LAPLACE’S EQUATION AND UNIQUENESS

General properties of solutions of $\nabla^2 V=0$

1) V has no local maxima or minima inside. Maxima and minima are located on surrounding boundary.

2) V is boring. (I mean “smooth & continuous” everywhere).

3) V(**r**) = average of V over any surrounding sphere:

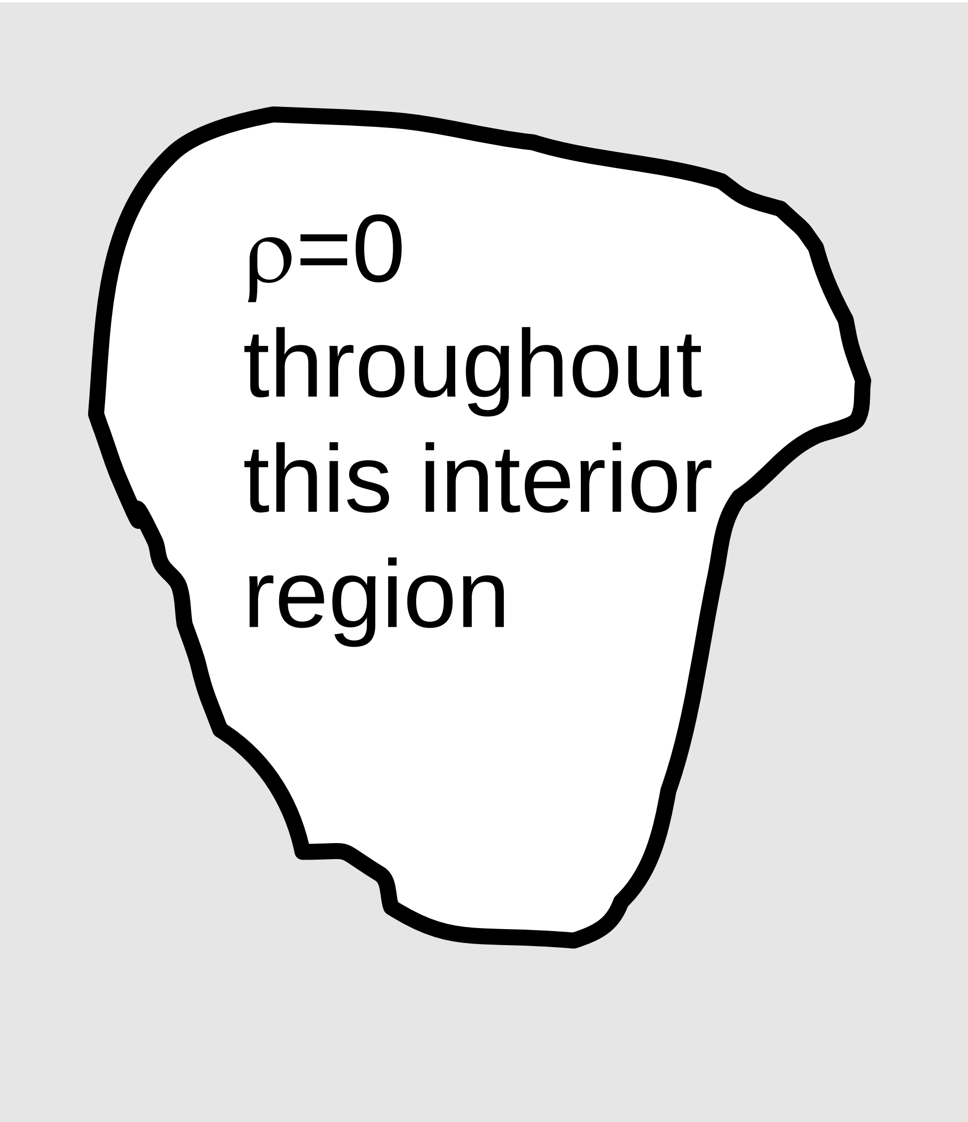
$$V(\vec{r}) = \frac{1}{4\pi R^2} \oint V \, da$$

Sphere with radius R around r

(4) V is unique: The solution of the Laplace eq. is uniquely determined if V is specified on the boundary surface around the volume.

