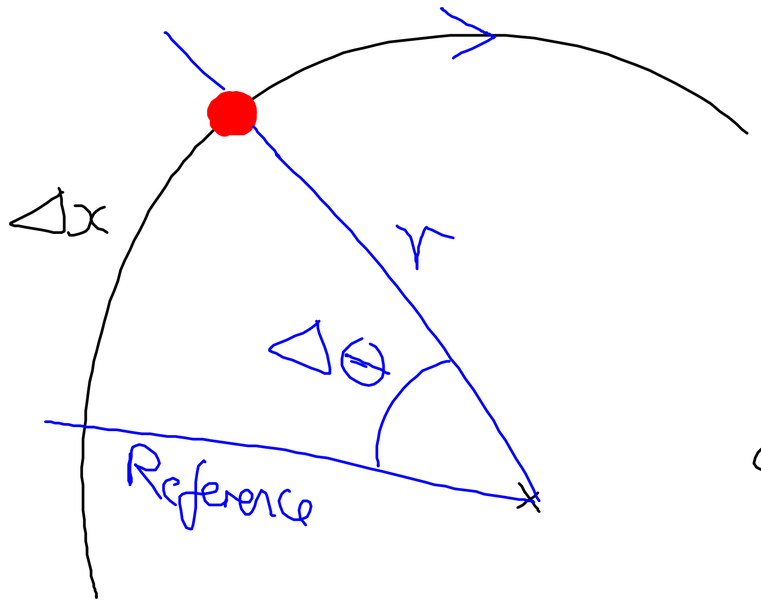
ROTATION

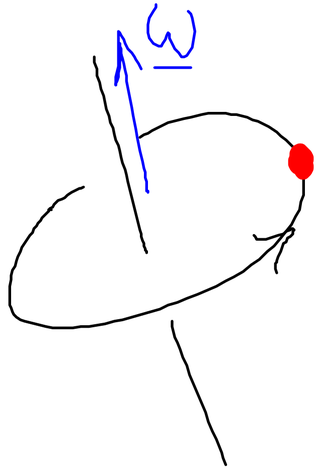


$$\Delta x = r\,\Delta\theta$$

radians

Linear speed $V = r\omega$

Linear	Angular
(x, y)	(r, θ)
Δx	$\Delta\theta$
velocity $V_x = \frac{dx}{dt}$	$\omega = \frac{d\theta}{dt}$ angular velocity
acceleration $a_x = \frac{dV_x}{dt}$	$\alpha = \frac{d\omega}{dt}$ angular acceleration

