Center of Mass

System of N particles

$$\underline{F}_i^{net} = m_i \underline{a}_i = m_i \frac{d^2 \underline{r}_i}{dt^2} \quad \text{each particle } i$$

$$\sum_{i=1}^{N} \underline{F}_i^{net} = M \frac{d^2}{dt^2}\left[\sum_{i=1}^{N} \frac{m_i \underline{r}_i}{M}\right] \quad \text{center of mass} \qquad M = \sum_{i=1}^{N} m_i$$

$$= M \frac{d^2}{dt^2} \underline{r}_{CM} = M \underline{a}_{CM}$$

$$\underline{r}_{CM} = \sum_i \frac{m_i \underline{r}_i}{M}$$

$$\frac{d\underline{r}_{CM}}{dt} = \underline{V}_{CM} = \sum_i \frac{m_i \underline{V}_i}{M}$$

Ex

2 particles

$$\underline{r}_{CM} = \frac{m_1 \underline{r}_1 + m_2 \underline{r}_2}{m_1 + m_2}$$

If $m_1 = m_2 = m \Rightarrow \underline{r}_{CM} = \frac{m\underline{r}_1 + m\underline{r}_2}{2m}$

$$= \frac{\underline{r}_1 + \underline{r}_2}{2}$$

a ||||b c| d e

Momentum & Impulse

Momentum

$$\underline{p} = m\underline{v}$$

Impulse

$$\underline{J} = \int \underline{F}\, dt = \underline{F}_{Av}\,\Delta t$$

(Δt: time over which force acts)

$$= \Delta \underline{p} = m\underline{v}_f - m\underline{v}_i$$

$$\sum m_i \underline{r}_i = M \underline{r}_{CM}$$

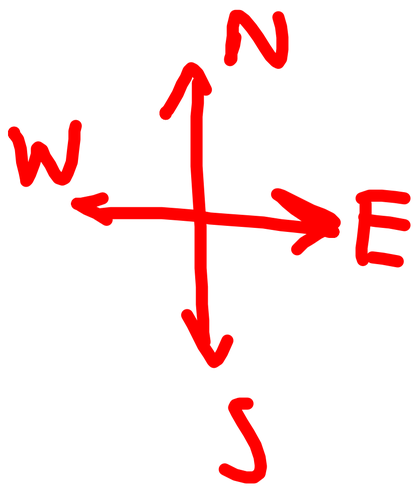
$$\sum m_i \underline{v}_i = M \underline{v}_{CM}$$

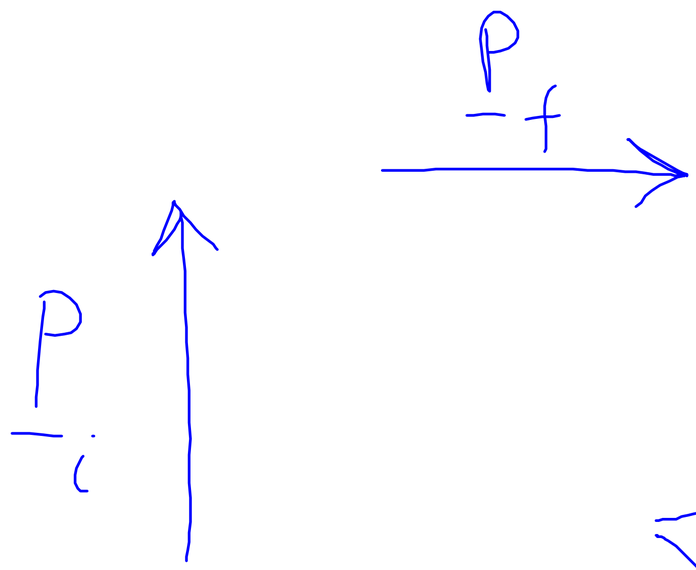
$$\sum \underline{p}_i = \underline{P}$$

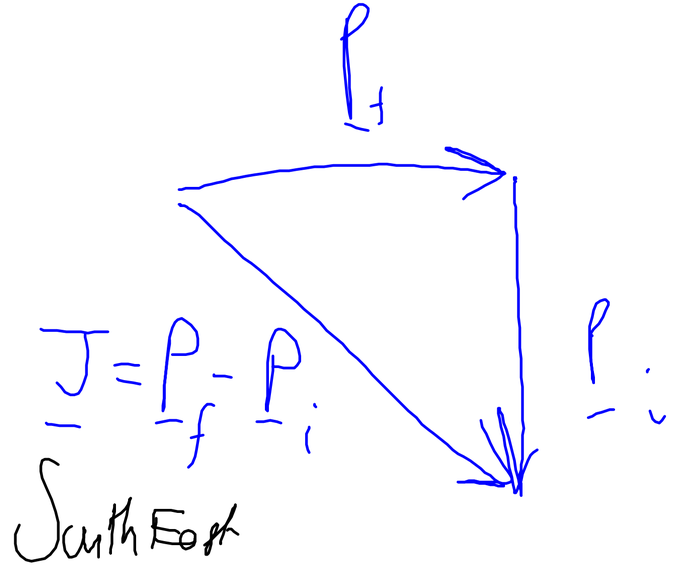
→ System momentum

For fixed impulse $\underline{J}$
F_{AV} is minimized by
maximizing Δt
→ crumple zone in car

