Introduction Sheet 3

Ordering Logic, Bracketing and Factoring

Rules to Remember:

Manipulating Brackets

$$a(b+c) = ab+ac \text{ e.g. } 84(X+Y) = 84X+84Y$$

$$(a+b)\times(c+d) = ac+ad+bc+bd \text{ e.g.} (3+e)(2-g) = 6-3g+2e-eg$$

Factorizing

$$\begin{aligned} 5a+10b &= 5(a+2b) \\ 3a^2-6ab &= 3a(a^2-2b) \\ x^2+6x+8 &= (x+2)(x+4) \\ 3b^2-20b+12 &= (3b-2)(b-6) \end{aligned}$$

Factorizing and Cancelling

$$\begin{aligned} \frac{2x+6y}{6x-8y} &= \frac{2(x+3y)}{2(3x-4y)} &= \frac{x+3y}{3x-4y} \\ \frac{2xy+5xz}{6x} &= \frac{x(2y+5z)}{6x} &= \frac{2y+5z}{6} \\ \frac{x^2-y^2}{(x+y)^2} &= \frac{(x+y)(x-y)}{(x+y)^2} &= \frac{x-y}{x+y} \end{aligned}$$

Practice Questions:

P1 Expand the following:

a) $4(3+a)$ b) $x(7-y)$ c) $x^3(xy+y^2)$ d) $(2+a)(3-b)$
e) $(z+x)^2$ f) $(a+2)(a-3)$ g) $(4a-2b)(b-3a)$

P2 Cancel to their simplest form the following:

a) $\frac{8y+2y}{4}$ b) $\frac{z^3+5z^2x}{2z}$ c) $\frac{4a-2b}{2a-b}$ d) $\frac{z^2-2z+1}{z-1}$ e) $\frac{(x^2-y^2)}{x+y}$

P3 A substance consists of equal numbers of two kinds of atom, with masses m_1 and m_2 respectively.
a) What is the mass of the substance if 2×10^6 atoms of each kind are present, where $m_1 = 3.4 \times 10^{-26}$kg and $m_2 = 1.6 \times 10^{-26}$kg?
b) How many atoms are there altogether (counting both kinds) in 1kg of the substance?

P4 A gas contains two kinds of atom: 4×10^5 atoms of type A and 8×10^5 atoms of type B. It is known that the mass of type-A atoms is 5×10^{-26}kg and the mass of the gas is 2.8×10^{-20}kg. What is the mass of a type-B atom?

P5 a) The maximum speed of a boat in still water is 8ms^{-1}. If it is travelling in the direction of flow of a river, whose water is moving at 2.5ms^{-1} relative to the bank,
(i) what is the maximum speed of the boat relative to the bank?
(ii) what is the maximum distance it can travel relative to the bank in 2 minutes?

b) Repeat the above for the case of the boat moving against the flow of the river.

P6 The vertical displacement s of an object after time t with initial velocity u is given by

$$s = ut - \frac{1}{2}gt^2$$

when subject to gravitational acceleration $g = 10\text{ms}^{-2}$. If a ball is thrown from ground level vertically into the air with initial velocity $u = 20\text{ms}^{-1}$, how long does it take to return to the ground? [Hint: displacement is always zero at ground level, then use factorization]