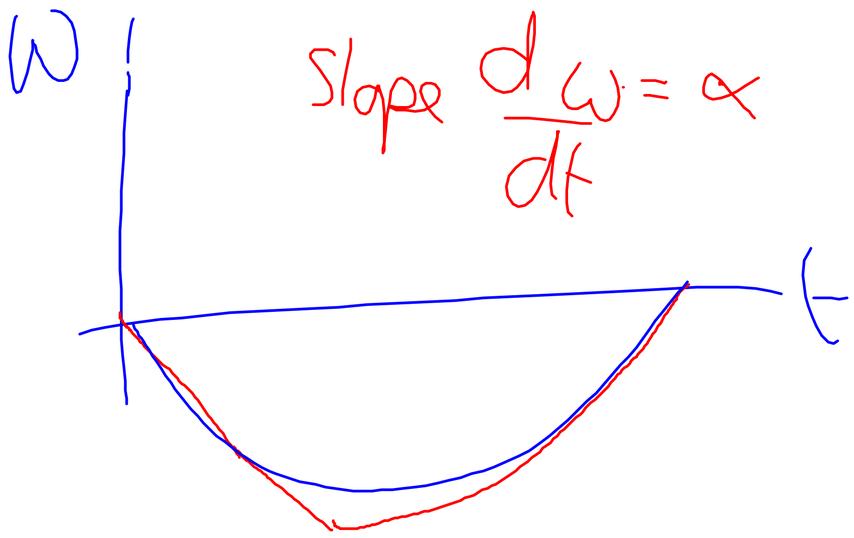


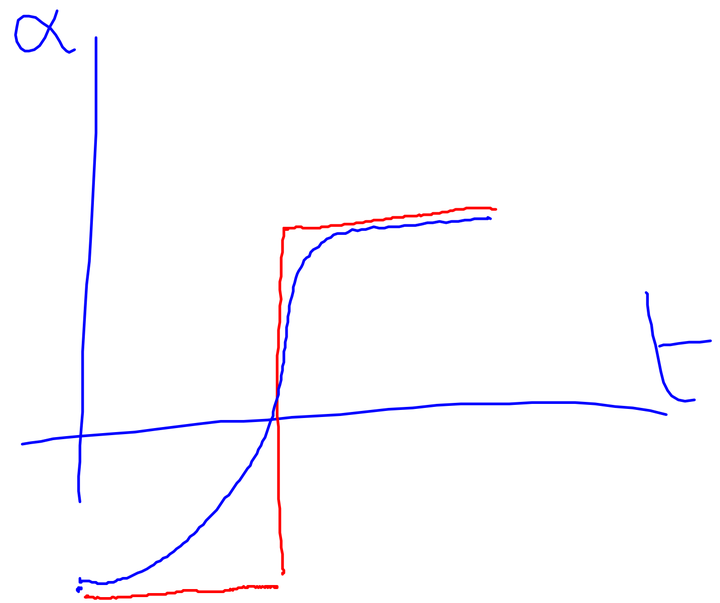
right hand rule

No axis wobbling

$\begin{cases} \alpha \text{ same } \omega \Rightarrow \text{speeding up} \\ \alpha \text{ opposite } \omega \Rightarrow \text{slowing down} \end{cases}$

