MAGNETIC VECTOR POTENTIAL

One of Maxwell's equations, $\nabla \times E = 0$ made it useful for us to define a scalar potential V, where $E = -\nabla V$

Similarly, another one of Maxwell's equations makes it useful for us to define the vector potential, **A.** Which one?

A) $\nabla \times E = 0$

B) $\nabla \cdot E = r / e_0$

C) $\nabla \times B = m_0 J$

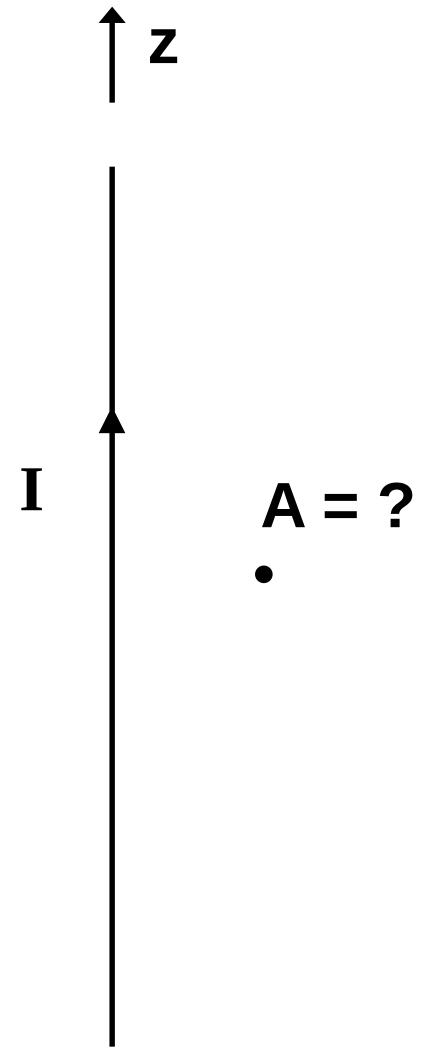
D) $\nabla \cdot B = 0$

E) something else!

What is $\oint \vec{\mathbf{A}}(\vec{r}) \cdot d\vec{\mathbf{l}}$

A) The current density **J**
B) The magnetic field **B**
C) The magnetic flux Φ_B
D) It's none of the above, but is something simple and concrete
E) It has no particular physical interpretation at all

The vector potential **A** due to a long straight wire with current I along the z-axis is in the direction parallel to:



A) $\hat{z}$

B) $\hat{j}$ (azimuthal)

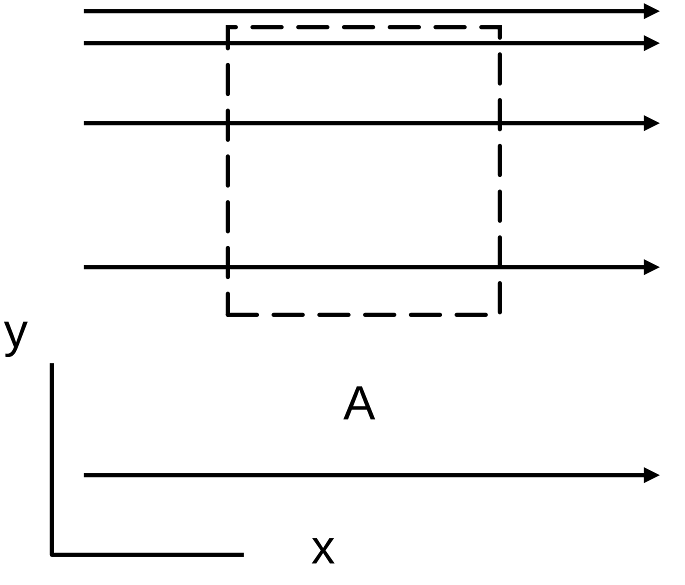
C) $\hat{s}$ (radial)

Assume Coulomb gauge

The vector potential in a certain region is given by

$$\vec{A}(x, y) = C\, y\, \hat{x}$$

(C is a positive constant) Consider the imaginary loop shown. What can you say about the magnetic field in this region?

