Introduction Sheet 8
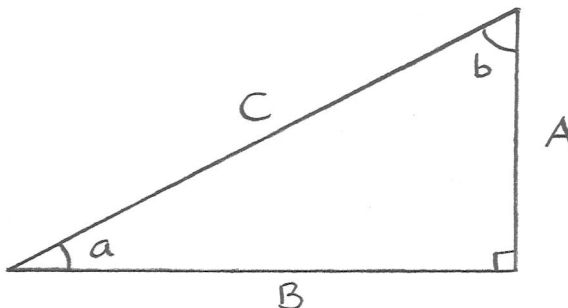## Special Functions 2: Trigonometric Functions

Rules to Remember:

Right-Angled Triangles:

$\dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{A}{C} = \cos a$

$\dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{B}{C} = \sin a$

$\dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{A}{B} = \tan a$
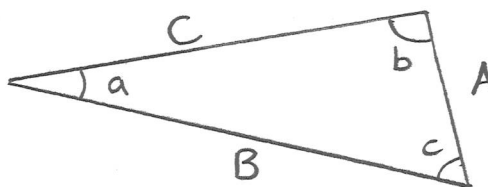
Pythagoras: $A^2 + B^2 = C^2 \rightarrow \sin^2 a + \cos^2 a = 1$

General Triangles:

Sine Rule: $\dfrac{\sin a}{A} = \dfrac{\sin b}{B} = \dfrac{\sin c}{C}$
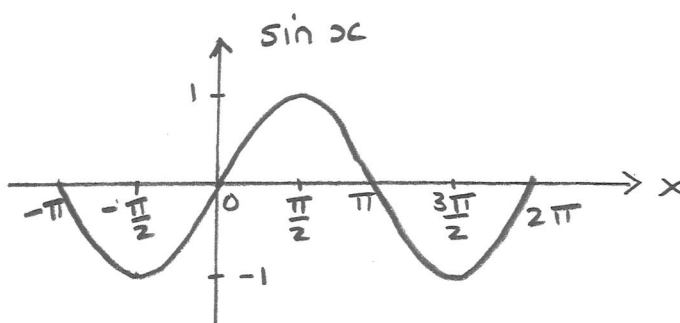
Cosine Rule: $C^2 = A^2 + B^2 - 2AB\cos c$

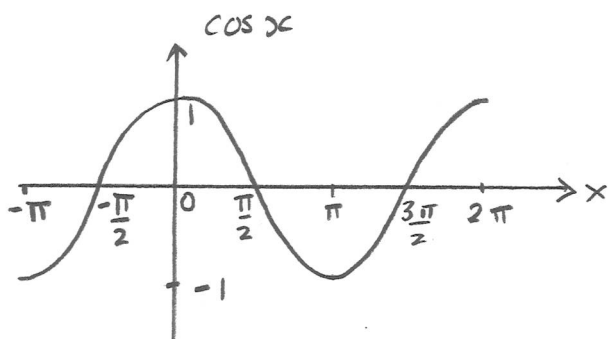**Radians:** To convert degrees to radians, multiply by $\frac{\pi}{180}$.

Properties of Trig Functions:

$\sin(-x) = -\sin x$ 'odd function' $\qquad \cos(-x) = \cos x$ 'even function'
$\sin(x+y) = \sin x \cos y + \sin y \cos x \qquad \cos(x+y) = \cos x \cos y - \sin x \sin y$
(Note: $\sin(x+y) \neq \sin x + \sin y$!)

Graphs:

θ	0°	30°	45°	60°	90°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0

Memorize this table!

Practice Questions:

P1 Using $\tan = \sin/\cos$ to help you if necessary, state the values of $\tan x$ for the following values of x:

a) $x = 0$ b) $x = \frac{\pi}{6}$ c) $x = \frac{\pi}{4}$
d) $x = \frac{\pi}{3}$ e) $x = \frac{\pi}{2}$ f) $x = -\frac{\pi}{2}$

P2 Sketch and label the graphs of the following functions for $-\pi < x < 2\pi$, labelling also the places where the axes are crossed (you may wish to use a pencil):

a) $y = \sin x$ and $y = \sin^2 x$ b) $y = \cos x$ and $y = \cos\frac{x}{2}$ c) $y = \tan x$

[Hint: For c) you could use $\tan = \sin/\cos$ and the graphs you've sketched for a) and c); note that $\tan x$ has gradient 1 whenever it crosses the x axis.]

P3 Obtain expressions for the following in terms of $\sin x$, $\sin y$, $\cos x$, and $\cos y$:

a) $\cos(x - y)$ b) $\sin 2x$ c) $\cos(2x + y)$

P4 Evaluate the following exactly (use $\sqrt{}$'s in your answers where necessary):

a) $\cos\pi$ b) $\sin -\pi$ c) $\sin\frac{\pi}{12}$ d) $\cos\frac{5\pi}{12}$ e) $\sin 135°$

[Hint: $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$.]

P5 A surveyor on the peak of mountain A, which has known height 3km, observes the peak of mountain B through a telescope inclined at 30° to the horizontal. Given that the peaks are estimated to be horizontal distance 6.8km apart, deduce the height of mountain B in km to 1 significant figure.

[Hint: substitute $\sqrt{3} \approx 1.7$ at the end of the calculation.]

P6 The straight tracks of three elementary particles are observed to cross so as to form a triangle. If the lengths of the sides of the triangle are 1cm, 1cm, and $\sqrt{3}$cm, calculate the acute ($\leq 90°$) angle between each pair of tracks.