PHYS 1301 IDEAS OF MODERN PHYSICS

Quantum Test

The following are practice questions for the test. You will need a simple calculator in the test (phones not accepted). Appended to this document is self-study material to help you master the problems.

The following formulas will be given on the test paper:

$P = h/\lambda$ $E = h\,f$ kinetic energy = $\frac{1}{2} m v^2$ (slow moving matter)

Activity = λ N $t_{1/2} = 0.693/\lambda$

h = 6.63×10^{-34} J s 1 eV = 1.6×10^{-19} J 1 u = 1.66×10^{-27} kg

1. Photons

1.1 Determine the energy in eV of a single photon in a beam of light of wavelength 450 nm.
[2.8 eV]

1.2 A laser produces 3.0 W = 3 J/s of light energy at wavelength 600 nm. How many photons per second are produced?
[9.1×10^{18}]

1.3 Photons of energy 5.0 eV strike a metal whose work function is 3.5 eV. Determine the maximum kinetic energy of the emitted electrons.
1.5 eV

2. De Broglie Wavelength

2.1 What kinetic energy must each neutron in a beam of neutrons have if their wavelength is 0.10 nm? The mass of a neutron is 1.67×10^{-27} kg.
[1.3×10^{-20} J]

2.2 Approximately, what is the de Broglie wavelength of an electron that has been accelerated through a potential difference of 150 Volts? The mass of an electron is 9.11×10^{-31} kg and the electric charge is 1.6×10^{-19} Coulombs. A charge q Coulombs gains energy q Joules when moving through a potential difference of 1 V.
[0.1 nm]

2.3 What is the kinetic energy of each electron in a beam of electrons if the beam produces a diffraction pattern which is similar to that of a beam of 1.00 eV neutrons?
Note: The electron mass is 9.11×10^{-31} kg; and the neutron mass is 1.67×10^{-27} kg.
[1830 eV]

3. Binding Energy

3.1 The binding energy of an isotope of chlorine is 298 MeV. What is the mass defect of this chlorine nucleus in atomic mass units?
[0.320 u]

3.2 The proton has a mass of 1.007 28 u; and the neutron has a mass of 1.008 67. Use this information to determine the binding energy per nucleon of $^{232}_{90}Th$ which has an atomic mass of 232.038 054 u.
[7.4 MeV]

3.3 How much energy is required to remove a neutron (mass = 1.008 665 u) from $^{15}_{7}N$ that has an atomic mass of 15.000 108 u to make $^{14}_{7}N$ that has an atomic mass of 14.003 074 u?
[10.83 MeV]

4. Radioactivity

4.1 An isotope of krypton has a half-life of 3 minutes. A sample of this isotope produces 1000 counts per minute in a Geiger counter. Determine the number of counts per minute produced after 15 minutes.
[30]

4.2 The same activity is measured for two different isotope samples. One sample contains 0.0450 kg of $^{230}_{92}U$ (atomic mass = 230.033 937 u, $t_{1/2}$ = 20.8 days). The second sample contains an unknown amount of $^{231}_{92}U$ (atomic mass = 231.036 264 u, $t_{1/2}$ = 4.3 days). What is the mass of the second sample?
[0.0093 kg]

4.3 The activity of carbon-14 in a sample of charcoal from an archaeological site is 0.04 Bq. Determine the age of the sample. The half-life of carbon-14 is 5730 years.
[14 500 yr]

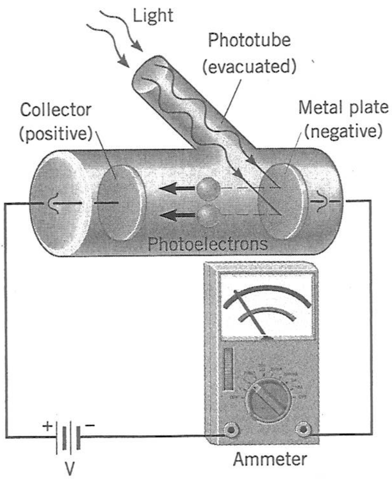


Figure 29.4 In the photoelectric effect, light with a sufficiently high frequency ejects electrons from a metal surface. These photoelectrons, as they are called, are drawn to the positive collector, thus producing a current.

Experimental evidence that light consists of photons comes from a phenomenon called the ***photoelectric effect,*** in which electrons are emitted from a metal surface when light shines on it. Figure 29.4 illustrates the effect. The electrons are emitted if the light being used has a sufficiently high frequency. The ejected electrons move toward a positive electrode called the *collector* and cause a current to register on the ammeter. Because the electrons are ejected with the aid of light, they are called ***photoelectrons.*** As will be discussed shortly, a number of features of the photoelectric effect could not be explained solely with the ideas of classical physics.

In 1905 Einstein presented an explanation of the photoelectric effect that took advantage of Planck's work concerning blackbody radiation. It was primarily for his theory of the photoelectric effect that he was awarded the Nobel Prize in physics in 1921. In his photoelectric theory, Einstein proposed that light of frequency f could be regarded as a collection of discrete packets of energy (photons), each packet containing an amount of energy E given by

Energy of a photon

$$E = hf \tag{29.2}$$

where h is Planck's constant. The light energy given off by a light bulb, for instance, is carried by photons. The brighter the bulb, the greater is the number of photons emitted per second. Example 1 estimates the number of photons emitted per second by a typical light bulb.

Example 1 Photons from a Light Bulb

In converting electrical energy into light energy, a sixty-watt incandescent light bulb operates at about 2.1% efficiency. Assuming that all the light is green light (vacuum wavelength = 555 nm), determine the number of photons per second given off by the bulb.

Reasoning The number of photons emitted per second can be found by dividing the amount of light energy emitted per second by the energy E of one photon. The energy of a single photon is $E = hf$, according to Equation 29.2. The frequency f of the photon is related to its wavelength λ by Equation 16.1 as $f = c/\lambda$.

