## PHYS $4392 \quad$ Practice Final Exam

Your Name

$\qquad$

## 3 hours

Open Book (hardcopy): Griffiths Introduction to Electrodynamics $4 e$

Each question is worth 10 points. Point breakdown for part question is shown [thus]

Partial Credit is given for working in case of an incorrect answer (so write and/or draw every step clearly)

Hand in this question sheet with your solutions

1. A perfectly conducting spherical shell of radius $R$ is filled with a uniform distribution of volume charge density:

$$
\rho=\rho_{0} \quad 0<r<R
$$

The conducting shell is grounded so that the scalar electric potential at radius $r=R$ is zero, $\mathrm{V}(\mathrm{r}=\mathrm{R})=0$.

(a) What is the electric field vector $\underline{\mathbf{E}}$ for $0<\mathrm{r}<\mathrm{R}$ ? [3]
(b) What is the scalar electric potential V for $0<\mathrm{r}<\mathrm{R}$ ? [4]
(c) What is the free surface charge density $\sigma$ induced on the inside surface of the conducting shell at $\mathrm{r}=\mathrm{R}$ ? [3]
2. A ring of radius $R$ carries uniform charge density $\lambda$ per unit length and sits in the $x-y$ plane with its center at the origir

(a) In spherical polar coordinates, write the volume charge density $\rho(\mathrm{r}, \theta, \varphi)$ in terms of delta functions. [3]
(b) Calculate the monopole [2] dipole [2] and quadrupole [3] contributions to the multipole expansion of the scalar electric potential V. [ $\left.\int_{0} 2 \pi \sin ^{2} \varphi^{\prime} \mathrm{d} \varphi^{\prime}=\int_{0} 2 \pi \cos ^{2} \varphi^{\prime} \mathrm{d} \varphi^{\prime}=\pi\right]$
3. A point charge $Q$ is placed in a small hole at the center of a sphere of radius $R$ made of linear dielectric material with permittivity $\varepsilon$.

(a) Calculate the electric displacement vector $\underline{\mathbf{D}}$ inside and outside the sphere. [2]
(b) Calculate the electric field vector $\underline{\mathbf{E}}$ inside and outside the sphere. [2]
(c) Calculate the polarization vector $\underline{\mathbf{P}}$ inside the sphere. [2]
(d) Calculate the volume bound charge density $\rho_{\mathrm{b}}$.[2]
(e) Calculate the surface bound charge density $\sigma_{b}$ on the outer surface at radius R. [2]
4. Find the magnetic field $\underline{\mathbf{B}}$ at point P due to the following current I distribution in the $\mathrm{x}-\mathrm{y}$ plane [10]

5. A solid, cylindrical permanent magnet of radius R is infinite in the $z$ direction and is centered on the $z$ axis. The cylinder has a uniform magnetization in the $y$ direction: $\underline{\mathbf{M}}=\mathrm{M} \underline{\hat{\mathbf{y}}}$

a) Calculate the bound volume current density $\underline{\mathbf{J}}_{\mathrm{b}}$. [3]
b) Calculate the bound surface current density $\underline{\mathbf{K}}_{\mathrm{b}}$. [3]
c) Calculate the magnetic field $\underline{\mathbf{B}}$ on the z-axis. [4]
6. A rectangular loop of wire with sides of length $a$ and $b$ sits in a uniform external magnetic field that is perpendicular to the plane of the loop and going into the page (see figure). The magnitude of the field is decaying linearly over time $\mathrm{B}=\mathrm{B}_{0}(1-\alpha t)$.
(a) If the resistance of the loop is R, find an expression for the current $I$ induced in the loop. [7]
(b) Does the current flow clockwise or anticlockwise (justify your answer)? [2]
(c) Why can back-emf be neglected? [1]