$$\underline{J} = \underline{F}_{AV}\,\Delta t = \Delta \underline{p} = \underline{p}_f - \underline{p}_i$$







$$\sum_{ext} \underline{F} = M \underline{a}_{CM}$$

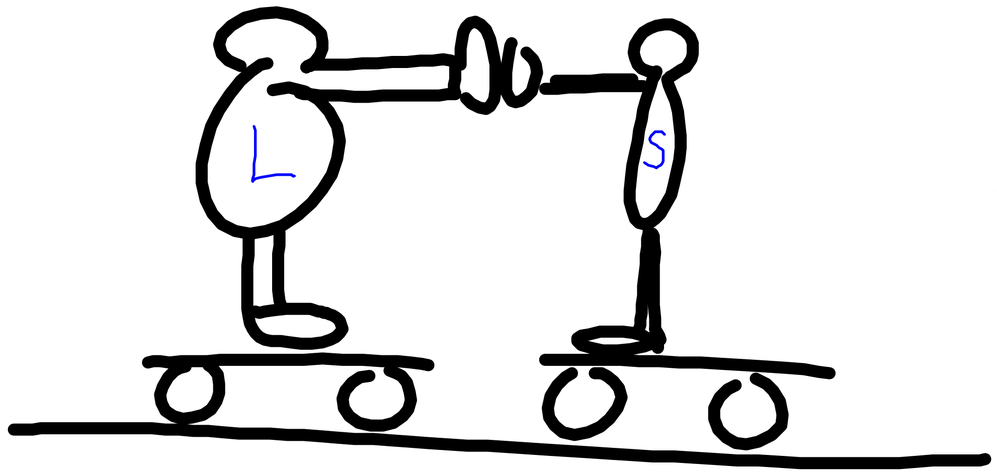
$$= M \frac{d \underline{v}_{CM}}{dt}$$

$$= \frac{d(M \underline{v}_{CM})}{dt}$$

$$\sum_{ext} \underline{F} = \frac{d \underline{p}}{dt}$$

Newton's 2nd law

if $\sum_{ext} \underline{F} = 0$ then $\Delta \underline{p} = 0$



System = 2 SMU Students

$\Sigma F = 0 \Rightarrow \Delta P = 0$

$\vec{P}_{Li} + \vec{P}_{Si} = \vec{P}_{Lf} + \vec{P}_{Sf}$

$-M_L \vec{V}_{Lf} = M_S \vec{V}_{Sf}$

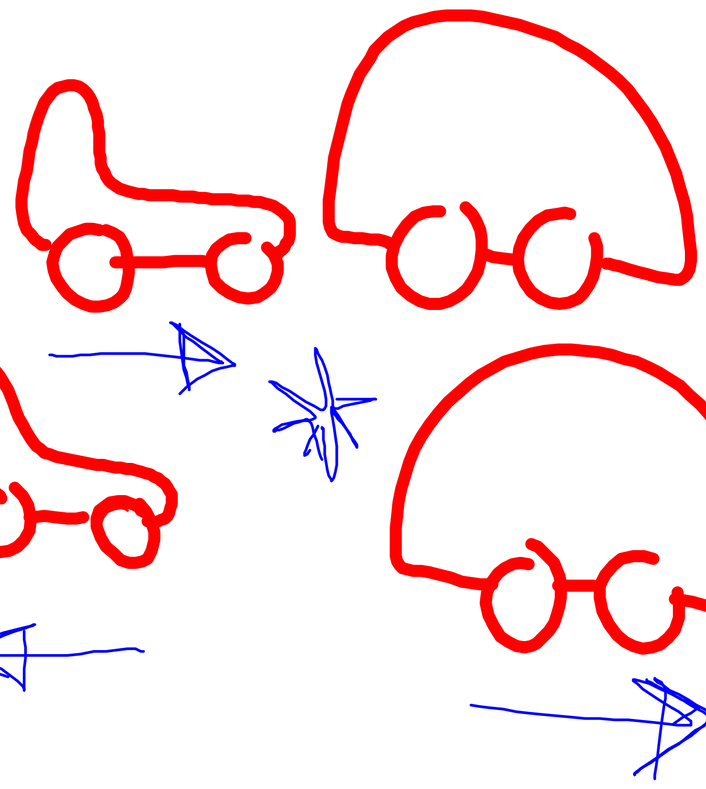
1 IIII

2

3 I

4

$$\vec{p}_{Li} + \vec{p}_{Si} = \vec{p}_{Lf} + \vec{p}_{Sf}$$



System $\Delta K = 0$

$\Rightarrow$ Elastic

Total Kinetic Energy of System Conserved

Elastic	$\Delta K = 0$
Inelastic	$\Delta K \neq 0$
totally inelastic	Δk is max (Stick together)

$$0 = \sum F = M \underline{a}_{CM}$$

$\Rightarrow$ center of mass of system does not move.