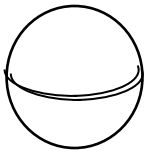
Warmup 6 – Separation of variables: Spherical coordinates

In class (and Griffiths) we frequently write down $\mathrm{V(r,\,\theta)} = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}}\right) P_l(\cos\theta)$. A student has several homework problems, in all of which she is being asked to solve for voltage throughout space. In each case, she needs to decide whether the equation (above) would be a **useful** step towards a solution. For which of the following cases can the student fruitfully use the above equation?



Case 1: Two infinitesimally thin hemispherical shells sit on top of each other, centered on the origin. The top is held at a constant potential of 0V and the bottom at a constant 20V all over its surface. Please choose one.

a) Yes, the equation above will be useful to find V in all regions of space.
b) Yes, the equation above will be useful to find V, but only in certain limited regions of space
c) No, the equation above is not particularly useful to find V anywhere (Well, except the shell itself, where V is already given!)

Please explain your reasoning for your answer to case 1 (If you qualified the answer, explain what region(s) it is NOT useful for, and why):

Case 2: A spherically symmetric distribution of charge $\rho(r) = \rho_0 e^{-r/\lambda}$ for all r. Please choose one.

a) Yes, the equation above will be useful to find V in all regions of space.
b) Yes, the equation above will be useful to find V, but only in certain limited regions of space
c) No, the equation above is not particularly useful to find V anywhere

Please explain your reasoning for your answer to case 2 (If you qualified the answer, explain what region(s) it is NOT useful for, and why):

turn over...

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Case 3: A line of uniform linear charge density λ, on the z axis extending from 0 to d, with no other charges. Please choose one.

a) Yes, the equation above will be useful to find V in all regions of space.
b) Yes, the equation above will be useful to find V, but only in certain limited regions of space
c) No, the equation above is not particularly useful to find V anywhere

Please explain your reasoning for your answer to case 3 (If you qualified the answer, explain what region(s) it is NOT useful for, and why):

Case 4: An infinitesimally thin hollow insulating sphere, centered on the origin, with a charge distribution $\sigma = \sigma_0 \cos\theta \sin\phi$ on the surface of the sphere. Please choose one.

a) Yes, the equation above will be useful to find V in all regions of space.
b) Yes, the equation above will be useful to find V, but only in certain limited regions of space
c) No, the equation above is not particularly useful to find V anywhere

Please explain your reasoning for your answer to case 4 (If you qualified the answer, explain what region(s) it is NOT useful for, and why):