to one another. **Question 3: Why is there no Tsao paradox? Note: the mistake was trying to combine observations from 2 different frames.**

7. Delete the train clocks and the runner clock. Now we will examine the twin "paradox." Suppose there is a star called Oasis at a distance such that it takes light traveling at c 10 years to get there, as measured in the Earth's reference frame. There isn't really such a star, we are pretending. We will let the Earth be represented by one clock and Oasis by a second. We will use a shuttle and program it for a round trip. You can imagine that the earth bound twin has the Earth clock on her wrist and that the shuttle clock is attached to the wrist of the traveling twin. The Oasis clock is there for clarity.

9. Create a shuttle at x=0 and with β=0.6. Create the Oasis clock at x=6.0 and β=0.0.

10. For now, the earth bound clock will be the reference clock. Advance this reference clock until the shuttle is at x=6.0. You will see the shuttle and the Oasis clock at the same position. Next, using the Program Object option in the Object Menu, program the shuttle to reverse its direction at this point. The shuttle should merely change the sign but not the magnitude of its β**.** This operation represents the traveling twin changing direction after she reaches Oasis.

11.

You should see 2 shuttles displayed at Oasis now, one just before and one just after turning around. **Question 4:** When the shuttle reaches Oasis, what time does the reference clock read and what time does the shuttle clock read?

12. Continue stepping the clock until the shuttle arrives back at Earth. **Question 5:** What time does the reference clock read and what time does the shuttle clock read? Decrement the reference clock until it reads t=0.0. The shuttle should be at x=0. Jump now to the shuttle's frame of reference. Now we will observe what the traveling twin observes. Increment the shuttle clock until the round trip is complete. Observe the apparent motion of the Earth. Does this make sense? Observe also the times on both the shuttle clock and the earth's clock. **Question 6:** What happens to the earth's clock just as the shuttle and Oasis have the same x position? Answer this question by watching the earth's clock just as the shuttle and Oasis meet and then start to recede. **Question 7:** Why do you think this is? Is the motion of the traveling twin and the earth bound twin really symmetric? (Hint: Imagine there is a third observer somewhere watching the earth and the shuttle move. What is different about the motion of the earth compared to the motion of the shuttle?) **Question 8:** Explain why there is no twin paradox and that indeed the traveling twin is younger than the earth twin.