A region of space contains no charges.

What can I say about V in the interior?

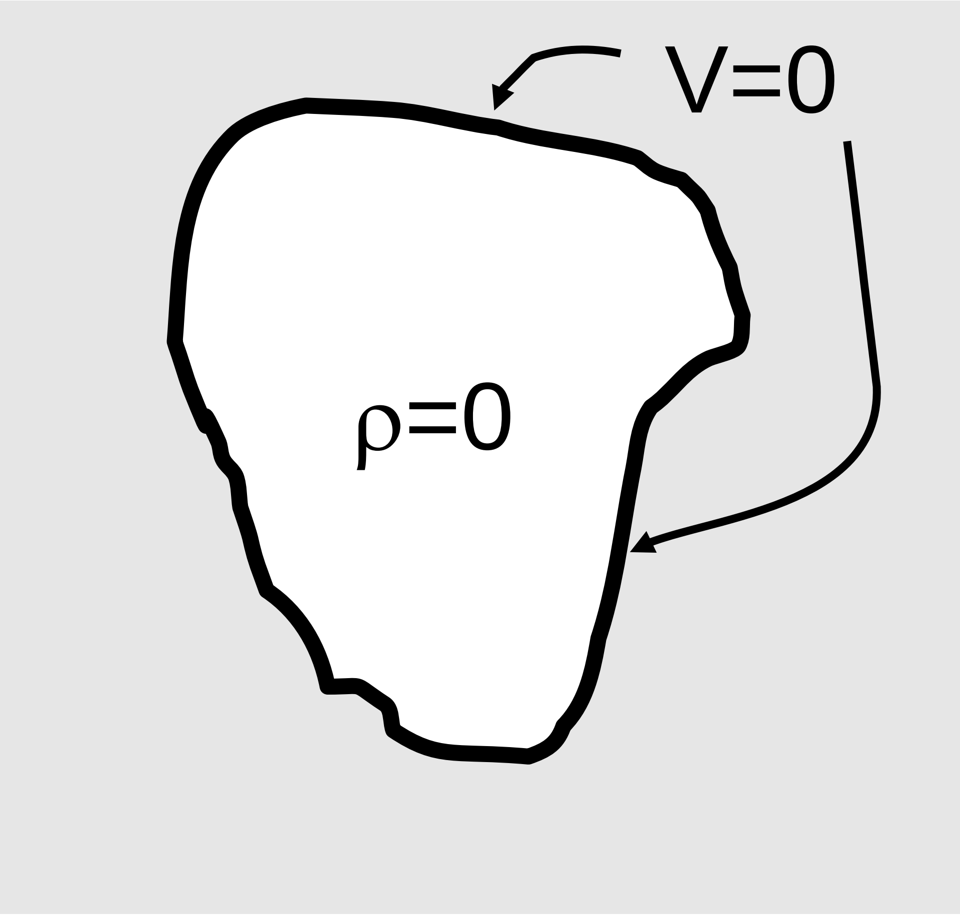


A) Not much, there are lots of possibilities for V(r) in there

B) V(r)=0 everywhere in the interior.

C) V(r)=constant everywhere in the interior

A region of space contains no charges.
The *boundary* has V=0 everywhere.
What can I say about V in the interior?



