PHYS 5382 Introduction to Quantum Mechanics **Practice Final**

Resources allowed:
Griffiths textbook, calculator, formulas from the covers of Griffiths hardback

Part I. Short Conceptual Questions. Time allowed 1 hour. 10% of course grade

1. A particle is described by a normalized wavefunction at time $t = 0$ on the x-axis

   $\Psi(x,0) = A \exp[-x^2 + (2\pi i\, x/\lambda)]$ where λ is a real parameter.

   Write down <u>explicitly</u> the x integral you would need to evaluate in order to calculate the following (you DO NOT need to evaluate any of the integrals):

(a) The constant A
(b) The probability of observing the particle in the interval (a , b) of the x axis.
(c) $\langle x^2 \rangle$
(d) $\langle p \rangle$
(e) The momentum space wavefunction $\Phi(p, 0)$ at time zero. **[½% each part]**

2. Consider a particle of mass m moving subject to the force of gravity (assumed constant) above the ground. With a clearly defined co-ordinate system, write down Schrodinger's equation for this problem and state the boundary conditions you would apply to its solutions. **[1%]**

3. Let $\psi_n(x)$ {n = 0,1,2,3,…} be a complete orthonormal set of eigenfunctions of the Hermitian operator Q with discrete eigenvalues q_n : $Q\,\psi_n = q_n\,\psi_n$. Consider the wavefunction defined at time zero as

$$\Psi(x,0) = \sum_{n=1}^{\infty} a_n \psi_n$$

for some constant coefficients a_n . In terms of these coefficients, write down expressions for the following
(a) The probability of measuring Q and getting the value q_3.
(b) After having measured Q and getting the value q_3, the probability of measuring Q immediately again and getting the value q_2.
(c) The probability of measuring Q and getting a value that is not one of the eigenvalues q_n.
(d) The normalization condition for Ψ
(e) $\langle Q \rangle$ **[½% each part]**

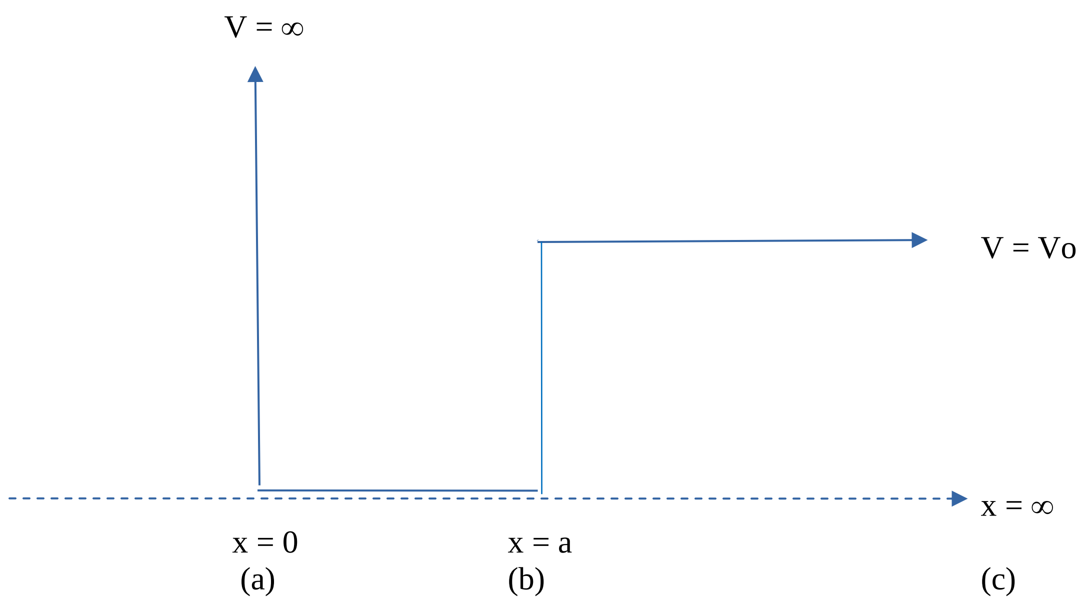
4. Write down the Hermitian conjugate of the following operators

(a) i d/dx

(b) [x , p] **[½% each part]**

5. A particle has an equal probability of being found anywhere on the interval (a, b) of the x axis, and probability ½ of being found outside this interval. What is the probability density in the interval (a,b)? **[1%]**

6. Write down all the boundary conditions you would apply at positions (a), (b), and (c) to a solution $\Psi(x)$ of Schrodinger's equation of total energy E < Vo for the following potential V(x). (you DO NOT need to solve the Schrodinger equation for $\Psi(x)$).



[2%]

Part II. Extended Problems. Time allowed 2 hours. 14% of course grade

1) A hydrogen atom is in the stationary state $\Psi_{nlm} = \Psi_{210.}$ Write out this normalized wavefunction <u>explicitly</u> in spherical polar coordinates and calculate the expectation value of the potential energy $< V >$, comparing it to the known total energy eigenvalue.

[5%]

2) An electron under the influence of a uniform magnetic field B_y in the y-direction has its spin initially (at $t = 0$) pointing in the positive x-direction. That is, it is in an eigenstate of S_x with eigenvalue $+½\, \hbar$. The Hamiltonian $H = -\boldsymbol{\mu} \cdot \mathbf{B} = - \gamma B_y S_y$ consists of the interaction of the magnetic dipole moment $\boldsymbol{\mu}$ due to spin and the magnetic field $\mathbf{B}$.

Show that the probability of finding the electron with its spin pointing in the positive z-direction at a later time t is $P(S_z = +½\, \hbar) = ½\, [1 + \sin (\gamma B_y t)]$.

[5%]

Hint: The eigenvectors of S_x are given in Griffiths [eq 4.151 2nd edition]; if you choose a method that uses the eigenvectors of S_y you will need to work those out yourself.

3) ??? **[4%]**