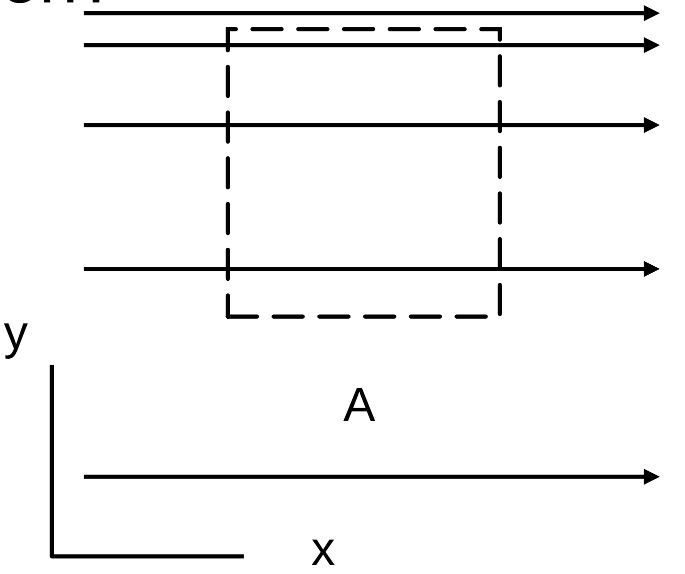


A. B is zero
B. B is non-zero, parallel to z-axis
C. B is non-zero, parallel to y-axis
D. B is non-zero, parallel to x-axis

The vector potential in a certain region is given by

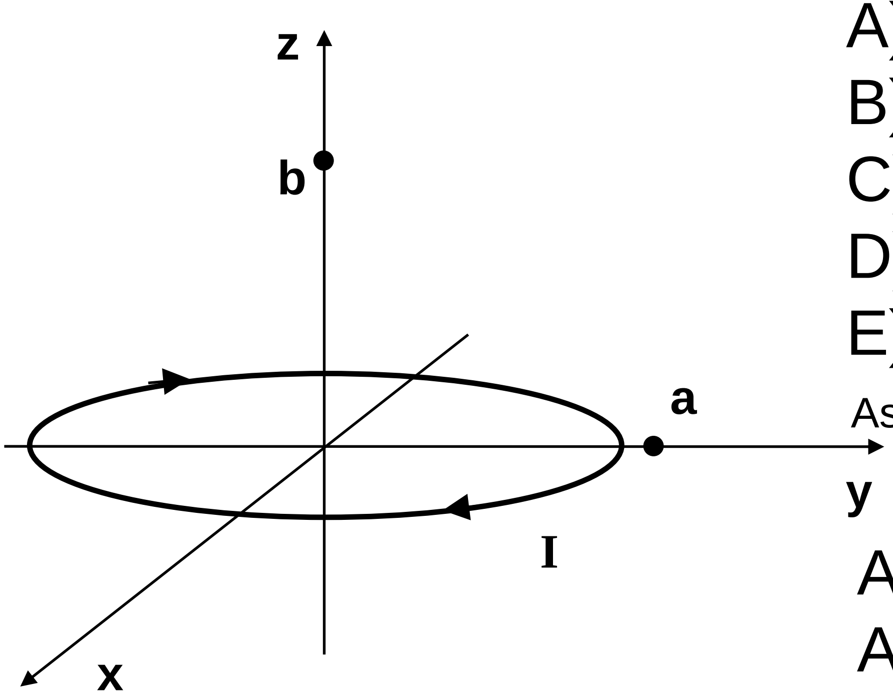
$$\vec{A}(x,y) = C\,y\,\hat{x}$$

(C is a positive constant) Consider the imaginary loop shown. What can you say about the direction of the magnetic field in this region?



A. Out of page
B. Into page

A circular wire carries current I in the xy plane. What can you say about the vector potential **A** at the points shown?



<u>At point a</u>, the vector potential **A** is:

A) Zero
B) Parallel to x-axis
C) Parallel to y-axis
D) Parallel to z-axis
E) Other/not sure…

Assume Coulomb gauge, and A vanishes at infinity

At point b, the vector potential **A** is:

A) Zero
B) Parallel to x-axis
C) Parallel to y-axis
D) Parallel to z-axis
E) Other/not sure

5.28

When you are done with p. 1:
Choose all of the following statements that are implied if $\oint\!\!\oint \vec{B}\cdot d\vec{a} = 0$ for any/all closed surfaces