A) Not much, there are lots of possibilities for V(r) in there

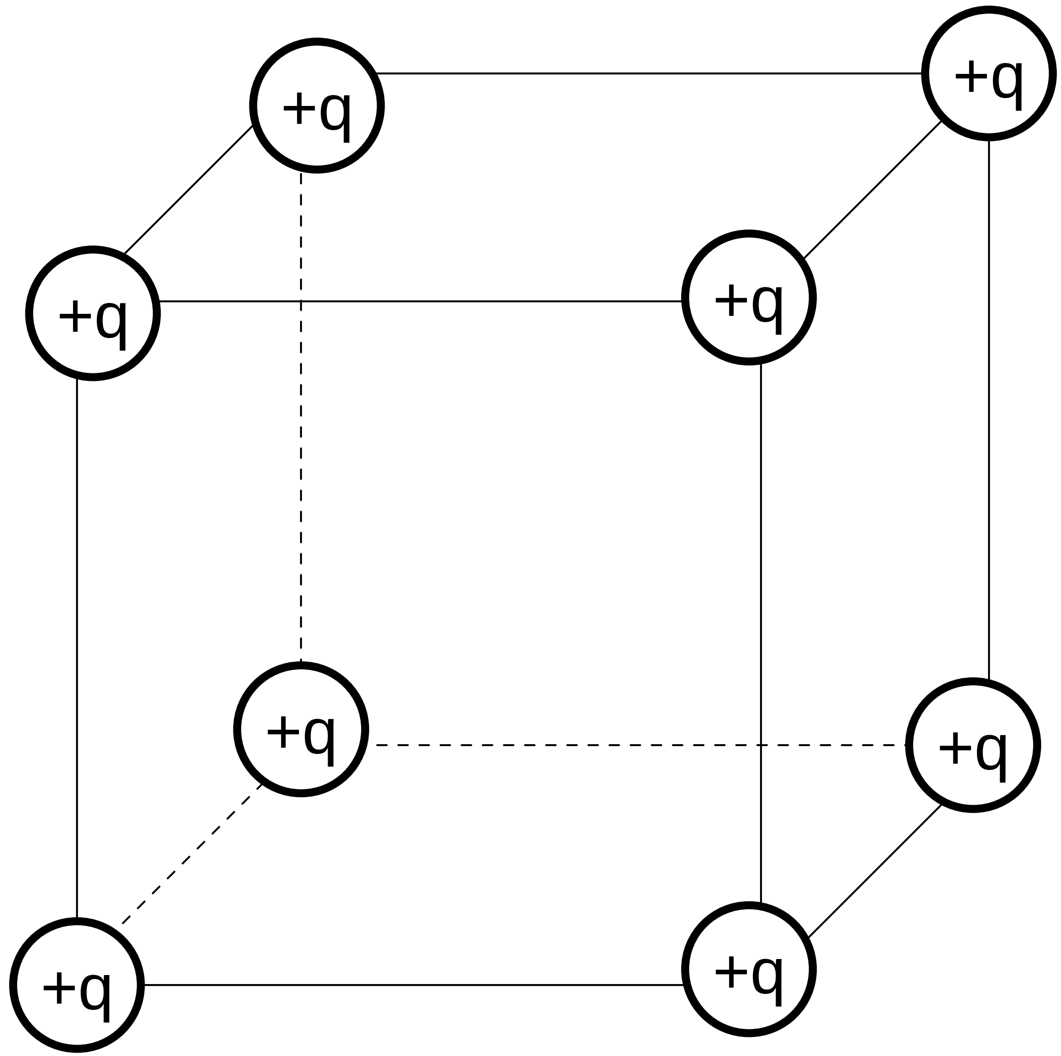
B) V(r)=0 everywhere in the interior.

C) V(r)=constant everywhere in the interior

If you put a + test charge at the *center* of this cube of charges, could it be in stable equilibrium?

A) Yes
B) No
C) ???

Earnshaw's Theorem



Pierre-Simon, marquis de Laplace

23 March 1749 – 5 March 1827

the “French Newton”

Mécanique Céleste (Celestial Mechanics)

Bayesian interpretation of probability
Laplace's equation (potential theory)
Laplace transform
Spherical Harmonics
Nebular of the origin of the Solar System
Black holes
Dynamic theory of tides
….

first articulation of **scientific determinism**

We may regard the present state of the universe as the effect of its past and the cause of its future.

