Oscillations

equlibrium $\sum F_z = 0$

$+z \uparrow$ displ. from equilibrium

$$\sum F_z = m a_z$$

ideal spring

k = stiffness

$$-kz = m \frac{d^2 z}{dt^2}$$

$-kz_m$ $\qquad$ $-m z_m \omega^2$

Simple Harmonic Motion

$z = 0$

$$z = z_m \cos(\omega t + \phi)$$

z_m, ω, ϕ const.

$$\omega = \sqrt{\frac{k}{m}}$$

$$v_z = \frac{dz}{dt} = -z_m \sin(\omega t + \phi)\, \omega$$

$$a_z = \frac{d^2 z}{dt^2} = -z_m \omega^2 \cos(\omega t + \phi)$$

$$Z = Z_m \cos(\underbrace{\omega t + \phi}_{\text{in radians}})$$

PHASE

Z_m = amplitude

ϕ = initial phase

$\omega = \sqrt{\frac{k}{m}}$

angular frequency

T = period (time 1 oscill)

$\omega T = 2\pi$

$\omega = \frac{2\pi}{T}$

(linear) frequency $f = \frac{1}{T}$

oscill per sec

Z_m $10T = 12.6s$

$T = 1.26s$

$2Z_m$ $10T = 12.8s$

$$\omega^2 = \frac{k}{m}$$

$$Z = Z_m \cos(\omega t + \phi)$$

$$V_z = -\omega Z_m \sin(\omega t + \phi)$$

$$E = K + U$$

$$= \frac{1}{2} m V_z^2 + \frac{1}{2} k Z^2$$

$$= \frac{1}{2} m \omega^2 Z_m^2 \sin^2(\omega t + \phi) + \frac{1}{2} k Z_m^2 \cos^2(\omega t + \phi)$$

$$= \frac{1}{2} k Z_m^2 \left(\underbrace{\sin^2(\omega t + \phi) + \cos^2(\omega t + \phi)}_{1} \right)$$

$$= \frac{1}{2} k Z_m^2$$

E is conserved

$$\boxed{-kz = m a_z}$$

$$\sqrt{\frac{k}{m}} = \omega = \frac{2\pi}{T} \qquad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

ang. freq.

$T = 1.3$ secs one spring

$T = 0.9$ two springs parallel

$\omega \to \sqrt{2}\,\omega$

lin freq $f = \frac{1}{T}$

z

Note: 2 springs in series $\omega \to \frac{\omega}{\sqrt{2}}$