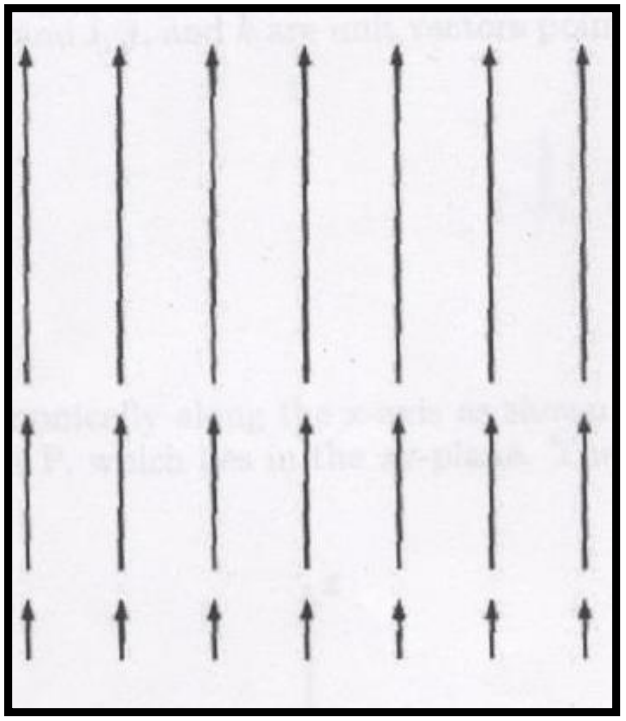
Chapter 1

Mathematical background

Differential Vector Calculus

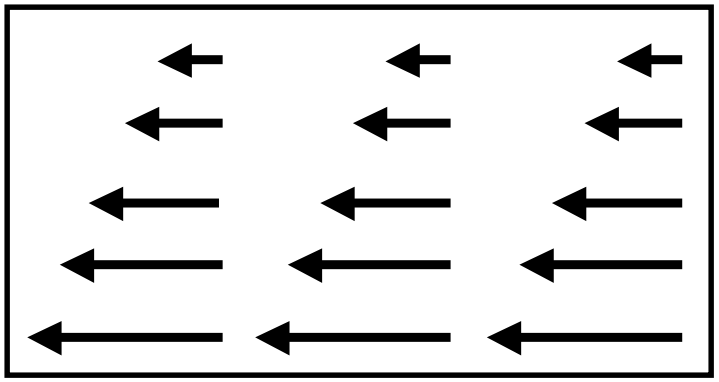
What is the divergence and curl of this vector field in the region shown?



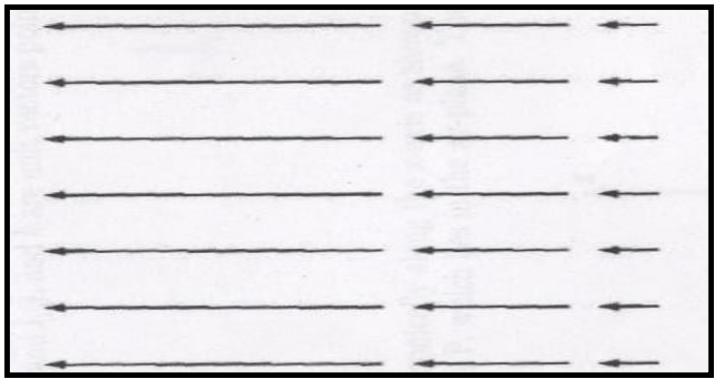
A) non-zero divergence, but zero curl.
B) zero divergence, but non-zero curl.
C) non-zero divergence, and non-zero curl.
D) zero divergence, and zero curl.
E) Impossible to predict, you need a formula!

Which of the following two fields has zero divergence?