+↑

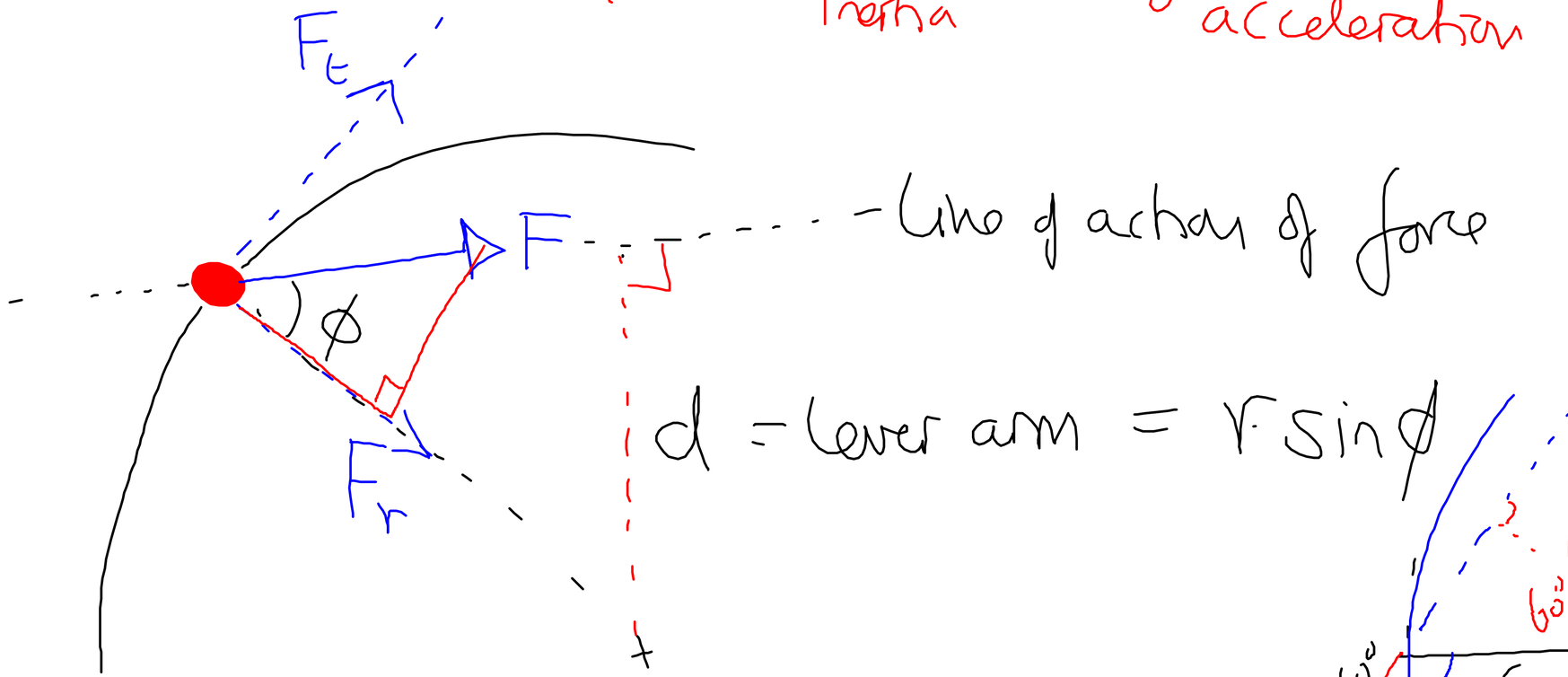




$$\sum \underline{\tau} = \underline{I}\,\underline{\alpha}$$

Torque — rotational inertia — angular acceleration

a |||
b ||
c
d



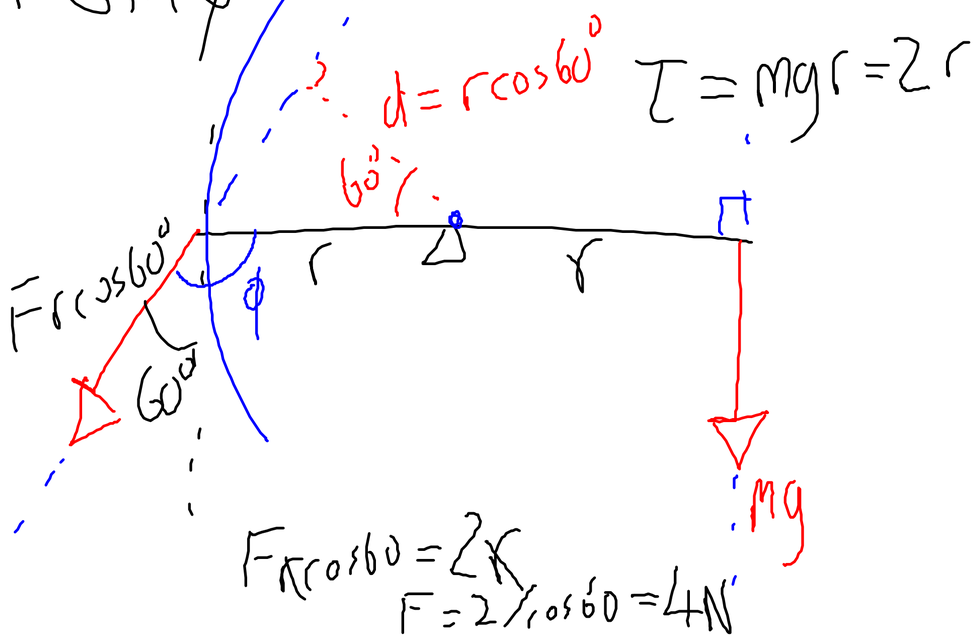
Line of action of force

use sign to indicate clockwise or anticlockwise

d = lever arm = $r \sin\phi$

$$\tau = r F_t = r F \sin\phi$$

$$\boxed{\tau = Fd}$$



Point mass $I = mr^2$

$\tau = I\alpha$

Several point masses $I = \sum_{i=1}^{N} m_i r_i^2$

Ex 2 masses

