Introduction Sheet 4

Symbolic Manipulation in Equations

Rules to Remember:

Since the two sides of an equation are equal, the equation is still true if you:

- add or subtract the same amount from each side
- multiply or divide by the same amount on each side (except you cannot divide by zero)
- raise both sides to a power

These rules can be used to solve equations for an unknown quantity by manipulating them to make the unknown quantity the **subject of the equation**, appearing on its own on one side.

Equations with one solution:

$x+5=4$
$\rightarrow x=-1$

$3p-9=18$
$\rightarrow 3p=27$
$\rightarrow p=9$

$\frac{4}{x}=2$
$\rightarrow 4=2x$
$\rightarrow 2=x$

$2z-3=7z$
$\rightarrow -3=5z$
$\rightarrow \frac{-3}{5}=z$

$\frac{4}{p+2}=\frac{2}{3p+1}$
$\rightarrow 4(3p+1)=2(p+2)$
$\rightarrow 12p+4=2p+4$
$\rightarrow 10p=0$
$\rightarrow p=0$

Sometimes there is more than one mathematical solution for the unknown quantity. We will always consider equations that can be solved by factorization in this case (you will not need any 'formula').

Examples with more than one solution:

$25x^2=16$
$\rightarrow x^2=\frac{16}{25}$
$\rightarrow x=\pm\frac{4}{5}$

$x^2+2=3x$
$\rightarrow x^2-3x+2=0$
$\rightarrow (x-1)(x-2)=0$
$\rightarrow x=1,2$

$\frac{2y-3}{y+3}=\frac{y-1}{y+1}$
$\rightarrow (2y-3)(y+1)=(y-1)(y+3)$
$\rightarrow 2y^2-y-3=y^2+2y-3$
$\rightarrow y^2-3y=0$
$\rightarrow y(y-3)=0$
$\rightarrow y=0,3$

The same ideas can be applied to sets of **simultaneous equations** with more than one unknown quantity. In this case, the amount that is added/subtracted from one equation is a multiple of another equation, the idea being to form an equation involving just one unknown quantity, which is then treated as above.

Example with more than one unknown (numbers in [] brackets label the equations):

$5a - 2b = 68$ [1]
$3a + b = 10$ [2] $\rightarrow$ add 2 x [2] to [1]

$$5a - 2b + 2(3a + b) = 68 + (2 \times 10)$$
$$\rightarrow 11a = 88$$
$$\rightarrow a = 8$$
$$b = 10 - 3a \text{ from rearranging [2]}$$
$$\rightarrow b = -14 \text{ using solution for } a$$

Practice Questions:

P1 Solve the following equations:

a) $x+7=4$ b) $3y=27$ c) $2p+3=8$ d) $10-2b=b+7$
e) $\frac{8}{x}=0.5$ f) $3(2x-1)=2(2x+3)$ g) $\frac{5}{3a-2}=5$ h) $\frac{2}{2a+1}=\frac{5}{3a-2}$

P2 Find **all** the solutions of the following equations:

a) $x^2=9$ b) $x^2-81=0$ c) $y^2+6y-16=0$ d) $a(a-6)+9=0$
e) $(x-3)^2=4$ f) $\frac{x-1}{x+1}=\frac{x+1}{x-1}$ g) $\frac{2x+4}{x+1}=\frac{x-8}{2x-1}$ h) $\frac{2}{y+1}+\frac{1}{y-1}=\frac{3}{y}$

P3 Solve the following simultaneous equations for the two unknown quantities:

a) $2x+3y=5$, $x-2y=6$ b) $5p-2q=9$, $2p+5q=-8$ c) $\frac{x}{3}-\frac{y}{2}+1=0$, $6x+y+8=0$ d) $\frac{6}{x}-\frac{2}{y}=\frac{1}{2}$, $\frac{4}{x}-\frac{3}{y}=0$

P4

(a) A droplet of liquid has mass $m = 6.4 \times 10^{-15}$kg. Calculate its weight. [Assume $g = 10\text{ms}^{-2}$]

(b) If the droplet carries electric charge Q, the electrostatic force on it between two parallel plates, distance d apart, and having electrical potential difference V, is QV/d. Assuming the droplet is held stationary between the horizontal plates, calculate the charge on the drop if $d = 5$mm and $V = 1000$V.

P5 The Swansea to London train cruises at 100 miles per hour before starting to decelerate uniformly at 240 miles per hour per hour as it passes Didcot. After passing Didcot, how many minutes does it take the train to travel the next 20 miles?

P6 Resistors of type A and B have unknown resistances R_A and R_B respectively. When two resistors of type A and one of type B are all connected in series to a supply voltage 12V, the current flowing is measured as 4mA. When one resistor of type A and three of type B are all connected in series to the same supply, the current flowing is measured as 3mA. What are the values of the resistances R_A and R_B?