I



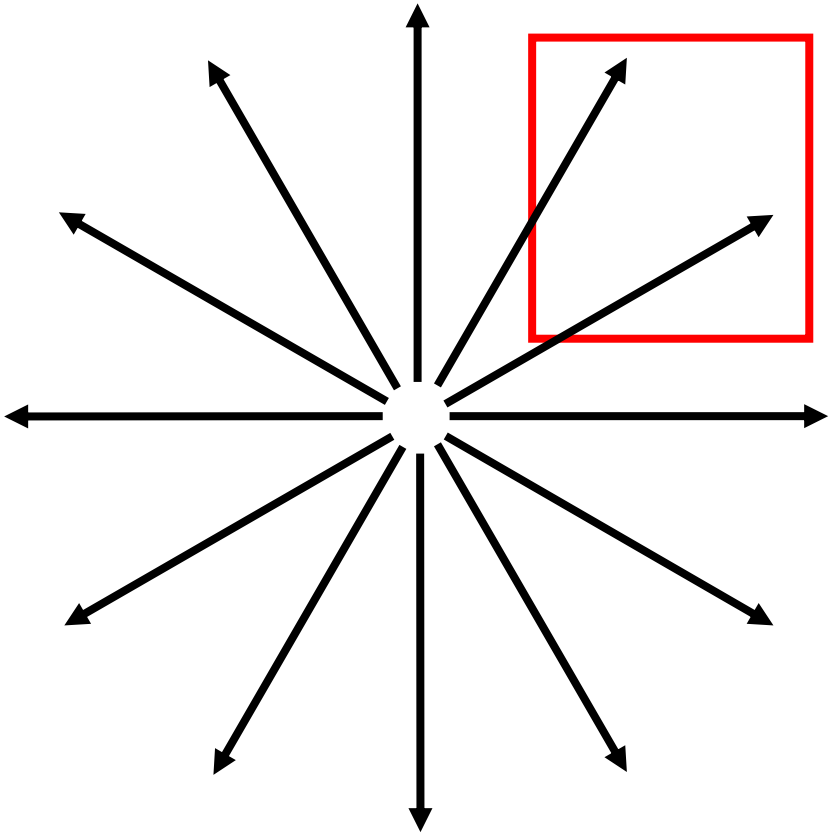
II



A) Both do
B) Only I is zero
C) Only II is zero
D) Neither is zero
E) ??

