SEPARATION OF VARIABLES

CARTESIAN COORDINATES

Say you have three functions f(x), g(y) and h(z).
f(x) depends on ‘x’ but not on ‘y’ or ‘z’.
g(y) depends on ‘y’ but not on ‘x’ or ‘z’.
h(z) depends on ‘z’ but not on ‘x’ or ‘y’.

If $f(x) + g(y) + h(z) = 0$ for all x, y, z, then:

A) All three functions are constants (i.e. they do not depend on x, y, z at all.)
B) At least one of these functions has to be zero everywhere.
C) All of these functions have to be zero everywhere.
D) All three functions have to be linear functions in x, y, or z respectively (such as $f(x)=ax+b$)
E) ???

Suppose $V_1(\mathbf{r})$ and $V_2(\mathbf{r})$ are linearly independent functions which *both* solve Laplace's equation, $\nabla^2 V = 0$

Does $aV_1(\mathbf{r})+bV_2(\mathbf{r})$ also solve it (with a and b constants)?

A) Yes. The Laplacian is a linear operator
B) No. The *uniqueness theorem* says this scenario is impossible, there are never two independent solutions!
C) It is a definite yes or no, but the *reasons* given above just aren't right!
D) It depends...

The X(x) equation in this problem involves the "positive constant" solutions:
A sinh(kx) + B cosh(kx)

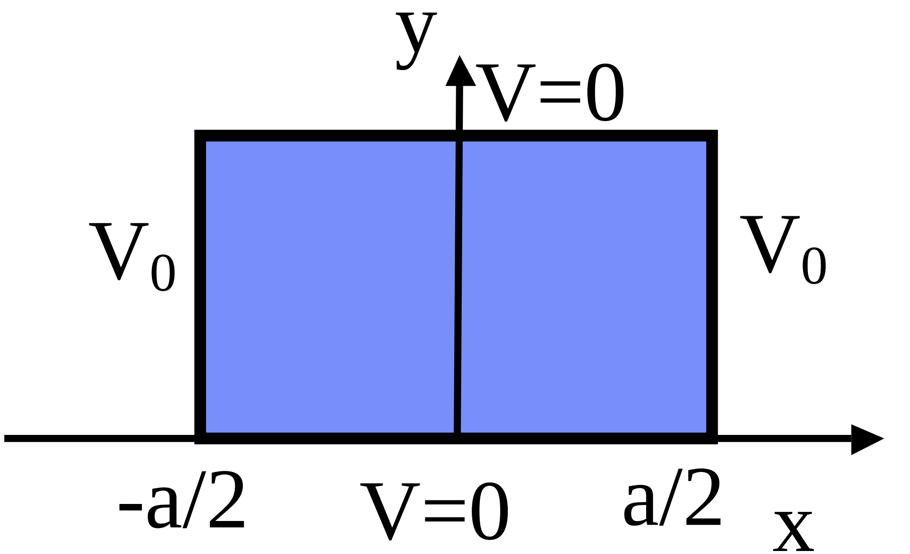
What do the boundary conditions say about the coefficients A and B above?

A) A=0 (pure cosh)

B) B=0 (pure sinh)

C) Neither: you should rewrite this in terms of $A' e^{kx} + B' e^{-kx}$!

D) Other/not sure?



Given the two diff. eq's:

$$\frac{1}{X}\frac{d^2X}{dx^2}=C_1 \qquad \frac{1}{Y}\frac{d^2Y}{dy^2}=C_2$$

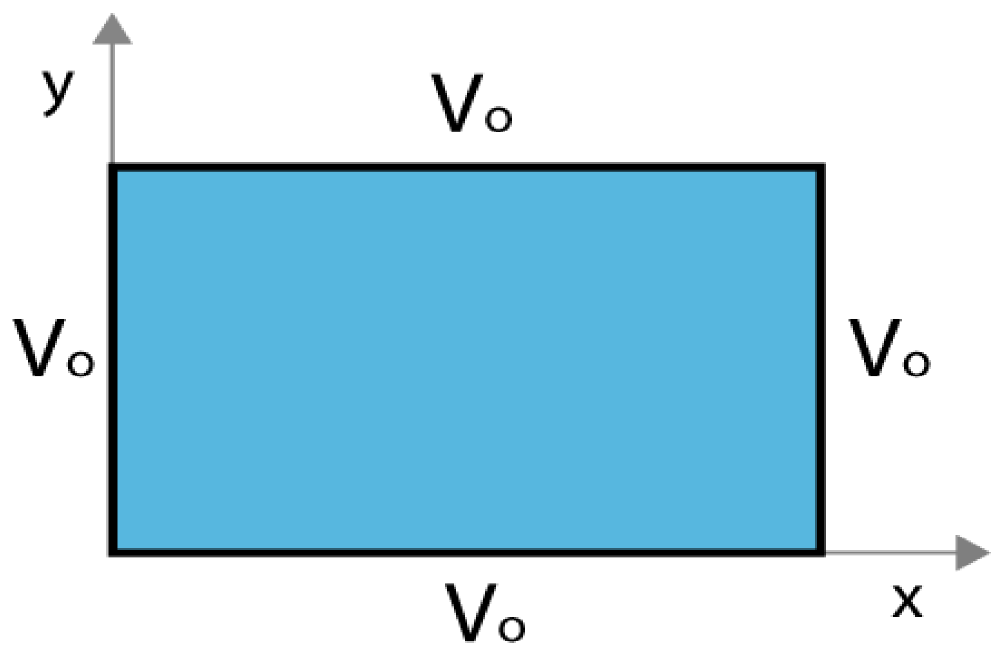
where $C_1+C_2 = 0$. Which coordinate should be assigned to the negative constant (and thus the sinusoidal solutions)?