METHOD OF IMAGES

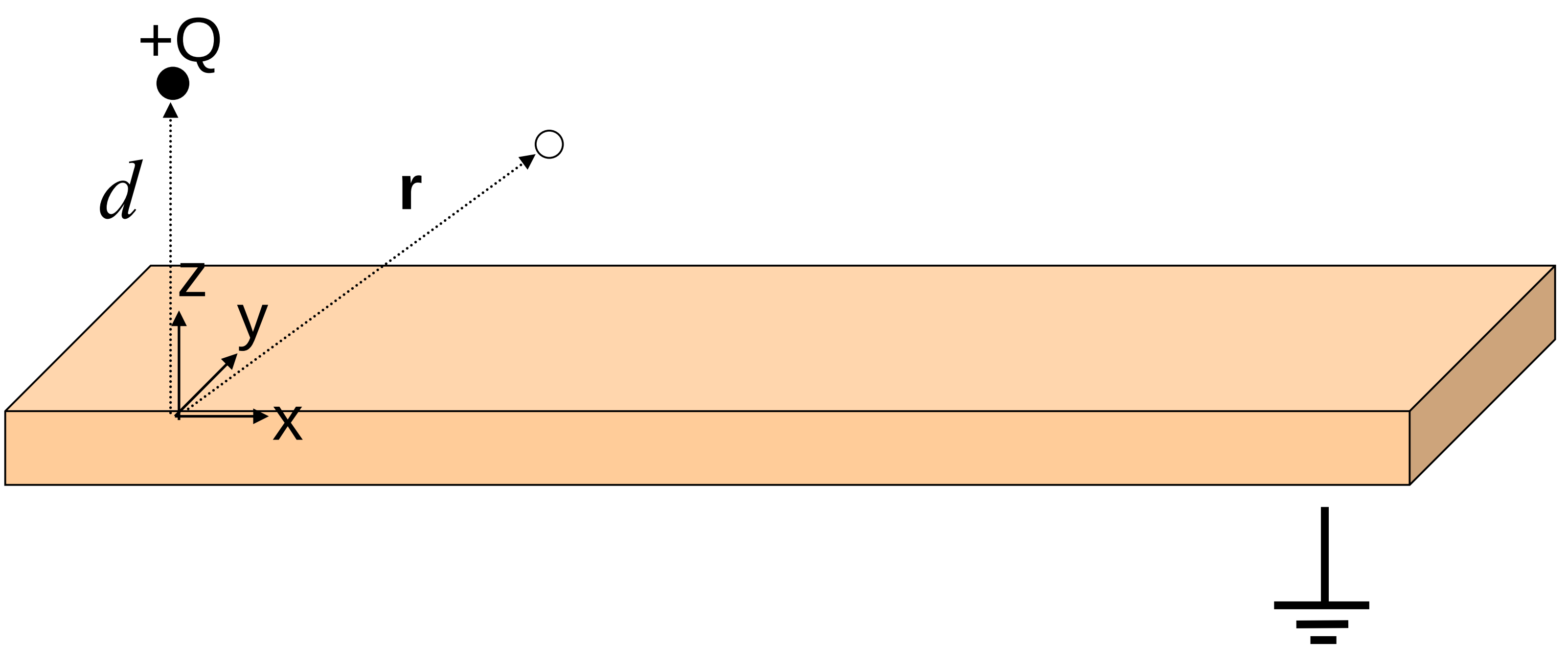
A point charge +Q sits above a *very large grounded* conducting slab.

What is **E**(**r**) for other points <u>*above*</u> the slab?