Solution At an efficiency of 2.1%, the light energy emitted per second by a sixty-watt bulb is (0.021)(60.0 J/s) = 1.3 J/s. The energy of a single photon is

$$E = hf = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34}\ \text{J}\cdot\text{s})(3.00 \times 10^{8}\ \text{m/s})}{555 \times 10^{-9}\ \text{m}} = 3.58 \times 10^{-19}\ \text{J}$$

Therefore,

$$\begin{array}{c}\text{Number of}\\ \text{photons emitted}\\ \text{per second}\end{array} = \frac{1.3\ \text{J/s}}{3.58 \times 10^{-19}\ \text{J/photon}} = \boxed{3.6 \times 10^{18}\ \text{photons/s}}$$

According to Einstein, when light shines on a metal, a photon can give up its energy to an electron in the metal. If the photon has enough energy to do the work of removing the electron from the metal, the electron can be ejected. The work required depends on how strongly the electron is held. For the *least strongly* held electrons, the necessary work has a minimum value W_0 and is called the ***work function*** of the metal. If a photon has energy in excess of the work needed to remove an electron, the excess appears as kinetic energy of the ejected electron. Thus, the least strongly held electrons are ejected with the maximum kinetic energy KE_{max}. Einstein applied the conservation-of-energy principle and proposed the following relation to describe the photoelectric effect:

$$\underbrace{hf}_{\substack{\text{Photon}\\\text{energy}}} = \underbrace{KE_{max}}_{\substack{\text{Maximum}\\\text{kinetic energy}\\\text{of ejected}\\\text{electron}}} + \underbrace{W_0}_{\substack{\text{Minimum}\\\text{work needed to}\\\text{eject electron}}} \qquad (29.3)$$

According to this equation, $KE_{max} = hf - W_0$, which is plotted in Figure 29.5, with KE_{max} along the y axis and f along the x axis. The graph is a straight line that crosses the x axis at $f = f_0$. At this frequency, the electron departs from the metal with no kinetic energy ($KE_{max} = 0$ J). According to Equation 29.3, when $KE_{max} = 0$ J the energy hf_0 of the incident photon is equal to the work function W_0 of the metal: $hf_0 = W_0$.

The photon concept provides an explanation for a number of features of the photoelectric experiment that are difficult to explain without photons. It is observed, for instance, that only light with a frequency above a certain minimum value f_0 will eject electrons. If the frequency of the light is below this value, no electrons are ejected, regardless of how intense the light is. The next example determines the minimum frequency value for a silver surface.

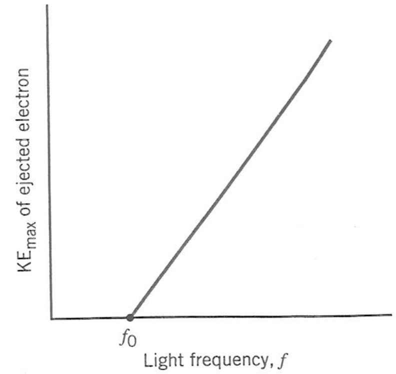


Figure 29.5 Photons can eject electrons from a metal when the light frequency is above a minimum value f_0. For frequencies above this value, ejected electrons have a maximum kinetic energy KE_{max} that is linearly related to the frequency, as the graph shows.

Example 2 The Photoelectric Effect for a Silver Surface

The work function for a silver surface is $W_0 = 4.73$ eV. Find the minimum frequency that light must have to eject electrons from this surface.

Reasoning The minimum frequency f_0 is that frequency at which the photon energy equals the work function W_0 of the metal, so the electron is ejected with zero kinetic energy. Since $1 \text{ eV} = 1.60 \times 10^{-19}$ J, the work function expressed in joules is $W_0 = (4.73 \text{ eV})[(1.60 \times 10^{-19} \text{ J})/(1 \text{ eV})] = 7.57 \times 10^{-19}$ J. Using Equation 29.3, we find

$$hf_0 = \underbrace{KE_{max}}_{= 0\,\text{J}} + W_0 \quad \text{or} \quad f_0 = \frac{W_0}{h}$$

Solution The minimum frequency f_0 is

$$f_0 = \frac{W_0}{h} = \frac{7.57 \times 10^{-19}\text{ J}}{6.63 \times 10^{-34}\text{ J}\cdot\text{s}} = \boxed{1.14 \times 10^{15}\text{ Hz}}$$

Photons with frequencies less than f_0 do not have enough energy to eject electrons from a silver surface. Since $\lambda_0 = c/f_0$, the wavelength of this light is $\lambda_0 = 263$ nm, which is in the ultraviolet region of the electromagnetic spectrum.

Problem solving insight
The work function of a metal is the minimum energy needed to eject an electron from the metal. An electron that has received this minimum energy has no kinetic energy once outside the metal.

Another significant feature of the photoelectric effect is that the maximum kinetic energy of the ejected electrons remains the same when the intensity of the light increases, provided the light frequency remains the same. As the light intensity increases, more photons per second strike the metal, and consequently more electrons per second are ejected. However, since the frequency is the same for each photon, the energy of each photon is also the same. Thus, the ejected electrons always have the same maximum kinetic energy.