A) x

B) y

C) $C_1 = C_2 = 0$ here

D) It doesn't matter



SEP OF VAR: LEGENDRE POLYNOMIALS

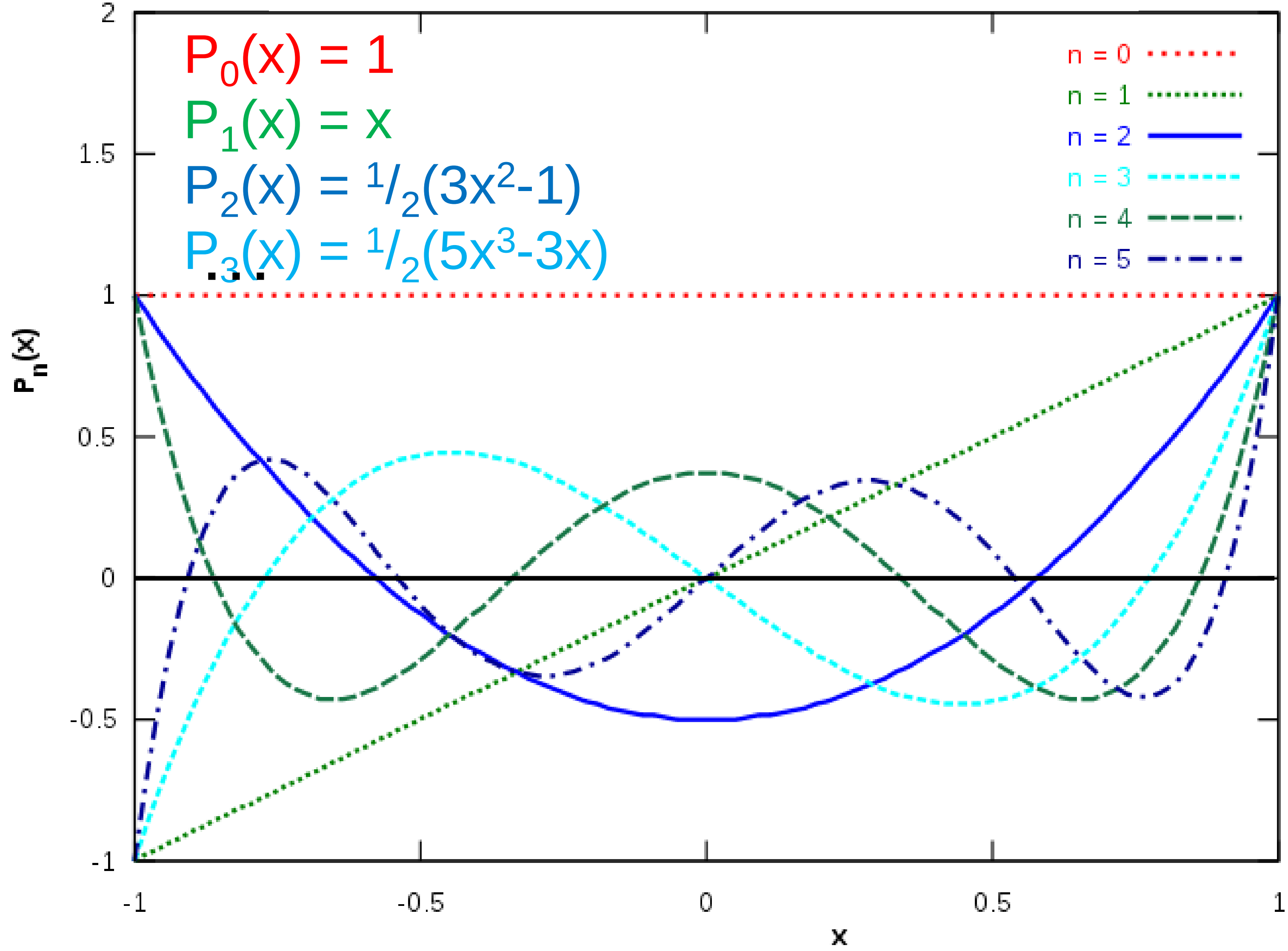
Given $\nabla^2 V = 0$ in Cartesian coords, we separated V(x,y,z) = X(x)Y(y)Z(z). Will this approach work in spherical coordinates, i.e. can we separate V(r,θ,φ) = R(r)P(θ)F(φ)?