A) Simple Coulomb's law:

$$\vec{E}(\vec{r}) = \frac{Q}{4\pi\varepsilon_0}\frac{\vec{Â}}{Â^3} \qquad \text{with } \vec{Â} = (\vec{r} - d\hat{z})$$

B)



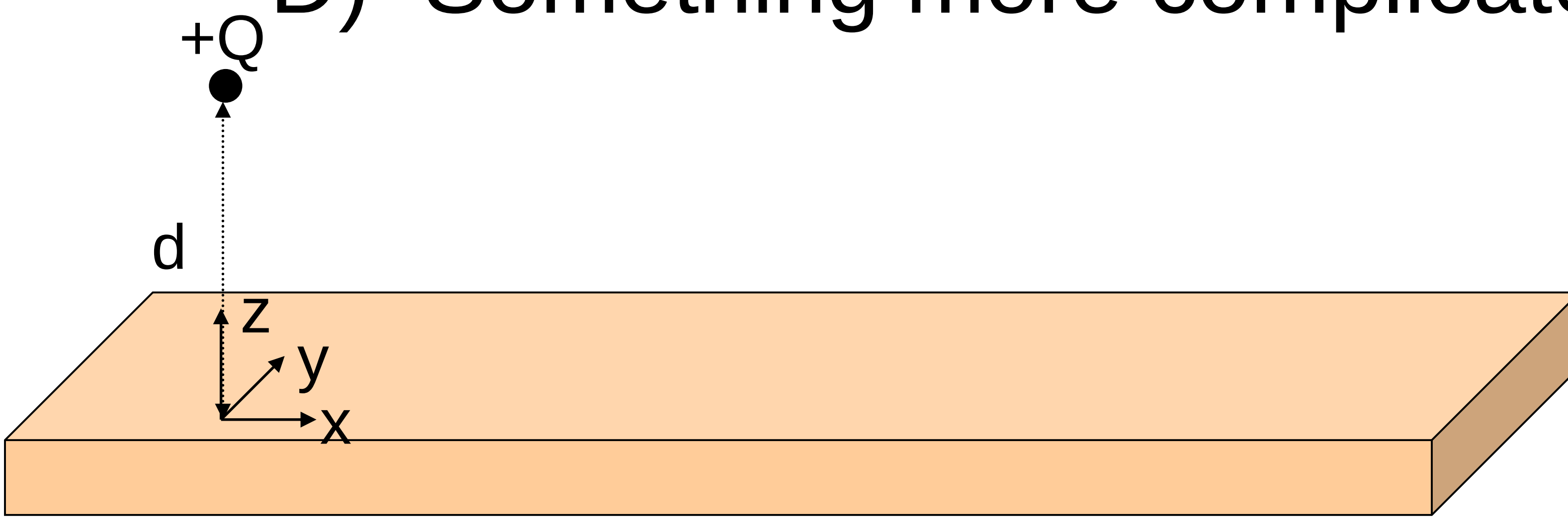
A point charge +Q sits above a *very large grounded* conducting slab. What is the electric force on this charge?