Whereas the photon model of light explains the photoelectric effect satisfactorily, the electromagnetic wave model of light does not. Certainly, it is possible to imagine that the electric field of an electromagnetic wave would cause electrons in the metal to oscillate and tear free from the surface when the amplitude of oscillation becomes large enough. However, were this the case, higher-intensity light would eject electrons with a greater maximum kinetic energy, a fact that experiment does not confirm. Moreover, in the electromagnetic wave model, a relatively long time would be required with low-intensity light before the electrons would build up a sufficiently large oscillation amplitude to tear free. Instead, experiment shows that even the weakest light intensity causes electrons to be ejected almost instantaneously, provided the frequency of the light is above the minimum value f_0. The failure of the electromagnetic wave model to explain the photoelectric effect does not mean that the wave model should be abandoned. However, we must recognize that the wave model does not account for all the characteristics of light. The photon

29.5 The de Broglie Wavelength and the Wave Nature of Matter

▶ **CONCEPTS AT A GLANCE** As a graduate student in 1923, Louis de Broglie (1892–1987) made the astounding suggestion that since light waves could exhibit particle-like behavior, particles of matter should exhibit wave-like behavior. De Broglie proposed that all moving matter has a wavelength associated with it, just as a wave does. The Concept-at-a-Glance chart in Figure 29.12, which is a continuation of the chart in Figure 29.3, shows that the notions of energy, momentum, and wavelength are applicable to particles as well as to waves. ◀

De Broglie made the explicit proposal that the wavelength λ of a particle is given by the same relation (Equation 29.6) that applies to a photon:

De Broglie wavelength

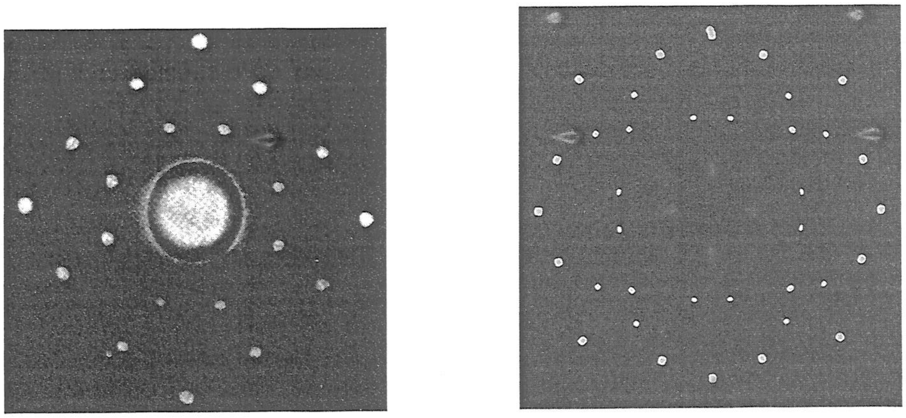
$$\lambda = \frac{h}{p} \tag{29.8}$$

where h is Planck's constant and p is the magnitude of the relativistic momentum of the particle. Today, λ is known as the ***de Broglie wavelength*** of the particle.

Confirmation of de Broglie's suggestion came in 1927 from the experiments of the American physicists Clinton J. Davisson (1881–1958) and Lester H. Germer (1896–1971) and, independently, those of the English physicist George P. Thomson (1892–1975). Davisson and Germer directed a beam of electrons onto a crystal of nickel and observed that the electrons exhibited a diffraction behavior, analogous to that seen when X-rays are diffracted by a crystal (see Section 27.9 for a discussion of X-ray diffraction). The wavelength of the electrons revealed by the diffraction pattern matched that predicted by de Broglie's hypothesis, $\lambda = h/p$. More recently, Young's double-slit experiment has been performed with electrons and reveals the effects of wave interference illustrated in Figure 29.1.

Particles other than electrons can also exhibit wave-like properties. For instance, neutrons are sometimes used in diffraction studies of crystal structure. Figure 29.13 compares the neutron diffraction pattern and the X-ray diffraction pattern caused by a crystal of rock salt (NaCl).

Although all moving particles have a de Broglie wavelength, the effects of this wavelength are observable only for particles whose masses are very small, on the order of the mass of an electron or a neutron, for instance. Example 4 illustrates why.



(a) (b)

Figure 29.13 (*a*) The neutron diffraction pattern (Wollan, Shull and Marney, *Phys. Rev.* 73:527, 1948) and (*b*) the X-ray diffraction pattern for a crystal of sodium chloride (NaCl). (Courtesy Edwin Jones, University of South Carolina)

Example 4 The de Broglie Wavelength of an Electron and of a Baseball

Determine the de Broglie wavelength for (a) an electron (mass $= 9.1 \times 10^{-31}$ kg) moving at a speed of 6.0×10^{6} m/s and (b) a baseball (mass $= 0.15$ kg) moving at a speed of 13 m/s.

Reasoning In each case, the de Broglie wavelength is given by Equation 29.8 as Planck's constant divided by the magnitude of the momentum. Since the speeds are small compared to the speed of light, we can ignore relativistic effects and express the magnitude of the momentum as the product of the mass and the speed.

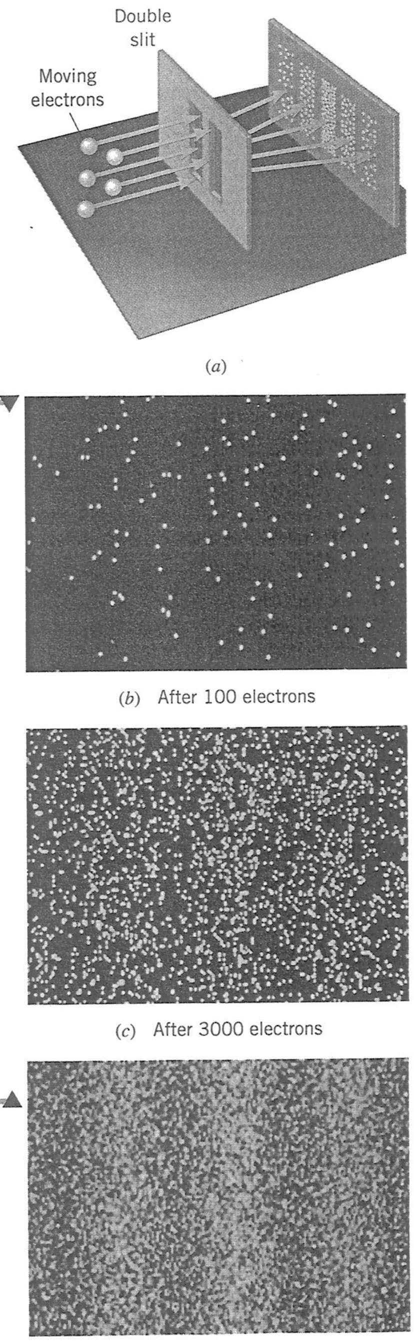
Solution

(a) Since the magnitude p of the momentum is the product of the mass m of the particle and its speed v, we have $p = mv$. Using this expression in Equation 29.8 for the de Broglie wavelength, we obtain

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63 \times 10^{-34}\ \text{J}\cdot\text{s}}{(9.1 \times 10^{-31}\ \text{kg})(6.0 \times 10^{6}\ \text{m/s})} = \boxed{1.2 \times 10^{-10}\ \text{m}}$$

A de Broglie wavelength of 1.2×10^{-10} m is about the size of the interatomic spacing in a solid, such as the nickel crystal used by Davisson and Germer, and, therefore, leads to the observed diffraction effects.