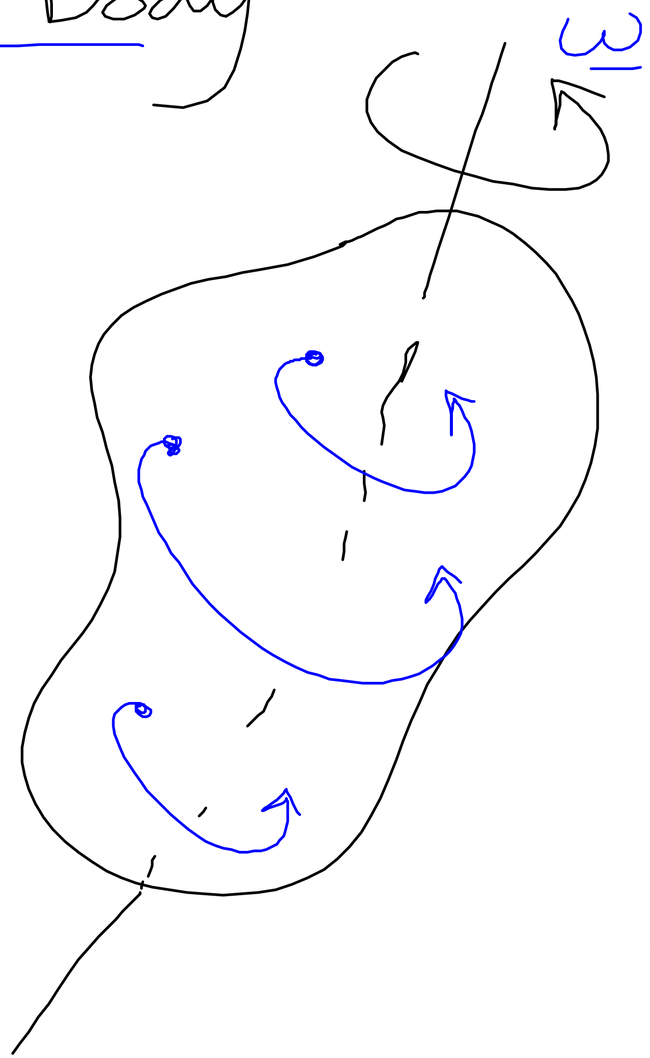
$$\tau = (mr^2 + mr^2)\alpha_1$$

Then $\tau = \left(m\left(\frac{r}{2}\right)^2 + m\left(\frac{r}{2}\right)^2\right)\alpha_2$

Rigid Body

$$\sum \underline{\tau} = I\underline{\alpha}$$

$$K_{rot} = \frac{1}{2} I \omega^2$$

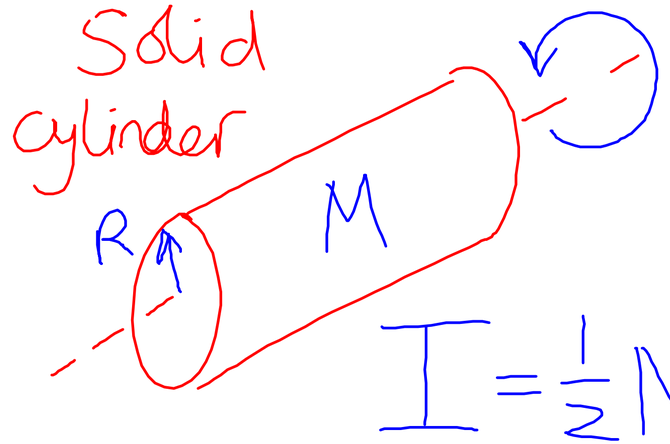


$$\sum \underline{F} = M\underline{a}_{CM}$$

$$K_{trans} = \frac{1}{2} M V_{CM}^2$$

$$K = K_{trans} + K_{rot}$$

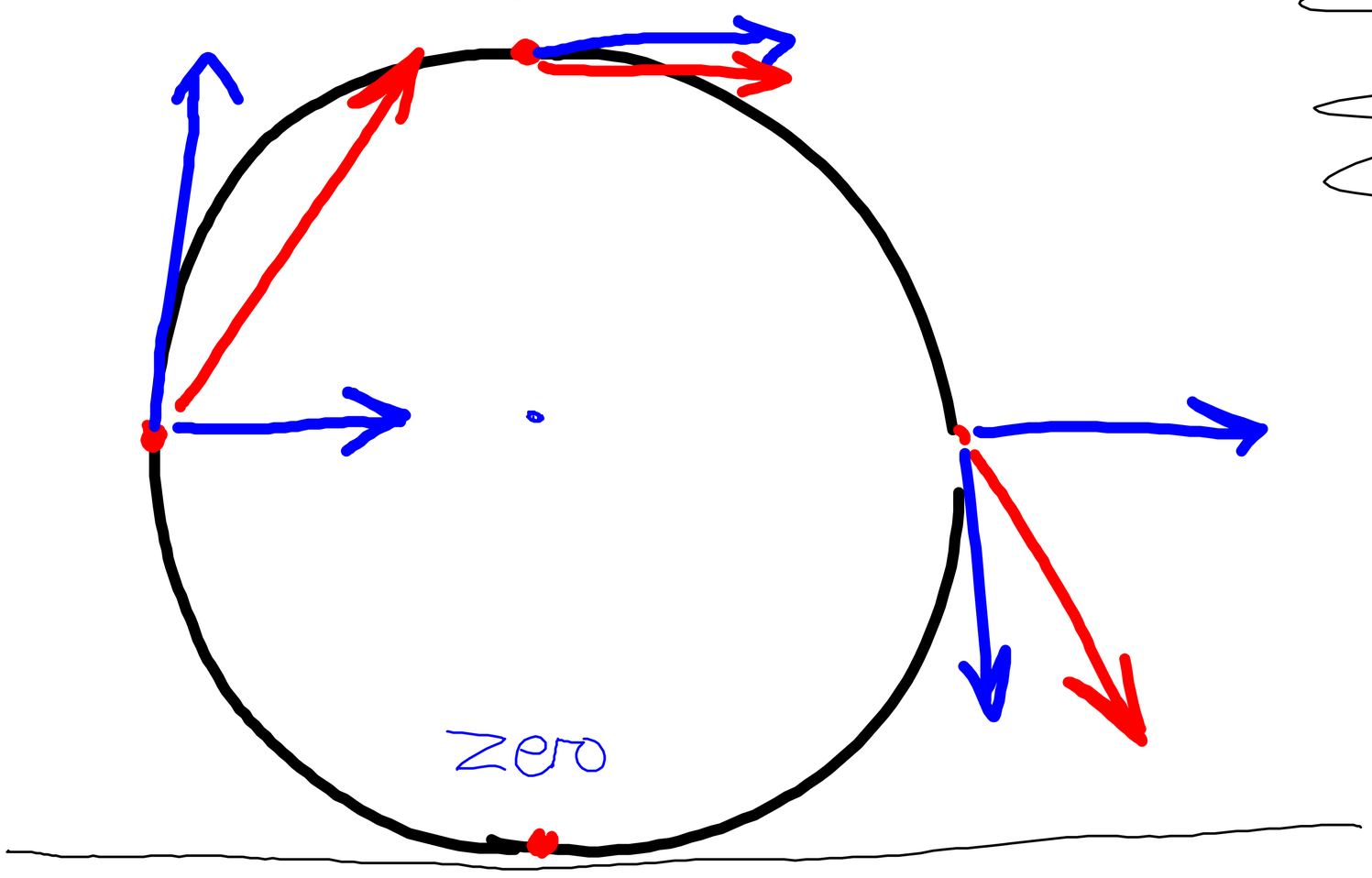
$$\sum \underline{\tau} = \underline{I}\,\underline{\alpha}$$