What is the divergence of this vector field *in the boxed region?*

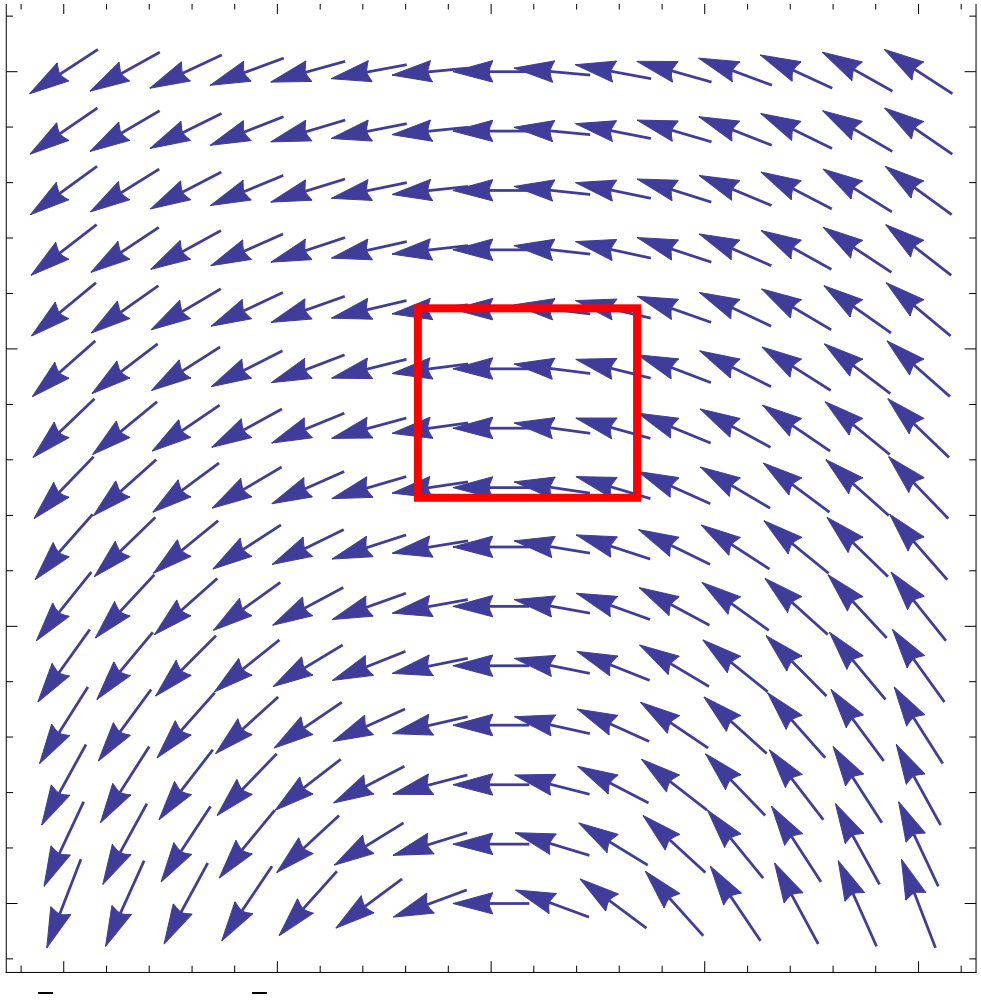
A) Zero
B) Not zero
C) Not enough info



What is the curl of this vector field, **V** in the region shown below?

$$\vec{\mathbf{V}} = c\,\hat{j}$$

A. non-zero everywhere
B. Zero at some points, non-zero others
C. zero curl everywhere shown



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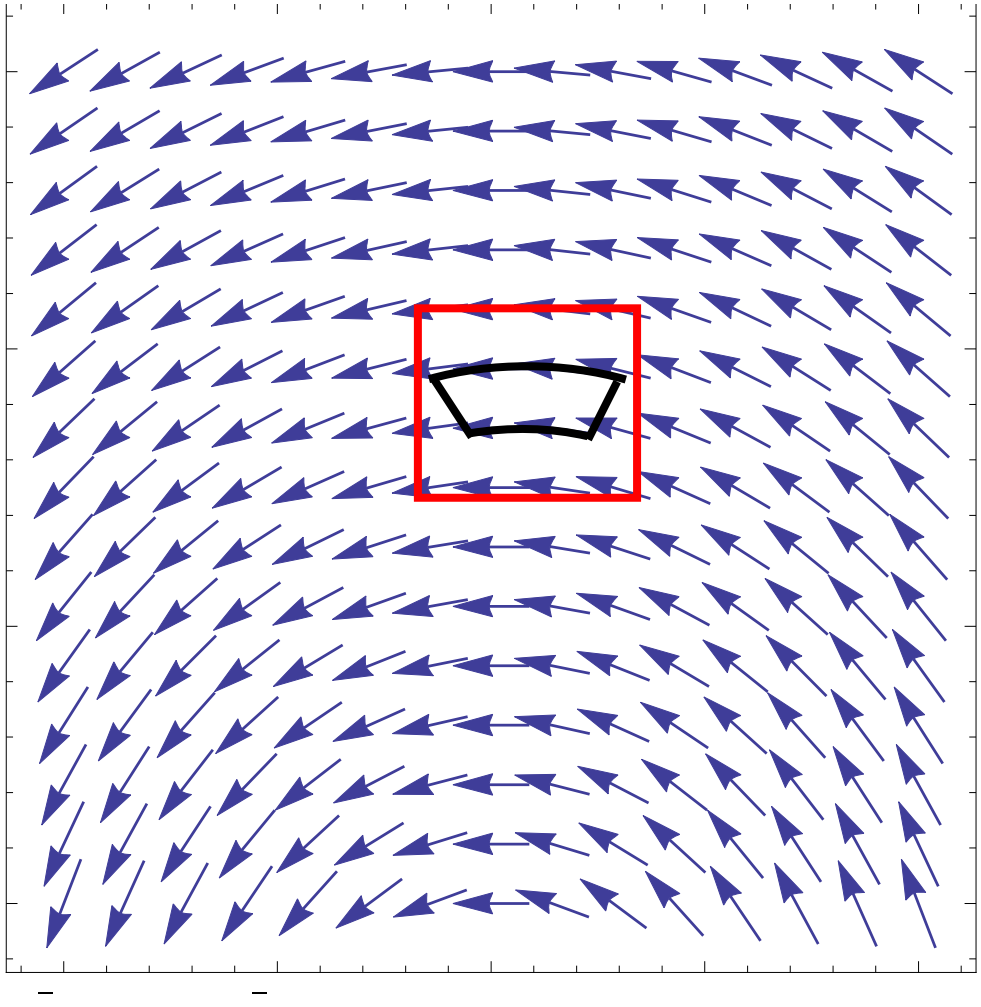
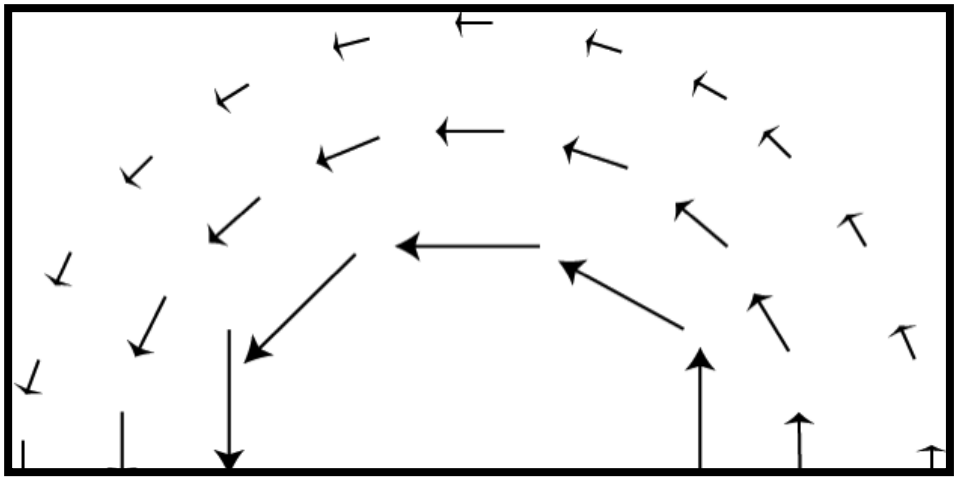