(b) A calculation similar to that in part (a) shows that the de Broglie wavelength of the baseball is $\boxed{\lambda = 3.3 \times 10^{-34}\ \text{m}}$. This wavelength is incredibly small, even by comparison with the size of an atom (10^{-10} m) or a nucleus (10^{-14} m). Thus, the ratio λ/W of this wavelength to the width W of an ordinary opening, such as a window, is so small that the diffraction of a baseball passing through the window cannot be observed.



(a)

(b) After 100 electrons

(c) After 3000 electrons

(d) After 70 000 electrons

Figure 29.14 In this electron version of Young's double-slit experiment, the characteristic fringe pattern becomes recognizable only after a sufficient number of electrons have struck the screen. (A. Tonomura, J. Endo, T. Matsuda, and T. Kawasaki, *Am. J. Phys.* 57(2): 117, Feb. 1989)

The de Broglie equation for particle wavelength provides no hint as to what kind of wave is associated with a particle of matter. To gain some insight into the nature of this wave, we turn our attention to Figure 29.14. Part *a* shows the fringe pattern on the screen when electrons are used in a version of Young's double-slit experiment. The bright fringes occur in places where particle waves coming from each slit interfere constructively, while the dark fringes occur in places where the waves interfere destructively.

When an electron passes through the double-slit arrangement and strikes a spot on the screen, the screen glows at that spot, and parts *b*, *c*, and *d* of Figure 29.14 illustrate how the spots accumulate in time. As more and more electrons strike the screen, the spots eventually form the fringe pattern that is evident in part *d*. Bright fringes occur where there is a high probability of electrons striking the screen, and dark fringes occur where there is a low probability. Here lies the key to understanding particle waves. ***Particle waves are waves of probability,*** waves whose magnitude at a point in space gives an indication of the probability that the particle will be found at that point. At the place where the screen is located, the pattern of probabilities conveyed by the particle waves causes

31.3 The Mass Defect of the Nucleus and Nuclear Binding Energy

Because of the strong nuclear force, the nucleons in a stable nucleus are held tightly together. Therefore, energy is required to separate a stable nucleus into its constituent protons and neutrons, as Figure 31.3 illustrates. The more stable the nucleus is, the greater is the amount of energy needed to break it apart. The required energy is called the ***binding energy*** of the nucleus.

▶ **CONCEPTS AT A GLANCE** As the Concepts-at-a-Glance chart in Figure 31.4 illustrates, two ideas that we have studied previously come into play as we discuss the binding energy of a nucleus. These are mass (Section 4.2) and the rest energy of an object (Section 28.6). In Einstein's theory of special relativity, mass and energy are equivalent. A change Δm in the mass of a system is equivalent to a change ΔE_0 in the rest energy of the system by an amount $\Delta E_0 = (\Delta m)c^2$, where c is the speed of light in a vacuum. Thus, in Figure 31.3, the binding energy used to disassemble the nucleus appears as extra mass of the separated nucleons. In other words, the sum of the individual masses of the separated protons and neutrons is greater by an amount Δm than the mass of the stable nucleus. The difference in mass Δm is known as the ***mass defect*** of the nucleus. ◀

As Example 2 shows, the binding energy of a nucleus can be determined from the mass defect according to Equation 31.3:

$$\text{Binding energy} = (\text{Mass defect})c^2 = (\Delta m)c^2 \qquad (31.3)$$

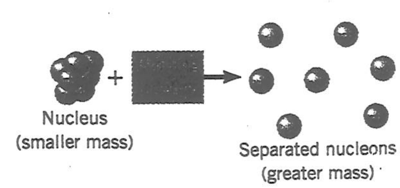


Figure 31.3 Energy, called the binding energy, must be supplied to break the nucleus apart into its constituent protons and neutrons. Each of the separated nucleons is at rest and out of the range of the forces of the other nucleons.