$(V = V_0 + at)$ $\omega = \omega_0 + \alpha t$

b

Rolling ✓



$$\sum \underline{\tau} = I\underline{\alpha}$$

$$\sum \underline{F} = M\underline{a}_{cm}$$

Angular Momentum

Linear momentum $\underline{P} = M \underline{V}_{CM}$

Angular momentum $\underline{L} = \underline{I}\,\underline{\omega}$

$$\frac{d\underline{P}}{dt} = \sum \underline{F} = M \underline{a}_{CM}$$

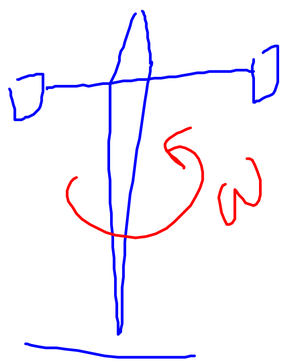
$$\frac{d\underline{L}}{dt} = \sum \underline{\tau} = \underline{I}\,\underline{\alpha}$$

$$K = \frac{1}{2} M v_{CM}^2 + \frac{1}{2} I \omega^2$$

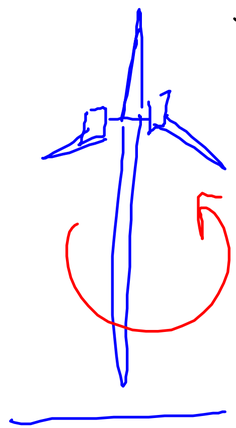
$\sum \underline{F} = 0 \Rightarrow \underline{P}$ conserved

$\sum \underline{\tau} = 0 \Rightarrow \underline{L}$ conserved

L constant $\Rightarrow$



I falls



ω grows

$$\frac{1}{2} I \omega^2 = \frac{1}{2} (I\omega)\omega$$

$$= \frac{1}{2} (\text{constant})\,\omega$$

$\Rightarrow K$ increases due to work done by student

wheel Bcr wheel Bcn

$\vec{L}_{tot}$ = . =

Conserved
(Bcn+wheel)

$\vec{L}$ of Bcn not conserved
$\vec{L}$ of wheel not conserved
} external forces applied on each other

$\rightarrow +$

$$\Sigma \underline{F} = +F - F = M \underline{a}_{CM}$$

$$\Rightarrow \underline{a}_{CM} = 0$$

$$\Rightarrow \underline{V}_{CM} = 0 \quad \text{if starts at rest}$$