A) 0

B) $\dfrac{Q^2}{4pe_0(2d)^2}$ downwards

C) $\dfrac{Q^2}{4pe_0 d^2}$ downwards

D) Something more complicated

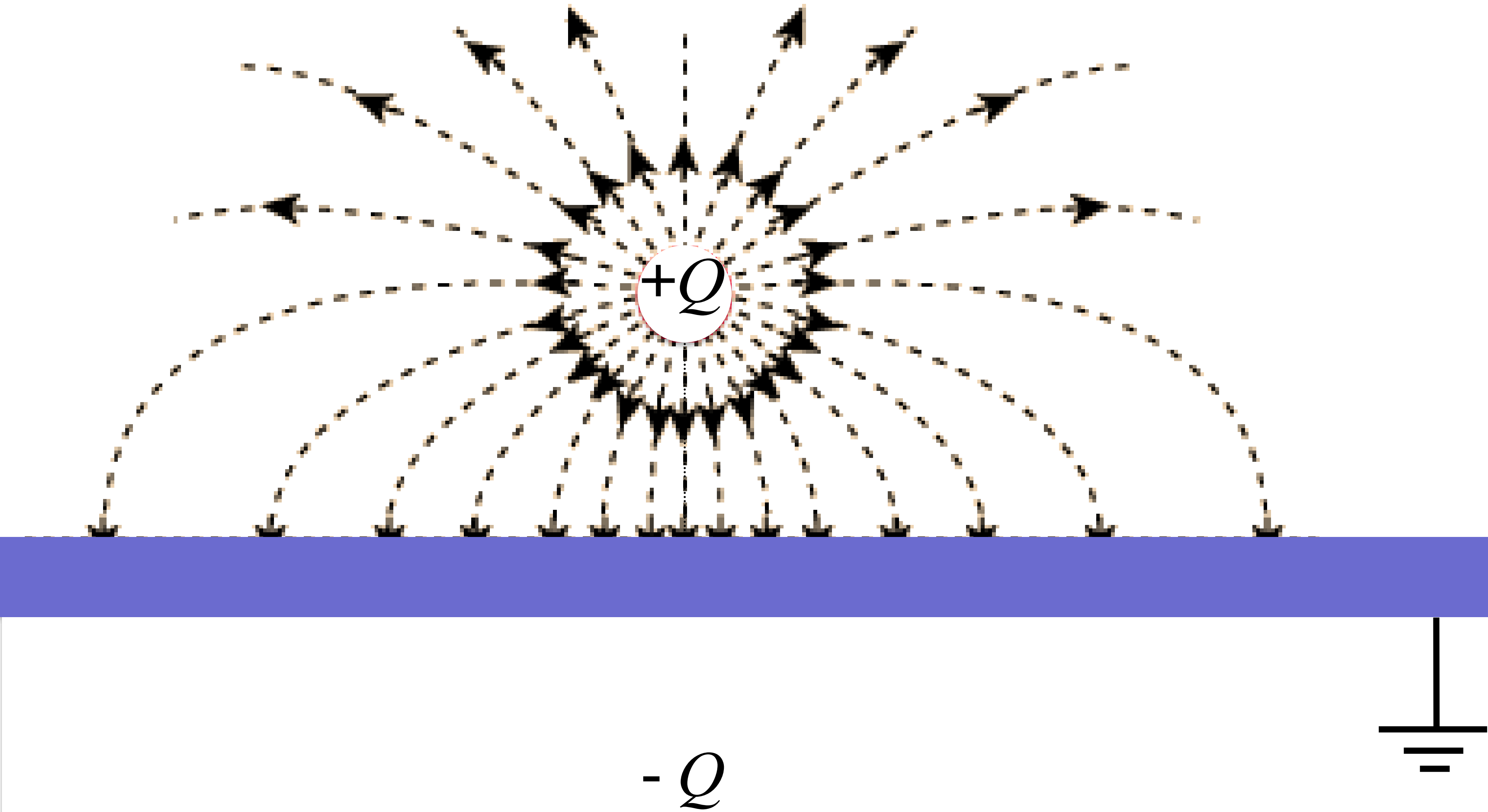


A point charge +Q sits above a *very large grounded* conducting slab.