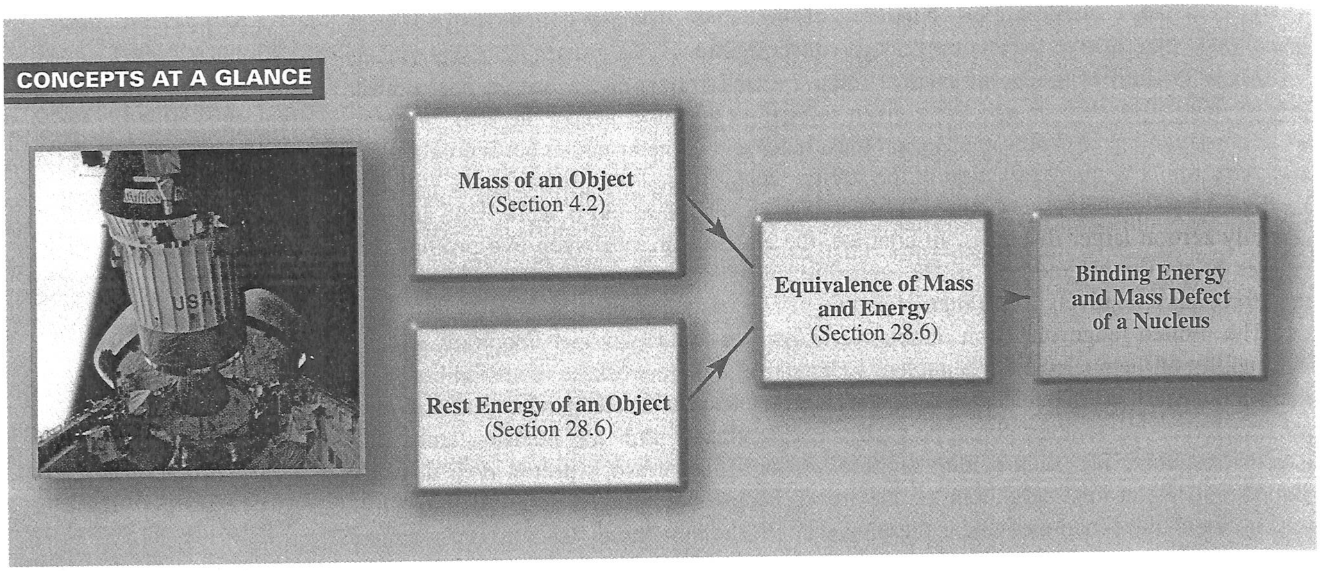


Figure 31.4 **CONCEPTS AT A GLANCE** The mass and rest energy of an object are equivalent, in the sense that if one increases (or decreases), the other does too. The binding energy of a nucleus is the mass of the separated nucleons minus the mass of the intact nucleus, expressed in units of energy. This photograph shows NASA's *Galileo* spacecraft, which uses a process called nuclear fission to generate its energy. This process depends on the fact that different nuclei have different binding energies and is discussed in Chapter 32. (Courtesy NASA)

Example 2 The Binding Energy of the Helium Nucleus

The most abundant isotope of helium has a ${}^{4}_{2}\text{He}$ nucleus whose mass is 6.6447×10^{-27} kg. For this nucleus, find (a) the mass defect and (b) the binding energy.

Reasoning The symbol ${}^{4}_{2}\text{He}$ indicates that the helium nucleus contains $Z = 2$ protons and $N = 4 - 2 = 2$ neutrons. To obtain the mass defect Δm, we first determine the sum of the individual masses of the separated protons and neutrons. Then we subtract from this sum the mass of the ${}^{4}_{2}\text{He}$ nucleus. Finally, we use Equation 31.3 to calculate the binding energy from the value for Δm.

Solution

(a) Using data from Table 31.1, we find that the sum of the individual masses of the nucleons is

$$\underbrace{2(1.6726 \times 10^{-27}\ \text{kg})}_{\text{Two protons}} + \underbrace{2(1.6749 \times 10^{-27}\ \text{kg})}_{\text{Two neutrons}} = 6.6950 \times 10^{-27}\ \text{kg}$$

This value is greater than the mass of the intact ${}^{4}_{2}\text{He}$ nucleus, and the mass defect is

$$\Delta m = 6.6950 \times 10^{-27}\ \text{kg} - 6.6447 \times 10^{-27}\ \text{kg} = \boxed{0.0503 \times 10^{-27}\ \text{kg}}$$

(b) According to Equation 31.3, the binding energy is

$$\text{Binding energy} = (\Delta m)c^2 = (0.0503 \times 10^{-27}\ \text{kg})(3.00 \times 10^8\ \text{m/s})^2 = 4.53 \times 10^{-12}\ \text{J}$$

Usually, binding energies are expressed in energy units of electron volts instead of joules ($1\ \text{eV} = 1.60 \times 10^{-19}$ J):

$$\text{Binding energy} = (4.53 \times 10^{-12}\ \text{J})\left(\frac{1\ \text{eV}}{1.60 \times 10^{-19}\ \text{J}}\right) = 2.83 \times 10^7\ \text{eV} = \boxed{28.3\ \text{MeV}}$$

In this result, one million electron volts is denoted by the unit MeV. The value of 28.3 MeV is more than two million times greater than the energy required to remove an orbital electron from an atom.

In calculations such as that in Example 2, it is customary to use the ***atomic mass unit*** (u) instead of the kilogram. As introduced in Section 14.1, the atomic mass unit is one-twelfth of the mass of a ${}^{12}_{6}\text{C}$ atom of carbon. In terms of this unit, the mass of a ${}^{12}_{6}\text{C}$ atom is exactly 12 u. Table 31.1 also gives the masses of the electron, the proton, and the neutron in atomic mass units. For future calculations, the energy equivalent of one atomic mass unit can be determined by observing that the mass of a proton is 1.6726×10^{-27} kg or 1.0073 u, so that

$$1\text{ u} = (1\text{ u})\left(\frac{1.6726 \times 10^{-27}\text{ kg}}{1.0073\text{ u}}\right) = 1.6605 \times 10^{-27}\text{ kg}$$

and

$$\Delta E_0 = (\Delta m)c^2 = (1.6605 \times 10^{-27}\text{ kg})(2.9979 \times 10^8\text{ m/s})^2 = 1.4924 \times 10^{-10}\text{ J}$$

In electron volts, therefore, one atomic mass unit is equivalent to

$$1\text{ u} = (1.4924 \times 10^{-10}\text{ J})\left(\frac{1\text{ eV}}{1.6022 \times 10^{-19}\text{ J}}\right) = 9.315 \times 10^8\text{ eV} = 931.5\text{ MeV}$$

Data tables for isotopes give masses in atomic mass units. Typically, however, the given masses are not nuclear masses. They are *atomic masses*—that is, the masses of neutral atoms, including the mass of the orbital electrons. Example 3 deals again with the ${}^4_2\text{He}$ nucleus and shows how to take into account the effect of the orbital electrons when using such data to determine binding energies.

Example 3 The Binding Energy of the Helium Nucleus, Revisited

The atomic mass of ${}^4_2\text{He}$ is 4.0026 u, and the atomic mass of ${}^1_1\text{H}$ is 1.0078 u. Using atomic mass units instead of kilograms, obtain the binding energy of the ${}^4_2\text{He}$ nucleus.

