Introduction Sheet 1

Manipulating Powers, Fractions, Symbols and Units

Rules to Remember:

Fractions

	Example
$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$	$\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$
$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$ (not $\frac{a+c}{b+d}$)	$\frac{2}{3} + \frac{4}{5} = \frac{22}{15}$
$\frac{a}{b\left(\frac{c}{d}\right)} = \frac{ad}{bc}$	$\frac{2}{3\left(\frac{4}{5}\right)} = \frac{10}{12} = \frac{5}{6}$ in simplest form

Powers

	Example
$a^n = a \times a \times a \times \cdots \times a$ (n lots of a)	$2^3 = 2 \times 2 \times 2 = 8$
$a^{-n} = \frac{1}{a^n}$	$2^{-3} = \frac{1}{8}$
$a^1 = a$, $a^0 = 1$	
$a^{\frac{1}{n}} = \sqrt[n]{a}$ (n^{th} root of a)	$8^{\frac{1}{3}} = 2$
$a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$	$27^{\frac{2}{3}} = (27^{\frac{1}{3}})^2 = (3)^2 = 9$
$a^m \times a^n = a^{n+m}$	$2^3 \times 2^2 = 32 = 2^5$
$\frac{a^m}{a^n} = a^{m-n}$	$\frac{2^3}{2^2} = 2 = 2^1$
$(a^m)^n = a^{mn}$	$(2^3)^2 = 8^2 = 64 = 2^6$

Units

The units of physical quantities can be treated by the same rules of algebra, e.g. if M is a mass in kilogrammes (kg), l a length in metres (m), t a time in seconds (s), the physical units of the quantity

$$\frac{\sqrt{M^2 l^3}}{t^2}$$

are $\text{kgm}^{3/2}\text{s}^{-2}$. When giving a numerical value for a physical quantity, units must always be specified. **Answers with no or incorrect units will be marked wrong!**

Practice Questions:

P1 Express as a single fraction in its simplest form:

a) $\frac{5}{8}+\frac{2}{3}$ b) $\frac{1}{2}-\frac{7}{6}$ c) $\frac{3}{4}+\frac{1}{5}+\frac{2}{7}$ d) $\frac{2}{3}-\frac{-5}{6}$ e) $\frac{2}{3}\times\frac{5}{4}$ f) $\frac{-6}{7}\times\frac{3}{2}$ g) $\frac{7}{3\left(\frac{3}{7}\right)}$ h) $\frac{\frac{9}{4}}{\frac{3}{2}}$

P2 Evaluate:

a) $16^{\frac{1}{2}}$ b) $27^{\frac{2}{3}}$ c) $32^{\frac{1}{5}}$ d) $27^{-\frac{1}{3}}$ e) $\left(\frac{1}{4}\right)^{-2}$ f) $\frac{3^3}{2^{-2}}$ g) $(2^2)^4$

P3 Simplify:

a) $10^1\times10^2$ b) $10^{14}\times10^6$ c) x^2x^9 d) $C^3D^2C^7D^8$

e) $\frac{a^5}{a^2}$ f) $\frac{\sqrt{a^{24}}}{a^{16}}$ g) $5x^7y^3\times2y^5x^{10}$ h) $\frac{4x^3y^{\frac{1}{3}}}{2x^3y^{-3}}$

i) $(y^2)^3$ j) $(z^4)^a$ k) $(x^3)^{-2}$ l) $(10^4)^{-2}$

m) $(y^4)^{\frac{1}{2}}$ n) $\sqrt{B^4}$ o) $\sqrt{\sqrt{C^4}}$ p) $\left(\frac{4y^{\frac{1}{3}}}{x^3}\sqrt{\frac{32x^2}{2\sqrt{y}}}\right)^{\frac{1}{2}}$

P4 A circuit contains two resistors, $R_1=6\Omega$, $R_2=4\Omega$. What is the total resistance if they are connected (a) in series (b) in parallel?

P5 For what range of positive values of x is
(a) $x^3<x^{1/3}$?
(b) $x^2>x^{-2}$?

P6 The time t it takes a ball, dropped from rest and falling under gravity, to travel distance s, is given by the formula

$$t=\sqrt{\frac{2s}{b}}\,.$$

Given that the units must be the same on both sides of the equation, what are the units of the constant b in terms of metres (m) and seconds (s)?