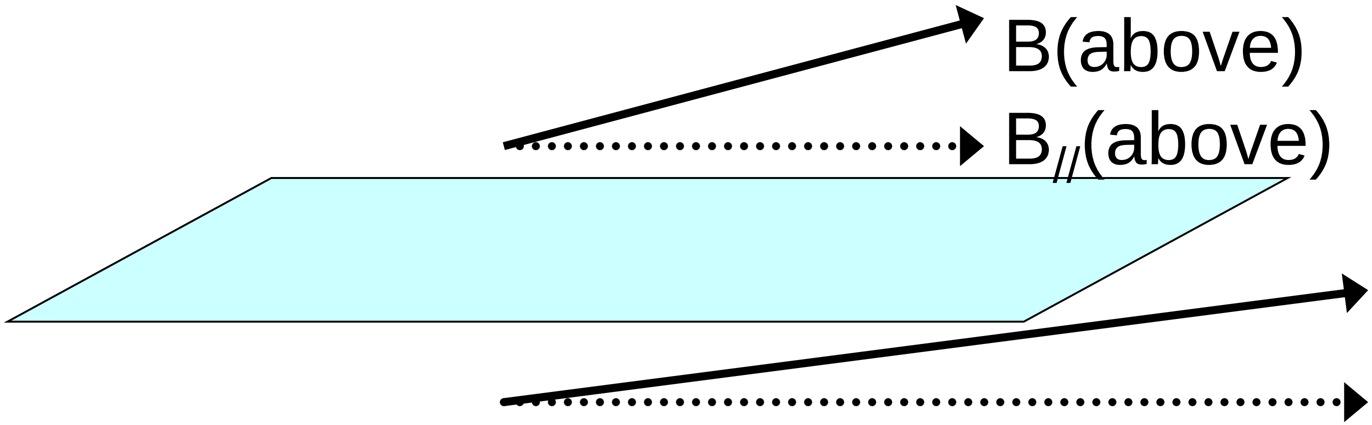
(I) $\vec{\nabla}\cdot\vec{B} = 0$

(II) $B^{//}_{above} = B^{//}_{below}$

(III) $B^{\perp}_{above} = B^{\perp}_{below}$

A) (I) only
B) (II) only
C) (III) only
D) (I) and (II) only
E) (I) and (III) only

I have a boundary sheet, and would like to learn about the change (or continuity!) of B(parallel) across the boundary.



Am I going to need to know about

A) $\nabla \times B$

B) $\nabla \cdot B$

C) ???

DIPOLES, MULTIPOLES

The leading term in the vector potential multipole expansion involves $\oint d\mathbf{l}'$

What is the magnitude of this integral?

A) R

B) 2π R

C) 0

D) Something entirely different/it depends!

The formula from Griffiths for a magnetic dipole at the origin is:

$$\vec{\mathbf{A}}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\hat{r}}{r^2}$$

Is this the *exact* vector potential for a flat ring of current with **m**=I**a**, or is it approximate?

A) It's exact

B) It's exact if |r| > radius of the ring

C) It's approximate, valid for large r

D) It's approximate, valid for small r

This is the formula for an ideal magnetic dipole:

$$\mathbf{B} = \frac{c}{r^3}(2\cos q\,\hat{r} + \sin q\,\hat{q})$$

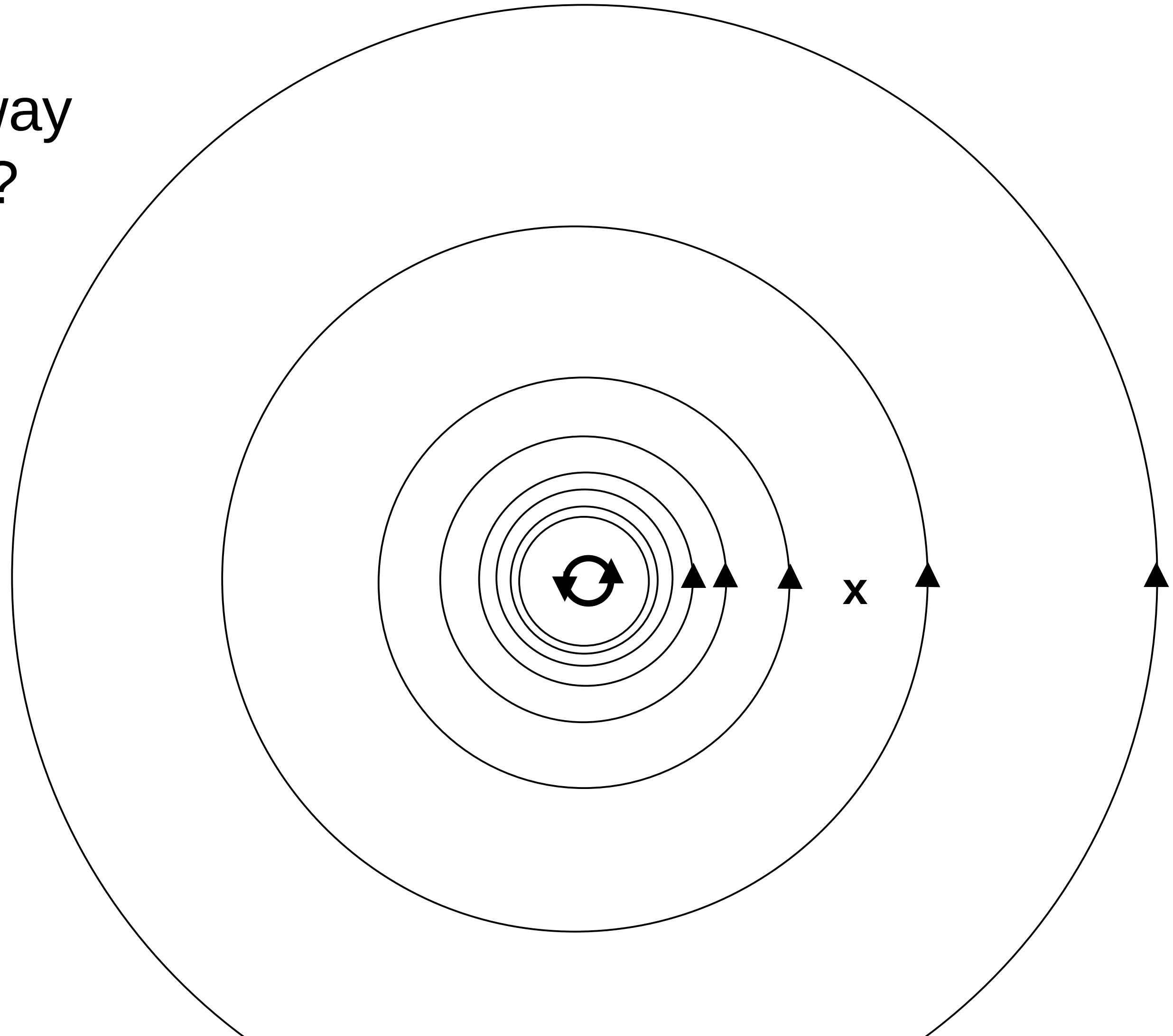
What is different in a sketch of a *real* (physical) magnetic dipole (like, a small current loop)?

In the plane of a magnetic dipole, with magnetic moment **m (out)**, the vector potential **A** looks kinda like this

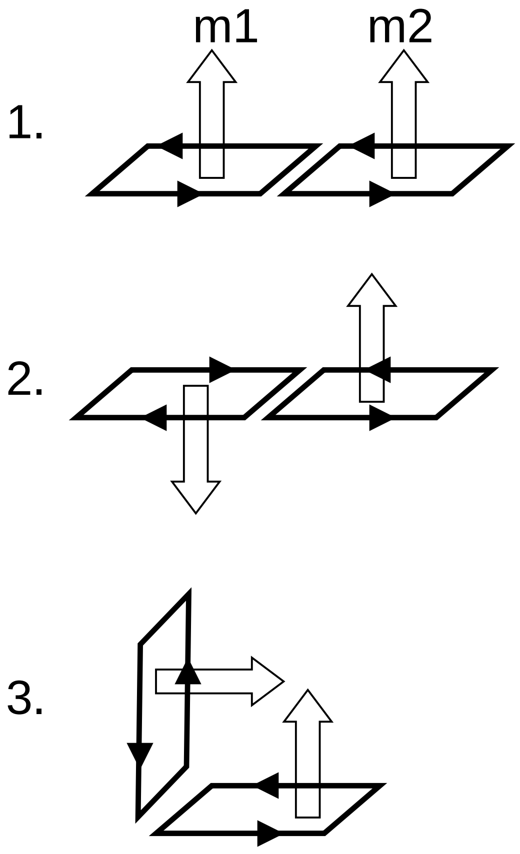
with A ~ $1/r^2$

At point x, which way does curl(**A**) point?

A) Right
B) Left
C) In
D) Out
E) Curl is zero



Two magnetic dipoles **m**1 and **m**2 (equal in magnitude) are oriented in three different ways.



Which ways produce a dipole field at large distances?

A) None of these

B) All three

C) 1 only

D) 1 and 2 only

E) 1 and 3 only