Reasoning To determine the binding energy, we calculate the mass defect in atomic mass units and then use the fact that one atomic mass unit is equivalent to 931.5 MeV of energy. The mass of 4.0026 u for ${}^4_2\text{He}$ *includes the mass of the two electrons in the neutral helium atom.* To calculate the mass defect, we must subtract 4.0026 u from the sum of the individual masses of the nucleons, including the mass of the electrons. As Figure 31.5 illustrates, the electron mass will be included if the masses of two hydrogen atoms are used in the calculation instead of the masses of two protons. The mass of a ${}^1_1\text{H}$ hydrogen atom is given in Table 31.1 as 1.0078 u, and the mass of a neutron as 1.0087 u.

Solution The sum of the individual masses is

$$\underbrace{2(1.0078\text{ u})}_{\text{Two hydrogen atoms}} + \underbrace{2(1.0087\text{ u})}_{\text{Two neutrons}} = 4.0330\text{ u}$$

The mass defect is $\Delta m = 4.0330\text{ u} - 4.0026\text{ u} = 0.0304$ u. Since 1 u is equivalent to 931.5 MeV, the binding energy is $\boxed{\text{Binding energy} = 28.3\text{ MeV}}$, which matches that obtained in Example 2.

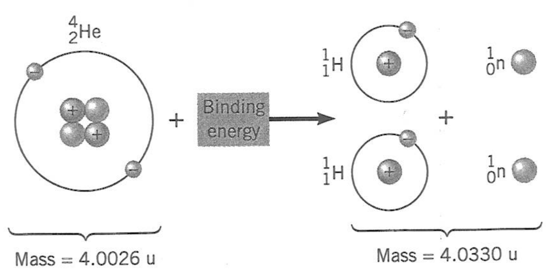


Figure 31.5 Data tables usually give the mass of the neutral atom (including the orbital electrons) rather than the mass of the nucleus. When data from such tables are used to determine the mass defect of a nucleus, the mass of the orbital electrons must be taken into account, as this drawing illustrates for the ${}^4_2\text{He}$ isotope of helium. See Example 3.

To see how the nuclear binding energy varies from nucleus to nucleus, it is necessary to compare the binding energy for each nucleus on a per-nucleon basis. The graph shown

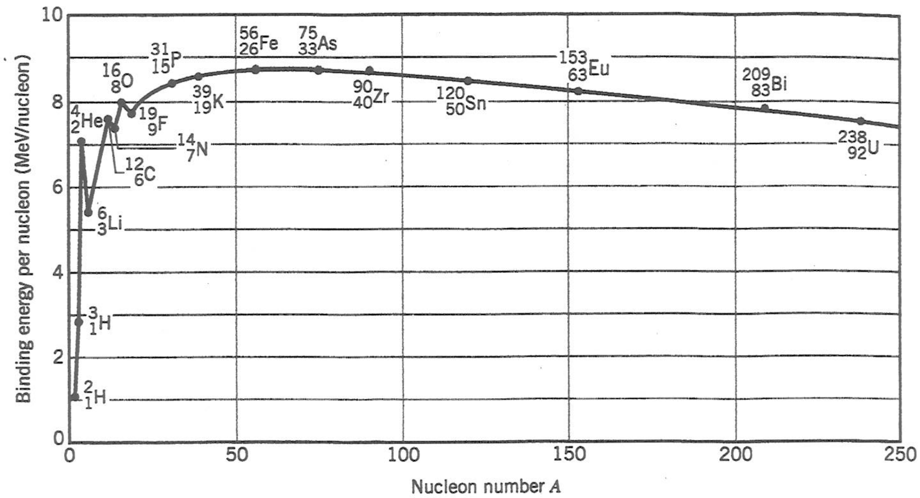


Figure 31.6 A plot of binding energy per nucleon versus the nucleon number *A*.

in Figure 31.6 shows a plot in which the binding energy divided by the nucleon number A is plotted against the nucleon number itself. In the graph, the peak for the $^{4}_{2}$He isotope of helium indicates that the $^{4}_{2}$He nucleus is particularly stable. The binding energy per nucleon increases rapidly for nuclei with small masses and reaches a maximum of approximately 8.7 MeV/nucleon for a nucleon number of about $A = 60$. For greater nucleon numbers, the binding energy per nucleon decreases gradually. Eventually, the binding energy per nucleon decreases enough so there is insufficient binding energy to hold the nucleus together. Nuclei more massive than the $^{209}_{83}$Bi nucleus of bismuth are unstable and hence radioactive.

The ***activity*** of a radioactive sample is the number of disintegrations per second that occur. Each time a disintegration occurs, the number N of radioactive nuclei decreases. As a result, the activity can be obtained by dividing ΔN, the change in the number of nuclei, by Δt, the time interval during which the change takes place; the average activity over the time interval Δt is the magnitude of $\Delta N/\Delta t$, or $|\Delta N/\Delta t|$. Since the decay of any individual nucleus is completely random, the number of disintegrations per second that occurs in a sample is proportional to the number of radioactive nuclei present, so that

$$\frac{\Delta N}{\Delta t} = -\lambda N \qquad (31.4)$$

where λ is a proportionality constant referred to as the ***decay constant.*** The minus sign is present in this equation because each disintegration decreases the number N of nuclei originally present.

The SI unit for activity is the *becquerel* (Bq); one becquerel equals one disintegration per second. Activity is also measured in terms of a unit called the *curie* (Ci), in honor of Marie (1867–1934) and Pierre (1859–1906) Curie, the discoverers of radium and polonium. Historically, the curie was chosen as a unit because it is roughly the activity of one gram of pure radium. In terms of becquerels,

$$1 \text{ Ci} = 3.70 \times 10^{10} \text{ Bq}$$

The activity of the radium put into the dial of a watch to make it glow in the dark is about 4×10^4 Bq, and the activity used in radiation therapy for cancer treatment is approximately a billion times greater, or 4×10^{13} Bq.