What is the curl of this vector field, in the region shown below?



A. non-zero everywhere
B. Non-zero at a limited set of points
C. zero curl everywhere shown
D. We need a formula to decide for sure

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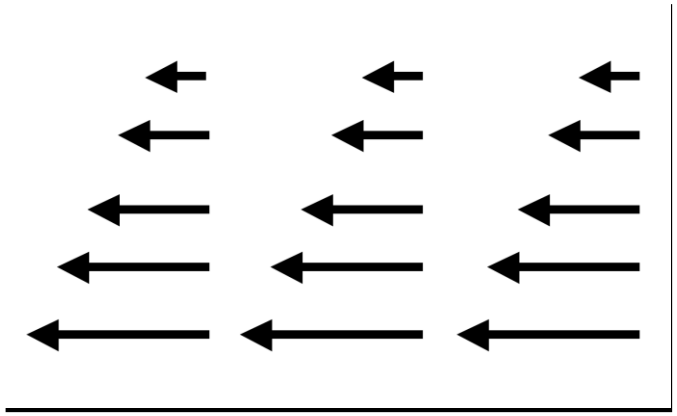

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Stokes' theorem

The modern form of Stokes' theorem is a vast generalization of a classical result that Lord Kelvin communicated to George Stokes in a letter dated July 2, 1850. Stokes set the theorem as a question on the 1854 Smith's Prize exam, which led to the result bearing his name.

Sir George Gabriel Stokes, Born in County Sligo, Ireland, spent all of his career at the University of Cambridge, where he was the Lucasian Professor of Mathematics from 1849 until his death.

seminal contributions to fluid dynamics, including the Navier-Stokes equation, and to physical optics