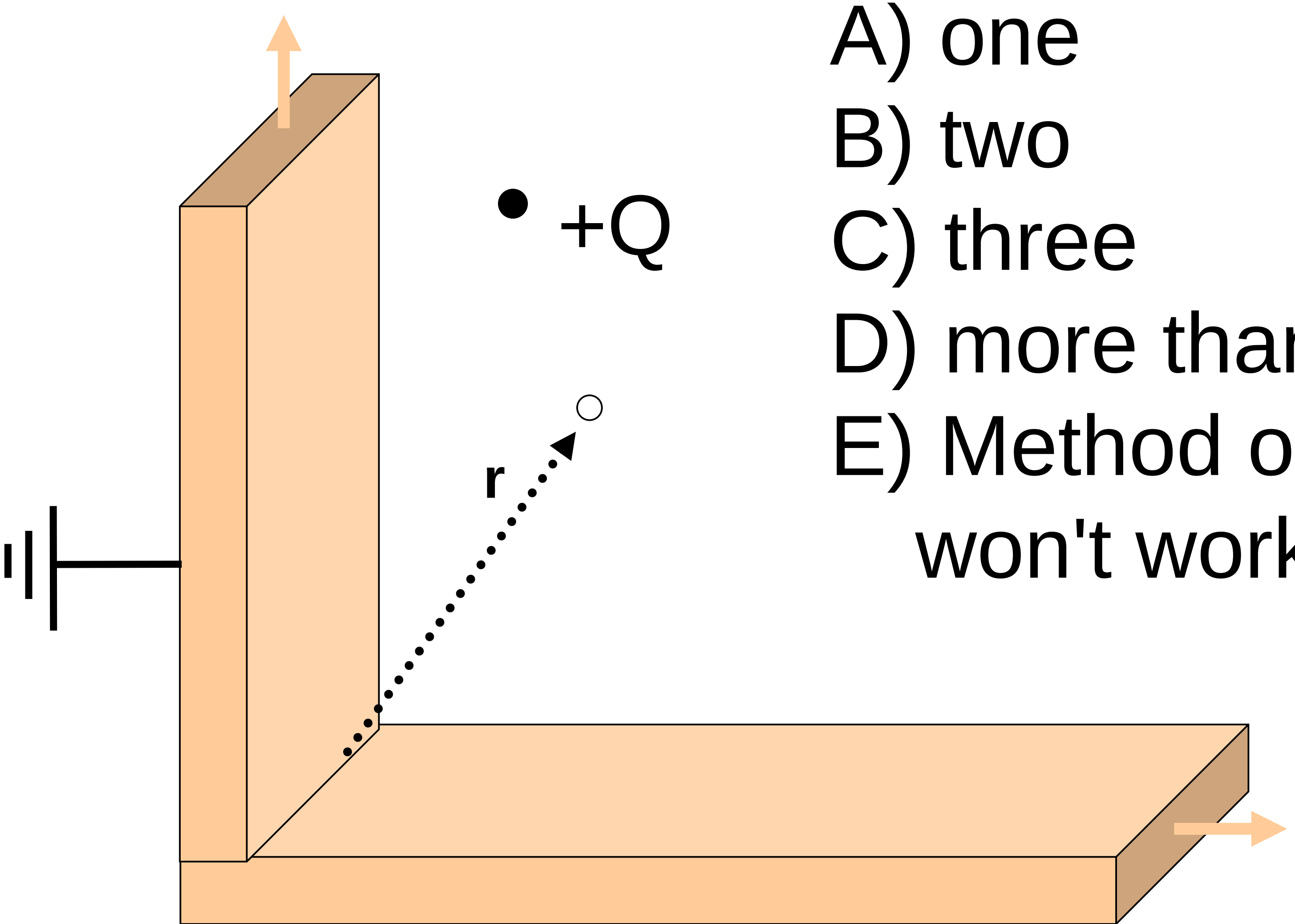
What's the energy of this system?

A) $$\frac{-Q^2}{4\pi\varepsilon_0(2d)}$$

B) Something else.