The mathematical expression for the graph of N versus t shown in Figure 31.15 can be obtained from Equation 31.4 with the aid of calculus. The result for the number N of radioactive nuclei present at time t is

$$N = N_0 e^{-\lambda t} \qquad (31.5)$$

assuming that the number at $t = 0$ s is N_0. The exponential e has the value $e = 2.718\ldots$, and many calculators provide the value of e^x. We can relate the half-life $T_{1/2}$ of a radioactive nucleus to its decay constant λ in the following manner. By substituting $N = \frac{1}{2}N_0$ and $t = T_{1/2}$ into Equation 31.5, we find that $\frac{1}{2} = e^{-\lambda T_{1/2}}$. Taking the natural logarithm of both sides of this equation reveals that $\ln 2 = \lambda T_{1/2}$ or

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda} \qquad (31.6)$$

The following example illustrates the use of Equations 31.5 and 31.6.

Example 9 The Activity of Radon $^{222}_{86}Rn$

As in Example 8, suppose there are 3.0×10^7 radon atoms ($T_{1/2} = 3.83$ days or 3.31×10^5 s) trapped in a basement. (a) How many radon atoms remain after 31 days? Find the activity (b) just after the basement is sealed against further entry of radon and (c) 31 days later.

Reasoning The number N of radon atoms remaining after a time t is given by $N = N_0 e^{-\lambda t}$, where $N_0 = 3.0 \times 10^7$ is the original number of atoms when $t = 0$ s and λ is the decay constant. The decay constant is related to the half-life $T_{1/2}$ of the radon atoms by $\lambda = 0.693/T_{1/2}$. The activity can be obtained from Equation 31.4, $\Delta N/\Delta t = -\lambda N$.

Solution

(a) The decay constant is

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{3.83 \text{ days}} = 0.181 \text{ days}^{-1} \qquad (31.6)$$

and the number N of radon atoms remaining after 31 days is

$$N = N_0 e^{-\lambda t} = (3.0 \times 10^7) e^{-(0.181 \text{ days}^{-1})(31 \text{ days})} = \boxed{1.1 \times 10^5} \qquad (31.5)$$

This value is slightly different from that found in Example 8 because there we ignored the difference between 8.0 and 8.1 half-lives.

(b) The activity can be obtained from Equation 31.4, provided the decay constant is expressed in reciprocal seconds:

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{3.31 \times 10^5 \text{ s}} = 2.09 \times 10^{-6} \text{ s}^{-1} \quad (31.6)$$

Thus, the number of disintegrations per second is

$$\frac{\Delta N}{\Delta t} = -\lambda N = -(2.09 \times 10^{-6} \text{ s}^{-1})(3.0 \times 10^7) = -63 \text{ disintegrations/s} \quad (31.4)$$

The activity is the magnitude of $\Delta N/\Delta t$, so initially $\boxed{\text{Activity} = 63 \text{ Bq}}$.

(c) From part (a), the number of radioactive nuclei remaining at the end of 31 days is $N = 1.1 \times 10^5$, and reasoning similar to that in part (b) reveals that $\boxed{\text{Activity} = 0.23 \text{ Bq}}$.

31.7 Radioactive Dating

The physics of radioactive dating.

One important application of radioactivity is the determination of the age of archeological or geological samples. If an object contains radioactive nuclei when it is formed, then the decay of these nuclei marks the passage of time like a clock, half of the nuclei disintegrating during each half-life. If the half-life is known, a measurement of the number of nuclei present today relative to the number present initially can give the age of the sample. According to Equation 31.4, the activity of a sample is proportional to the number of radioactive nuclei, so one way to obtain the age is to compare present activity with initial activity. A more accurate way is to determine the present number of radioactive nuclei with the aid of a mass spectrometer.

The present activity of a sample can be measured, but how is it possible to know what the original activity was, perhaps thousands of years ago? Radioactive dating methods entail certain assumptions that make it possible to estimate the original activity. For instance, the radiocarbon technique utilizes the ${}^{14}_{6}\text{C}$ isotope of carbon, which undergoes β^- decay with a half-life of 5730 yr. This isotope is present in the earth's atmosphere at an equilibrium concentration of about one atom for every 8.3×10^{11} atoms of normal carbon ${}^{12}_{6}\text{C}$. It is often assumed* that this value has remained constant over the years because ${}^{14}_{6}\text{C}$ is created when cosmic rays interact with the earth's upper atmosphere, a production method that offsets the loss via β^- decay. Moreover, nearly all living organisms ingest the equilibrium concentration of ${}^{14}_{6}\text{C}$. However, once an organism dies, metabolism no longer sustains the input of ${}^{14}_{6}\text{C}$, and β^- decay causes half of the ${}^{14}_{6}\text{C}$ nuclei to disintegrate every 5730 years. Example 10 illustrates how to determine the ${}^{14}_{6}\text{C}$ activity of one gram of carbon in a living organism.

This mummy, with a sun-bleached skull and surrounded by burial artifacts, was found in the arid highlands of southern Peru. The extreme dryness helped to preserve the remains, thought to be around 2000 years old. Radioactive dating is one of the techniques used to determine the age of such artifacts. (© David Nunuk/Photo Researchers)

* The assumption that the ${}^{14}_{6}\text{C}$ concentration has always been at its present equilibrium value has been evaluated by comparing ${}^{14}_{6}\text{C}$ ages with ages determined by counting tree rings. More recently, ages determined using the radioactive decay of uranium ${}^{238}_{92}\text{U}$ have been used for comparison. These comparisons indicate that the equilibrium value of the ${}^{14}_{6}\text{C}$ concentration has indeed remained constant for the past 1000 years. However, from there back about 30 000 years, it appears that the ${}^{14}_{6}\text{C}$ concentration in the atmosphere was larger than its present value by up to 40%. As a first approximation we ignore such discrepancies.