A) Sure.

B) Not quite - the angular components cannot be isolated, e.g. f(r,θ,φ) = R(r)Y(θ,φ)

C) It won't work at all because the spherical form of Laplace's Equation has cross terms in it (see the front cover of Griffiths)

Legendre Polynomials $P_n(x)$



Given $V_0(\theta) = \sum_l C_l P_l(\cos(\theta))$ we want to get to the integral: $\int_{-1}^{1} P_l(u) P_m(u)\,du = \begin{bmatrix} \frac{2}{2l+1}, l = m \\ 0, l \neq m \end{bmatrix}$

we can do this by multiplying both sides by:

A) $P_m(\cos\theta)$

B) $P_m(\sin\theta)$

C) $P_m(\cos\theta)\sin\theta$

D) $P_m(\sin\theta)\cos\theta$

E) $P_m(\sin\theta)\sin\theta$

$$V(r,\theta) = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos\theta)$$

V everywhere on a spherical shell is a given constant, i.e. $V(R, \theta) = V_0$.

There are no charges inside the sphere.

Which terms do you expect to appear when finding V(inside) ?

A) Many A_l terms (but no B_l's)

B) Many B_l terms (but no A_l's)

C) Just A_0

D) Just B_0

E) Something else!

$$V(r,\theta) = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos\theta)$$

Suppose V on a spherical shell is constant, i.e. $V(R, \theta) = V_0$.

Which terms do you expect to appear when finding V(outside) ?

A) Many A_l terms (but no B_l's)

B) Many B_l terms (but no A_l's)

C) Just A_0

D) Just B_0

E) Something else!!

$$P_0(\cos\theta) = 1, \qquad P_1(\cos\theta) = \cos\theta$$

$$P_2(\cos\theta) = \frac{3}{2}\cos^2\theta - \frac{1}{2}, \qquad P_3(\cos\theta) = \frac{5}{2}\cos^3\theta - \frac{3}{2}\cos\theta$$

Can you write the function $V_0(1+\cos^2\theta)$ as a sum of Legendre Polynomials?

$$V_0(1+\cos^2\theta) \overset{???}{=} \sum_{l=0}^{\infty} C_l P_l(\cos\theta)$$

A) No, it cannot be done

B) It would require an infinite sum of terms

C) It would only involve P_2

D) It would involve all three of P_0, P_1 AND P_2

E) Something else/none of the above

$$V(r,\theta) = \sum_{l=0}^{\infty}\left(A_l r^l + \frac{B_l}{r^{l+1}}\right)P_l(\cos\theta)$$

Suppose V on a spherical shell is

$$V(R,\theta) = V_0(1+\cos^2\theta)$$

Which terms do you expect to appear when finding V(inside) ?

A) Many A_l terms (but no B_l's)

B) Many B_l terms (but no A_l's)

C) Just A_0 and A_2

D) Just B_0 and B_2

E) Something else!

$$V(r,\theta) = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos\theta)$$

Suppose V on a spherical shell is

$$V(R,\theta) = V_0 (1 + \cos^2\theta)$$

Which terms do you expect to appear when finding V(outside) ?

A) Many A_l terms (but no B_l's)

B) Many B_l terms (but no A_l's)

C) Just A_0 and A_2

D) Just B_0 and B_2

E) Something else!

Suppose that applying boundary conditions to Laplace's equation leads to an equation of the form:

$$\sum_{l=0}^{\infty} C_l P_l(\cos\theta) = 4 + 3\cos\theta$$

$(x \equiv \cos\theta)$

$P_0(x) = 1$

$P_1(x) = x$

$P_2(x) = (3x^2 - 1)/2$

Can you solve for the coefficients, the C_ℓ's ?

A) No, you need at least one more equation to solve for any the C's.

B) Yes, you have enough info to solve for all of the C's

C) Partially. Can solve for C_0 and C_1, but cannot solve for the other C's.

D) Partially. Can solve for C_o, but cannot solve for the other C's.

$$P_0(\cos\theta) = 1, \qquad P_1(\cos\theta) = \cos\theta$$

$$P_2(\cos\theta) = \frac{3}{2}\cos^2\theta - \frac{1}{2}, \qquad P_3(\cos\theta) = \frac{5}{2}\cos^3\theta - \frac{3}{2}\cos\theta$$

Can you write the function $\sin^2\theta$ as a sum of Legendre Polynomials?

$$\sin^2\theta \overset{???}{=} \sum_{l=0}^{\infty} C_l P_l(\cos\theta)$$

A) No, it cannot be done

B) Yes, It would require an infinite sum of terms

C) Yes, only C_2 would be nonzero

D) Yes, but only C_0 and C_2 would be nonzero

E) Something else/none of the above