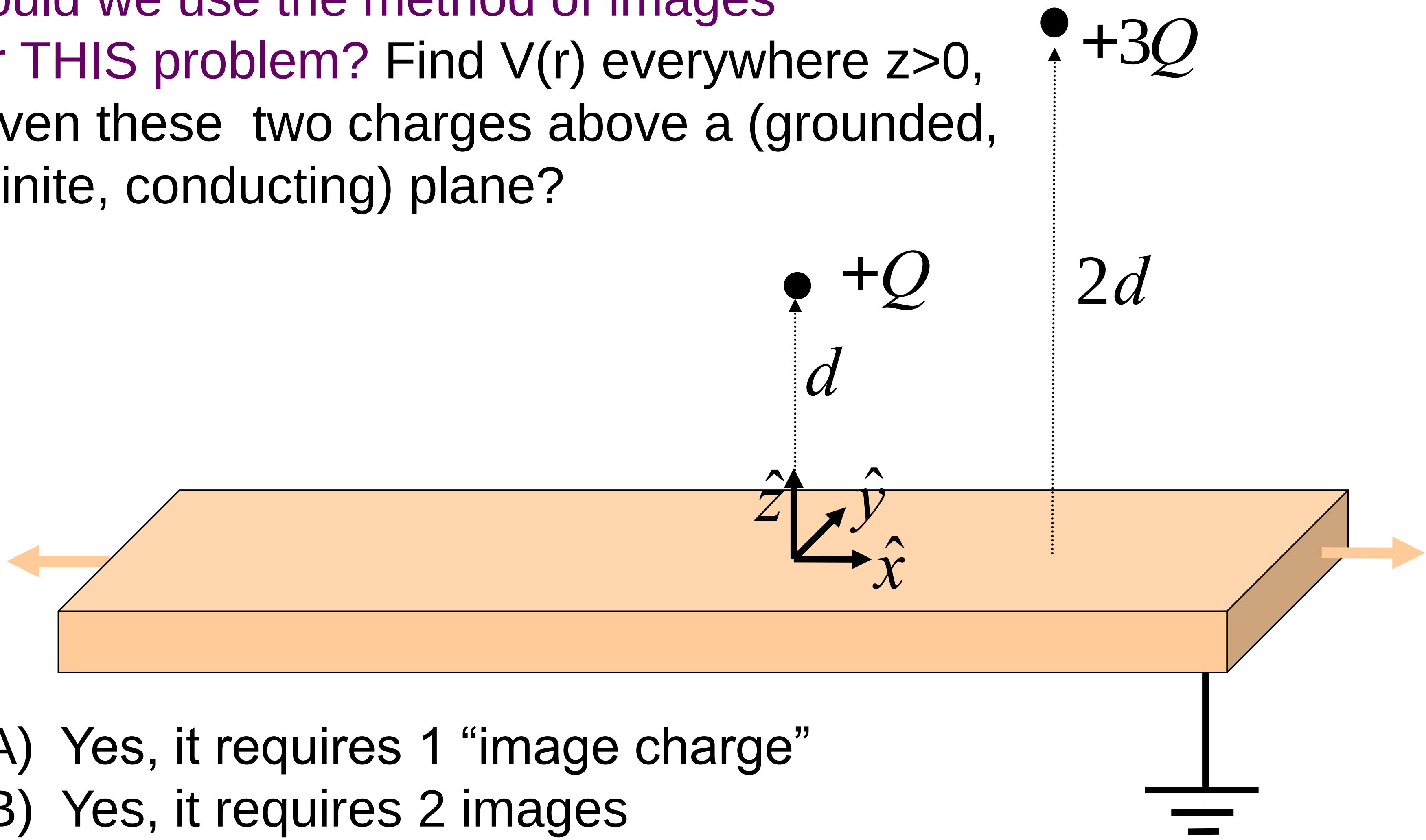

+Q
- Q

Two ∞ grounded conducting slabs meet at right angles. How many image charges are needed to solve for V(**r**)?



A) one
B) two
C) three
D) more than three
E) Method of images won't work here

Could we use the method of images for THIS problem? Find V(r) everywhere z>0, Given these two charges above a (grounded, infinite, conducting) plane?



A) Yes, it requires 1 "image charge"
B) Yes, it requires 2 images
C) Yes, more than 2 image charge
D) No, this problem can NOT be solved using the "trick" of image charges…

Could we use the method of images for THIS problem? Find V(r) everywhere z>0, Given these two charges above a (grounded, infinite, conducting) plane?