Example 10 $^{14}_{6}C$ Activity Per Gram of Carbon in a Living Organism

(a) Determine the number of carbon $^{14}_{6}C$ atoms present for every gram of carbon $^{12}_{6}C$ in a living organism. Find (b) the decay constant and (c) the activity of this sample.

Reasoning The total number of carbon $^{12}_{6}C$ atoms in one gram of carbon $^{12}_{6}C$ is equal to the corresponding number of moles times Avogadro's number (see Section 14.1). Since there is only one $^{14}_{6}C$ atom for every 8.3×10^{11} atoms of $^{12}_{6}C$, the number of $^{14}_{6}C$ atoms is equal to the total number of $^{12}_{6}C$ atoms divided by 8.3×10^{11}. The decay constant λ for $^{14}_{6}C$ is $\lambda = 0.693/T_{1/2}$, where $T_{1/2}$ is the half-life. The activity is equal to the magnitude of $\Delta N/\Delta t$, which is equal to the decay constant times the number of $^{14}_{6}C$ atoms present, according to Equation 31.4.

Solution

(a) One gram of carbon $^{12}_{6}C$ (atomic mass = 12 u) is equivalent to 1.0/12 mol. Since Avogadro's number is 6.02×10^{23} atoms/mol and since there is one $^{14}_{6}C$ atom for every 8.3×10^{11} atoms of $^{12}_{6}C$, the number of $^{14}_{6}C$ atoms is

$$\begin{array}{c}\text{Number of } ^{14}_{6}\text{C} \\ \text{atoms for every 1.0} \\ \text{gram of carbon } ^{12}_{6}\text{C}\end{array} = \left(\frac{1.0}{12}\text{ mol}\right)\left(6.02 \times 10^{23}\,\frac{\text{atoms}}{\text{mol}}\right)\left(\frac{1}{8.3 \times 10^{11}}\right) = \boxed{6.0 \times 10^{10}\text{ atoms}}$$

(b) Since the half-life of $^{14}_{6}C$ is 5730 yr (1.81×10^{11} s), the decay constant is

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{1.81 \times 10^{11}\text{ s}} = \boxed{3.83 \times 10^{-12}\text{ s}^{-1}} \tag{31.6}$$

(c) Equation 31.4 indicates that $\Delta N/\Delta t = -\lambda N$, so the magnitude of $\Delta N/\Delta t$ is λN.

$$\begin{array}{l}\text{Activity of } ^{14}_{6}\text{C for} \\ \text{every 1.0 gram of} \\ \text{carbon } ^{12}_{6}\text{C in a} \\ \text{living organism}\end{array} = \lambda N = (3.83 \times 10^{-12}\text{ s}^{-1})(6.0 \times 10^{10}\text{ atoms}) = \boxed{0.23\text{ Bq}}$$

An organism that lived thousands of years ago presumably had an activity of about 0.23 Bq per gram of carbon. When the organism died, the activity began decreasing. From a sample of the remains, the current activity per gram of carbon can be measured and compared to the value of 0.23 Bq to determine the time that has transpired since death. This procedure is illustrated in Example 11.

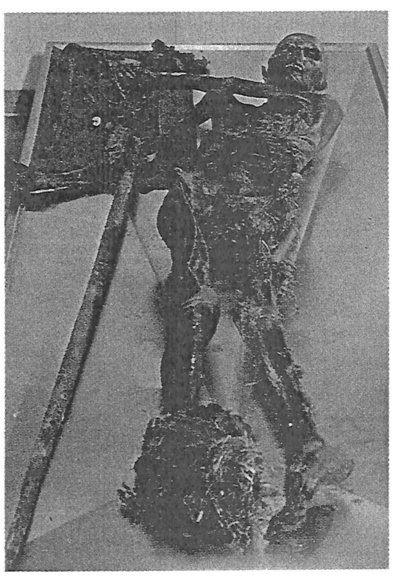


Figure 31.16 These remains of the Iceman were discovered in the ice of a glacier in the Italian Alps in 1991. Radiocarbon dating reveals his age (see Example 11). (© Corbis Sygma)

Example 11 The Iceman

On 19 September 1991, German tourists on a walking trip in the Italian Alps found a Stone Age traveler, later called the Iceman, whose body had become trapped in a glacier. Figure 31.16 shows the well-preserved remains that were dated using the radiocarbon method. Material found with the body had a $^{14}_{6}C$ activity of about 0.121 Bq per gram of carbon. Find the age of the Iceman's remains.

Reasoning According to Equation 31.5, the number of nuclei remaining at time t is $N = N_0 e^{-\lambda t}$. Multiplying both sides of this expression by the decay constant λ and recognizing that the product of λ and N is the activity A, we find that $A = A_0 e^{-\lambda t}$, where $A_0 = 0.23$ Bq is the activity at time $t = 0$ s for one gram of carbon. The decay constant λ can be determined from the value of 5730 yr for the half-life of $^{14}_{6}C$, using Equation 31.6. With known values for A_0 and λ, the given activity of $A = 0.121$ Bq per gram of carbon can be used to find t, the Iceman's age.

Solution For $^{14}_{6}C$, the decay constant is $\lambda = 0.693/T_{1/2} = 0.693/(5730\text{ yr}) = 1.21 \times 10^{-4}\text{ yr}^{-1}$. Since $A = 0.121$ Bq and $A_0 = 0.23$ Bq, the age can be determined from

$$A = 0.121\text{ Bq} = (0.23\text{ Bq})e^{-(1.21 \times 10^{-4}\text{ yr}^{-1})t}$$

Taking the natural logarithm of both sides of this result gives

$$\ln\left(\frac{0.121\text{ Bq}}{0.23\text{ Bq}}\right) = -(1.21 \times 10^{-4}\text{ yr}^{-1})t$$

which gives an age for the sample of $\boxed{t = 5300\text{ yr}}$.